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Exploring how two assessment tools evaluated six learners' approaches to solving base ten additive tasks

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Background: Base-ten thinking (BTT) – children's ability to reason in tens and ones is a crucial measure of Foundation Phase learners' mathematical performance in South Africa.

Aim: The study looks at the six learners using BTT to solve additive tasks through two different assessments.

Setting: Six purposely selected Grade 3 learners in Johannesburg township school.

Methods: The study used two different assessments to look at six learners' mental strategies for solving additive tasks. The first assessment analysed how additive tasks were solved by learners before and after an intervention (Mental Starter Assessment Project [MSAP]). The second assessment instrument (Learning Framework in Number [LFIN]) focussed on how learners (two high achievers, two average achievers and two low achievers) solved particular counting activities when solving additive problems.

Results: The findings demonstrate that learners who can count efficiently, partition ones and tens, work with groups of ten and understand number relationships when solving addition problems, operate with high levels of BTT.

Conclusion: The study showed that well-designed test items are crucial for assessing and enhancing learners' understanding of BTT.

Contributions: This research offers insights into assessment practices that assist teachers in identifying BTT in resource-constrained settings.

Keywords: Mathematics Assessment Tools; MSAP; LFIN; Base-Ten Thinking; Additive Tasks.

Introduction

Learners' abilities to carry out mental mathematics are key in establishing their level of mathematical proficiency as it reflects the learners' shifts from concrete forms of counting into abstraction. The Mental Starter Assessment Project (MSAP) pre-and post-test and the Adapted Learning Framework in Number (LFIN) are both assessment instruments in South Africa that can be employed to determine learners' levels of calculation strategies and to scaffold further development. Both assessments consider base-ten thinking (BTT) in the design of some of the test items. As such, a deeper investigation into establishing the structure of the test items in the two assessment instruments in terms of BTT can yield great benefits for the South African education landscape fraught with many systemic challenges.

This article will detail the problem context that led to this research focus, the literature and theory base underpinning the study, the methodology employed to generate data on test items and learner performance, the findings and the analysis of the findings. It answers the question: What insights can two different assessment instruments test items reveal about the nature of six Grade 3 learners' use of BTT when solving additive tasks?

Background

Results of the Annual National Assessment (ANA) that was administered between 2011 and 2014 indicated that young learners were consistently improving in a number of related mathematical tasks (DBE 2011, 2012). However, when a comparative study conducted by Weitz and Venkat (2013) in which a group of Grade 3 learners demonstrated year-on-year improvement in the ANA was

Note: Additional supporting information may be found in the online version of this article as Online Appendix 1.

assessed through the employ of the LFIN, and the findings of the LFIN were completely different from those obtained through the ANA. The main findings regarding the performance of Grade 1 learners on the ANA compared to the diagnostic test by Wright, Martland and Stafford (2006) indicate a significant discrepancy between the two assessments. The study found that learners generally scored higher on the ANA than on Wright et al.'s (2006) diagnostic tests. Specifically, the analysis revealed that many learners who performed well on the ANA (considered 'high' performers) did so by using rudimentary counting strategies, which were not adequately assessed by the ANA (Weitz & Venkat 2013). In contrast, the diagnostic test required more sophisticated strategies for higher scores, leading to lower performance on that assessment. For instance, a summary of the performance groups showed that the largest group consisted of learners who scored high on the ANA but low on the Wright et al. (2006) tests, suggesting that the ANA may not effectively measure deeper understanding or the use of advanced number strategies. This discrepancy raises concerns about the validity of the ANA as a measure of true numeracy skills, as it may allow for the acceptance of low-level counting strategies that do not promote the development of more sophisticated mathematical thinking. In administering the LFIN assessment, learners selected from the same sample group demonstrated slow, rudimentary and error-prone concrete forms of unit counting when solving additive relation problems. These findings raised concern about a lack of correlation between learners' ANA and LFIN performance (Weitz & Venkat 2013). The ANA assessment was concerned with correct answers, while the LFIN assessment was concerned with the sophistication of the strategies. One of the insights of the study was the use of two assessment tools to determine learners' ways of solving additive tasks. Likewise, this article looks at two assessments to capture learners' responses concerning additive tasks from the perspective of BTT.

Literature review

In this literature review, I will delve into the concept of BTT, exploring its significance in mathematical understanding. I also examine the pivotal role of math assessments in gaining deeper insights into mathematical concepts. Additionally, I discuss the structure and characteristics of both the LFIN and the MSAP assessment tools.

Base-ten thinking

Much of the literature on early arithmetic relies on base-ten relations and number patterns, which display essentially algebraic properties. Thus, developing an algebraic appreciation of number structure and base-ten structure is important but also critical for developing arithmetic proficiency within mental mathematics. One concern about poor performance is learners' lack of work with algebraic thinking as opposed to arithmetic. By algebraic thinking, I refer to the idea that learners can relationally see the structure of numbers, while the arithmetic approach concerns learners' ability to be prone to counting. Somewhat overlooked in the general arithmetic literature is the specific attention to base-ten number relations within algebraic approaches. Evidence suggests that there are connections between understanding and utilising base-ten structure and comprehension of the relative magnitude of numbers (Ellemor-Collins & Wright 2009). Similarly, Van den Heuvel-Panhuizen's (2008) transition from counting to structuring relies on developing BTT. Several mental calculation strategies described in the literature as flexible and efficient (Beishuizen 1993) are based on awareness and use of base-ten number relations. Bridging across 10, utilising Jump Strategies, compensation doubling and halving all rely on a strong grasp of informal BTT and place value decomposition.

Bobis and Bobis (2005) assert that the Jump Strategy is straightforward; the main characteristic of the strategy is that the first addend (in an addition calculation) or minuend (in subtraction) is kept whole and the second addend or subtrahend is added or subtracted from the former in place value chunks of tens and ones. Wright et al. (2012b) described the strategy as a sequence of jumps on a straight line, and for this reason, the Jump Strategy is described as a sequential or linear approach to computation referred to in the literature as the N10 approach. As opposed to the 1010 computational procedure in which the numbers are decomposed into tens and ones processed separately and then put back together again upon completion of the process (Torbeyns & Verschaffel 2013). In other words, the N10 approach makes use of jumps to find the answer by keeping the first number whole and splitting the second number into tens and ones. For example, 24 + 13 =, in using the N10 approach, the learner would plot 24 on the empty number line and split 13 into 10 and 3, then jump from 24 to 34 in 10 and then jump from 34 to 37 in three. The 1010 procedure splits both the numbers in the additive task to find the answer. So, in 24 + 13, the learner will split 24 into 20 and 4 and 13 into 10 and 3, then add 20 plus 10 to make 30 and then add 4 and 3 to make 7; thereafter, 30 and 7 are added together to produce the answer of 37. The N10 approach is most suitable for the Jump Strategy on the empty number line instead of the 1010 procedure (Blöte et al. 2000).

The material designers used the notion of base-ten to assist them in designing the MSAP material. According to them, Molina and Castro's (2021) study provided a useful instance of algebraic attention to decimal structure alongside number relations more generally. This was seen in their inclusion of number sentence tasks asking about the truth of statements such as: True or False 257 – 34 = 257 – 30 – 4. Given the South African evidence of lack of movement from calculating-by-counting into calculating-by-structuring, underpinned by base-ten oriented strategies like bridging through 10 and Jump Strategies, attention to base-ten structure was particularly imperative for the designers of the MSAP material. Similarly, while patterns form a core focus of attention in the early algebra literature, the designers found it useful to view the number-oriented attention to patterns seen within decimal structure calculation strings such as 3 + 5 = 8; 13 + 5 = 18; 43 + 5 = 48 in algebraic terms. This helped the material designers to incorporate attention to generalised expressions (any number with three units that has five units added to it will end up with eight units, leaving the other parts unchanged) within the Mental Starters materials, noting that this would be useful to support efficient production of answers in higher number ranges without reversion to counting in ones (Graven & Venkat 2021).

In Morrison's (2018) use of LFIN within her doctoral study, she noted the importance of BTT. Morrison (2018) focussed on the learners' oral communication and problem-solving strategies in different situations to gain insight into the emerging emphasis of BTT within the LFIN framework. This increased focus led to further insights into the base-ten aspect of the LFIN, particularly in how learners progress and revert in their use of base-ten, as inferred from their oral communication and ability to apply base-ten strategies in various situations flexibly. Wright et al. (2012a) state that to progress in mathematics, children must be inducted into BTT, developing a skilful habit of organising numbers and calculations into 1s, 10s and 100s. In conclusion, because of the insights gained from the doctoral study, it is evident that the design of the Jump Strategy Intervention prioritised BTT as a critical aspect of solving additive tasks. As a result of this finding, the role of BTT became important when unpacking some of the LFIN tasks within this article.

Maths assessment

Assessment is used worldwide, either within the classroom or through an external large-scale assessment, to gauge learners' performance. Teachers can use these activities to pinpoint valuable feedback that can help learners progress in their learning and inform future instruction, and this is essential for formative assessment. Van den Heuvel-Panhuizen and Becker (2003) propose that assessment tasks within classroom assessment be constructed with the following principles in mind: The first principle could be tasks have multiple solutions to encourage learner choice and critical thinking. The solutions could include multiple pathways to a single solution or multiple solutions. The next principle looks at dependent tasks, which involve using solutions from earlier parts to make progress on subsequent parts. This helps assess whether the learner understands the relationship between problems and can apply it (p. 709). The final principle looks at tasks that could also have a task solution strategy focus rather than the actual answer. Teachers can observe how students identify and use relationships to find efficient solutions, distinguishing between efficient and convoluted approaches (Van den Heuvel-Panhuizen & Becker 2003). This article flags up to the reader the importance of how assessments can be used to find efficient solutions for solving additive tasks by focusing on BTT.

Wright et al. (2006) Learning Framework in number assessment

Wright et al. (2006) formulated and developed an extensively elaborate intervention programme known as the LFIN; a mathematics recovery programme aimed at remediating underperforming learners in the mathematics classroom. The LFIN is administered in the format of conducting a one-on-one interview comprising 50 possible number-related problem items. These problem items include addition and subtraction type problems suitable to the grade. The learner is encouraged to demonstrate their solutions by writing them. The learner is posed with one question at a time, whereafter the learner can demonstrate the strategy employed to solve the given problem item. Each problem item is completed as the learners progress through the interview schedule. In this manner, the interviewer can profile the learner according to their abilities in terms of calculation strategies. Thereafter, remedial work can commence suited to the needs of each learner.

The LFIN includes both the notions of calculating-by-counting and calculating-by-structuring within its assessment framework, as its primary function is to assess a learner's ability to demonstrate their existing levels of calculation strategies. It includes the most elementary aspects of concrete forms of unit counting and the most sophisticated forms of mental calculating strategies. Although there are generally two broadly different approaches to computational work, with each side having proponents that stress the importance of each, the LFIN only seeks to establish the level of learner's proficiencies. Firstly, as such, the LFIN will only highlight the absence or presence of the learner's proficiencies. Once these levels of proficiencies, as well as the lack thereof, have been established, remedial work can be commenced in such a manner as to meet each learner at their particular level. Secondly, as the LFIN assesses learner levels of proficiencies, it incorporates elements that seek to accentuate any form of sophistication in strategies employed in several related tasks. Some of the aspects of the LFIN are described as Stages of Early Arithmetical Learning (SEAL) and Base Arithmetical Strategies.

For the article, I will focus only on how the learners worked with the base-ten Arithmetical Strategies (Three Levels) as expressed in certain test items in the adapted LFIN test:

Level 1: Initial Concept of Ten. The child does not see 10 as a unit. The child focusses on the individual items that make up the 10. One 10 and 10 ones do not exist for the child at the same time. In additive tasks (addition and subtraction) involving tens, children at this level count forwards or backwards by ones.

Level 2: Intermediate Concept of Ten. Ten is seen as a unit composed of 10 ones. The child is dependent on representations (like a mental replay or recollection) of units of 10 such as hidden 10-strips or open hands of 10 fingers. The child can perform addition and subtraction tasks involving tens where these are presented with materials such as covered strips of tens and ones. The child cannot solve addition and subtraction tasks involving tens and ones when presented as a written number sentence.

Level 3: Facile Concept of Ten. The child can solve addition and subtraction tasks involving tens and ones without using materials or representations of materials. The child can solve written number sentences involving tens and ones by adding or subtracting units of ten and ones (Wright et al. 2006:22). These ideas underpinned some of the tasks within the adapted LFIN assessment. In other words, I analysed all the test items in the adapted LFIN test instrument. Then, I selected test items that were underpinned by one or more of the three Base-Ten Arithmetical Strategies (Three Levels) that were evident in the design of the test items with a particular focus on the use of multiples of 10 and ones.

Mental Starter Assessment Project (2021) assessment

The Mental Starters Assessment Project was one of the iterations of a host of research intervention programmes being undertaken by the University of the Witwatersrand (Wits) Number Sense Maths Primary Project team in collaboration with the University of the Witwatersrand, Rhodes University, Department of Basic Education (DBE) Provincial Education Department and the work of Askew, Venkat et al. (2021) that focussed on the development and enhancement of mental mathematics in Foundation Phase learners. Their focus is on formulating and developing intervention programmes within the South African education landscape that advocate a calculating-by-structuring approach instead of a calculating-by-counting approach to number work. The Mental Starter Assessment Project is grounded in a base-ten approach that emphasises and foregrounds the notions of de-composing and re-composing strategies that furthermore emphasise the relational aspect of numbers (Ekdahl 2019). The MSAP intervention results have indicated a 15% increase from pre- to post-test for participating learners. This shift from the pre-test to the post-test has also been supported by data observed in the space of pre-service teachers who have implemented the Jump Strategy (Venkat and Mathews 2024). In the formulation and development of the MSAP intervention programme, three strands came into focus, namely *Fluency, Strategic Calculations and Strategic Thinking,* which were used as the building blocks in the design of the MSAP assessment tool. The completed MSAP intervention programme includes the design of a written pre- and post-test comprised of 30 additive relation questions. These three calculating strategies were categorised and divided into Questions 1–20 as the *Fluency (F)* items (rapid recall facts). Questions 21–25 have been classified as *Strategic Calculation (SC)* test items (involve finding solutions to additive tasks through problem-solving). Test items 26–30 have been classified as *Strategic Thinking (ST)* items (using the structure of numbers to solve additive tasks). Furthermore, the MSAP is designed to be a 2-week intervention programme that commences with a pre-test that is comprised of 15 addition test items and 15 subtraction test items. Thereafter, a series of eight intervention lessons on additive reasoning is conducted with the class of Grade 3 learners. The Jump Strategy on the Empty Number Line was selected for the intervention programme. The test items start with relatively easy test items and progress into more complex test items in the schedule of eight lessons. The test items are furthermore comprised of low- to mid-number range. Graven and Venkat (2021) note that the intervention employs two types of representations: part-part-whole and the empty number line (ENL). The MSAP assessment use of Jump Strategies evaluates learners' ability to enact working with tens and ones, and in so doing, promotes the use of BTT.

Theoretical framework

The knowledge of place value forms the foundational blocks of multi-digit number sense. Place value requires that learners know the relation between the number name and objects being counted together with numbers; for example, 14 is made up of 1 ten and 4 ones. What is fundamental to place value understanding and BTT is that 10 must be viewed as a unit. As soon as 10 has been internalised as a unit then the requisites for place value are in place. Place value is thus weaved into the four constructs that appear to be central to the development of multi-digit numbers through working with counting, partitioning, grouping and number relationships. Multi-digit numbers refer to any two more-digit numbers. These categories are placed in a multi-digit number sense framework designed by Jones et al. (1996). The first category is entitled counting. *Counting* explores the significance of counting as the foundation for understanding single and multiple units. These researchers have emphasised the transition from using single units to using groups of 10, and finally using a strategy of counting in tens and ones. The second category is *partitioning,* which looks at the concept of unique partitioning, which arises at the early stage of learning multi-digit numbers when children express a number as a combination of a tens value and one's value in standard form; for example, 23 is seen as two tens and three ones. Then it speaks about multiple partitioning in that the learner can use non-standard forms of partitioning for example seeing 43 as three tens with 13 ones. In this article, because the focus is on the Jump Strategy, the second addended or subtrahend would be partitioned into tens and ones. The third category *grouping* noted many children struggle with understanding how to use grouping when working with multi-digit numbers. Some children can only group objects in the context of counting (e.g. by grouping into tens), and very few understand how to use grouping to solve problems involving multi-digit numbers. In other words, few learners can do and undo groups to solve a multi-digit number problem. In terms of this article, grouping was determined only by the way they worked on the adapted LFIN. The last category is called *number relationships*, as children first learn the linear order of numbers, understand the sequence and increase in magnitude, learners can solve addition and subtraction numeric problems up to 1000. They can use many strategies required to make sense of multi-digit whole numbers. In other words, what value does each of the digits represent? In this article, number relationships were particularly seen in solving additive tasks with either missing second addends or subtrahends together with number sentences with missing addends at the start of the number sentence. These questions are known as strategic calculation and strategic thinking type questions in the MSAP pre- and post-tests.

Across the four categories, the number sense framework inserted five levels of thinking. The first level (pre-place value) involves using single units. Level 2 (initial place value) is characterised by the transition from using only single units to using 10 as a composite unit. In Level 3 (developing place value), students use two-digit numbers to practice mental addition. Level 4 expands the focus to three-digit numbers. Level 5 (fundamental place value) requires a strong grasp of numbers, including mental addition and subtraction up to 1000. Apart from experiences with larger numbers and rounding, these five levels encompass the key understandings required for the successful use of multi-digit whole numbers. For the study, while I adhere to Jones et al.'s (1996) notions of levels, I have adapted the levels to suit the data of my study. In both assessments, the number range does not extend into three-digit numbers. The classification of the different levels is based on the post-test results for both assessments. So, level 0 was included to illustrate that some learners could not count, partition, group or enact number relationships when solving additive tasks. Level 1 focuses predominantly on additive tasks on counting multiples of 10 and tens on ENL, with materials and in number sentences. Level 3 highlights predominantly the tasks related to subtraction in terms of counting multiples of 10 and tens on ENL with materials and in number sentences. Level 4 continues to focus on subtraction items which shows more difficulties. Level 5 speaks to learners who can do all the related tasks and could maybe get one to two of the items incorrect for counting in similar tasks (see Table 3). Furthermore, for partitioning, grouping and number relationships, the learners should get all the related tasks correct. For a learner to be at a higher level assumes that the learner can enact the lower levels of additive tasks within a particular category.

Research methods and design

Drawn from a master's project at an urban university, a non-traditional mixed methods approach was used, emphasising the qualitative aspect but by using the numbers generated by the pre- and post-test results and results of learners' responses in the adapted LFIN test. This approach allows for richer data and has gained attention in recent years. Hesse-Biber (2015) refers to this non-traditional approach as *qualitatively driven mixed methods*. In this design, qualitative data are the primary focus, and quantitative data are used in a supporting or secondary role. This approach is ideal when the primary goal is to explore in-depth qualitative insights, and the quantitative component helps to enhance or corroborate the findings. So, for this article, I continued to use this qualitative-driven mixed method approach to assist me in crafting how the two assessments promoted or inhibited the use of BTT in learners' responses.

The first phase of gathering data entailed administering the MSAP intervention programme with a cohort of 42 Grade 3 learners in a township school in Johannesburg. All the ethical processes were granted before the intervention began. The intervention commences with a pre-test and continues with a schedule of eight intervention lessons followed by a post-test. The pre- and post-test consisted of two parts: the first part needed to be completed in 2 min and the second part needed to be completed in 3 min (see Online Appendix 1 – A1). The results of the pre- and post-test were analysed to establish the gains made from pre- to post-test. Based on the learners' scores, six learners were selected to form part of the qualitative data analysis that involved interviewing six learners through the employ of the adapted LFIN interview schedule (see Online Appendix 1 – A2). These six learners were selected from across the three attainment bands with the following: two strong (achieved 30% and 23% mean), two average (achieved 6% and 3% mean) and low-performing learners (achieved 0% and –30% mean) from the pre-test to the post-test.

To create clarity for the reader, I will only include in Table 1 the items in which any of the six learners had attempted to answer. Nitko (2001) speaks about the researcher's ability to exclude test items. He notes the need for accurate interpretation of test scores and how excluding irrelevant or non-discriminating items, including unattempted ones, can provide more meaningful results. Test items in the MSAP pre- and post-11, 15, 18, 23, 24, 27, 29 and 30 were not included to assist with focussing on how learners provided correct and incorrect answers. Table 1 shows how the test items of the six learners were captured and categorised.

Column 1 uses counting to look at the structure of test items (1, 2, 13, 5, 7, 6, 9, 3, 14, 4, 10 and 16). Counting is identified as the ability to produce answers quickly either verbally, on the ENL or as an answer to a written number sentence. In other words, no splitting of numbers or working with missing addends or subtrahends. The counting could be seen in finding the next multiple of 10 either before or after a number (Test items 1 and 2). Then the counting was expressed finding the next multiple of 10 on an ENL (Test item 13). Another way counting was observed by jumping forwards and backwards on an ENL in 10 or multiples of tens (Test Items 5, 7, 6 and 9). Finally, counting can be seen in written number sentences with result unknown additive tasks in which addends and subtrahends are added to or subtracted from with multiples of 10 and in multiples of tens (Test Items 3, 14, 4, 10 and 16).

Next, I assessed partitioning (understanding of tens and ones in standard form) within test items 21, 22, 25 and 26. Test items 21 and 22 showed how the second number in the additive task needed to be split into tens and ones on an ENL to establish the answer. Test items 25 and 26 were addition and subtraction number sentences in which the answer needed to be established by partitioning the second number into a standard form of tens and one and some tasks were solved through the use of bridging through 10. For example, bridging through 10 in the following number sentence 8+ 5 = is, the learner would split 5 into 2 and 3 and then add 2 to 8 and then find the number 10 and then add 3 to 10 to make 13. The idea of bridging through 10 is seen in the crossing over of the 10.

For number relationships, I looked at questions 8, 19, 17, 28 and 20. These test items required the learners to think flexibly about numbers. Test items 19 and 17 required learners to know how to find answers for additive tasks with the second addend and subtrahend missing (see Test items 8, 19, 17 and 28). In test item 28, the learner needed to place 47 and 62 on an ENL, then the learner could jump backwards in 10 landing on 10 and then bridge through 50 and make the final jump to 47, the answer is the number of jumps being 15. Test item 20 required learners to find the missing addend at the start of a number sentence.

In Table 1 (an example of the MSAP pre- and post-test item), column 1 speaks to the clustering of test items by the number of the test item in the original tests, column 2 shows the test item and column 3 provides the anticipated answer from column 4 to column 8 is the pre- and post-test codes used to describe the answers of each test item of each of the six learners. Three codes were used to analyse their answers, the first code (1) indicates the correct answer, and the second code (x) indicates the incorrect answer listed alongside the answer given in brackets, followed by the third code (B) which showed that learners did not provide any answers.

Thereafter, I focussed on the test items within the adapted LFIN test created by the Wits Maths Connect primary team – interview schedule (Table 2 – an example of the adapted LFIN test item) that focussed on Base–Ten Arithmetical Strategies with the focus on the use of groups and determined the level of performance of the six learners in terms of the BTT. In other words, only questions 10, 11, 12 and 13 will be analysed because they make use of groupings of tens and ones from the adapted LFIN. Table 2 shows the test items in the first column. Then, the next columns indicated the six learners. Beneath the first row in the table, the authors used 'a' and 'b' to show that some questions required only one answer while some questions required two answers 'i' and 'ii'. '1' indicated the correct answer while 'x' indicated the incorrect answer.

As mentioned earlier, I needed to create a table which provided ways in which to classify items, in terms of working with tens and ones from the MSAP instrument and the adapted LFIN test items (see Table 3).

I analysed six learners by looking at their results from the MSAP post-tests and the adapted LFIN results. This helped me understand their level of BTT and place value decomposition understanding. I focussed on specific groups of questions to evaluate their knowledge of tens and ones in different areas. I assigned each learner a level from 0 to 5 based on their results from the two assessments. For example, a learner at level 0 in counting cannot provide any answers for the identified tasks, while a learner at level 4 in partitioning could solve two-digit numbers, by splitting the second number and bridging through 10. For grouping, a learner at level 1 can count a 4-dot strip and add a 10-dot strip, counting in tens from 24 to 84. For number relationships, learners at level 5 needed to answer most of the tasks and were allowed to get one task wrong in a test item with the same structure. For example, if 31 – 20 = and 79 – 40 = they are similar in structure, a learner at Level 5 could get one of them wrong.

Ethical considerations

Ethical clearance to conduct this study was obtained from the University of the Witwatersrand School of Education Ethics Committee (reference no.: 2022ECE021M).

Results and discussion

Two tables became central to the findings and discussion of the data. The first table (Figure 1) indicates how all six learners answered their pre- and post-tests. Then, I will discuss the second table (Figure 2), which will focus on the overall impression and what each learner did in terms of the adapted LFIN test. Thereafter, I will discuss

TABLE 1: Pre-and post-test items from the Mental Starter Assessment Project.

Clustering of the test items in terms of counting, partitioning, grouping and number relationships	Test item examples	Anticipated answer	Learner 1		Learner 2		Learner 3		Learner 4		Learner 5		Learner 6	
			Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
13	Fill in the missing number 14, 24, 34, 44, [ ]	60	1	1	B	1	B	B	B	B	B	x (56)	B	B

Note: Mathematical Task from MSAP assessment

TABLE 2: Adapted Learning Framework in Number Test items.

Test items (adapted LFIN)	Learner1		Learner 2		Learner 3		Learner 4		Learner 5		Learner 6	
	a	b	a	b	a	b	a	b	a	b	a	b
12. Base-ten thinking (using strips) a. Place a 10 dot strip on the desk. *How many dots are there?*	1	-	1	-	1	-	1	-	x	-	1	-
b. Place a 3 dot strip to the right of the 10 dot strip. *How many dots now?*	1	-	1	-	1	-	1	-	x	-	1	-
c. Repeat with 10 and 8	1	-	1	-	1	-	x	-	x	-	1	-
d. *I have [total] dots on this paper. Here are [] dots* (uncover one side), *how many are under here?* 10 + [ ] = 13 6 + [ ] = 16	1	1	1	1	x	x	1	x	x	x	x	x
e. Place out a 10 dot strip. *How many dots are there?* Place a 5 dot-strip to the right of it. *How many dots do we have now?* If the child answered 15, then ask how many more dots are needed to make 25. 10 + 5 = [ ] 15 + [ ] = 25	1	1	1	1	1	1	1	x	x	x	1	x

Source: Jones, G.A., Thornton, C.A., Putt, I.J., Hill, K.M., Mogill, A.T., Rich, B.S. et al., 1996, 'Multidigit number sense: A framework for instruction and assessment', *Journal for Research in Mathematics Education* 27(3), 310–336. https://doi.org/10.2307/749367

LFIN, Learning Framework in Number.

TABLE 3: Adapted from the framework of nurturing and assessing multi-digit number sense.

Level	Counting	Partitioning	Grouping	Number relationships
0	Cannot count verbally forwards or backwards in ten OR tens on an ENL OR to the next multiple of ten on ENL OR add from a one-digit number OR add and subtract from a two-digit in multiples of ten or tens.	No partitioning of two-digit numbers into tens and ones.	Cannot identify and work with 10-dot strips and single-dot strips in screened collections and number sentences.	No evidence of solving tasks involving missing numbers or using answers from previous number sentences to solve consequent number sentences.
1	Can either count verbally forwards OR backwards in a multiples of ten. Can add multiples of ten to a one-digit number (6 + 30 =) and can subtract in multiples of ten from two-digit numbers in number sentences (57 − 10 =).	Can split the first and second addend into tens and ones.	Can recognise a 4-dot strip and add a 10-dot strip to make 14 and then count from 24 to 84.	Can find the missing addend to make the next multiple of ten (e.g. 36 + _ = 40)
2	All Level 1 descriptors achieved. Can count verbally forwards and backwards in multiples of ten.	All Level 1 descriptors achieved. Can keep the first number whole and split the second addend into tens and ones and jumps forwards on the ENL	All Level 1 descriptors achieved using strips. Can recognise a 4-dot strip and add a 10-dot strip to make 14 dots; this is done for 13 and 15 OR 18 OR 13, 15 and 18; however, cannot count the total number of ten-dot strips or total number of dots (90).	Same as Level 1 indicators achieved and can find the missing addend – multiples of 10 (e.g. 37 + _ = 77)
3	All Levels 1, 2 descriptors achieved. On an ENL can jump forwards and backwards in a multiple of ten from one-digit numbers. On an ENL can jump forwards OR backwards in multiples of ten from two-digit numbers	All Levels 1, 2 descriptors achieved. Can keep the first number whole and can split the second subtrahend into tens and ones and jump backwards on the ENL	All Levels 1, 2 descriptors achieved. Can identify a 4-dot strip and add a 10-dot strip and continues to add 10-dot strips until 84; can add 3 and 5-dot strip to a 10-dot strip, can recognise the 10-dot strip in a screened collection of 10-dot strips and 3-dot strips, can count 10-dot strips with 3, 4 and 2 dot strips.	All Levels 1, 2 descriptors achieved. Same as Level 2 indicators achieved and can find the missing subtrahend consisting of multiples ten (20) OR 15 (ten and five), ten (e.g. 38 − _ = 18; 62-_= 47)
4	All Levels, 1, 2 and 3 were achieved On an ENL, can jump forwards and backwards in multiples of ten from two-digit numbers	All Level 1, 2 and 3 descriptors were achieved. Can split the second addend into ten and ones together with bridging through ten to find the answer	All Level 1, 2 and 3 descriptors were achieved and can count in tens with the aid of 10-dot strips, they 10 to 90 and know the total number of 10-dot strips and the total number of 10-dots strips, they cannot work with the screened collection in (10+3= and 6 + 10 = 16) OR 10 dot strips added with 3, 4 and 2 dot strips.	All Level 1, 2 and 3 descriptors were achieved. Can find missing minuend number sentences (e.g. _ + 20 = 66)
5	All Levels 1, 2, 3 and 4 descriptors were achieved. Can identify the next multiple of ten afterwards. Can subtract multiples of ten from two-digit numbers (31−20 = or 79−40 =).	All Levels 1, 2, 3 and 4 descriptors were achieved and can split the second subtrahend into ten and ones together bridging through ten to find the answer.	All Levels 1, 2, 3 and 4 descriptors were achieved	All Levels 1, 2, 3 and 4 descriptors were achieved

Note: Mathematical tasks from MSAP and Adapted LFIN assessment.

Source: Jones, G.A., Thornton, C.A., Putt, I.J., Hill, K.M., Mogill, A.T., Rich, B.S. et al., 1996, 'Multidigit number sense: A framework for instruction and assessment', *Journal for Research in Mathematics Education* 27(3), 310–336. https://doi.org/10.2307/749367

how each learner performed in terms of BTT based on their MSAP pre- and post-test results together with how they solved tasks within the adapted LFIN interview.

In the description given in Figure 1, the only difference is that the last row of the table shows all the answers that were correct and incorrect for both the pre- and post-test. The data from this table have been used to inform the levels of each learner in terms of the three categories counting, partitioning and number relationships.

Figure 2 consists of seven columns. The first column includes the test items that have been taken from the adapted LFIN test. Then columns 2–7 show each learner's responses to the interview questions. These results reflect the concept of grouping using the Adapted Framework (Jones et al. 1996) to nurture and assess BTT. Important to note that question 12 was split into two tasks for analysis, the first task focussed on using 10-dot strips while the second task used the screening of 10-dot strips.

Table 4 provides a summarised overview of Learner 1 MSAP and LFIN results.

Learner 1 (30% improvement from pre-test to post-test)

Counting, partitioning and number relationships

Learner 1 (L1) demonstrated the most significant improvement between the pre-test and post-test. After the post-test, L1 showcased the ability to make jumps in multiples of 10, moving forwards from 14 to 54 and backwards from 79 to 39. Additionally, he was able to identify the next multiple of 10 consistently. His work on the number line reflected his proficiency in making both forwards and backwards jumps in multiples of 10 or a multiple of 10. L1 also illustrated his ability to solve various number sentences. He successfully tackled problems with single-digit first addends, such as (6+ 30 =), as well as with two-digit first addends, for example (16 + 30 =), where the second addend was multiples of ten/s. Moreover, he was capable of solving two-digit subtraction problems involving subtrahends that were a multiple of ten or multiples of ten, like (57 – 10 =) and (79 – 40 =). Overall, L1's performance indicates a solid understanding of number concepts, particularly in working with a multiple of ten or multiples of ten.

Partitioning is a learner's ability to break down the second addend or subtrahend into tens and ones to arrive at the

Clustering of the test items in terms of counting, partitioning, grouping and number relationships	Test item examples	Anticipated answer	Learner 1		Learner 2		Learner 3		Learner 4		Learner 5		Learner 6	
			Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
1	Fill in the missing number 14, 24, 34, 44, □	54	1	1	1	1	1	1	1	1	1	x (24)	1	x (45)
2	Fill in the missing number 79, 69, 59, 49, □	39	1	1	1	1	x (59)	1	1	1	1	x (67)	1	1
13	What is the next multiple of 10? 56 □	60	1	1	B	1	B	B	B	B	B	x (56)	B	B
5	+10; 7 □	17	1	1	1	1	B	B	x (13)	1	X 10	x (60)	1	B
7	−10; □ 54	44	x (64)	1	x (64)	1	B	B	x (64)	1	1	x (20)	1	B
6	+10 +10 +10; 23 □	53	1	1	1	1	B	B	x (30)	1	B	x (10)	1	B
9	−10 −10 −10 −10; □ 71	31	x (111)	1	B	1	B	B	B	x (57)	B	x (40)	B	B
3	6 + 30 = □	36	1	1	1	1	1	1	x (34)	1	X (10)	x (22)	1	1
14	16 + 30 = □	46	1	1	B	1	B	B	B	B	B	x (66)	B	B
4	57 – 10 = □	47	1	1	1	1	x (40)	1	1	x (67)	1	x (95)	1	1
10	31 – 20 = □	11	1	1	B	1	B	B	B	B	B	x (57)	B	B
16	79 – 40 = □	39	1	B	B	x (54)	B	B	B	B	B	x (54)	B	B
21	+20 +3; 36 □	59	1	1	x (58)	B	x (536)	x (40)	1	x (59)	1	x (76)	1	1
22	−2 −10; □ 72	60	x (84)	1	1	B	x (607)	x (49)	x (90)	x (84)	1	x (20)	1	1
25	57 + 26 = □	83	B	1	B	B	B	x (67)	B	x (78)	1	x (42)	1	1
26	83 – 24 = □	59	B	1	B	B	B	B	B	B	x (64)	x (57)	x (64)	B
8	36 + □ = 40	4	1	1	x (12)	1	B	B	B	B	1	x (39)	1	B
19	37 + □ = 77	40	B	1	B	B	B	B	B	B	B	x (15)	B	B
17	38 – □ = 18	20	B	1	B	x (14)	B	B	B	B	B	x (14)	B	B
28	62 – □ = 47	15	B	1	B	B	B	B	B	B	B	x (37)	B	B
20	□ + 20 = 66	46	B	1	B	B	B	B	B	B	B	x (98)	B	B
Total correct (C) /incorrect (I) answers			**C -12 I - 3**	**C -20 I -0**	**C -7 I -3**	**C -12 I -2**	**C -2 I - 4**	**C -4 I -3**	**C -4 I -5**	**C -6 I -5**	**C -8 I -3**	**C -0 I -21**	**C -11 I- 1**	**C- 6 I -1**

FIGURE 1: Results of the pre-and post-test items from the mental starter assessment project.

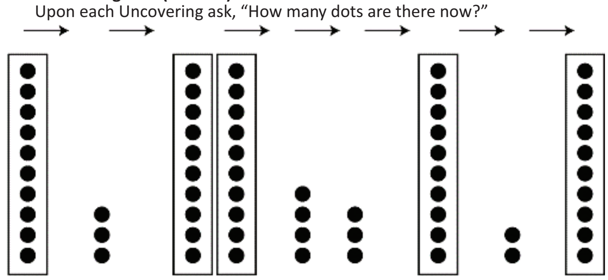
correct answer. For example, L1 successfully utilised a number line to aid in this process, demonstrating partitioning with calculations such as 36 + 20 + 3 = and 72 − 10 – 2 =. According to Blöte, Klein and Beishuizen (2000), learners who can efficiently jump on a number line in increments of tens and ones showcase proficiency in the N10 approach. In this context, the learner effectively solved number sentences by separating both the second addend (57 + 26) and the subtrahend (83 − 24) into their tens and ones components. In both cases, the learner also needed to employ bridging through 10 to achieve the result.

The use of number relationships became evident when L1 solved number sentences in which he was required to find the missing addends (36 + □ = 40; 37 + □ = 77) and the missing subtrahend (38 – □ = 18). Together with finding the missing addends and subtrahends, he managed to provide the answer to a number sentence which did not have the first addend (□ + 20 = 66). According to the adapted Jones et al. (1996) framework, L1 could be categorised as working with counting, partitioning and number relationships at level 5 (see Table 4). Graven and Venkat (2019) note that this learner demonstrated more sophisticated strategic calculation and thinking strategies.

Grouping

In terms of the Adapted LFIN test, the L1 can identify a strip of 4 and add a 10 dot strip to it to make 14 and proceed to follow the 10 dot strips and count from 10 to 84. He then added tens from 10 to 90 after the interviewer placed down strips of 10, followed by knowing how many 10-dot strips were there and how many dots altogether. L1 can produce the correct answer after the interviewer places down a 10 dot strip together with a 5 to make 15 and when asked if another 10 dot strip is added what would the answer be, L1 could say 25. Thereafter, he could identify one strip of 10 and add 3 to make 13 and then add 8 to the 10 dots strip to make 18. The interviewer presents screened dots of 13 (one 10 dots strip and 3 ones) and 16 (one 10 dots strip and 6 ones). He then reveals the 3, dots the learner say that the 10 dots are still screened. Then the interviewer reveals the 10 dots screen of the 16 and the learners say that 6 dots are still screened. Finally, L1 can verbalise the correct sequence as the interviewer reveals the following sequence 10, 13, 23, 33,

Test items (adapted LFIN)		Learner1		Learner 2		Learner 3		Learner 4		Learner 5		Learner 6	
		a	b	a	b	a	b	a	b	a	b	a	b
10. Incrementing by ten off the decuple (using strips) a. Place the 4 dot strip on the desk. Can you count how many dots there are? b. Place a 10 dots strip to the right of the four strip. How many dots are there altogether? If the child starts counting remind her that a strip has ten dots – we do not need to count it. a. Continue placing 10 dots strips to the right of the 4 dots strips. How many dots are there altogether? 24 34 44 54 64 74 84	a	1	-	1	-	1	-	1	-	1	-	1	-
	b	1	-	1	-	1	-	1	-	1	-	1	-
	c	1	-	1	-	1	-	1	-	1	-	X	-
11. Counting by ten on the decuple (using strips) ***** **Only if Q10 is incorrect** **** Explain to the child that each strip like this (show ten strip) has ten-dots. She does not have to count it. a. Put down a 10 dots strip. *How many dots do we have?* If the child counts in one remind her that there are ten dots on each strip b. Put down one 10 dots strip at a time to 9 strips, asking *How many altogether?* 10 20 30 40 50 60 70 80 90 c. Pick up at the strips. How many dots do we have? d. How many strips are here?	a	1	-	1	-	1	-	1	-	X	-	X	-
	b	1	-	1	-	1	-	X	-	X	-	X	-
	c	1	-	1	-	1	-	X	-	X	-	X	-
	d	1	-	1	-	1	-	X	-	X	-	X	-
12. Base-ten thinking (using strips) a. Place a 10 dots strip on the desk. *How many dots are there?* b. Place a 3 dots strip to the right of the ten-strip. *How many dots now?* c. Repeat with 10 and 8 d. Place out a 10 dots strip. *How many dots are there?* Place a 5 dots strip to the right of it. *How many dots do we have now?* If the child answered 15, then ask how many more dots are needed to make 25. **10+5 = [ ]** **15+[ ] = 25**	a	1	-	1	-	1	-	1	-	X	-	1	-
	b	1	-	1	-	1	-	1	-	X	-	1	-
	c	1	-	1	-	1	-	X	-	X	-	1	-
	d	1	1	1	1	1	1	1	X	X	X	1	X
12. Base-ten thinking (using strips) Screened Collection *a. I have [total] dots on this paper. Here are [] dots* (uncover one side), *how many are under here?* **10 + [ ] = 13** **6 + [ ] = 16**	a	1	1	1	1	X	X	1	X	X	X	X	X
13. Uncovering Task (Board 1) Upon each Uncovering ask, "How many dots are there now?"	a	1	-	X	-	1	-	1	-	X	-	X	-



LFIN, Learning Framework in Number.

FIGURE 2: Results of the adapted learning framework in number assessment.

TABLE 4: Learner's performance across Mental Starter Assessment Project pre- and post-test and adapted Learning Framework in Number.

Learner	Improvement	Counting level	Partitioning level	Grouping Level	Number relationships level
Learner 1 (L1)	30% increase	Level 5	Level 5	Level 5	Level 5

Note: Mental Starter Assessment Project pre- and post: All the test items were answered correctly. Adapted Learning Framework in Number: All the test items were answered correctly.

37, 40, 50, 52 and 62. In other words, L1 could work with 10 dots strips together with 3, 4 and 2 dot strips. In terms of grouping L1 was placed at Level 5.

Overall insights from both assessments on base-ten thinking

The MSAP pre-and post-test and adapted LFIN test items emphasise counting – a fundamental component of base-ten. L1's performance in counting not only reflects their ability to sequentially identify numbers but also their understanding of how numbers are grouped within the base-ten frameworks. The tests encourage learners to explore counting in multiple ways, fostering a deeper connection with the numerical system. The assessments prompt L1 to engage in partitioning and decomposing numbers. These skills are vital for base-ten, as they require learners to break down larger numbers into smaller, more manageable parts (e.g. understanding that 23 can be viewed as 20 + 3). L1's growing proficiency in this area indicates a shift from using concrete strategies to employing more abstract reasoning, essential for adeptly navigating the base-ten system. L1's ability to work with groups of numbers, such as identifying groups of 10, highlights the assessments' role in reinforcing understanding of number organisation. This organisation is key to efficient calculations within the base-ten system, enabling learners to recognise patterns and relationships between numbers. Wright et al. (2012a:16) argue that for children to advance in mathematics, they must be introduced to BTT and develop the ability to organise numbers into ones, tens and hundreds. The tests promote exploration of these groupings, laying a foundation for understanding larger quantities and operations. Higher use of fluencies allows learners to perform calculations with a deeper understanding of the underlying number structures, fostering readiness for more complex mathematical concepts like algebra.

Learner 2 (23% increase from pre- to post-test)

Counting, partitioning and number relationships

Learner 2 (L2) operated at the same level as L1 in terms of counting except that she worked with a lower number range in terms of the multiples of 10. In other words, L1 could find the solution to a subtraction number sentence with a subtrahend that contained 4 multiples of 10 (79−40 =), and L2 could only find the solution to a number sentence with a subtrahend with two multiples of 10 (31−20 =). L2 operated at Level 0 as she could not do any test items which promoted partitioning. Furthermore, L2 could only do one test item associated with number relationships (36+ __ = 40). In terms of the adapted Jones et al. (1996) framework, the learners could be described as operating in categories of counting at Level 5 while partitioning at Level 0, and number relationships at Level 1 (see Table 5).

Grouping

L2 like L1 could operate until Level 4, however, could not count a sequence of numbers containing 10-dot strips and 3, 4 and 2 dot strips. In terms of grouping L2 is categorised at Level 4.

Overall insights from both assessments on base-ten thinking

The insights from the MSAP pre- and post-tests underscore the necessity for L2 to develop fluency in basic operations using base-ten structures before progressing to more advanced mathematical concepts. L2's ability to complete initial test items successfully indicates that she possesses foundational skills in mathematics. However, difficulties encountered with more complex operations (e.g. 79 − 40 and 57 + 26) suggest that proficiency may diminish as the complexity of problems increases. L2's struggles with tasks involving missing addends and subtrahends (e.g. solving equations like 38 − □ = 18) illustrate limitations in understanding the relationships between numbers. This gap is critical as they progress because being able to manipulate numbers and recognise relationships between them is foundational for algebraic thinking and higher-order mathematics. With L2 operating at Level 5 for counting and Level 0 for partitioning and number relationships at Level 1, it is evident that L2 has a competent grasp of counting but requires more development regarding their understanding of number operations and relationships, especially when it comes to manipulating two-digit numbers. Combining insights from both assessments, while L2 shows some foundational understanding of mathematics, there are several areas requiring attention for enhanced BTT. L2 demonstrates a good grasp of counting and grouping but exhibits substantial challenges with partitioning and understanding number relationships. L2's difficulties with missing addends and subtrahends suggest gaps in algebraic thinking that must be addressed to build a solid foundation for future mathematics.

Learner 3 (6% increase from pre- to post-test)

Counting, partitioning and number relationships

Learner 3 (L3) could count forwards and backwards in 10s and then add 30 to 6 and subtract 10 from 57. This resulted in L3 being placed at Level 2 of counting, in that the learner provided four correct answers from 12 test items that were associated with counting. In other words, the learner was not fluent in counting in a multiple of 10 or multiples of 10 either on an ENL or in number sentences. L3 operated at Level 0 for both partitioning and number relationships that the learner could not get (see Table 6). The learner could not answer any questions with test items that required splitting numbers into tens and ones and worked with missing addends and subtrahends.

Grouping

L3 could do what L1 did in terms of grouping. However, the difference is that L3 could not work with screened collections to identify a 3 and a 10. So, while the framework suggests that he should be at Level 5, the learner's inability to work with screened collections places L3 at Level 4 in that L3 appears to be able to work generally with grouping flexibly.

Overall insights from both assessments for base-ten thinking

Being classified at Level 0 for partitioning indicates significant challenges in breaking down numbers into their parts (e.g. decomposing 57 into 50 and 7). This difficulty reflects a lack of understanding of how to manipulate numbers effectively for calculations based on the base-ten system. L3's apparent struggle with tasks involving missing addends and subtrahends – where they were unable to find unknown quantities – demonstrates a critical gap in understanding how to relate numbers within arithmetic operations. This inability to tackle basic unknown problems implies they may not yet grasp the foundational concepts of addition and subtraction that are necessary for advanced arithmetic and algebraic reasoning. L3 demonstrates some ability in

TABLE 5: Learner's performance across Mental Starter Assessment Project pre- and post-test and adapted Learning Framework in Number.

Learner	Improvement	Counting level	Partitioning level	Grouping level	Number relationships level
Learner 2 (L2)	23% increase	Level 5	Level 0	Level 4	Level 1

Note: Mental Starter Assessment Project pre- and post: Correct test items, questions, 1, 2, 13, 5, 7, 6, 9, 3, 14, 4 and 10. Adapted Learning Framework in Number: Correct test items, 10 abc, 11 abcd, 12abcd, 12 ab (screened).

TABLE 6: Learner's performance across Mental Starter Assessment Project pre- and post-test and adapted Learning Framework in Number.

Learner	Improvement	Counting level	Partitioning level	Grouping level	Number relationships level
Learner 3 (L3)	6% increase	Level 2	Level 0	Level 4	Level 0

Note: Mental Starter Assessment Project pre- and post: Correct test items, questions, 1, 2, 3, 4. Adapted Learning Framework in Number: Correct test items, 10 abc; 11 abcd; 12abc; 13.

grouping, comparable to Learner 1 (L1). Despite having the potential to place L3 at Level 5 for grouping skills, significant challenges in working with screened collections led to a classification at Level 4 overall. Van den Heuvel-Panhuizen (2008) notes that the shift from counting to structuring numbers depends on the advancement of BTT. This indicates that although L3 can form and recognise groups, he struggles significantly with applying those grouping strategies in practical scenarios. Developing a solid foundation in grouping and relationships will be essential for L3 as they advance in mathematics, particularly in understanding and applying base-ten concepts effectively. The minimal progress noted in the MSAP pre-and post-test emphasises a significant need for engaging and varied practice with foundational skills, particularly in counting and partitioning. Difficulty in working with missing addends and understanding number relationships highlights a more profound misunderstanding that may impede L3's success in more complex arithmetic tasks and algebraic reasoning. Results suggest that L3 would benefit from focussed interventions that build both their confidence and competency in applying BTT to addition and subtraction contexts. In conclusion, the assessment of L3 illustrates a complex profile of strengths and weaknesses in their mathematical understanding related to BTT. L3 shows a potential foundation in the grouping but struggles with essential operations involving counting, number relationships and partitioning.

Learner 4 (3% increase from pre- to post-test)

Counting, partitioning and number relationships

Learner 4 (L4) could provide the answer to the next multiple of 10 either by counting forwards or counting backwards. He could jump forwards and backwards on an ENL in a multiple of ten. Furthermore, L4 could only jump forwards in multiples of ten on the ENL. He also could only add multiples of tens to one-digit numbers to find the answer to a number sentence. L3 was placed at Level 3. In terms of partitioning and number relationships, L4 is the same as L3 and is classified at Level 0 in that they did not answer any of these identified tasks correctly (see Table 7). According to the adapted Jones et al. (1996) framework, the learner is at Level 3 for counting and Level 0 for partitioning and number relationships.

Grouping

L4 appears to be inconsistent in terms of how the learner counts in tens. In question 10 of the adapted LFIN, the learner could add a 10 to 4 dots strip and could count in tens from 14 to 84. While the learner could identify a 10-dots strip, he could add a 3 to make 13 and add a 5 to make 15. Then the learner could count in 10 dots strip together with 3, 4 and 2 dots strips. However, this learner does not count in tens from 10 to 90 or count the number of 10 dots strips and the total number of dots. I would categorise L4 at Level 3 because the learner does not consistently count in tens from a multiple of 10 together with counting the total number of nine 10-dots strips and the total number of dots altogether which is 90.

Overall insights from both assessments for base-ten thinking

L4's ability to count forwards and backwards in multiples of 10 indicates a foundational skill in recognising and working within the base-ten number system. This skill is crucial for higher-level arithmetic operations, reinforcing the concept of grouping by tens. L4's proficiency in adding multiples of ten to single-digit numbers demonstrates an emerging grasp of basic addition within the base-ten framework. While L4 shows understanding in specific tasks, their inconsistency – such as failing to count from 10 to 90 or counting 10 dots strips – suggests a fragmented understanding. This variability indicates that L4 may lack the fluency required to confidently apply base-ten concepts across different contexts. L4's reliance on mental images, like jumping on a number line, reflects an attempt to understand addition concretely. However, the mixed success in transferability of counting by tens (for instance, counting up to various endpoints) may imply that the learner is still in the process of internalising these base-ten concepts fully. L4's classification at Level 0 in partitioning reveals significant challenges in understanding how to break down numbers effectively. Jones et al. (1996) note that learners will struggle with multi-digit numbers when they cannot express a number as a combination of tens value and one's value in a standard form. This gap indicates a need for targeted instruction, as partitioning is fundamental for performing more complex operations within the base-ten system. The inability to solve any tasks related to number relationships (Level 0 classification) highlights a critical area for development. Understanding how numbers interact through addition and subtraction is essential for future concepts in algebra and mathematical problem-solving. The modest 3% increase in scores from pre-test to post-test across both assessments signals some progress but indicates that L4 has not yet effectively mastered key base-ten concepts. The mixed performance across counting and addition tasks suggests sufficient foundational skills exist but require greater consistency and fluency.

Learner 5 (0% shift from pre- to post-test)

Counting, partitioning and number relationships

Learner 5 (L5) in the post-test answered all the answers incorrectly. As seen in the adapted Jones et al. (1996) framework, the learner would be categorised at Level 0 for counting, partitioning and number relationships (see Table 8).

TABLE 7: Learner's performance across Mental Starter Assessment Project pre- and post-test and adapted Learning Framework in Number.

Learner	Improvement	Counting level	Partitioning level	Grouping level	Number relationships level
Learner 4 (L4)	3% increase	Level 3	Level 0	Level 3	Level 0

Note: Mental Starter Assessment Project pre- and post: Correct test items, questions, 1, 2, 5, 7, 6, 3. Adapted Learning Framework in Number: Correct test items, 10 abc, 11a, 12 abd – i, 12 a – i, 13a.

TABLE 8: Learner's performance across Mental Starter Assessment Project pre- and post-test and adapted Learning Framework in Number.

Learner	Improvement	Counting level	Partitioning level	Grouping level	Number relationships level
Learner 5 (L5)	0% shift	Level 0	Level 0	Level 1	Level 0

Note: Mental Starter Assessment Project pre- and post Correct test items, questions, none. Adapted Learning Framework in Number Correct test items, 10 abc.

TABLE 9: Learner's performance across Mental Starter Assessment Project pre- and post-test and adapted Learning Framework in Number.

Learner	Improvement	Counting level	Partitioning level	Grouping level	Number relationships level
Learner 6 (L6)	−30% shift	Level 1	Level 3	Level 2	Level 0

Note: Mental Starter Assessment Project pre- and post: Correct test items, questions, 2, 3 4, 21, 22 and 25. Adapted Learning Framework in Number: Correct test items, 10 ab; 12, abc.

Grouping

L5 could only identify a 4-dots strip and then add 10-dots strips until 84. This learner could not do any other tasks regarding grouping so the learner should be classified as Level 1 in the framework.

Overall insights from both assessments on base-ten thinking

Analysing MSAP pre-and post-tests reveals important insights into L5's classification at Level 0 indicating a significant struggle with counting, partitioning and understanding number relationships. These areas are crucial for developing a solid foundation in BTT, which relies on the understanding of place value and how numbers are constructed from tens and ones. The absence of correct answers suggests that L5 may not even recognise numerical patterns or sequences that are fundamental to base-ten concepts. This lack of recognition is a barrier to understanding how numbers operate within the base-ten system. L5's ability to identify a four-dot strip shows the beginnings of quantity recognition but is still very limited in scope. The recognition of groups of dots is a foundational skill for understanding larger numbers and how they relate in the context of BTT. The capability to add 10 dots strips up to 84 indicates that L5 has made some progress in recognising the value of tens as a grouping strategy. This partial understanding suggests that they can manipulate quantities in a limited way but lack fluency and flexibility, as they struggle to generalise beyond this specific task. L5's inability to complete other tasks related to grouping in the adapted LFIN test highlights a critical gap in their understanding of how numbers interact (which is fundamental in BTT). Wright et al. (2012a) note that these types of children rely on visual aids, like 10 dots strips or fingers, to solve addition and subtraction tasks.

Learner 6 (−30% shift from pre- to post-test)

Counting, partitioning and number relationships

Learner 6 (L6) performed the worst among the six learners in terms of performance on the MSAP pre-and post-tests and adapted LFIN assessments. This learner could count backwards in multiples of 10 and add multiples of 10 to a single digit and subtract a multiple of 10 from a two-digit number. So, for counting, L6 was placed at Level 1. However, I placed L6 at Level 3 for partitioning because the learner is showing some evidence of working with splitting two-digit numbers. On an ENL, they could keep the first number whole and split the number into tens and ones while jumping forwards and backwards. Then, L6 managed to work with tasks that required more flexibility in terms of solving the addition of two-digit numbers (57+26 =) which required the use of bridging through ten to find the answer. In terms of number relationships, I have placed L6 at Level 0 in that they could answer none of the tasks requiring finding the missing, second addend and subtrahend (see Table 9).

Grouping

While L6 scored the lowest score from the pre-test to the post-test the learner showed evidence of working with groups in the following way. The learner identified a 4-dots strip, added a 10-dots strip and then could not count in tens from 24 to 84. The learner could identify 10-dots strips, then could identify and add a 3, 8 and 5 dots strips to it to make 13, 18 and 15. This learner was placed at Level 2.

Overall insights from both assessments for base-ten thinking

L6's ability to count backwards in tens and add multiples of ten to single digits reflects some foundational competency in recognising and manipulating groups of 10, a crucial aspect of BTT. However, this is overshadowed by significant gaps in other foundational skills, particularly in more complex number relationships. L6 was assigned a Level 3 for partitioning, indicating some ability to decompose two-digit numbers. The ability was linked particularly to the jump strategy. Bobis and Bobis (2005) state that the Jump Strategy keeps the first addend or minuend whole, adding or subtracting the second addend or subtrahend in place value chunks of tens and ones. Base-ten thinking relies heavily on the ability to break numbers down into tens and ones. The Level 0 classification in number relationships also highlights a significant gap in understanding how numbers interact. For effective BTT, learners must grasp not only the quantities themselves but also how those quantities can be manipulated and related to one another. L6's struggle with tasks that require more fluent and strategic calculations points to a lack of depth in their comprehension of number relationships. Fluency in counting and understanding the structure of numbers within the base-ten frameworks is essential for progress in mathematics. The analysis shows that L6 can successfully add specific dot strips but has difficulty employing various counting strategies (e.g. counting by 3s, 4s and 2s) and finding missing addends. This inflexibility indicates that the learners' understanding of how to manipulate numbers within the base-ten systems is limited and not yet fully developed.

TABLE 10: Comparison of learner's performance.

Learner	Improvement	Counting level	Partitioning level	Grouping level	Number relationships level
Learner 1 (L1)	30% increase	Level 5	Level 5	Level 5	Level 5
Learner 2 (L2)	23% increase	Level 5	Level 0	Level 4	Level 1
Learner 3 (L3)	6% increase	Level 2	Level 0	Level 4	Level 0
Learner 4 (L4)	3% increase	Level 3	Level 0	Level 3	Level 0
Learner 5 (L5)	0% shift	Level 0	Level 0	Level 1	Level 0
Learner 6 (L6)	−30% shift	Level 1	Level 3	Level 2	Level 0

The analysis of all of the learners' performances linked to levels (see Table 10) reveals a compelling connection between mastery of number relationships and proficiency in BTT. Learners who achieved higher scores across counting, partitioning, grouping and number relationships exhibited a clear understanding of the place value system, particularly in their ability to manipulate tens and ones. For instance, Learner 1's 30% improvement at Level 5 across all categories highlights the advantage of a solid foundation in base-ten concepts, which facilitated more effective problem-solving strategies. In contrast, learners with lower levels in partitioning, grouping and number relationships, such as L5 and L6, showed some elements that would be used to develop BTT. While L6 achieved the lowest mark, he had more elements which pointed and being better at working with BTT. The analysis indicates that although learners may obtain higher scores, the emphasis on their approach to working with BTT offers teachers opportunities to help them improve in specific areas such as counting, partitioning, grouping and number relationships.

Conclusion

The MSAP pre-and post-test and adapted LFIN assessments featured elements of BTT, seen in the learner's ability to count, partition, group and see number relationships with tens and ones. The levels within these categories were determined based on how learners approached additive tasks, either through counting or algebraic methods (Molina & Castro 2021). Learners who excelled at higher levels in both assessments showed their ability to transition from counting to recognising the structure of numbers (L1 and L2). Conversely, learners (L5 and L6) who struggled with tasks in both the MSAP pre-and post-tests and adapted LFIN tests tended to operate at lower levels of BTT.

The analysis of data shows the importance methodologically of clustering test items in terms of their mathematical structure and how the test item speaks to BTT. The results indicate that learners with high levels of BTT (counting, partitioning, grouping and number relationships) will operate at higher levels in terms of their mental strategies to solve additive tasks. Conversely, learners with low levels of BTT will work with smaller number ranges, struggle to partition numbers into tens and ones, use groups to solve tasks and work with more complex number relationships. This study has provided insights into the importance of looking at the design of mathematics test items in the FP from the perspective of BTT. In other words, the analysis of the test items in terms of BTT provides teachers with insights into how to support learners in counting, partitioning, grouping and understanding number relationships, therefore supporting the development of fluent and effective mathematical learning.

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