An Economic Order Quantity Model for Imperfect and Deteriorating Items with Freshness and Inventory-Level-Dependent Demand

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ABSTRACT

The sale of many products is influenced by the characteristics of the products and the behaviour of the consumers. This is particularly true of food items. Consumers of most food items want them fresh; thus, while items that are edible but have lost some of their freshness might be sold, the demand usually drops, and the closer the expiry date, the lower the demand for such products. Another factor that is known to affect the sale of many items is the attractiveness of the stocking pattern. Seeing a large stock of consumable items tends to attract consumers; hence the stock level dependence of demand. An additional characteristic of such products is that they continue to deteriorate; thus affects both the quality and quantity of such items, as the overly deteriorated ones are removed from the stock of saleable items. Consequently, the stock level reduces owing to both demand and deterioration. This description fits many agricultural products such as fruits and vegetables. Another characteristic of such products is the possibility of their being imperfect when they are received, as some stock might be damaged by harvesting, handling, or transportation. In this research, we develop a mathematical model to determine the optimal lot size for an item having all these characteristics, with the profit function as the model objective. We used a case of banana as the numerical illustration. Sensitivity analyses of important model parameters were conducted to demonstrate the model's robustness. The findings were that the date of expiry, the scale of demand, and the selling price of perfect products were the main constituents that affected their profitability. This model should find ready application in most stores where fresh consumable food items such as fruits and vegetables are managed.

OPSOMMING

Die verkope van baie produkte word beïnvloed deur die eienskappe van die produkte en die gedrag van die verbruikers. Dit is veral waar vir voedselprodukte. Verbruikers van die meeste voedselprodukte wil dit vars hê; dus, terwyl items wat eetbaar is, maar van hul varsheid verloor het, verkoop kan word, daal die vraag gewoonlik, en hoe nader aan die vervaldatum, hoe laer is die vraag na sulke produkte. Nog 'n faktor wat bekend is om die verkope van baie items te beïnvloed, is die aantreklikheid van die voorraadpatroon. Om 'n groot voorraad verbruikbare items te sien, lok verbruikers; vandaar die voorraadvlakafhanklikheid van vraag. 'n Bykomende kenmerk van sulke produkte is dat hulle aanhou versleg; dit beïnvloed dus beide die kwaliteit en hoeveelheid van sulke items, aangesien die oormatig verslegde items uit die voorraad verkoopbare items verwyder word. Gevolglik verminder die voorraadvlak as gevolg van beide vraag en verslegting. Hierdie beskrywing pas by baie landbouprodukte soos vrugte en groente. Nog 'n kenmerk van sulke produkte is die moontlikheid dat hulle onvolmaak is wanneer hulle ontvang word, aangesien sommige produkte beskadig kan word deur oes, hantering of vervoer. In hierdie navorsing ontwikkel ons 'n wiskundige model om die optimale lotgrootte vir 'n item met al hierdie eienskappe te bepaal, met die winsfunksie as die modeldoelwit. Ons het 'n geval van piesangs as die numeriese illustrasie gebruik. Sensitiwiteitsanalises van belangrike modelparameters is uitgevoer om die model se robuustheid te demonstreer. Die bevindinge was dat die vervaldatum, die omvang van vraag en die verkoopprys van perfekte produkte die hoofkomponente was wat hul winsgewendheid beïnvloed het. Hierdie model behoort maklike toepassing te vind in die meeste winkels waar vars verbruikbare voedselprodukte soos vrugte en groente bestuur word.

1. INTRODUCTION

Most recent studies on inventory modelling consider factors such as product freshness, imperfect quality, deterioration, and inventory-level-dependent demand. These extensions are part of efforts to create models that are suitable for many practical environments and purposes. These characteristics particularly suit agricultural produce, which has features that are different from many of the basic assumptions of the classic lot-sizing models. Most agricultural products have limited lifespans, deteriorate during storage, and sometime even grow (i.e., they increase in weight or size) when kept; and it may be necessary to consider the environmental impact of the production and logistics decisions, among others. The problem considered in this research is typical of most agricultural supply chains. We consider items that are imperfect on receipt, that deteriorate when stored, and the demand for which may be influenced by the perceived level of their freshness and the level of stock displayed based on shelf management principles in the store. Before presenting the model, we start with a brief review of the various characteristics of the products.

Products in a supply chain are not always perfect, and so there is a need to account for defective products. Salameh and Jaber [1] were the first researchers to investigate the idea that items delivered to the customer may not be of perfect quality. Khan *et al.* [2] collated the extensions to [1] in a literature review. They noted the published adaptations and extensions of the original model that address features such as supply chain coordination, quality improvement, and yield management for that period. In [1], the model assumed that screening would be done by the customer, even though it would have been the supplier who provided the imperfect product. Rezaei and Salimi [3] derived a model in which the management of the screening process switched from the customer to the supplier – a departure from the prior norm. A version of the economic production quantity (EPQ) model was also investigated by Yassine *et al.* [4]. They evaluated two scenarios of the delivery of inferior quality products, namely through aggregation and through disaggregation. In the aggregation scenario, inferior quality products are collected during a number of production runs and shipped as a single consignment. In the disaggregation scenario, inferior quality items are deemed to have been sold during each production cycle.

The demand rate of items may also be affected by pricing and marketing considerations [5]. However, these factors are seldom considered when inventory models are developed. Sadjadi *et al.* [6] considered an EPQ model for goods of inferior quality, which took into account the effect of the marketing efforts of a company. The model considered variables such as the marketing budget allocation, the costs of maintenance, the costs of production, warehousing availability, and the available machine hours. These were deemed to be constraints or limiting factors in the overall process. Lin and Hou [7] presented an inventory model with overlapping and advanced receiving attributes in which the supplier would provide a discount factor based on the procurement cost. This discount factor would compensate the buyer for holding returnable stock, thus maintaining a collaborative relationship. Khan *et al.* [8] considered a vendor-buyer system of inventory in which the buyer might be provided with items that are not all of perfect quality, but in which an agreement is made with the vendor to keep the stocks at the warehouse of the buyer, who would be responsible for managing that stock. An increasing number of manufacturers elect to have retailers oversee that type of inventory. Sebatjane and Adetunji [9] presented a four-echelon model of agricultural system consisting of farming, processing, screening, and retail operations in which the processor exchanges the processed inferior quality products at a lower price in the secondary market. Hauck *et al.* [10] assumed that the speed at which the screening is done would be a decision variable together with the order quantity.

Prior inventory models, starting with that of Harris [11], had assumed that products could be stored indefinitely without deteriorating. However, this was not realistic, as many perishable items such as fruit, vegetables, medication, and certain liquids can deteriorate or degrade over time. Ghare and Schrader [12] recognised this and proposed an economic order quantity (EOQ) model that assumed inventory decays, which follow an exponential profile. Studies of different types of deterioration have since emerged. Raafat [13] collated various such studies into a comprehensive synopsis of deterioration models. Thereafter, Goyal and Giri [14] recorded further progress in another review of deteriorating inventory, in which the shelf-life attributes of inventory items in particular were considered. Yadavalli *et al.* [15] examined a two-product continuous review inventory system in which customer arrivals were modelled as a Markovian arrival process and the demand for the second product was assumed to be a random variable. More recently, studies of inventory systems that deteriorate have been compiled in a report by Bakker *et al.* [16]. Jaggi *et al.* [17] investigated a dual-warehouse model for inventory that simultaneously considered imperfect quality items, deterioration, and certain trade credit terms. In their study, total profit was maximised through the optimisation of the order quantity. Rahman *et al.* [18] evaluated a model with interval-based parameters for items that deteriorate. Two situations were considered, namely those in which shortages are applicable, and those without shortages in a discounted environment. The rate of deterioration was assumed to be interval valued, while the carrying cost was a function of the length of time that the items had been stored, and the interval-valued purchase cost. Sebatjane [19] proposed an integrated and sustainable production–inventory model for a three-echelon supply chain for deteriorating items with a circular economy indicator under various carbon emissions regulations. Inaniyan and Kumar [20] presented an inventory model in which items had an initial linear deterioration rate that transitioned to a non-linear pattern in a dual-warehouse inventory system.

Products such as fruits and vegetables also have expiry dates. Fujiwara and Perera [19] investigated the effect of product freshness on consumer demand and found that freshness was an important consideration in the purchasing decisions of consumers. Wu *et al.* [20] determined the replenishment cycle time and ending-stock level for a retailer when the demand is based on the freshness of the product and the stock displayed. Chen *et al.* [21] expanded this concept to find the optimal lot size, ending-stock levels, and shelf space for a retailer by considering shelf-space size as an extra variable in optimisation. According to Giuseppe *et al.* [22], the food supply chain can be affected by the loss of products as the expiry date approaches. In order to manage expiring products such as food at the retail stage, their study investigated the optimal time that would be required for the retailer to withdraw that food from the shelf. The issue of expiry dates when shortages are permitted for deteriorating items had previously not been much considered. Khan *et al.* [23] suggested an EOQ model to cater for this in a situation where the end-user demand relied on the sales price of the products. Perishable food products may be influenced by the selling price and the age of those items. As perishable products have little brand identification, factors such as freshness and age become significant determinants of demand. Sebatjane and Adetunji [24] used this concept to investigate perishable inventory. This resulted in a model to manage perishable goods in the supply chain, which starts with farming operations and ends with the consumption of that inventory at its destination. In effect, there are three stages in this specific supply chain: production (or farming), processing, and retail distribution. Their study investigated jointly optimising pricing and inventory policies.

It has also been noted that the display of a larger number of products can influence demand for that product. Thus, the configuration and quantity of product that is displayed becomes an additional attraction for its end users. Baker and Urban's [25] analysis sought to develop a scenario in which the demand for the product of a company would be a function of the on-hand inventory of that particular product. They confirmed that large exhibits or amounts of inventory were able to stimulate market demand. Complementary studies by Urban and Baker [26] and by Teng and Chang [27] considered the scenario in which the rate of demand depended on the selling price and the level of inventory. Urban and Baker [26] also investigated a positive or non-zero ending-stock level together with a reduction in price at the end of the replenishment cycle. Teng and Chang [27] extended Urban and Baker's [26] work by incorporating item deterioration into this scenario. Goyal and Chang [28] presented a model that determined the best order quantity for a retailer and the rate of transfer from the warehouse to the retailer per order. The display space was constrained, and the demand relied on the level of inventory on exhibit. The study maximised the mean profit over time that the customer would be able to realise. Two inventory control models were considered by Duan *et al.* [29]. One of the models considered back-ordered shortages, while the other did not. They examined items that deteriorated with a rate of demand that relied on the actual stock level. Sargut and Işık [30] studied a dynamic EOQ scenario for a single item that perished in a production environment. The objective was to identify the production, inventory, and back-ordering decisions that would be necessary during the planning horizon. The parameters were deterministic but changed over time, and the producer had a constant production capacity that limited production in each period. Outstanding demand would be met at some time in the future. Pando *et al.* [31] studied a stock system that sought to establish the maximum profit/cost ratio. They assumed that the rate of demand would depend on the amount of stock and that the stock-holding cost would be non-linear. Demand at the consumer end relied on the stock level and on the expiry date of the products. Sebatjane and Adetunji [32] derived a model to ascertain the effectiveness of a mechanism that enhanced profit in a situation that removed the zero ending-stock policy in the supply chain to enhance profitability. The assumption was that inventory was kept at the retailer, and that it was replenished when a certain minimum level had been reached. Clearance sales were held to ensure that the required level of inventory was maintained at an acceptable level of freshness. De *et al.* [35] developed an inventory model for non-instantaneous deteriorating commodities under inflation in which the demand for the item depended on price and stock. Shortages were permitted and were assumed to be partially backlogged. Limi *et al.* [36] developed an inventory model for non-instantaneous deteriorating items with quadratic demand and time-dependent holding costs with shortages over a finite time horizon.

As could be seen from the above literature review, previous studies have contributed to different aspects of these reviewed product features – i.e., imperfect quality of delivery, product having a lifespan, the demand depending on the perceived level of freshness of the product, and the demand depending on the level and structure of stock displayed, in some combinations, but not comprising all these features. It is apparent, however, that all these features can be readily found in many agricultural products such as bananas and apples. This suggests that this research area has not been fully investigated; hence the need for this paper. The objective funtion was developed to combine all these features to develop a mathematical model that would be suitable for managing the fresh produce that is widely consumed in people's daily lives.

This article is developed in five sections. It begins with an introduction, which is followed by four more sections. Section 2 defines the notations and then specifies the assumptions. An EOQ model is developed in Section 3 for imperfect and deteriorating items with demand that depends on the freshness and inventory level of the product. Section 4 provides numerical examples and a sensitivity analysis. Section 5 concludes the study with a discussion of the findings and the implications, and suggestions for possible further investigations.

2. NOTATIONS AND ASSUMPTIONS

2.1. Notations

Table 1: Notations used in the formulation of the mathematical model.

Symbol	Description
Q	Order size per cycle (item unit, e.g., kg).
D	Demand rate (item unit per time, e.g., kg/day).
x	Screening rate (item unit per time, e.g., kg/day).
p_s	Percentage rate of imperfect items in Q (dimensionless proportion, or %).
$I(t)$	The instantaneous state of inventory level at time t (item unit, e.g., kg).
t_1	Screening time (time dimension, e.g., day).
T	Cycle length (time dimension, e.g., day).
$F(t)$	Products freshness index of the inventory at time t, which is a function of the shelf-life (a dimensionless parameter).
L	The or shelf-life of the product (time dimension, e.g., day).
θ	Rate of deterioration for each unit of time (item unit per time, e.g., kg/day).
a	Scaling parameter for the demand function (per time, e.g., per day).
b	The shape parameter representing the elasticity of demand (a dimensionless parameter).
h	Holding cost per unit of the perfect product for each unit of time (monetary unit per item unit per per time, e.g., R/kg/day).

Symbol	Description
h_s	Holding cost per unit of the imperfect product for each unit of time (monetary unit per item unit per per time, e.g., R/kg/day).
K	Ordering cost (monetary unit per batch, simply written as the monetary unit, e.g., R/batch, or simply R).
C_d	Deterioration cost per item (monetary unit per item unit, e.g., R/kg).
C_s	Screening cost per unit item screened (monetary unit per item unit, e.g., R/kg).
C_g	Cost price per unit of each product (monetary unit per item unit, e.g., R/kg).
S_g	Selling price per unit of each perfect product (monetary unit per item unit, e.g., R/kg).
S_d	Selling price per unit of each imperfect product (monetary unit per item unit, e.g., R/kg).

2.2. Assumptions

Levin *et al.* [33] noted that an important determinant of demand is the inventory level of the item that is on display. It has been found that the extent to which inventory is displayed has the effect of inducing customers to increase their levels of purchase. Various functions can be applied to portray this observation. The power function is a common representation of the rate of demand, as also used by Baker and Urban [25] and shown in Equation (1).

$$D = a[I(t)]^b \tag{1}$$

In this instance a would be the scaling parameter for the rate of demand. Alternatively, it is the asymptotic level of demand that would be attainable whenever an inventory level is considered optimal for consumer inducement. Similarly, b is a shape parameter that represents the elasticity of the demand rate with respect to the inventory level that is being displayed, where $a > 0$ and $0 \leq b < 1$. From Equation (1), the relationship shows that the rate of demand for inventory increases with higher inventory level, although non-linearly.

The dependence of the rate of demand on the expiry date is introduced with the concept of a freshness index. In an environment where a product has a finite shelf-life, the consumers of that product are likely to make their purchases after considering the age of the product to be purchased. Wu *et al.* [20] applied the date of expiry of an item to define the freshness index of that product, as in Equation (2).

$$F(t) = \frac{L - t}{L} \tag{2}$$

The expiry date of the product is represented by L. Over time, the product becomes less fresh and is less attractive to consumers. The inventory would be freshest just after it had been delivered to the customer. At that moment in time, $t = 0$, $F(0) = 1$. The product becomes the least fresh when it has reached its expiry date – i.e., at $t = L$ with $F(L) = 0$. This means that the inventory cycle time of the retailer, T, cannot exceed the shelf-life, i.e., $T \leq L$.

From studies by Chen *et al.* [21] and Feng *et al.* [34], Equation (1) and Equation (2) are integrated to compute the demand rate as being a multiplication function of inventory level function together with the freshness attributes of that inventory. This is shown in Equation (3).

$$D = a[I(t)]^b \left(\frac{L - t}{L}\right) \tag{3}$$

Given that this study models imperfect quality, item deterioration, and inventory- and freshness-dependent demand, the analysis assumes that Q units of the product get delivered and are stored at their destination. This is based on the assumption that it is a single product type, but with that product being of varying quality. On arrival, the items are screened to determine their quality. The screening process examines each item individually for defects. This results in two separate categories of the product but of different quality. The first category would comprise the perfect items, while the second category would comprise the items that are of imperfect quality. The rate of screening is assumed to be much higher than the demand rate, such that the quantity of perfect items that is available at a particular point in time is always greater than the demand rate. During the screening phase, when an item has been classified as imperfect, it is separated from the perfect items and is then stored separately. Immediately after the completion of the screening process (t_1 in Figure 1), the imperfect quality items are sold at a price that is lower than the price of the perfect items. During the cycle, deterioration occurs, and an intervention is initiated to remove the deteriorated items from the items that are for sale. This deterioration occurs relative to the size of the inventory level with the function $\theta I(t)$, where $0 < \theta < 1$.

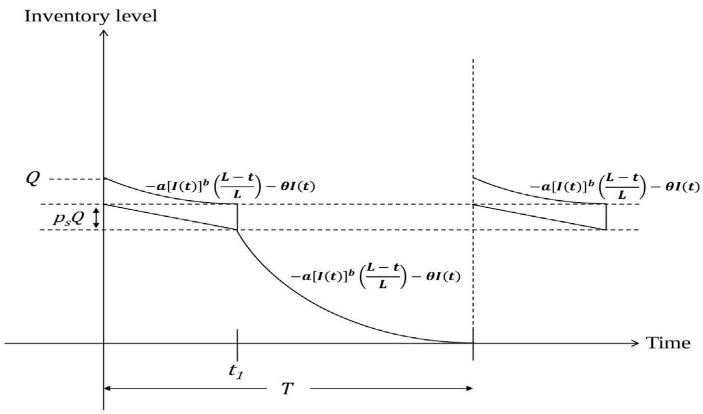

Figure 1 : Graph of the stock position against time during a cycle

Figure 1 illustrates the instantaneous demand profile of the system being discussed, where the stock level drops based on the demand (which is not constant, but depends on the stock level and the time since the delivery of the stock), and where the deteriorated quantity depends on the instantaneous inventory level; hence the non-linear shape. At the end of the cycle (T in Figure 1) the inventory level reaches the minimum and another order of Q items arrive. The process then repeats itself for the next cycle. A practical example would be fresh fruit such as bananas.

3. MATHEMATICAL MODEL

The inventory throughout the replenishment cycle would be depleted because of demand and deterioration. It depends on the inventory level, the freshness index of that inventory, and the way in which product deterioration occurs. Thus, the level of the inventory is determined by the differential Equation (4).

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -D - \theta I(t),\ 0 \le t \le T \tag{4}$$

By substituting Equation (3) in Equation (4) and rearranging it becomes Equation (5).

$$\mathrm{d}I(t) = \left[-a[I(t)]^b\left(\frac{L-t}{L}\right) - \theta I(t)\right]\mathrm{d}t,\ 0 \le t \le T \tag{5}$$

Integrating both sides of Equation (5) results in Equation (6).

$$e^{(1-b)\theta t}[I(t)]^{1-b} = \left(ae^{(1-b)\theta t}\right)\left[\left(\frac{t}{L}-1\right)\left(\frac{1}{\theta}\right) - \left(\frac{1}{(1-b)\theta^2}\right)\left(\frac{1}{L}\right)\right] + C \tag{6}$$

The amount of the inventory reduces to zero at the completion of each cycle of replenishment (alternatively stated, $I = 0$ at $t = T$). Therefore, the boundary condition $I(T) = 0$ is binding. The boundary condition $I(T) = 0$ is then used to solve for C from Equation (6) and the result is Equation (7).

$$C = -\left(ae^{(1-b)\theta T}\right)\left[\left(\frac{T}{L}-1\right)\left(\frac{1}{\theta}\right) - \left(\frac{1}{(1-b)\theta^2}\right)\left(\frac{1}{L}\right)\right] \tag{7}$$

An expression for the inventory level at any time is determined by substituting Equation (7) into Equation (6) and rearranging the terms. The result is Equation (8).

$$[I(t)] = \left[\frac{a}{\theta L}\left[t - L - \frac{1}{(1-b)\theta} - e^{(1-b)\theta(T-t)}\left[T - L - \frac{1}{(1-b)\theta}\right]\right]\right]^{\frac{1}{1-b}} \tag{8}$$

When a replenishment cycle begins, the retailer receives an order of Q items from the supplier. This implies that the boundary condition $I(0) = Q$ is binding. The lot size, or the amount of inventory at the beginning of the cycle, is determined by substituting the boundary condition into Equation (8). It is depicted in Equation (9).

$$Q = \left[\frac{a}{\theta L}\left[-L - \frac{1}{(1-b)\theta} - e^{(1-b)\theta(T)}\left[T - L - \frac{1}{(1-b)\theta}\right]\right]\right]^{\frac{1}{1-b}} \tag{9}$$

3.1. Total cost function

The total cost function (TCF) that is being investigated in this study focuses only on the costs that are defined by Equation (10).

$$TCF = HC + HC_s + DC + SC + PC + OC \tag{10}$$

TCF is the total cost, HC is the holding cost of the perfect product, HC_s is the holding cost of the imperfect product, DC is the deterioration cost, SC is the screening cost, PC is the purchasing cost of the product, and OC is the ordering cost. Figure 2 is a graphical illustration of this.

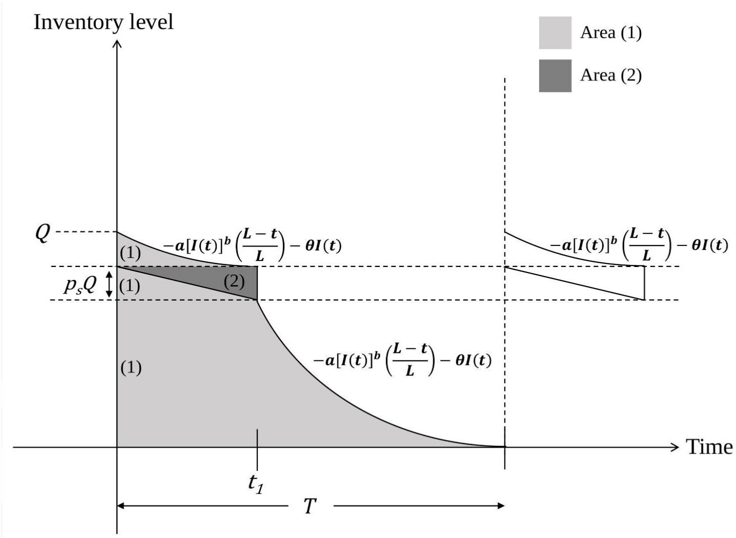

Figure 2: Representation of the holding cost structure of the model

3.1.1. Holding cost of the good product

To determine the holding cost of the perfect product, the Area (1) in Figure 2 is multiplied by the holding cost per unit of the perfect product for each unit of time. Stated mathematically, the area is represented by Equation (11).

$$HC = h\int_0^T I(t)dt + h\left(\frac{1}{2}\right)(t_1)(p_s Q) \tag{11}$$

where Equation (8) is substituted into Equation (11). This is shown in Equation (12).

$$HC = h \int_0^T \left[\frac{a}{\theta L}\left[t - L - \frac{1}{(1-b)\theta} - e^{(1-b)\theta(T-t)}\left[T - L - \frac{1}{(1-b)\theta}\right]\right]\right]^{\frac{1}{1-b}} \mathrm{d}t + h\left(\frac{1}{2}\right)(t_1)(p_s Q) \tag{12}$$

To make the model tractable, Equation (12) is simplified by the expansion of the exponential function using Maclaurin's expansion for $I(t)$ in Equation (8) and Q in Equation (9).

The Maclaurin expansion of the exponential function in $I(t)$ is

$$e^{\theta(1-b)(T-t)} = \sum_{i=1}^{\infty} \frac{\theta^i(1-b)^i(T-t)^i}{i!} = 1 + \frac{\theta(1-b)(T-t)}{1!} + \frac{\theta^2(1-b)^2(T-t)^2}{2!} + \frac{\theta^3(1-b)^3(T-t)^3}{3!} + \frac{\theta^4(1-b)^4(T-t)^4}{4!} + \cdots \tag{13}$$

For small values of θ and $(1-b)$, the expansion shown in Equation (13) is approximated and represented by Equation (14).

$$e^{\theta(1-b)(T-t)} \approx 1 + \theta(1-b)(T-t) \tag{14}$$

By substituting Equation (14) into $I(t)$ from Equation (8) and simplifying, Equation (15) is obtained.

$$I(t) = \left[-\left(\frac{a}{L}\right)(1-b)(T-t)(T-L)\right]^{\frac{1}{1-b}} \tag{15}$$

In a similar way, using the Maclaurin expansion for the exponential function in Q from Equation (9) for small values of θ and $(1-b)$, the approximation is represented by Equation (16).

$$e^{\theta(1-b)(T)} \approx 1 + \theta(1-b)(T) \tag{16}$$

Substituting Equation (16) into Q from Equation (9) and simplifying, Equation (17) is obtained.

$$Q = \left[-\left(\frac{a}{L}\right)(1-b)(T)(T-L)\right]^{\frac{1}{1-b}} \tag{17}$$

Substituting Equation (15) and Equation (17) into Equation (12) yields the holding cost of perfect products.

$$HC = h \int_0^T \left[-\left(\frac{a}{L}\right)(1-b)(T-t)(T-L)\right]^{\frac{1}{1-b}} \mathrm{d}t + h\left(\frac{1}{2}\right)(t_1)(p_s)\left[-\left(\frac{a}{L}\right)(1-b)(T)(T-L)\right]^{\frac{1}{1-b}} \tag{18}$$

Using integration and by simplifying Equation (18), the holding cost of the perfect product is obtained, as shown in Equation (19).

$$HC = h\left[-\left(\frac{a}{L}\right)(1-b)(T-L)\right]^{\frac{1}{1-b}}\left[\frac{1-b}{2-b}(T)^{\frac{2-b}{1-b}}\right] + h\left(\frac{1}{2}\right)(t_1)(p_s)\left[-\left(\frac{a}{L}\right)(1-b)(T)(T-L)\right]^{\frac{1}{1-b}} \tag{19}$$

3.1.2. Holding cost of the imperfect product

The Area (2) in Figure 2 is similarly multiplied by the holding cost per unit of the imperfect product to determine the holding cost of the imperfect product. Stated mathematically, the area is represented by Equation (20).

$$HC_s = h_s\left(\frac{1}{2}\right)(t_1)(p_sQ) \tag{20}$$

By substituting Q from Equation (17) into Equation (20), Equation (21) is obtained, which is the holding cost of the imperfect product.

$$HC_s = h_s\left(\frac{1}{2}\right)(t_1)(p_s)\left[-\left(\frac{a}{L}\right)(1-b)(T)(T-L)\right]^{\frac{1}{1-b}} \tag{21}$$

3.1.3. Deterioration cost

This is a cost that is attributed to the products that deteriorate. It is derived from the effort applied to identify and remove those deteriorated products from the remaining inventory that is still adequate for sale. The derivation of the cost is based on the level of the inventory. It is the cost of the deterioration multiplied by the quantity of deteriorated stock for the cycle, as shown in Equation (22).

$$DC = C_d\int_0^T \theta I(t)\mathrm{d}t \tag{22}$$

By substituting the approximation of $I(t)$ given in Equation (15) into Equation (22), the result becomes Equation (23).

$$DC = C_d\int_0^T \theta\left[-\left(\frac{a}{L}\right)(1-b)(T-t)(T-L)\right]^{\frac{1}{1-b}}\mathrm{d}t \tag{23}$$

On integration and simplification, the result from Equation (23) is shown in Equation (24).

$$DC = C_d\theta\left[-\left(\frac{a}{L}\right)(1-b)(T-L)\right]^{\frac{1}{1-b}}\left[\frac{1-b}{2-b}(T)^{\frac{2-b}{1-b}}\right] \tag{24}$$

3.1.4. Screening cost

A certain component of those items, p_s, would be of imperfect quality. The screening is conducted up until the period t_1, to separate the items of perfect quality from those of imperfect quality. The costs of screening a unit item is C_s. The cost to screen all of the items in each cycle is represented by Equation (25).

$$SC = C_sQ \tag{25}$$

By substituting Q from Equation (17) into Equation (25), Equation (26) is obtained, which is the screening cost of the cycle.

$$SC = C_s\left[-\left(\frac{a}{L}\right)(1-b)(T)(T-L)\right]^{\frac{1}{1-b}} \tag{26}$$

3.1.5. Purchasing cost of the product

The cost of the delivered product per unit multiplied by the numerical quantity ordered is shown in Equation (27).

$$PC = C_gQ \tag{27}$$

By substituting Q from Equation (17) into Equation (27), Equation (28) is obtained, which is the purchasing cost of that product.

$$PC = C_g\left[-\left(\frac{a}{L}\right)(1-b)(T)(T-L)\right]^{\frac{1}{1-b}} \tag{28}$$

3.1.6. Ordering cost

Ordering costs are the expenses incurred to acquire the batch of the products that is ordered. This is a fixed cost per order, and is shown in Equation (29).

$$OC = K \tag{29}$$

From TCF represented in Equation (10) and by substituting in the holding cost of the perfect product (HC) from Equation (19), the holding cost of the imperfect product (HC_s) Equation (21), the deterioration cost (DC) from Equation (24), the screening cost (SC) from Equation (26), the purchasing cost of the products (PC) from Equation (27) and the ordering cost (OC) from Equation (29), the TCF in Equation (30) is obtained after simplification.

$$\begin{aligned} TCF = {} & (h + C_d\theta)\left[-\left(\frac{a}{L}\right)(1-b)(T-L)\right]^{\frac{1}{1-b}}\left[\frac{1-b}{2-b}(T)^{\frac{2-b}{1-b}}\right] \\ & + \left[(p_s)(h+h_s)\left(\frac{1}{2}\right)(t_1) + C_s + C_g\right]\left[-\left(\frac{a}{L}\right)(1-b)(T)(T-L)\right]^{\frac{1}{1-b}} \\ & + K \end{aligned} \tag{30}$$

3.2. Total revenue function

The total revenue function (TRF) is the sum of TRG, the revenue from the perfect product, and TRD, the revenue from the imperfect/deteriorated product, as shown in Equation (31).

$$TRF = TRG + TRD \tag{31}$$

The selling price of the perfect product is multiplied by the number of perfect products that are sold. The quantity of the perfect product is made up of the quantity delivered, less the imperfect/deteriorated products that have been removed from the system. This is the revenue of the perfect product. The goods that have been removed are made up of imperfect and deteriorated products. Those goods are then multiplied by the discounted selling price of the deteriorated/ imperfect products. This represents the revenue of the imperfect/deteriorated product. The total revenue function is shown in Equation (32).

$$TRF = S_g\left[Q - \theta\int_0^T I(t)dt - (p_sQ)\right] + S_d\left[\theta\int_0^T I(t)dt - (p_sQ)\right] \tag{32}$$

By substituting $I(t)$ from Equation (15) and Q from Equation (17) and then integrating and simplifying, Equation (33) is obtained) as the TRF.

$$\begin{aligned} TRF = {} & (S_g + S_d)\left[-\left(\frac{a}{L}\right)(1-b)(T-L)\right]^{\frac{1}{1-b}}\left[\frac{1-b}{2-b}(T)^{\frac{2-b}{1-b}}\right] \\ & + \left(S_g - p_s(S_g - S_d)\right)\left[-\left(\frac{a}{L}\right)(1-b)(T)(T-L)\right]^{\frac{1}{1-b}} \end{aligned} \tag{33}$$

3.3. Total profit per unit of time

The total profit function (TPF) is the total revenue function TRF less the total cost function TCF, as shown in Equation (34).

$$TPF = TRF - TCF \tag{34}$$

By substituting TRF from Equation (33) and TCF from Equation (30) into Equation (34), we get Equation (35).

$$TPF = (S_g + S_d)\left[-\left(\frac{a}{L}\right)(1-b)(T-L)\right]^{\frac{1}{1-b}}\left[\frac{1-b}{2-b}(T)^{\frac{2-b}{1-b}}\right] + \left(S_g - p_s(S_g - S_d)\right)\left[-\left(\frac{a}{L}\right)(1-b)(T)(T-L)\right]^{\frac{1}{1-b}}$$
$$-\left[\begin{array}{c}(h + C_d\theta)\left[-\left(\frac{a}{L}\right)(1-b)(T-L)\right]^{\frac{1}{1-b}}\left[\frac{1-b}{2-b}(T)^{\frac{2-b}{1-b}}\right] \\ + \left[(p_s)(h+h_s)\left(\frac{1}{2}\right)(t_1) + C_s + C_g\right]\left[-\left(\frac{a}{L}\right)(1-b)(T)(T-L)^{\frac{1}{1-b}}\right]^{\frac{1}{2}} + K\end{array}\right] \tag{35}$$

TPF in Equation (35) is divided by time T and simplified to arrive at the TPU, the total profit per unit time, as in Equation (36).

$$\begin{aligned} TPU \quad &= \left[-h + \theta\left(-S_g + S_d - C_d\right)\right] \\ &+ \left[\frac{1-b}{2-b}\right]\left[-\left(\frac{a}{L}\right)(1-b)(T)(T-L)\right]^{\frac{1}{1-b}} \\ &\left[S_g - C_s - C_g + p_s\left[-S_g + S_d - (h+h_s)\left(\frac{1}{2}\right)(t_1)\right]\right] \\ &- \left[\frac{b}{1-b}\left[-\left(\frac{a}{L}\right)(1-b)(T-L)\right]^{\frac{1}{1-b}}\right. \end{aligned} \tag{36}$$

3.4. Solution procedure

The model seeks to optimise the total profit per unit time (TPU) with the cycle time T as the decision variable. The first derivative of Equation (36) determines the optimal value of the cycle time T, while the second derivative shows that the the TCF is a concave at this value. Using Equation (36), the first order derivative with respect to the cycle time T is calculated, as shown in Equation (37), to locate the optimum point.

$$\begin{aligned} \frac{d(TPU)}{dT} = \quad & \left[-h + \theta\left(-S_g + S_d - C_d\right)\right]\left[\frac{1-b}{2-b}\right] \\ + \quad & \left[-\left(\frac{a}{L}\right)(1-b)(T)(T-L)\right]^{\frac{b}{1-b}}\left[-\left(\frac{a}{L}\right)(2T-L)\right] \\ & \left[S_g - C_s - C_g + p_s\left[-S_g + S_d - (h+h_s)\left(\frac{1}{2}\right)(t_1)\right]\right] \\ & \left[\left[\frac{b}{1-b}T^{\frac{2b-1}{1-b}}\right]\left[-\left(\frac{a}{L}\right)(1-b)(T-L)\right]^{\frac{1}{1-b}}\right. \\ & \left. + \left[-\frac{a}{L}\right]\left[-\left(\frac{a}{L}\right)(1-b)(T)(T-L)\right]^{\frac{b}{1-b}}\right] \\ + \quad & \left[\frac{K}{T^2}\right] \end{aligned} \tag{37}$$

The second order derivative with respect to T is represented in Equation (38).

$$\begin{aligned}\frac{d^2(TPU)}{dT^2} = {} & [-h+\theta(-S_g+S_d-C_d)]\left[\frac{1-b}{2-b}\right] \\ & \left[\left[-\left(\frac{a}{L}\right)(1-b)(T)(T-L)\right]^{\frac{b}{1-b}}\left[-\frac{2a}{L}\right] \right.\\ & \left. + \left[-\left(\frac{a}{L}\right)(1-b)(T)(T-L)\right]^{\frac{2b-1}{1-b}}\left[\left(\frac{a}{L}\right)^2(b)(2T-L)^2\right]\right] \\ + \Big[S_g - {} & C_s - C_g + p_s\left[-S_g+S_d-(h+h_s)\left(\frac{1}{2}\right)(t_1)\right]\Big] \\ & \left[\left[\frac{b}{1-b}T^{\frac{2b-1}{1-b}}\right]\left[-\frac{a}{L}\right]\left[-\left(\frac{a}{L}\right)(1-b)(T-L)\right]^{\frac{b}{1-b}} \right.\\ & + \left[\frac{(b)(2b-1)}{(1-b)^2}T^{\frac{3b-2}{1-b}}\right]\left[-\left(\frac{a}{L}\right)(1-b)(T-L)\right]^{\frac{1}{1-b}} \\ & \left. + \left[\left(\frac{a}{L}\right)^2(b)(2T-L)\right]\left[-\left(\frac{a}{L}\right)(1-b)(T)(T-L)\right]^{\frac{2b}{1-b}}\right] \\ - \left[\frac{2K}{T^3}\right] & \end{aligned} \tag{38}$$

The complex nature of Equation (38) means that it is difficult, from a practical perspective, to show the concavity; thus, an alternative method is required. To address this, an iterative procedure was followed to show the concavity of the TPU function (Equation (36)). The iterative procedure is as follows. We first set $T = 1$. Thereafter, the value of T is substituted into Equation (36) to compute TPU. By increasing the value of T, the corresponding values of TPU can be determined. The expectation is that TPU ought to increase until a turning point (or a maximum value) is attained, after which TPU starts to decline. Figure 3 is a graphical realisation of this procedure, and it can be seen that Figure 3 is concave. The data for the numerical examples has been used for this illustration, and is discussed in the next section.

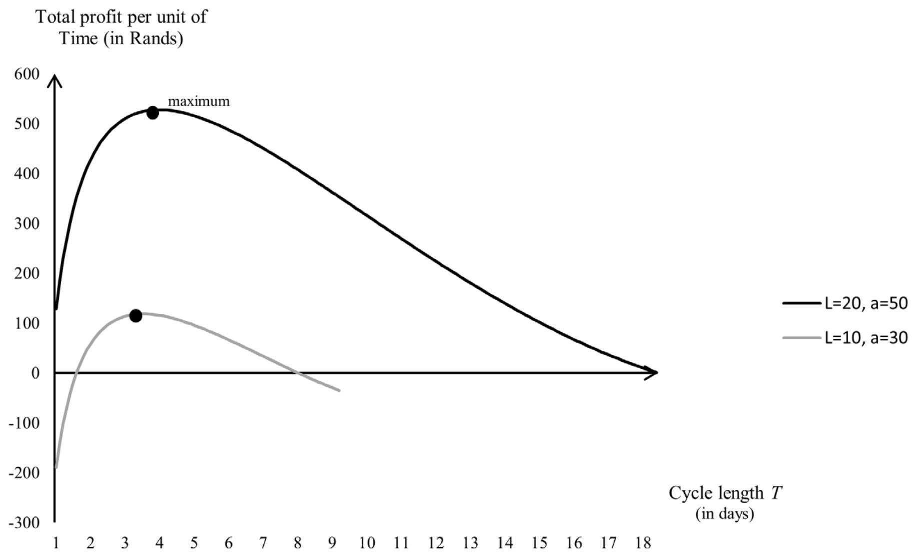

Figure 3: Concavity of the TPU function

4. NUMERICAL EXAMPLES AND SENSITIVITY ANALYSIS

4.1. Numerical examples

Numerical examples are used to illustrate the use of the model and the solution procedure to compute the optimum cycle length (time) T^* from the TPU – i.e., Equation (36). This was implemented using the solver function that is available in Microsoft Excel. The parameter values in Table 2 were used as inputs for the computation in the first example, while two input parameters (the product life, L, and the asymptotic demand level, the scaling factor, a) were changed for the second example. The results from these two examples are shown together in Figure 3. However, the discussion focuses mainly on the insight from the first example. The drastic shift in the daily profit rate is apparent from the changes in these two parameters, and this is expected because TPU is highly sensitive to both. This is discussed further in the sensitivity analysis section.

Table 2: Numerical input parameters

Symbol	Value	Symbol	Value
a	50 / day	S_d	5.00 Rand /kg
L	20 Days	C_d	0.01 Rand /kg/ day
b	0.2	p_s	0.15
h	0.1 Rand /kg/ day	t_1	3 Days
h_s	0.05 Rand /kg/ day	K	500 Rand
θ	0.05	C_s	0.05 Rand /kg
S_g	12.00 Rand /kg	C_g	4.00 Rand /kg

After substitution of the parameters from Table 2 into Equation (36), the optimal T^* value is computed. The optimal cycle length, T^*, is 3.98 days, and the TPU obtained is 527.85 Rands/day. Thereafter, substituting the input parameters of Table 2 and T^* into the Equation (17), the optimal quantity, Q^*, is 428.75 kg. Note that the second example gives us the optimal cycle time, T^*, of 3.5 days, the optimum order quantity, , of 148 kg, and the optimal TPU of 118.52 Rands/day, although the sensitivity analysis takes the first example as the basis input set.

4.2. Sensitivity analysis

The parameters' data in Table 2 was taken as the base set, and from this, the sensitivity of the decision variables and model objective (hereafter, together also called model outputs) to changes in the parameters was evaluated to understand which parameters were more important for inventory control purposes. The value of each of those parameters in Table 2 was increased and/or decreased by certain percentages, as shown in Table 3 in Appendix A, to identify how sensitive the outputs were to each of those parameters. The resultant percentage changes in cycle length, T, because of variation in these parameters, is presented in Figure 4. Similarly, Figure 5 shows the percentage changes in order quantity Q, and Figure 6 shows the percentage changes in TPU. From Figures (4), (5), and (6), certain trends emerge, which are discussed below.

A review of Figures (4), (5), and (6) indicates that certain variables have greater impacts on the TPU, T, and Q values. In discussing the observed trends, the sensitivity of the three outputs to all parameters should first be mentioned; however, the objective of the next discussion is to emphasise the parameters that have the greatest impacts on the output of interest.

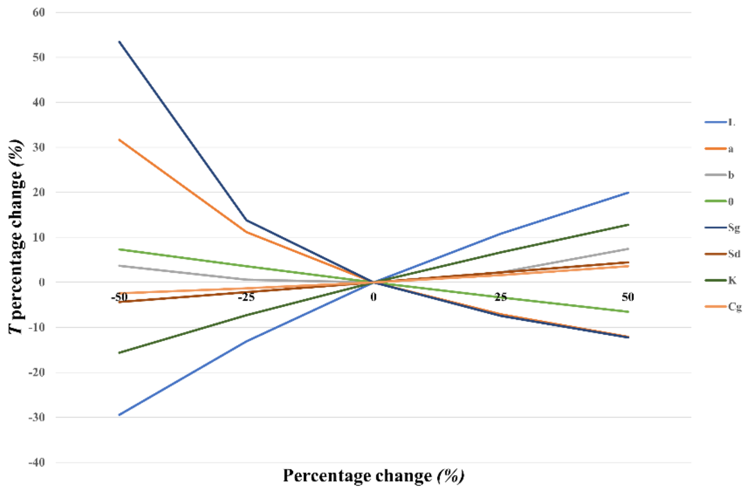

Figure 4: Sensitivity analysis of cycle length (T)

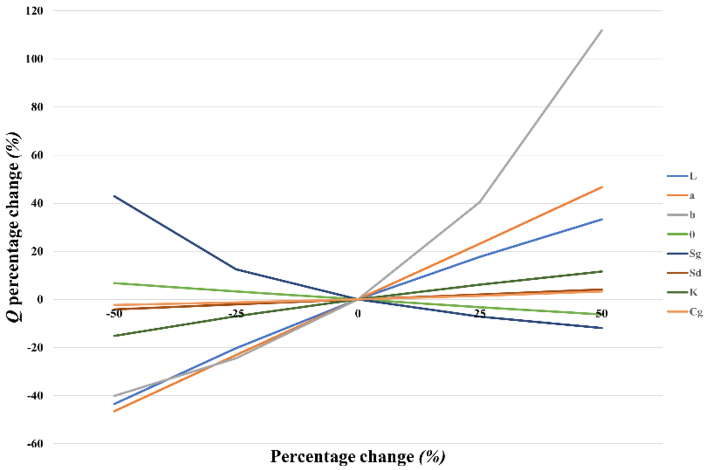

Figure 5: Sensitivity analysis of order quantity (Q)

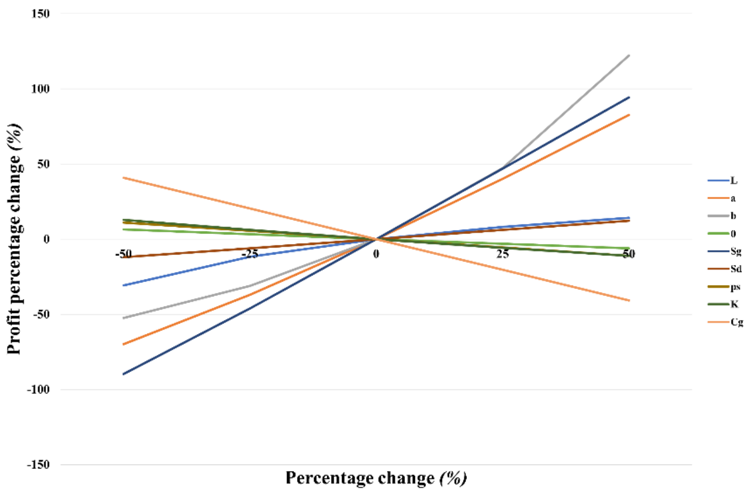

Figure 6: Sensitivity analysis of TPU

From the results in Table 3, the following observations may be made:

- The cycle length, T, is almost insensitive to changes in h_s, C_d, p_s, t_1, and C_s.
- The cycle length, T, is moderately sensitive to changes in b, h, θ, S_d, and C_g.
- The cycle length, T, is highly sensitive to changes in a, L, S_g, and K.
- The order quantity, Q, is almost insensitive to changes in h_s, C_d, p_s, t_1, and C_s.
- The order quantity, Q, is moderately sensitive to changes in b, h, θ, S_d, and C_g.
- The order quantity, Q, is highly sensitive to changes in a, L, S_g, and K.
- Total profit per unit of time, TPU, is almost insensitive to changes in h_s, C_d, t_1, and C_s.
- Total profit per unit of time, TPU, is moderately sensitive to changes in b, h, θ, S_d, and C_g.
- Total profit per unit of time, TPU, is highly sensitive to changes to changes in a, L, S_g, P_s, and K.

4.3. Managerial insights

When the scale parameter of the demand rate a decreases, the cycle length T increases. This is because fewer products are required, and goods do not have to be delivered as frequently. Similarly, if a increases, the cycle length T decreases. However, when the scale parameter of the demand rate a is decreased, there is a corresponding decrease, so that the quantity ordered Q also decreases. Similarly, for increases in the demand rate a, it is found that Q also increases. If the quantity Q increases, the ordering cost K per item actually decreases because more items per shipment have been delivered; and this has a positive impact on the TPU. For a constant value of K, the price per unit (or the Rand price per kilogram) of stock ordered has declined. However, this also has an adverse impact on the holding cost of both the perfect h and the imperfect/ deteriorated items h_s. It would thus have a negative impact on the TPU. The reverse applies if the quantity ordered for each shipment of the goods reduces. The scale parameter a is used to determine the scale or size of the demand. It implies that a large value for a would have the effect of appropriately increasing demand. With more goods to sell, the TPU would increase. The reverse is also true when the value of a declines.

When the expiry date of the inventory L is reduced, the optimal solution of the model would be shown in a reduction in the cycle length T. A reduction in the shelf-life L implies that the quantity Q is reduced. This leads to reduced holding costs, because the inventory spends less time as stock on hand. Alternatively, the fixed costs of placing an order K increases because the frequency of the number of orders has also increased. Since the rate of demand is a function of the freshness index and the stock level of that inventory, reduced shelf-life dates L will have a negative effect on the rate of demand. When the shelf-life L increases, the adverse effects of smaller order sizes Q on the demand, combined with the fixed increased cost K and the reduced holding costs (h and h_s where applicable) has the overall effect of reducing the TPU. The TPU increases when the shelf-life L increases, even though the order size K and holding costs (h and h_s) move in the opposite direction. Management may be able to apply this outcome practically to increase the TPU by extending the cycle length T of that inventory. Freshness needs to be protected; and this is possibly attained by investing in technology such as refrigeration and other preservation methods.

When the shape parameter b decreases, which incorporates elasticity of demand, the optimal solution responds by decreasing the cycle length T, but by a small amount. With a big decrease in the quantity Q ordered, the holding costs (h and h_s) are decreased, and this increases the per unit ordering cost K. As the shape parameter b is decreased, the impact results in a lower demand rate, and so the TPU is decreased. The reverse is also applicable when b is increased. Essentially, b has the most significant impact on the TPU and the quantity Q.

There is another tier of variables that have an impact on specific aspects of the TPU calculation in respect of how they influence the overall process in the comprehensive TPU scenario.

The unit rate of deterioration for each item θ provides a quantitative method to assess deterioration. If the rate of deterioration is low, additional quantity Q is available for a longer cycle length T. This has a positive impact by increasing the TPU, even if the holding costs (h and h_s) increase in tandem, because there is more quantity Q. This also has an associated reduction in the ordering cost K per unit. Similarly, the reverse applies for increased deterioration rates θ.

With no other change, the greater the selling price S_g of a perfect product, the greater the influence on the TPU. The reverse applies when prices are reduced. However, this might have an impact on demand, because the price may no longer be competitive in the marketplace. It also applies to the selling price S_d of imperfect/deteriorated items. Attempts should always be made to maximise these prices, because that would have a direct influence on the TPU.

The amount of imperfect product affects profitability because, with a higher p_s, more products become available for sale at the deteriorated/imperfect price S_d, even if this price is less than the price that the perfect products are sold for. Low levels of deteriorated/imperfect goods means that there are more perfect products to be sold at better prices S_g, even if a small cost is associated with the storage h of the additional volume of the perfect product.

The ordering cost K has a direct influence on the TPU because it is a fixed cost per order that is placed. If the ordering cost K increases, the unit cost attributable to K increases. If the ordering cost K decreases, the unit cost attributable to K decreases. Ordering cost K has a marginal impact on cycle length L and quantity ordered Q.

C_g is the cost price per unit of the goods. The lower the C_g, the bigger the TPU purchase scenario outcome. A purchaser buys goods from the same supplier at the lowest possible price. Buyers must strive for the lowest possible price to optimise the TPU. Buyers may elect to find alternate suppliers if the C_g can be sourced elsewhere.

5. CONCLUSION AND SUGGESTION FOR FUTURE WORK

The objective of this research is to develop a lot-sizing model that is particularly suited to some types of agricultural product, such as the sale of fruits in a retail store. The model considers a deteriorating stock of items that, when received, may contain some damaged stock (items of imperfect quality), and the demand for which depends on the level of freshness of the item and how the item is stacked for advertisement or marketing purposes to attract customers into the store. This model is important, because agricultural supply chains are some of the most pervasive chains in everyday life. Moreover, many such products are commodity items, meaning that the profit margin might be narrow, and that any opportunity to save costs or to increase profits may have a huge effect on the very survival of the business.

We have developed a mathematical model that optimises the profit function of the inventory system, having the cycle time as the decision variable. We have shown the concavity of the profit function, and developed a numerical solution procedure for the problem. From the optimal cycle time, we retrieved the optimal order quantity and calculated the corresponding system profit. Two numerical examples were provided, with a discussion based on the main data set. We analysed the sensitivity of the model output variables (i.e., the optimal cycle time, order quantity, and system profit) to the system parameters by observing the level of changes in the percentage values of the output variables to some percentage changes in the input parameters. The model was found to be almost insensitive to certain parameters, moderately sensitive to some, and highly sensitive to others. All the model outputs (i.e., the optimal cycle time, order quantity, and system profit) were found to be highly sensitive to the scaling factor of the demand, the lifespan of the product, the ordering cost, and the selling price of the good items. The system profit was found to also be sensitive to the ratio of the imperfect items to the good items, in addition to the prior four model input parameters.

This suggests that the manager should pay particular attention to three main parameters: the product lifespan (expiry date from the stock receipt), the scale parameter of the demand, and the selling price of the perfect products; these are the main factors that appear to affect the profitability of the system significantly. This information may be quite useful to the supply chain procurement manager. For instance, it may imply paying a little more for products from vendors whose produce is known to last longer than that of others. It may also mean that investing in safe and efficient preservation technology is worthwhile; this could be done after a relevant engineering economy analysis with a proper cash flow projection. Understanding the sensitivity of the profit to the sales price means that the manager could easily determine how better to sell the products, not just as the commodities that they are, but through some value-added activities that may help to boost the margins. Thus the selling price would be found to be a high leverage input parameter. These are valuable insights, as we have seen from the model that these key inputs leveraged the overall profitability of the system quite significantly.

This work also presents opportunities for further research. It may be important to consider other possible operating inputs as model parameters or decision variables in future studies. For instance, the impacts of quantity discount, incorporation of the circular system of return, reuse, and repurposing of stocks, the implications of shipment size on the environmental impact of the business, the possibility of sharing various costs related to the imperfect items with the vendor, and the impacts of offering clearance sale on the items when the expiry date is close in order to circumvent the zero-inventory property of the ending stock are among the model adaptations that might be possible. All these would assist the supply chain manager to make even better decisions. However, the properties that have been considered in this model would, on their own, have helped the manager to improve their profit significantly, especially when compared with the models that are currently applied in the industry.

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APPENDIX A: SENSITIVITY ANALYSIS VALUES

Table 3: Sensitivity analysis

	Parameter		Cycle length (T)		Quantity (Q)		TPU	
Symbol	**Value**	**% change**	**Days**	**% change**	**kgs**	**% change**	**Rands**	**% change**
Base			3.98		428.75		527.85	
	25	-50	5.24	+31.66	229.47	-46.48	159.20	-69.84
	37.5	-25	4.43	+11.16	329.77	-23.09	332.51	-37.01
a	50	0	3.98	0	428.75	0	527.85	0
	62.5	+25	3.70	-7.12	528.18	+23.19	739.54	+40.10
	75	+50	3.50	-12.08	628.78	+46.65	964.38	+82.70
	10	-50	2.81	-29.47	242.23	-43.50	365.63	-30.73
	15	-25	3.46	-13.14	341.95	-20.24	465.75	-11.76
L	20	0	3.98	0	428.75	0	527.85	0
	25	+25	4.41	+10.81	504.69	+17.71	570.96	+8.17
	30	+50	4.77	+19.82	571.32	+33.25	602.92	+14.22
	0.10	-50	4.13	+3.70	256.84	-40.10	251.14	-52.42
	0.15	-25	4.01	+0.60	324.07	-24.42	363.54	-31.13
b	0.20	0	3.98	0	428.75	0	527.85	0
	0.25	25	4.07	+2.20	602.20	+40.45	776.83	+47.17
	0.30	50	4.28	+7.40	908.14	+111.81	1172.58	+122.14
	0.050	-50	4.06	+1.99	436.74	+1.86	538.67	+2.05
	0.075	-25	4.02	+0.99	432.73	+0.93	533.24	+1.02
h	0.100	0	3.98	0	428.75	0	527.85	0
	0.125	+25	3.94	-0.97	424.82	-0.92	522.5	-1.01
	0.150	+50	3.91	-1.93	420.93	-1.82	517.2	-2.02
	0.0250	-50	3.97	-0.01	428.72	-0.01	528.45	+0.11
	0.0375	-25	3.98	0	428.74	0	528.15	+0.06
h_s	0.0500	0	3.98	0	428.75	0	527.85	0
	0.0625	+25	3.98	0	428.77	0	527.55	-0.06
	0.0750	+50	3.99	+0.01	428.78	+0.01	527.24	-0.11
	0.0250	-50	4.27	+7.33	457.76	+6.77	562.36	+6.54
	0.0375	-25	4.12	+3.57	443.00	+3.32	544.82	+3.22
θ	0.0500	0	3.98	0	428.75	0	527.85	0
	0.0625	+25	3.85	-3.37	415.08	-3.19	511.42	-3.11
	0.0750	+50	3.72	-6.54	402.00	-6.24	495.51	-6.13
	6	-50	6.11	+53.46	612.74	+42.91	54.29	-89.71
	9	-25	4.53	+13.80	482.41	+12.51	284.71	-46.06
S_g	12	0	3.98	0	428.75	0	527.85	0
	15	+25	3.68	-7.47	398.14	-7.14	775.49	+46.91
	18	+50	3.49	-12.27	377.98	-11.84	1025.39	+94.26
	2.50	-50	3.81	-4.37	410.95	-4.15	464.1	-12.08
	3.75	-25	3.89	-2.20	419.86	-2.07	495.86	-6.06
S_d	5.00	0	3.98	0	428.75	0	527.85	0
	6.25	+25	4.07	+2.21	437.59	+2.06	560.05	+6.10
	7.50	+50	4.16	+4.42	446.36	+4.11	592.47	+12.24
	0.0050	-50	3.99	+0.01	428.79	+0.01	527.90	+0.01
	0.0075	-25	3.98	0	428.77	0	527.87	0
C_d	0.0100	0	3.98	0	428.75	0	527.85	0
	0.0125	+25	3.98	0	428.73	0	527.82	0
	0.0150	+50	3.97	-0.01	428.71	-0.01	527.8	-0.01

	Parameter		Cycle length (T)		Quantity (Q)		TPU	
Symbol	**Value**	**% change**	**Days**	**% change**	**kgs**	**% change**	**Rands**	**% change**
Base			3.98		428.75		527.85	
	0.0750	-50	3.95	-0.75	425.74	-0.70	586.20	+11.06
	0.1125	-25	3.97	-0.38	427.21	-0.36	557.02	+5.53
p_s	0.1500	0	3.98	0	428.75	0	527.85	0
	0.1875	+25	4.00	+0.40	430.37	+0.38	498.68	-5.53
	0.2250	+50	4.02	+0.83	432.08	+0.78	469.52	-11.05
	1.50	-50	3.98	-0.02	428.65	-0.02	529.66	+0.34
	2.25	-25	3.98	-0.01	428.70	-0.01	528.76	+0.17
t_1	3.00	0	3.98	0	428.75	0	527.85	0
	3.75	+25	3.98	+0.01	428.80	+0.01	526.94	-0.17
	4.50	+50	3.98	+0.02	428.85	+0.02	526.03	-0.34
	250	-50	3.36	-15.63	363.61	-15.19	595.87	+12.89
	375	-25	3.69	-7.32	398.77	-6.99	560.42	+6.17
K	500	0	3.98	0	428.75	0	527.85	0
	625	+25	4.25	+6.64	455.05	+6.13	497.47	-5.75
	750	+50	4.49	+12.77	478.56	+11.62	468.86	-11.17
	0.0250	-50	3.96	-0.04	428.61	-0.03	530.54	+0.51
	0.0375	-25	3.97	-0.02	428.68	-0.02	529.19	+0.25
C_s	0.0500	0	3.98	0	428.75	0	527.85	0
	0.0625	+25	3.99	+0.02	428.82	+0.02	526.50	-0.25
	0.0750	+50	4.00	+0.04	428.90	+0.03	525.16	-0.51
	2	-50	3.89	-2.43	418.90	-2.30	743.34	+40.82
	3	-25	3.93	-1.32	423.41	-1.24	635.56	+20.41
C_g	4	0	3.98	0	428.75	0	527.85	0
	5	+25	4.05	+1.60	435.18	+1.50	420.24	-20.39
	6	+50	4.13	+3.59	443.10	+3.35	312.75	-40.75