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Pairs trading of nearly identical twin stocks: The case of GOOGL versus GOOG

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Purpose: As computer technology and statistical methods advance, statistical arbitrage research focuses on developing more sophisticated trading methods. Twin stocks are stock pairs such as GOOGL and GOOG that trade as different stocks but are identical except one has voting rights. The two stocks should be identical, and standard arbitrage trading using these twin stocks should produce little if any profits. The purpose of this study is to test whether this hypothesis is true and whether the novel statistical arbitrage methods can be used on twin stocks to produce profits.

Design/methodology/approach: This article uses the Ornstein-Uhlenbeck process method to calculate the entry and exit timing and expected return under different transaction costs. We also use the factor model to test the impact of long- and short-term factors on the expected returns obtained by our statistical arbitrage trades.

Findings/results: Our test results show evidence of positive arbitrage profits, indicating that the hypothesis concerning twin stocks is not true. In addition, we find that the arbitrage profits are not affected by short-term risk factors.

Practical implications: Our research shows that profitable statistical arbitrage can be applied to asset pairs other than an asset and its derivative. Applying statistical arbitrage to twin stocks may be a good alternative for investors interested in arbitrage trading.

Originality/value: This is the first study that applies statistical arbitrage to twin stocks. We show investors can profit when applying statistical arbitrage to even highly correlated assets such as twin stocks still obtain excess returns.

Keywords: twin stocks; synthetic portfolio; Ornstein-Uhlenbeck process; Pairs trading.

Introduction

Statistical arbitrage refers to a set of trading strategies that use mean regression analysis to invest in different portfolios of several securities in a short period. Pairs trading is a type of statistical arbitrage, and its methods range from basic weighted moving average to complex dynamic factor analysis. Pairs trading can be divided into two categories: statistical arbitrage and risk arbitrage. A pair trade involves two stocks in the same industry or with similar characteristics when their relative pricing deviates from equilibrium. It is linked to co-integration (Bossaerts 1988; Bossaerts & Green, 1989) and correlations of stock prices, mean reversions, overreactions (De Bondt & Thaler, 1985; Lo & MacKinlay, 1990), contrarian selection (Jegadeesh & Titman 1993) and price trends. Pairs trading profits, however, are largely determined by the modelling and prediction of the spread time-series of two stocks (or index or commodities). Past research on mean-reversion and contrarian strategies included Poterba and Summers (1988), Lehmann (1990) and Lo and MacKinlay (1990). A common approach is to construct a stationary, mean-reversion synthetic asset as a linear combination of securities.

Essentially, pairs trading involves three steps. Firstly, look for homogeneous securities with similar historical price trajectories. Secondly, find the price difference between the two securities during subsequent trading. Thirdly, if the two securities exist in a balanced relationship, the investors can construct a long (short) dollar-neutral portfolio. When the paired stock price difference (spreads) deviates from the historical average, sell higher-priced stocks and buy lower-priced stocks at the same time and wait for them to return to a long-term equilibrium relationship, thereby earning a return on the price convergence of the two stocks. Consequently, if investors wish to maximise profits, they should search for spreads on pairs that have both high variance and strong mean-reversion.

There are very few examples of stock-to-stock arbitrages, mainly because few such securities exist in the market. Even if such pairs of stocks exist, the price is too close to let investors be willing to carry out arbitrage. The shares issued by the same company in different markets often have the same rights. For example, every shareholder has the same voting rights and dividend rights. This kind of stock is called twin stock. In theory, because the cash flow corresponding to each stock is the same, their stock price should also be the same. Moreover, the factors that allow for the construction of the replicating asset may not fully exist in the real market. The most important feature of replicating assets is that they have the same trend. Among the physical assets, twins stock can best show this characteristic.

The only difference between GOOGL and GOOG is that the GOOGL shares are voting shares, while the GOOG shares are not. Therefore, GOOG should trade at a discount to GOOGL, but because both ticker symbols have the same stake in Alphabet, their up and down movements should for all practical purposes be identical. In the light of this, arbitrage profits between these two stocks should be small, if not non-existent after accounting for transaction costs.

The results of this study show that this hypothesis of non-existent arbitrage profits after accounting for transaction costs is not true. Specifically, we show that if we combine GOOGL and GOOG to construct a synthetic portfolio, the synthetic portfolio would follow a mean-reverting Ornstein-Uhlenbeck (OU) process. This study makes use of the methodologies of Bertram (2010) to analyse the arbitrage pair formed by GOOGL and GOOG and the OU process to form the optimal statistical arbitrage. Robustness tests using another pair of twin stocks further show that applying the optimal statistical arbitrage strategy to pair trade RDS.A and RDS.B shares of Royal Dutch Shell. We also modify our tests to include the NASDAQ, VIX, excess market return (market return minus free rate), term spread and default spread to test the relationship of this arbitrage strategy to long-term and short-term economic effects. We show that this strategy is not affected by short-term risk factors.

In summary, our findings challenge the hypothesis that twin stocks such as GOOGL and GOOG offer no arbitrage opportunities after transaction costs. By modelling the price spread as an OU process, we show that statistically significant profits (8.71% – 8.36% expected returns, Sharpe ratios of 5.638–5.363) can be achieved, with robustness confirmed across another twin-stock pair (RDS.A/RDS.B). Crucially, these returns are insulated from short-term market risks, underscoring the strategy's appeal for long-term investors. This study's key contributions include: (1) the first application of optimal statistical arbitrage to twin stocks and (2) empirical evidence that such strategies can exploit even minimal pricing inefficiencies between near-identical assets.

The remainder of the article is organised as follows. Section 'Related literature' presents the related literature on statistical arbitrage trading. Section 'Data and methodology', discusses data and methodology. Section 'Empirical results and Robustness tests' presents the empirical results and robust. Section 'Conclusion' concludes and summarises the main results of the article.

Related literature

Statistical arbitrage trading is a market-neutral strategy based on the concept of relative pricing. When a strategy is neutral with respect to market returns, the strategy return is unrelated to the market return. The basic idea of relative pricing is that stocks with similar characteristics are sometimes overvalued, sometimes undervalued or the same. Therefore, under this concept, the spread can be seen as the degree of mispricing of each other. Pairs trading is the original market-neutral strategy. This strategy involves simultaneously establishing long and short positions when the price difference deviates significantly from the mean. The success of statistical arbitrage depends on finding two suitable securities and then modelling and forecasting the time-series of the price difference.

However, statistical arbitrage strategies must rely on constructing a predictable mean-reversion spread model. Vidyamurthy (2004) uses Arbitrage Pricing Theory (APT) to find stocks with similar common return components. They developed a framework for forecasting using co-integration and analysed the mean-reversion of the residuals. In short, statistical arbitrage strategies, pairs trading and related strategies rely on constructing mean-reverting spreads with a certain degree of predictability.

Pairs trading usually uses two securities of similar characteristics with price imbalances for arbitrage. When the spread expands, short the high priced stock and long the low priced stock simultaneously, and the investor will profit from when the spread eventually shrinks in the future. In addition, pairs trading is also applicable to commodity trading. Commodities trading includes commodity index investment and the proliferation of commodity-based hedge funds (Cummins & Bucca, 2012; Gregeriou et al., 2009; Mou, 2010). Arbitrage trading studies usually use securities and their derivative futures or commodities of similar nature.

In arbitrage trading, investors want the price to deviate from the mean as much as possible, but at the same time have the mean-shift degree to be as small as possible. However, in the arbitrage method described in Gatev et al. (2006), both are minimised; so the distance method is not an optimal solution but a suboptimal solution. Vidyamurthy (2004) constructed a Pearson correlation coefficient based on common factors to measure the absolute value of the distance between stock values. The higher the absolute value, the better the co-integration pairing. However, there is a problem in his method: how to define the APT model factor. According to Avellaneda and Lee (2010), at least 30

factors are needed to explain only about 50% of the change in yield. Moreover, their model is not actually tested with real financial assets.

Elliott et al. (2005) use a mean-reverting Gaussian Markov chain model, which is observed in Gaussian noise, for the spread. They compare the model with subsequent observations of the spread to determine appropriate investment decisions. They show this approach can be used in any financial market and gain wealth, even though it is at times out of equilibrium. Huck and Afawubo (2014) explore the performance of a pairs trading system based on various pairs selection methods. They use the components of the S&P 500 index as an observation target. They show that when the stock price deviates from equilibrium, it becomes feasible to enter the trade (long or short) after controlling for risk and transaction costs.

Most of the financial information is continuous time-series, non-stationary and non-Gaussian. It is necessary to deal with the stability of the time-series data. However, the OU process is a stationary Gauss Markov process and is temporally homogeneous. It satisfies the stability condition. Boguslavsky and Boguslavskaya (2004) also based their research on the OU process. Bertram (2009, 2010) proposed an applicable mathematical framework for statistical arbitrage trading.

Bertram (2009) gives a continuous-time trading strategy and provides a framework for constructing and analysing mathematical, statistical arbitrage methods. Their results show that in this framework, the best strategy is to strike a balance between revenue per transaction and transaction cost and the frequency of random transactions. Bertram's (2010) method is based on transforming asset pairs into a synthetic asset that satisfy the OU process. This has the advantage of producing closed-form optimal solutions and can also be used to model high-frequency trading.

Ekström et al. (2011) explored optimal liquidation of pairs trading in the framework of the OU process and analysed the sensitivity of the model parameters. Cummins and Bucca (2012) use Bertram's method to discuss the arbitrage process of oil and its derivative. They observe 861 energy futures spreads from 2003 to 2010. They assumed that a rational investor would take advantage of high-profit opportunities. As a result, pairs should combine the lowest drift in spread mean and the highest spread variance features. They show that Bertram's (2010) approach is potentially profitable in non-Gaussian processes.

There are other studies on the application of OU process in arbitrage, such as Göncü and Akyildirim (2016) assume an OU process with the noise term driven by a Lévy process with generalised hyperbolic distributed marginals. Their article uses the commodity futures data to give trading signals and ranks all potential pairs for trade priority in terms of the distance to the expected profit-maximising thresholds. Empirical evidence from commodity futures indicates the existence of significant mean-reversion together with high peak and fat tails for the distribution of spread residuals. Their result shows strong evidence for the model's profitability even in the presence of transaction costs.

Chen and Yang (2021) use Berkshire-A and its replicating assets for statistical arbitrage trading. The replicating assets are constructed from a portfolio that simulates the return of a factor model. Their results indicate that the statistical arbitrage method of Bertram (2010) is profitable using the replicating assets. Bertram (2010), Cummins and Bucca (2012) and Chen and Yang (2021) provide evidence for the profit potential of Bertram's approach. It could be conceptually enhanced by applying Bertram's method to non-Gaussian processes, thus better reciting the stylised facts of financial data.

Caldeira and Moura (2013) applied the univariate co-integration method to the 50 most liquid stocks in the Brazilian stock index Bovespa. They follow the two-step method of Engle and Granger (1987) to test the co-integration relationship of all pairs formed during a year. They show that this strategy generates excess returns of 16.38% per year. Moreover, it also gives a Sharpe ratio of 1.34.

In addition to assets such as commodity stocks, Rösch (2021) used American Depositary Receipt (ADR) to study the impact of arbitrage on liquidity. He uses trading data in the ADR market between 2001 and 2016. His results showed that price deviations lasted an average of 12 min and were mainly caused by price pressure. The results indicate that impulse response functions estimated at 1-min intervals find positive shocks to arbitrage. These findings are confirmed using the regime of exogenous changes in arbitrage barriers. His findings suggest that arbitrage reduces price pressure and provides liquidity.

Kovbasyuk and Marco (2022) studied the effects of short-term investment and private information on arbitrage. Their studies suggest that arbitrageurs with shorter investment horizons can profit from accelerating price discovery by publicising their private information. They propose that risk-averse arbitrageurs can advertise private information about mispriced assets to rational investors with limited attention, while selecting their portfolios to exploit price adjustments induced by such advertising. However, advertising many assets can fragment investors' focus, leading to a smaller number of informed traders per asset and slowing price discovery. Therefore, arbitrageurs will concentrate advertising on a few assets. The results show that as long as advertising succeeds in attracting the attention of rational investors, it reduces the risk that arbitrageurs will have their positions liquidated by noise traders.

Renault et al. (2023) develop an extension of the APT framework to study the pricing of squared returns and volatility. The study analysed the interaction between return factors and characteristic variance and found that

there is a common characteristic variance factor between the two, but it cannot be proved that this represents a risk factor that causes missing return levels. However, studies have shown that the price of a special variance factor determined using squared returns is smaller than the price of market variance risk.

Dávila et al. (2024) conducted research from the perspective of social value and arbitrage gap. The results show that the arbitrage gap corresponds exactly to the social marginal value of executing the arbitrage trade. In addition, the arbitrage gap and price impact indicators are the total social value generated by closing the arbitrage gap. Arbitrage gaps may arise due to non-monetary benefits of holding a specific asset. At a given arbitrage gap, the total social value of arbitrage is higher when market liquidity is high.

While prior research has extensively explored statistical arbitrage in asset-derivative pairs (e.g. Cummins & Bucca, 2012) or replicating portfolios (Chen & Yang, 2021), the application of these methods to twin stocks – near-identical assets with minimal structural differences – remains unexamined. Existing studies either focus on highly divergent pairs (Gatev et al., 2006) or complex factor-modelled replicas, leaving a critical gap in understanding whether arbitrage opportunities persist even for assets with near-perfect correlation. Our study fills this gap by (1) demonstrating that twin stocks such as GOOGL/GOOG exhibit statistically exploitable deviations despite their theoretical parity and (2) providing the first empirical framework to optimise such trades using the OU process. By bridging this oversight in the literature, we expand the scope of statistical arbitrage to a new class of assets while challenging assumptions about market efficiency in near-identical securities.

Methodology

In this article, we examine the statistical arbitrage trading between the GOOGL stock and the GOOG stock. We obtain the price of the stock from the DataStream. Our article also uses the Chicago Board Options Exchange (CBOE) Volatility index (VIX). The VIX is an index designed to measure the market's expectations for volatility over the following 30 days. It does this using the CBOE's S&P 500 index derivatives contracts. The VIX has become the primary benchmark for measuring the volatility of the US stock market. Because of the rapid flow of funds in markets worldwide and the mutual influence between markets, investors worldwide pay attention to its changes. It is cited by numerous financial media outlets as the fear index. The VIX data are collected from the Federal Reserve Economic Data (FRED) website. The data period is from 01 January 2015 to 31 December 2024.

The OU process differs from the Geometric Brownian motion process. The standard workhorse and the most widely used stock model for stock behaviour, for the vast majority of stochastic stock price modelling simulation research, especially in the option pricing literature, is the assumption that the underlying stock prices follow the Geometric Brownian motion process or some extension of it to account for stochastic volatility and or jumps. This assumption is reasonable and fits the behaviour pattern of actual stock price patterns nicely. However, assets that follow the Geometric Brownian motion based stochastic process cannot be traded successfully under the standard pairs trading framework, because assets that follow the Geometric Brownian motion process do not mean revert and may drift away either up or down from the initial price forever. For successful pairs trading, the traded asset needs to mean revert so that the investor can go long or short the asset whenever its price drifts too far away from its long-term mean value.

In this article, because the GOOGL and GOOG are for all practical purposes identical assets, differing only in voting rights, the difference between these two stock prices should be just white noise, with a long-term mean slightly above zero to account for the voting rights of the GOOGL compared to GOOG, which does not have the voting rights. This means if we construct a synthetic asset X by forming a portfolio that simultaneously has a long position in GOOGL and a short position in GOOG; this synthetic asset should be white noise and thus should mean revert. Formally, this mean-reverting white noise process can be described mathematically using the PU process.

Specifically, under the OU process statistical arbitrage trading model, a continuous trading strategy can be formed by a series of separate transactions executed on the continuous-time mean-reverting OU stochastic process. As a result, several important factors influencing the profitability of the trading strategy can be specified as functions of the frequency by which these trades take place. The trading frequency, for example, can be defined by how many times the pairs trading strategy under analysis trades per unit of time. This value depends on how long it takes in total to go from the entry point of the trade to the next entry point, transiting through the exit point of the trade during the cycle.

Mathematically, we model the price of our synthetic asset as Equation 1:

$$P_t = e^{x_t};\ X_{t0} = x_0 \qquad \text{[Eqn 1]}$$

X_t denotes the synthetic asset, which is constructed as a portfolio that is simultaneously long GOOGL and short GOOG. Thus, the spread between two assets' log-price series should follow a zero-mean OU process. Using Equation 1, the synthetic asset price at time t can be written as: synthetic asset price(t) = log (GOOGL price(t))-log (GOOG price(t)). X_t, therefore, can be written as the following stochastic differential equation (Equation 2):

$$dX_t = -\delta X_t dt + \rho dW_t \qquad \text{[Eqn 2]}$$

where $\delta > 0$ and $\rho > 0$, W_t is a Wiener process. Defining the entry and exit levels of the trading strategy by a and b, the entry levels of the trading strategy by $X_t = a$ and exiting the trade at $X_t = b$. A trade cycle is the time taken for the process

from a to b and then return back to a. A completed trading cycle time can be written as Equation 3:

$$T = T_{a\to b} + T_{b\to a} \quad \text{[Eqn 3]}$$

where $T_{a\to b}$ is the time to transition from a to b and $T_{b\to a}$ is the time to transition from b to a. Because the transaction follows the Markovian property of the OU process, $T_{a\to b}$and $T_{b\to a}$ are independent.

Given this framework, the return from one full round of trade can be written as a function of exit and entry minus the transaction costs c. Equation 1 and Equation 2 show that when X_t represents the log-price of the synthetic asset, the function $r(a, b, c) = (a - b - c)$ will be the continuously compounded rate of return for a single trade, after accounting for transaction cost. When the return exceeds the transaction cost from entry to exit, we can obtain the following equations for the mean and variance of the return (Equation 4 & Equation 5):

$$\mu(a,b,c) = r(a,b,c)/E(T) \quad \text{[Eqn 4]}$$

$$\sigma^2(a,b,c) = r(a,b,c)^2 V(T)/E(T)^3 \quad \text{[Eqn 5]}$$

where $E(T)$ denotes the complete expected trade cycle time and $V(T)$ denotes the variance of the complete expected trade cycle time. In Equation 2, the process X_t is stationary, so the return for each transaction is deterministic. However, the time frame for realising returns is stochastic. According to the properties of X_t and T, a trade may have an uncertain duration of time and experience significant deviations in the exit level. This problem can be solved by taking the first-passage time of the OU process.

Given this framework and Bertrams' (2010) results, the optimal entry-level and expected return can be specified as a function of the transaction cost c (Equation 6 & Equation 7):

$$a = -\frac{c}{4} - \frac{c^2\delta}{4\left(c^3\delta^3 + 24c\delta^2\rho^2 - 4\sqrt{3c^4\delta^5\rho^2 + 36c^2\delta^4\rho^4}\right)^{\frac{1}{3}}} - \frac{\left(c^3\delta^3 + 24c\delta^2\rho^2 - 4\sqrt{3c^4\delta^5\rho^2 + 36c^2\delta^4\rho^4}\right)^{\frac{1}{3}}}{4\delta} \quad \text{[Eqn 6]}$$

$$\mu^*(a,c) = \frac{\delta(2a+c)}{2\pi \mathrm{Erfi}\,(a\sqrt{\delta/\rho})} \quad \text{[Eqn 7]}$$

where a is the optimal entry-level, c is the transaction cost and μ^* is the maximum expected return.

Macroeconomic conditions can influence the capital market and the primary motivation for using arbitrage methods is to mitigate unfavourable macroeconomic conditions. Therefore, in addition to detecting the entry and exit points and the profits of synthetic asset arbitrage, we want to know whether applying the optimal arbitrage strategy can also circumvent the influence of macroeconomic conditions. To answer this question, we also examine the relationship between several macroeconomic variables and profits from optimal pairs trading. We use the term spread and default spread variables to proxy long-term economic sentiment and NASDAQ return, VIX return and market risk premium (market return minus free rate) to proxy short-term economic sentiment.

These proxies have been used in several studies. Cook and Tang (2010), for example, used the macroeconomic variables term spread and default spread to test the capital structure. The term spread is the difference between the 10-year Treasury bill rate and the 3-month government bond yield rate. Default spread is the difference between the average yield of bonds rated Baa and the average yield of bonds rated AAA, each rated by Moody's based on bonds with maturities 20 years and above. Term spreads usually show a positive value. Larger values of this variable mean that the economy is doing better. Default spreads are also typically positive.

We obtain the data from the FRED database. *Rm-Rf* is the excess return on the market. R_m denotes the value-weight return of all Center for Research in Security Prices (CRSP) firms in the US that are listed on the New York Stock Exchange (NYSE), AMEX or NASDAQ with a CRSP share code of 10 or 11 at the beginning of month t with good shares and price data at the beginning of t and good return data for t. *Rf* is the 1-month Treasury bill rate (from Ibbotson Associates).

To examine the relationship between the macroeconomic indicators and the stock returns of GOOGL, we use the following Generalised Autoregressive Conditional Heteroskedasticity (GARCH) model specifications; Equation 8 and Equation 9 are the conditional mean equations of the two models.

$$\begin{aligned} Googl_t &= a_i + m(R_{mt} - R_{ft}) + s(r_{Nasdaq})_t \\ &+ h(r_{Vix})_t + u(default)_t \\ &+ b(term)_t + e_{it} \end{aligned} \quad \text{[Eqn 8]}$$

$$\begin{aligned} synthetic_t &= a_i + m(R_{mt} - R_{ft}) + s(r_{Nasdaq})_t \\ &+ h(r_{Vix})_t + u(default)_t \\ &+ b(term)_t + e_{it} \end{aligned} \quad \text{[Eqn 9]}$$

The conditional variance equation of the models is specified in Equation 10 and Equation 11:

$$h_t^2 = \gamma_0 + \gamma_1 e_{t-1}^2 + \gamma_2 h_{t-1}^2 \quad \text{[Eqn 10]}$$

$$e_t = h_t v_t, v_t \sim N(0,1) \quad \text{[Eqn 11]}$$

In Equations 8 and Equation 9, $Googl_t$ denotes the price return of GOOGL at day t. $synthetic_t$ denotes the price return of the

synthetic asset at day t. $R_{mt} - R_{fi}$ is the risk premium at day t. r_{nasdaq} is the return of the NASDAQ index, and r_{Vix} is the return of VIX. The returns are computed by using the following function (Equation 12):

$$r_index_t = in(index_t / index_{t-1}) \quad \text{[Eqn 12]}$$

Results

Empirical results

In this section, we present the results of the optimal statistical arbitrage trading of GOOGL and GOOG. We also provide the results of robustness tests that use the RDS-A and RDS-B. In the research of Chen and Yang (2021), factor models are used to form replicating assets and conduct statistical arbitrage. Their research proves that a replicating asset can be formed by using various assets with similar properties to conduct arbitrage. Nevertheless, there are two problems: where to find so many assets of similar nature; the composition of the portfolio may be too subjective.

This study explores whether twin stocks can meet this nature. Using twin stocks can eliminate the use of factor models and allow investors to use existing stocks in the stock market for arbitrage directly. However, existing methods, such as the distance approach of Gatev et al. (2006), are not easy to apply in twin stocks. Because the prices of twin stocks may be too close to each other (e.g. GOOGL and GOOG) or maybe too far apart from each other (e.g. Berkshire-A and B), the traditional statistical arbitrage methods require extensive restrictions and modifications.

Table 1 lists the comparison of GOOGL and GOOG's stock prices. In the table, we find that the statistics of the two stocks are very similar and the two stocks have the same trend. To express this relationship, we have compiled the price return trend chart of GOOGL and GOOG from 01 January 2015 to 31 December 2024 in Figure 1 and Figure 2.

In Figure 1 and Figure 2, we can see that the two stocks have the same trend. Having a similar trend meets the primary conditions for pairs trading. We can see that during 2020 (the start of coronavirus disease 2019 [COVID-19]), share price returns fell to close to –15%, be it GOOGL or GOOG. The primary purpose of this article is to use twin stocks to conduct statistical arbitrage trading. When twin stocks temporarily show different prices, an arbitrage opportunity will exist.

TABLE 1: Descriptive statistics of price and price return for GOOGL and GOOG.

Variables	Stock price		Price return	
	GOOGL	GOOG	GOOGL	GOOG
MAX	198.16	196.66	0.151	0.149
MIN	24.56	24.85	-0.124	-0.118
AVG.	82.79	82.71	0.001	0.001
STD.	44.54	43.73	0.018	0.018

Note: In Table 1, we list the maximum, minimum, mean and standard deviation of the stock price and stock price return of GOOGL and GOOG, respectively.

For this study, we apply the OU process to arbitrage strategies and find the optimal entry and exit points for twin stocks. We compute the log-price of the GOOGL and GOOG to form a synthetic asset and calculate the parameters for the OU process. We use Equation (1) and get δ = 0.0032568, ρ = 0.012784 for the synthetic asset constructed from GOOGL and GOOG. The computed mean and variance then allow us to further determine the properties of the strategy in terms of the trade entry and exit levels for different transaction costs.

Because transaction costs may have different bargaining power depending on the conditions of investors (such as transaction size or institutional investors), a cost range is used for subsequent calculations. Figure 3 shows the distribution of expected returns of the synthetic asset constructed using GOOGL and GOOG. It shows the distribution of expected returns of the synthetic asset under different transaction costs and entry levels. Specifically, the figure shows the expected returns under different combinations of 'a' and 'c', where the transaction cost 'c' ranges from 0.001 to 0.007 and entry-level 'a' range from –0.03376 to –0.06591. The values for 'a' and 'c' are computed using Equation 6. Figure 3 shows that a larger 'a' makes the expected return smaller under the same transaction cost 'c'. In addition, higher transaction costs make the expected return smaller for a given entry-level 'a'. Large values for 'a' and transaction costs are unfavourable for the expected

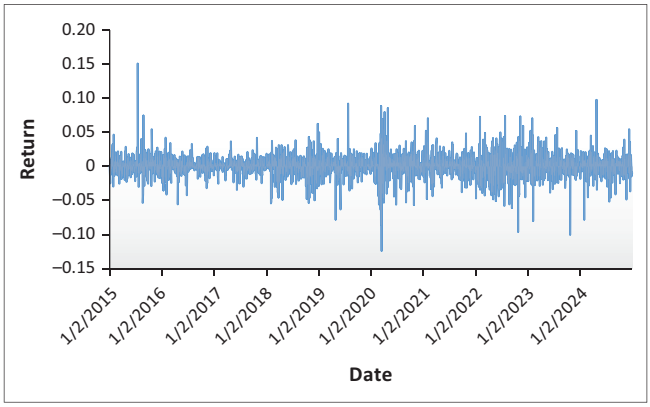


FIGURE 1: This figure shows the stock price return of GOOGL; the sample period is from 01 January 2015 to 31 December 2024.

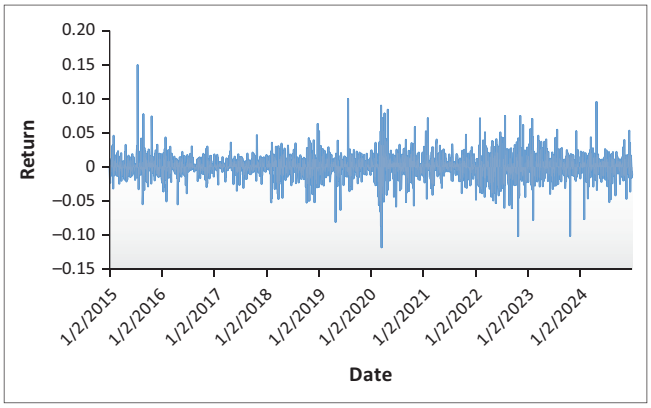


FIGURE 2: This figure shows the stock price return of GOOG; the sample period is from 01 January 2015 to 31 December 2024.

return of the trading strategy. So a suitable return mean and variance allows us to determine the trading range for the optimal trading strategy. Equation 3 helps us to obtain the optimal 'a' for $a < 0$. When $a > 0$, the expected return is equal to 0.

Table 2 shows the expected returns of using the optimal statistical arbitrage strategies for GOOGL and GOOG pair for different transaction costs 'c' and 'a'. This table also presents the Sharpe ratios. The Sharpe ratio describes how well an asset's returns are compensated relative to the risk investors take. Assets with a higher Sharpe ratio represent better returns under the same risk. For ordinary investors, it is a relatively easy-to-understand indicator to measure of risk and return. The optimal solution for 'a' is from Equation 6. The results show 'a' and expected return will become smaller as the transaction costs increase. From the table, as the transaction cost increases from 0.001 to 0.007, the expected return decreases from 8.71% to 8.36%. The Sharpe ratio also decreases from 5.638 to 5.363 as the transaction cost increases (This article assumes that the risk-free interest rate is 1.5%.).

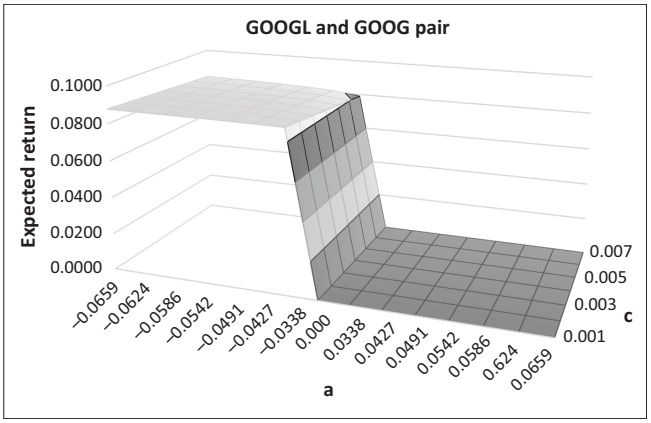
Robustness tests

To test for the robustness of our method, we use RDS.A and RDS.B. These two stocks are ADRs of Royal Dutch Shell. Royal Dutch Shell is the second-largest oil company globally, registered in the UK and headquartered in The Hague, Netherlands. The differences between RDS.A and RDS.B as follows: RDS.A is listed in the United States (NYSE) in the form of an ADR and complies with Dutch regulations where dividends are calculated in US dollars using a 15% dividend withholding tax. RDS.B is listed in the US (NYSE) in the form of an ADR and complies with UK regulations where dividends are calculated in US dollars with no withholding tax. The data period for analysis is from 01 January 2010~31 December 2021. These two ADRs are issued by the same company that issued GOOGL and GOOG. Shell USA, Inc.'s RDS-A and RDS-B were merged into a new stock named 'The Greenbrier Companies (GBX)' in January 2022, so the sample period for Shell Oil is only taken until 31 December 2021.

We construct the synthetic asset from RDS.A and RDS.B. The parameters for the OU process were computed using Equation 1. We get $\delta = 0.0056797$, $\rho = 0.014507256$. The trading band test results shown in Figure 4 are similar to those in Figure 3 for GOOGL and GOOG. The pattern shows that at a given transaction cost 'c', an increase in entry-level 'a' reduces the expected return. Moreover, at a given 'a', an increase in transaction costs hurts the expected return.

Table 3 shows the expected returns of using the optimal statistical arbitrage strategies for the RDS.A and RDS.B pair under different transaction costs 'c'. The table displays four types of information: entry-level 'a', transaction cost 'c', expected return and Sharpe ratio. Table 3 shows that 'a' and expected return will become smaller as the transaction costs increase. In the table, the transaction cost increases from 0.001 to 0.007; the expected return decreases from 8.63% to 8.26%. Sharpe ratio also decreases from 1.4167 to 1.3992 when the transaction cost increases.

Figure 5 shows graphically the results of how hypothetically investing $100 000.00 into the optimal pairs trading strategy would have grown when applied to the Google synthetic asset and RDS synthetic asset. It is important to notice that the true returns the investor would obtain in the real world



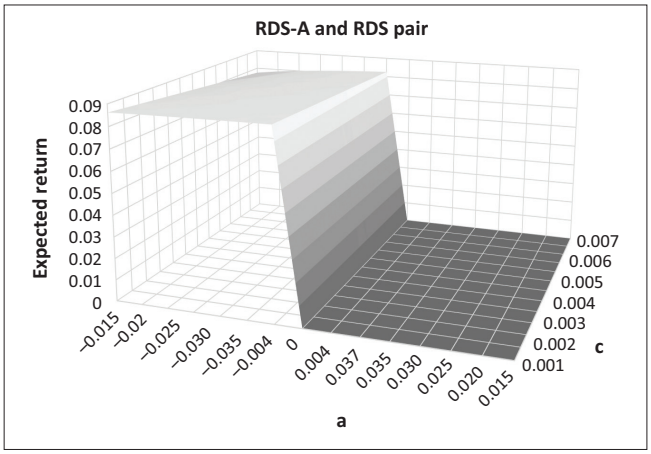
Note: This figure presents the relationship of expected return, a, and transaction cost for the optimal statistical arbitrage of the (GOOGL and GOOG) synthetic asset. The parameters are $\delta = 0.003268$, $\rho = 0.012784$ and $c = 0.001$~0.007.

FIGURE 3: The band of expected returns for GOOGL and GOOG pair.

TABLE 2: Expected returns and Sharpe ratios for GOOGL and GOOG pair.

c	GOOGL & GOOG (2015–2024)		
	a	Return (%)	Sharpe ratio
0.001	-0.03376	8.71	5.638
0.002	-0.04273	8.63	5.577
0.003	-0.04909	8.56	5.525
0.004	-0.05422	8.51	5.480
0.005	-0.05858	8.45	5.438
0.006	-0.06243	8.40	5.399
0.007	-0.06591	8.36	5.363

Note: This table presents the expected returns and Sharpe ratios from using the optimal arbitrage strategy under the OU process for the (GOOGL and GOOG) pair. The table lists the expected returns under different transaction costs. The optimal solution for 'a' is from Equation 6. The results of this table were computed using $\delta = 00.003268$ and $\rho = 0.012784$. 'c' is the transaction cost, and 'a' represents the entry level.



Note: This figure presents the relationship of expected return, a, and transaction cost for the optimal statistical arbitrage of the (RDS.A and RDS.B) synthetic asset. The parameters are $\delta = 0.0056797$, $\rho = 0.014507256$ and c = 0.001~0.007.

RDS, Royal Dutch Shell.

FIGURE 4: The band of expected returns for RDS.A and RDS.B pair.

should be quite a bit larger than that shown in Figure 5, depending on her creditworthiness; because in the real world, the long-short portfolio of the synthetic asset is actually a zero investment portfolio, and, thus, the true initial investment required by the investor would be her minimum margin deposit required for short selling and not the actual cost of purchasing GOOGL, because simultaneously shorting GOOG will credit the investor's account and offset the debit in her account associated with purchasing GOOGL. Here, we are making the most conservative assumption that the initial investment is the cost to purchase GOOG and that the investor's account is not credited when she shorts GOOGL.

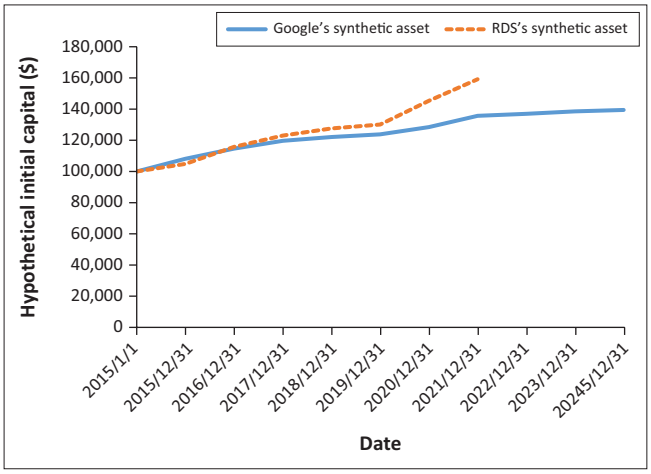
We use this method to calculate the annualised return for each year from 2015 to 2024. It is of interest to observe that the optimal pairs trading arbitrage continues to be profitable, even during the COVID-19 period of 2020–2021. In Figure 5, we additionally assume that the transaction cost is $0.001 per year, and the initial capital is invested starting in 2015. For Google, at the end of 2024, the arbitrage portfolio would have grown to $139 448.00. For the RDS arbitrage portfolio, its value would have grown to $159 146.00 at the end of 2021.

TABLE 3: Expected returns and Sharpe ratio for RDS.A and RDS.B pair.

c	RDS.A & RDS.B (2010–2021)		
	a	Return (%)	Sharpe ratio
0.001	-0.0305401	8.63	1.4167
0.002	-0.0386698	8.54	1.4130
0.003	-0.044449	8.47	1.4099
0.004	-0.0491037	8.41	1.4070
0.005	-0.0530761	8.36	1.4043
0.006	-0.0565825	8.31	1.4017
0.007	-0.0597469	8.26	1.3992

Note: This table presents the expected return and Sharpe ratio from using the optimal arbitrage strategy under the OU process of the (RDS.A and RDS.B) pair. The table lists the expected returns under different transaction costs. The optimal solution for 'a' is from Equation 6. For the RDS.A and RDS.B pair, δ = 0.0056797, ρ = 0.014507256, 'c' is the transaction cost and 'a' represents the entry level.

RDS, Royal Dutch Shell.



Note: This graph shows the hypothetical growth of $100 000.00 under the optimal pairs trading strategy for the synthetic asset formed from the GOOGL and GOOG pair and the synthetic asset formed from the RDS-A and RDS-B pair. This figure assumes the transaction cost is $0.001 per year, and the initial capital of $100 000.00 is invested starting in 2015. The figure also makes the most conservative assumption that the cost for forming the zero investment portfolio is not zero, but instead will be the entirety of the cost of the long leg of the theoretical zero investment portfolio.

RDS, Royal Dutch Shell.

FIGURE 5: Hypothetical growth of optimally trading the synthetic assets.

Finally, we use Equation 8 to Equation 11 to test whether the synthetic asset of the optimal arbitrage strategy is affected by the economic state. We use the term spread and default spread to proxy long-term economic sentiment. NASDAQ return, VIX return and market risk premium (market return minus free rate) proxy short-term economic sentiment. The sample period is 01 January 2015 – 31 December 2024. The results are presented in Table 4 and Table 5.

Table 4 shows that the stock returns of Google are affected by short-term factors. These short-term factors are either favourable or unfavourable factors. For example, the NASDAQ returns and the market risk premiums show significant adverse effects at the 1% statistical levels, respectively. However, long-term factors (term spread and default spread) are not as significant as short-term factors. These results show that Google stock is easily affected by short-term factors in the market, and investors must spend more time dealing with short-term risks.

In Table 4, we find that the returns of an individual stock such as Google are affected by economic conditions. Would the optimal pairs trading arbitrage portfolio also be affected by economic conditions and show the same result? Table 5 shows the relationship between the macroeconomic indicators and pairs trading arbitrage profits. The pairs trading arbitrage profits are returns of the synthetic asset portfolio formed from simultaneously long GOOGL and short GOOG. The model estimated in Table 5 corresponds to the GARCH model specified by Equation 9 through Equation 11, where the dependent variable in the conditional mean equation is the synthetic asset return. Unlike the results shown in Table 4, in Table 5, the terms spread and default spread are significantly positive. It means that when the long-term economy is good, the pairs trading of GOOGL and GOOG will be more profitable.

TABLE 4: The relationship of Google's return and economic factors.

Variable	Coefficient	SE	*z*-Statistic	Probability
Mkt-rf	-0.004989	0.000532	-9.373201	0.0000***
lnnasdaq	1.426608	0.037339	38.20725	0.0000***
lnvix	-0.003403	0.002992	-1.137321	0.2554
default	0.000168	0.000817	0.205406	0.8373
term	0.000105	0.000131	0.800040	0.4237

Note: This table shows the relationship of Google's return and economic factors. In table, the coefficients of NASDAQ return and market risk premium are significantly negative at the 1% statistical levels, respectively. The significance of term spread and default spread is not obvious and the influence is very low (the coefficient is very small).

SE, standard error.

***, significant at the 1% statistical level.

TABLE 5: The relationship of Google's synthetic asset returns and economic factors.

Variable	Coefficient	SE	*z*-Statistic	Probability
Mkt-rf	-1.66E-05	7.98E-05	-0.207991	0.8352
lnNASDAQ	0.001692	0.006230	0.271581	0.7859
lnvix	0.000799	0.000410	1.949017	0.0513*
default	0.008038	0.000129	62.30629	0.0000***
term	0.002393	2.11E-05	113.5103	0.0000***

Note: In the table, both term spread and default spread are significantly positive at the 1% significance level or better. NASDAQ return and market risk premiums are all insignificant. Although both term spread and default spread have significant positive effects, the associated coefficients are not large. When default spread increased 1% and the Google's synthetic portfolio will only increase 0.008%. The results in the table also show that Google's synthetic asset is not significantly affected by short-term fluctuations in the market.

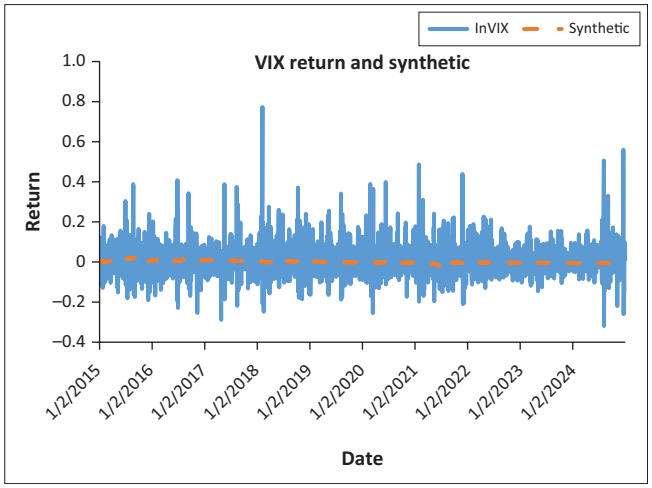
SE, standard error.

*, significant at the 10% statistical level;

***, significant at the 1% statistical level.

On the contrary, the NASDAQ return and the market risk premium are not statistically significant. From Table 5, we find that the optimal arbitrage strategy is not affected by short-term fluctuations in the market. The results suggest that the optimal arbitrage strategy is beneficial to investors who adopt a long-term investment strategy.

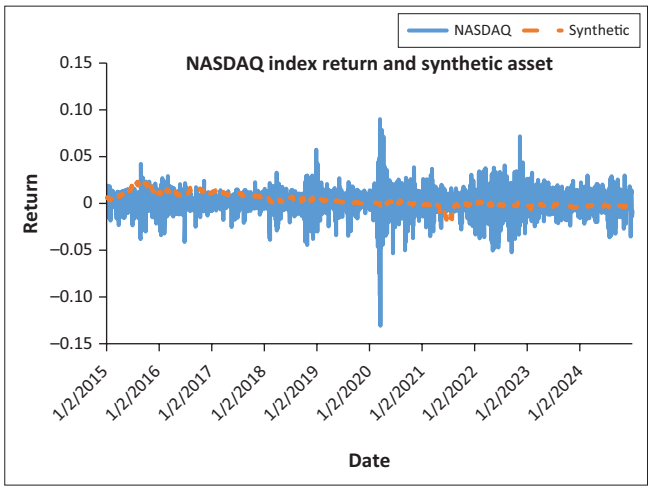
To reinforce the results in Table 5, we plot and compare the returns of the synthetic asset, VIX and the NASDAQ index in Figure 6 and Figure 7. From the two figures, we can visually see that the returns obtained from the synthetic asset are highly stable (almost as if it is a risk-free asset) and do not show any correlation with the returns of the VIX and the NASDAQ index. These plots confirm that the results of Table 5 are correct. The period shown in Figure 6 and Figure 7 spans from 01 January 2015 to 31 December 2024.



Note: This figure shows the relationship between VIX returns and Google's synthetic asset returns. The table shows that the synthetic asset is quite stable compared to the VIX (01 January 2015–31 December 2024).

VIX, Volatility Index.

FIGURE 6: The relationship between VIX returns and the Google's synthetic asset returns.



Note: This figure shows the relationship between NASDAQ index returns and Google's synthetic asset returns. This table presents similar results as that shown in Figure 6 and shows the synthetic asset to be much more stable than the NASDAQ index (01 January 2015–31 December 2024).

FIGURE 7: The relationship of NASDAQ index returns and Google's synthetic asset returns.

Discussion

This section presents three key findings from our study. Firstly, the literature on optimal statistical arbitrage and its applications is expanding. Bertram (2010) developed a closed-form solution for determining the entry and exit points for statistical arbitrage, where the synthetic asset follows the OU process. Cummins and Bucca (2012) applied this method to the arbitrage of crude oil and its derivatives. Chen and Yang (2021) demonstrated the feasibility of using a replicating asset, estimated from a factor model, to construct a synthetic asset for statistical arbitrage, specifically using Berkshire Hathaway stock. These studies provide empirical evidence supporting the use of the OU process in statistical arbitrage.

Our study builds on this foundation by showing that twin stocks with nearly identical prices can be effectively arbitraged using this framework to generate significant returns, even after accounting for transaction costs. We demonstrate that a synthetic portfolio combining GOOGL and GOOG follows a mean-reverting OU process. By applying the optimal entry and exit formulas from Bertram (2010) and Cummins and Bucca (2012), we found that trading this synthetic portfolio can yield expected returns between 8.71% and 8.36%, with Sharpe ratios ranging from 5.638 to 5.363 under various transaction costs. Our robustness tests with another pair of nearly identical twin stocks, RDS.A and RDS.B from Royal Dutch Shell, indicated even higher annualised returns of at least 8.26% after transaction costs.

Secondly, we examined market factors and economic indicators to assess the sensitivity of our arbitrage method to short-term market risks. Our findings suggest that this statistical arbitrage strategy is not significantly affected by short-term market fluctuations, making it suitable for investors with a long-term investment horizon. This method can be applied across different financial markets to achieve stable, long-term investment returns. Future research could explore other nearly identical twin assets beyond the stock market to validate and expand the applicability of this strategy.

Thirdly, we emphasise the practical implications of our findings. Our research indicates that investors can successfully apply statistical arbitrage to twin stocks, achieving profitable results. This expands the range of tools available for market-neutral investment strategies, providing a reliable alternative to traditional arbitrage methods. By highlighting the robustness of our approach across different pairs of twin stocks, we offer a valuable contribution to both academic research and practical investment strategies.

In conclusion, our study reinforces the potential of the OU process for statistical arbitrage in twin stocks. While recognising the limitations because of the limited availability of such stocks and the impact of transaction costs, our findings provide a solid foundation for future research and practical applications in statistical arbitrage.

Conclusion

This study on pairs trading between nearly identical twin stocks GOOGL and GOOG provides significant insights into the potential of such strategies in enhancing portfolio returns while managing risk. The findings indicate that despite their similarities, there are discernible opportunities for arbitrage, which investors can exploit. This research contributes to the broader discourse on market efficiency and the potential inefficiencies that can arise even in highly liquid and closely related stocks.

Practical implications

The results provide significant practical implications for investors and financial analysts interested in market-neutral trading strategies. The application of statistical arbitrage between GOOGL and GOOG demonstrates the potential for consistent profits with minimal risk exposure. By utilising the OU process, investors can capitalise on price discrepancies between these twin stocks, achieving returns of 8.71% to 8.36% with Sharpe ratios ranging from 5.638 to 5.363, even after accounting for transaction costs. The robustness tests with RDS.A and RDS.B further validate this strategy, showing annualised returns of at least 8.26%. These findings suggest that this approach is not only viable but also scalable across different market conditions and asset classes, providing a valuable tool for diversifying investment portfolios and managing market volatility.

Limitations

Despite the promising results, this study is subject to several limitations. The primary limitation is the scarcity of twin stocks in the market, which restricts the generalisability of the findings. Moreover, the study's focus on specific market conditions and a particular timeframe may not fully capture the dynamic nature of financial markets. Additionally, the impact of transaction costs, while accounted for, could vary significantly depending on market liquidity and trading volume, potentially affecting the profitability of the strategy. Future studies should consider a broader range of market conditions and include other financial instruments to better understand the applicability of statistical arbitrage strategies.

Recommendations for future research

Building on the insights gained from this study, future research could explore the integration of big data analytics and machine learning techniques to enhance the identification and validation of arbitrage opportunities. This could involve developing algorithms that dynamically adjust to market changes and optimise entry and exit points in real time. Further research could also investigate the impact of different economic indicators and market volatility on the effectiveness of statistical arbitrage strategies. Furthermore, examining the applicability of this strategy across various asset classes, including commodities, bonds and cryptocurrencies, would provide a more comprehensive understanding of its potential.

Contribution to the literature

This research contributes to the existing literature on statistical arbitrage by extending its application to twin stocks, demonstrating that even highly correlated assets can offer profitable trading opportunities. The study highlights the effectiveness of the OU process in capturing mean-reverting price behaviours, providing a theoretical and empirical foundation for future research in this area. Moreover, the findings underscore the potential for market-neutral strategies to offer stable returns in diverse market conditions, thereby enhancing the toolkit available to investors and financial analysts. This study thus provides both theoretical insights and practical applications, enriching the discourse on market efficiency and investment strategies.

This article has two main contributions. Firstly, our article found that twin stocks can also be arbitraged. This may provide a feasible research direction for subsequent research. For example, can stocks with the same trend in the same industry be arbitraged? Secondly, another finding is that the arbitrage method of the OU process is not affected by short-term risks. This method is suitable for institutional investors or pension funds that make long-term investments.

Our findings advance arbitrage theory in three key ways. Firstly, we demonstrate that the OU process – previously applied primarily to commodity spreads and asset-derivative pairs – can successfully model price deviations between theoretically perfect substitutes, expanding its theoretical utility in statistical arbitrage. Secondly, by proving that persistent arbitrage opportunities exist even in near-identical twin stocks (contrary to efficient market assumptions), we contribute to the growing body of evidence challenging strict interpretations of the Law of One Price under real-world market frictions. Thirdly, our methodology provides a replicable framework for detecting and exploiting minimal pricing inefficiencies between highly correlated assets, offering researchers a new tool to test market efficiency boundaries in similar asset classes. These theoretical insights complement our empirical results, bridging the gap between arbitrage theory's traditional focus on heterogeneous assets and the underexplored domain of near-identical securities.

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Competing interests

The authors declare that they have no financial or personal relationships that may have inappropriately influenced them in writing this article.

Authors' contributions

C.-M.Y. was responsible for the theoretical framework, writing of the article, data collection, application of methods, and analysis of results. A.-S.C. assisted in the development of the theoretical framework, provided suggestions during the writing process, and assisted in writing and proofreading the manuscript.

Ethical considerations

This article followed all ethical standards for research without direct contact with human or animal subjects.

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Data availability

The data that support the findings of this study are available from the corresponding author, C.-M.Y. upon reasonable request.

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