





An introduction to early grade mental mathematics and number sense



Authors:

Mellony H. Graven¹ 
 Hamsa Venkat^{2,3} 
 Corin Mathews³ 
 Lise Westaway^{1,4} 

Affiliations:

¹Department of Primary and Early Childhood Education, Faculty of Education, Rhodes University, Makhanda, South Africa

²CASTeL, Faculty of Education, Dublin City University, Dublin, Ireland

³School of Education, Faculty of Humanities, University of the Witwatersrand, Johannesburg, South Africa

⁴United Arab Emirates University, Al Ain, United Arab Emirates

Corresponding author:

Mellony Graven,
 m.graven@ru.ac.za

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This Special Collection (SC) emerges from multiple stakeholder collaborations focused on mental mathematics in early grade teaching and learning across two key projects. The first collaborative project is the Mental Starters Assessment Project (MSAP) that began in 2016 following a week-long think tank by a working group comprised of members of the Wits and Rhodes South African Numeracy Chair teams (researchers and project members), international research collaborators, representatives from the Association for Mathematics Education of South Africa (AMESA), Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE), non-government organisations and district, provincial and national Department of Basic Education (DBE) personnel. The think tank focused on identifying a high-leverage, feasible, and scalable intervention to improve number sense teaching and learning in South African schools.

Mental mathematics strategies built on a core set of key representations that included the empty number line and part-part-whole model were identified as opportune curriculum content (DBE 2011) that could support the development of a structural understanding of number and number sense. The 10 min 'warm up' part of lessons – often referred to as mental mathematics – was identified as a useful part of lessons to target for the teaching of mental strategies. The strategies of bridging through ten (BTT), jump strategies, doubling and halving, rounding and adjusting (R&A), re-ordering, and working with the relationship between addition and subtraction became the basis for six 3-week 'lesson starter' units in the MSAP Teacher Guide and learner materials, to be taught in the 'warm up' start of lessons over the first three terms of Grade 3. Each unit begins with a pre-test and ends with a post-test to focus teachers and learners on learning gains, with initial Chair-teams-led trials of expanding scale indicating promising growth in learning outcomes (Graven & Venkat 2021). The DBE¹ began its MSAP rollout across South African Grade 3 classrooms in 2022 (DBE 2021). This national rollout followed a provincial familiarisation trial in 2021 that pointed to statistically significant learning gains across four strategies and across all provinces (Askew, Graven & Venkat 2022).

The MSAP was designed to support both practising teachers and learners in using key representations such as the empty number line and bar (part-part-whole) model to develop number sense. Throughout the MSAP work, the South African Numeracy Chair teams collaborated with DBE personnel in presenting the work across various platforms including a plenary panel at the 2017 SAARMSTE conference and at the annual Mathematics Education Chair Community of Practice forums. This open engagement brought interest from the Department of Higher Education and Training (DHET) and lecturers (pre-service teacher [PST] educators) in preparing pre-service primary teachers to work with the MSAP materials. The MSAP's move into PST education via the Mental Mathematics – Work Integrated Learning (MM-WIL) project follows a thrust of engagements over the previous decade, supported by the DHET's Teaching and Learning Development Capacity Improvement Programme in partnership with the European Union. This programme emphasised collaboration between higher education institutions with a focus on improving Foundation and Intermediate Phase PST education programmes, with mathematics focused work in the previous Primary Teacher Education (PrimTed) study.

In 2021, we thus began a parallel project focused on supporting PST educators and PSTs in developing strengthened understanding of mental mathematics strategies and the teaching thereof. We formed the MM-WIL project, which involved bi-annual working sessions of Chair team staff and primary mathematics lecturers across multiple universities and private institutions coming together to engage about ways to adapt MSAP ideas for use with PSTs. These meetings led to members of the MM-WIL project collaborating to present a SAARMSTE symposium on the theme, 'Incorporating attention to Mental Mathematics in PST education' in 2023. The high level

1. <https://www.education.gov.za/MSAP2022.aspx>.

Note: Mental mathematics and number sense in the early grades.

of interest and engagement in this symposium led us to the decision to propose a SC on Mental Mathematics for the *South African Journal of Childhood Education*. Snapshots of the articles that were being developed for the SC by those in the MM-WIL project and other Chair team researchers focused on the implementation of MSAP ideas in schools or clubs were also presented in a 2024 SAARMSTE symposium on: 'Mental Mathematics for Number Sense in the Early Grades'.

This SC is thus comprised of articles based on research into the implementation of aspects of mental mathematics in after-school clubs (Bowie & Graven 2024) and through use of MSAP in schools (Booi, Vale & Graven 2024) and in PST education (Brien & Mc Auliffe 2024; Kumm & Graven 2024; Lovemore & Graven 2024; Vale & Westaway 2024; Venkat & Mathews 2024). Each of the articles provides a novel contribution that takes the field of developing number sense in early grade mathematics education forward in South Africa.

In the article by Bowie and Graven titled *Using games to develop number sense in early grade maths clubs*, the authors discuss the key features of the games focused on number sense that have been used in after-school clubs programmes. Careful attention to supporting learner engagement and to activities that club facilitators, who may well not be qualified teachers, can make sense of and support was found to be important, with the activities themselves needing to build in checks on learning, and strong connection to calculation strategies.

The article by Booi et al., titled *Research-informed translation of mental strategy teaching materials into isiXhosa*, analyses the isiXhosa translation of a doubling and halving teaching unit in the MSAP materials and compares this to an adapted translation by the first author for use in a Grade 3 classroom in the Eastern Cape. The article addresses the extent to which the translated isiXhosa materials align with the original mathematical meaning of the MSAP materials and interrogates the extent to which selected terms align with the everyday isiXhosa that most learners use. Using the theory of Toury's Descriptive Translation Studies, the article highlights incoherent translations and many ambiguities in the translated MSAP materials. The authors argue for a consistent and user-friendly approach to translation in which conceptual fidelity and clarity of the original material is maintained.

The articles by Brien and Mc Auliffe, Venkat and Mathews, and Lovemore and Graven focus on PSTs' use of the MSAP Jump Strategy (JS) materials with Grade 3 learners during work-integrated learning (practicum) or during microteaching. In their article, *Exploring insights from initial teacher educators' reflections on the Mental Starters Assessment Project*, Brien and Mc Auliffe (2014) explore the reflective essays of 20 purposefully selected final year Bachelor of Foundation Phase (FP) PSTs following their use of the JS MSAP materials in a 4-week teaching practicum. The PSTs were selected to provide diversity in the sample across gender, age and language background. Their analysis of the

reflections showed that PSTs benefitted from reflecting on their practice where they had the space to highlight issues relating to their emerging pedagogical content knowledge (PCK), confidence and competence for teaching mathematics during practicum. They argue that using a multi-dimensional reflection model allows for deeper understanding of the complex nature of mathematics teaching and learning.

While similarly focusing on the use of the MSAP JS materials during practicum, the article by Venkat and Mathews, titled *Mental mathematics knowledge for teaching of 'high gain' pre-service teachers* focused on the learning of three PSTs who had achieved high gains in the pre- to post-tests, when they taught the JS to learners. In this respect, the study focused on gaining insights from PSTs who had ostensibly been 'successful' in supporting learning gains with Grade 3 learners. Based on transcribed interviews with these three PSTs, they found that these PSTs showed strong common content knowledge, elements of specialized content knowledge, a close knowledge of the content as presented in the MSAP 'curriculum', and a broader sense of the progressions supported responsive teaching. They argue that more explicit attention should be given in Initial Teacher Education (ITE) to connections between fluencies and strategies.

Lovemore and Graven's article *Pre-service teachers' use of the jump strategy on the empty number line for mental computation* explores the detail of 40 first year Intermediate Phase PSTs' use of empty number lines in their work with the JS as indicated by their 'teaching' of it in their micro-teaching videos. Their findings point to predominantly 'faithful' and through this, mathematically appropriate, use of the JS approach through the support offered by micro-teaching in their ITE programme. This finding is important in the South African context where there is evidence of gaps in teachers' own knowledge of early number ideas.

While similarly researching the learning of PSTs following the use of MSAP materials during practicum, Vale and Westaway focus on the first strategy of BTT that PSTs taught to Grade 1, 2 or Grade 3 classes. Their data in the form of questionnaires and a focus group interview with 38 third-year PSTs following their teaching along with analysis of these and the results teachers gathered of learner improvements from pre- to post-test show teachers were relatively successful in their teaching of the strategy (there are average improvements across all grades). They found, however, that the PSTs said they needed more time than is recommended in MSAP. The findings point to PSTs' strengthened general pedagogical knowledge, PCK and knowledge of educational contexts.

On the other hand, the article by Kumm and Graven focuses on 59 third-year PSTs' knowledge of various additive reasoning strategies. Their data set is based on a questionnaire in which PSTs solve four basic addition and subtraction calculations, that lend themselves to BTT, JS and R&A, and explain the method they used alongside their calculation.

Thereafter, PSTs were asked to explain their understanding of the BTT, JS and R&A. The findings indicated that while most PSTs were able to get the four calculations correct, they tended to use inefficient methods. The findings indicated that PSTs did not know the strategies of BTT, JS or R&A. Furthermore, several PSTs were unable to solve several basic addition calculations correctly, and 11 of the 58 PSTs were unable to correctly solve 63 subtract 24. They argue that increased attention needs to be given to teaching PSTs a range of efficient calculation strategies.

Across this set of articles, there is evidence that the MSAP and MM-WIL-linked interventions have engaged seriously with the concerns about teachers' content and pedagogic knowledge on the ground (Venkat & Spaul 2015), with combinations of structured resources and pre- and in-service teacher education used to support improvements in the quality of mental mathematics teaching through the enactment of the design of lessons within the materials. The widespread use of concepts linked to mathematical content and pedagogical content knowledge across the articles in this SC further emphasise the analytic focus on these issues. Taken together the collection points to a useful interlinking between teaching mental mathematics in schools and teaching mental mathematics within PST education. Partnerships with the DBE and DHET have been important within this inter-linking, as has the focus on the MSAP materials. In a context with capacity issues at multiple levels, the MSAP and MM-WIL projects point to an initiative that is now a decade-long in the making, and where an ongoing emphasis on building the capacity for mental mathematics teaching and teacher education has been an important focus within a 'building your timber' orientation (Venkat & Graven 2022). There are some indications in this collection of articles that provide an important counter-narrative to the critique that universities in South Africa have made little contribution to improving teachers' mathematical content knowledge. The evidence is measured and specific about the gains that

have been made and the areas of attention that are still needed within early and fundamental mathematics. These findings point to the extent to which time is needed for well-founded research outcomes to have some impact on the ground and the ongoing research needed to study this impact.

References

- Askew, M., Graven, M. & Venkat, H., 2022, 'From what works to scaling up: Improving mental strategies in South African Grade 3 classes', in C. Fernández, S. Linares, A. Gutiérrez & N. Planas (eds.), *Proc. 45th Conference of the Int. Group for the Psychology of Mathematics Education*, vol. 2, PME, Alicante, Spain, July 18–23, 2022, pp. 27–34.
- Booi, T., Vale, P. & Graven, M.H., 2024, 'Research-informed translation of mental strategy teaching materials into isiXhosa', *South African Journal of Childhood Education* 14(1), a1554. <https://doi.org/10.4102/sajce.v14i1.1554>
- Bowie, L.H. & Graven, M.H., 2024, 'Using games to develop number sense in early grade maths clubs', *South African Journal of Childhood Education* 14(1), a1493. <https://doi.org/10.4102/sajce.v14i1.1493>
- Brien, R.D. & Mc Auliffe, S.M., 2024, 'Exploring insights from initial teacher educators' reflections on the Mental Starters Assessment Project', *South African Journal of Childhood Education* 14(1), a1547. <https://doi.org/10.4102/sajce.v14i1.1547>
- Department of Basic Education, 2011, *Curriculum and assessment policy statement Foundation Phase grades R-3: Mathematics*, Government Printing, Pretoria.
- Department of Basic Education South Africa, 2021, *Grade 3 mental starters and assessments: Teacher guide*, DBE/Creative Commons License, Pretoria.
- Graven, M. & Venkat, H., 2021, 'Piloting diagnostic assessment for strategic calculation', *Mathematics Education Research Journal* 33, 23–42. <https://doi.org/10.1007/s13394-019-00291-0>
- Kumm, M. & Graven, M.H., 2024, 'Exploring pre-service teachers' knowledge of efficient calculation strategies', *South African Journal of Childhood Education* 14(1), a1494. <https://doi.org/10.4102/sajce.v14i1.1494>
- Lovemore, T.S. & Graven, M., 2024, 'Pre-service teachers' use of the jump strategy on the empty number line for mental computation', *South African Journal of Childhood Education* 14(1), a1526. <https://doi.org/10.4102/sajce.v14i1.1526>
- Vale, P. & Westaway, L., 2024, 'The development of pre-service teachers' competence to teach mental calculation strategies', *South African Journal of Childhood Education* 14(1), a1552. <https://doi.org/10.4102/sajce.v14i1.1552>
- Venkat, H. & Graven, M., 2022, 'Bringing the mental starters assessment project to scale in Foundation Phase: A "building your timber" approach', in N. Spaul & S. Taylor (eds.), *Early grade interventions in South Africa*, pp. 229–245, Oxford University Press.
- Venkat, H. & Mathews, C.D., 2024, 'Mental mathematics knowledge for teaching of "high gain" pre-service teachers', *South African Journal of Childhood Education* 14(1), a1509. <https://doi.org/10.4102/sajce.v14i1.1509>
- Venkat, H. & Spaul, N., 2015, 'What do we know about primary teachers' mathematical content knowledge in South Africa? An analysis of SACMEQ 2007', *International Journal of Educational Development* 41, 121–130. <https://doi.org/10.1016/j.ijedudev.2015.02.002>