

One-dimensional Reactor Model for the Integrated Simulation of the PBMR Power Plant

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The Pebble Bed Modular Reactor (PBMR) power plant is currently being developed by PBMR (Pty) Ltd in South Africa together with ESKOM and other industrial partners. This high temperature gas cooled reactor (HTGR) plant is based on a three-shaft Brayton cycle with helium gas as coolant. The complexity associated with the thermal-hydraulic design of the cycle calls for the use of a variety of analysis techniques and simulation tools. One of the most prominent of these is the Flownet thermal-hydraulic network simulation software. Flownet allows detailed steady-state and transient thermal-hydraulic simulations of all components in the plant, fully integrated with core neutronics and controller algorithms. This paper describes the theory and integration of the neutronics and thermal-hydraulic models for the reactor core and presents sample calculations to illustrate the results obtained.

Nomenclature

A	Cross-sectional area of flow element (m^2)
C_i	Normalised precursor atom density for group i
c_p	Specific heat capacity of fluid (J/kg K)
D	Hydraulic diameter of fluid element (m)
f	Darcy-Weisbach friction factor
g	Gravitational acceleration (m/s^2)
J	Normalised Iodine concentration
L	Length of flow element (m)
m_i, m_e	Inlet and outlet mass flow rates (kg/s)
Nu	Nusselt number
p	Static pressure (Pa)
P_{oi}, P_{oe}	Inlet and outlet total pressures (Pa)
P_n	Normalised reactor power
Pr	Prandtl number
Q	Heat addition to flow element (W)
Q_{ex}	Reactivity addition due to external sources (nile)
Re	Reynolds number
T	Static temperature ($^{\circ}C$)
T_{oi}, T_{oe}	Inlet and outlet total temperatures ($^{\circ}C$)
V	Fluid velocity (m/s)
∇	Volume of flow element (m^3)
X	Normalised Xenon concentration
z_i, z_e	Inlet and outlet elevations (m)

Greek Letters

α	Weighting factor for implicit time-wise integration
β_i	Delayed neutron fraction for group i

Δt	Size of time step (s)
ε	Void fraction
Λ	Average neutron lifetime (s)
Γ	Feedback coefficients
λ_i	Decay constant for delayed neutron group i (s^{-1})
λ_J	Decay constant for Iodine (s^{-1})
λ_X	Decay constant for Xenon (s^{-1})
ρ	Reactivity (nil)
$\bar{\rho}$	Average fluid density (kg/m^3)
σ, ϕ_0	Absorption cross section times average equilibrium neutron flux (s^{-1})
ΣK	Sum of the loss coefficients for flow element

Introduction

The pebble bed reactor core is made up of fuel spheres and pure graphite spheres. Each fuel sphere consists of an inner fuel region with a 50 mm outer diameter made up of coated UO_2 particles set inside a graphite matrix. The fuel region is covered by a fuel-free graphite protective layer with an outer diameter of 60 mm.

A schematic three-dimensional representation of the geometry of the reactor core is shown in Fig 1. The inner core region contains pure graphite spheres while the outer active core region is filled with fuel spheres. Helium gas enters the top of the reactor core at approximately $500^{\circ}C$.

The gas is heated primarily through the active core region where heat is generated inside the fuel spheres. Upon leaving the core at the bottom the hot gas is mixed with gas from the pure graphite region to obtain a fully mixed exit temperature of approximately $900^{\circ}C$.

Integration of core neutronics and power conversion unit (PCU) thermal-hydraulics has been done successfully before for HTGR systems. One example of this is the coupling of Panther.

Thermix-Direkt and Relap 5 by Verkerk¹ for the simulation of the pebble bed reactor for the Dutch Incogen and Acacia² studies. Panther was used to solve the neutron diffusion equation in Cartesian coordinates, Thermix-Direkt took care of

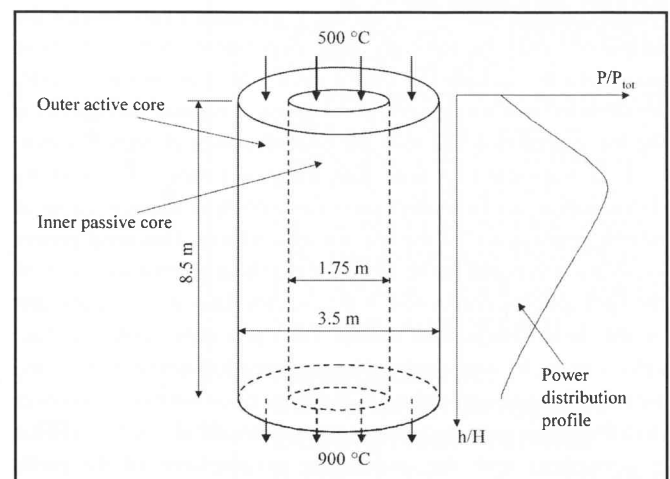


Fig 1: Schematic three-dimensional representation of the pebble bed reactor core

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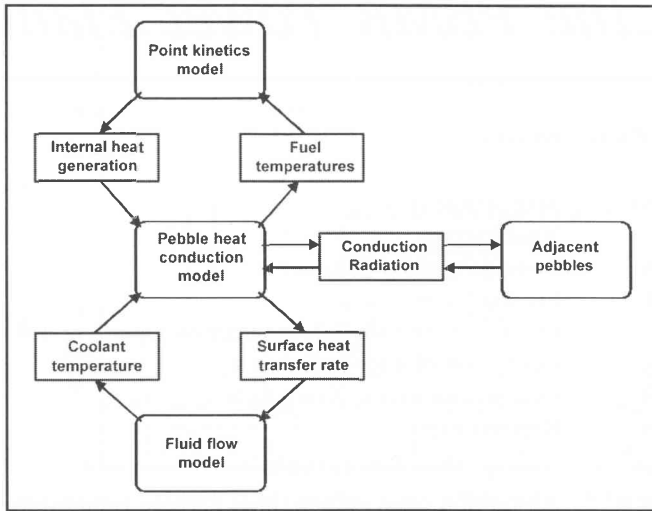


Fig 2: Schematic illustration of the interaction between the three models

the core thermal-hydraulics and Relap 5 of the PCU thermal-hydraulics. This allowed the integrated simulation of thermal-hydraulics and 3-D neutronics of a pebble bed HTGR together with its PCU. Note however, that the PBMR system differs from the Dutch systems in the sense that it is based on a multi-passage fuelling scheme as opposed to a bit-by-bit fuelling scheme.

Since most transient reactor phenomena are at least two-dimensional in nature, it is clear that the development of an accurate one-dimensional model for detailed reactor design and optimisation is not a realistic goal. However, the main aim of Flownet is to simulate the integrated system performance, focusing on the detail of the PCU, while still ensuring acceptable calculation times. Therefore, a fully integrated, simplified reactor model was called for that could supply sufficiently accurate values of pressure drop and heat transfer across the reactor. The calculation of the heat generated within the fuel spheres is therefore based on a point kinetics neutronics and decay heat generation model that requires as input the temperatures within the fuel spheres inside the reactor. Point kinetics refers to the approach, whereby the global reactor behaviour is simulated dynamically, as a single point having certain weighted average properties that may be assumed to be constants over time. This simplification is valid when the reactor is sufficiently small so that it is well-coupled and the space and time variables are essentially separable. This means that the spatial neutron flux shape changes negligibly during a transient even though the amplitude may be strongly time-dependent. Although these assumptions exclude the use of such a heat generation model for detailed reactor design calculations, it is quite sufficient for the integrated simulation of the overall power plant behaviour.

It is important to note that the point kinetics model by definition at any time step, provides as output a single value of power generation for the reactor as a whole. This total power generation is equal to the total internal heat generation in all of the fuel spheres contained in the reactor. In order to apply this in the heat conduction model for each representative fuel sphere in each layer, the total heat generation is first distributed among the layers according to the prescribed normalised power distribution profile that was shown schematically in Fig 1. This is consistent with the underlying assumptions of the point kinetics model. The normalised power distribution profile is typically obtained from stand-alone calculations with more

detailed three-dimensional reactor simulation models for the specific reactor geometry and fuel characteristics.

The model is further based on a one-dimensional approach for the thermal-hydraulics, consistent with the overall approach followed in the Flownet software. The one-dimensional approach implies that the reactor core is divided into a number of layers along its height, each with two separated flow elements representing, the pure graphite inner-leg and an active outer-leg respectively. Consistent with the one-dimensional approach it is assumed that all spheres contained in a single core section have exactly the same temperature distribution and internal heat generation per unit volume. Each reactor section is therefore characterized by a single representative sphere. Fluid flow in the radial direction is assumed to be negligible. However, the combined effect of radial and axial contact conduction and radiant heat transfer between the surfaces of spheres in the different reactor sections are taken into account via an effective thermal conductivity.

The temperature distribution within each representative sphere is calculated in detail based on a transient heat conduction model taking into account the thermal storage, the internal heat generation, conduction between material layers at different radii as well as the convective heat transfer to the coolant gas flowing over the sphere. This results in a representative sphere surface temperature, which when applied to the total sphere surface area contained in the core section, can be used to calculate the heat transfer rate to the gas passing through that section. The one-dimensional reactor model therefore consists of the three sub-models shown in Fig 2. These are:

- The transient point kinetics neutronics and decay heat generation model. It requires as input the temperatures within the fuel spheres and provides as output the total internal heat generation within all the fuel spheres contained in the reactor core.
- The detailed transient internal heat conduction for each representative sphere in each core section. It requires as input the heat generation density within the fuel, as well as the temperature of the gas surrounding the spheres. It provides as output the temperature distribution within the spheres, as well as the heat transfer through convection between the surfaces of the spheres and the surrounding coolant.
- The fully transient fluid flow model that determines the temperature and pressure variations in the gas contained in each core section. It requires as input the surface heat transfer

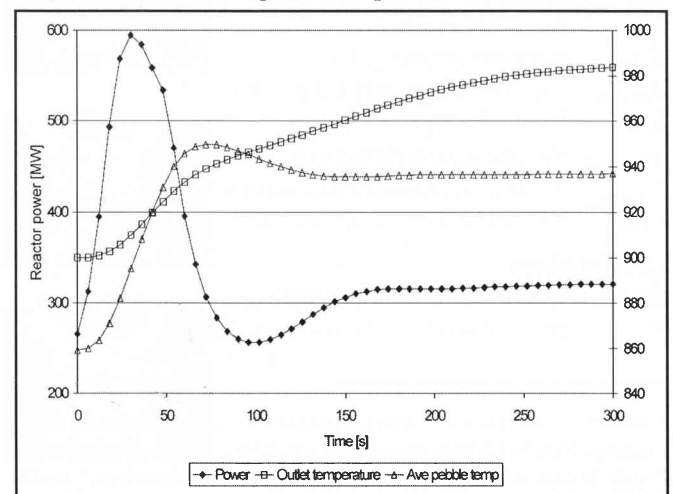


Fig 3: Results of reactivity transient simulation

rate and provides as output the coolant temperatures and pressures.

The approach outlined above allows for fully transient simulations of temperature distributions along the height of the reactor as well as the calculation of the radial temperature distribution within each representative sphere in each of the core sections.

The purpose of this paper is to describe the theory and integration of the neutronics and thermal-hydraulic models for the reactor core and to present sample calculations to illustrate the results obtained.

Point Kinetics and Decay Heat Model

The model consists of twelve coupled differential equations that must be solved simultaneously through time-wise integration. A good explanation of the fundamentals can be found in Glasstone and Sesonske³. The first seven equations are collectively known as the point kinetics equations and describe the prompt and delayed neutron balance due to precursor atoms. There are numerous fission products that can lead to delayed neutron emission. However, the net effect of all precursors can be adequately represented by six effective groups of precursors.

Decay heat generation is the result of unstable fission fragments produced during fission that undergo a series of beta decays liberating additional energy. Decay heat is a complex function of the time and power history of the fuel elements. However, the simulation will make use of a rough approximation by means of three exponentially decaying functions which is only valid for approximately three days after shutdown.

It is also essential to track the concentration of Xenon in the reactor since it effectively acts as a neutron poison. Since Xenon is mostly produced during the decay of Iodine, the Iodine concentration in the reactor must also be tracked. The rate of Iodine production is proportional to the reactor power level.

The rate of change of Xenon concentration equals the rate of Xenon production minus the rate of Xenon loss. The absorption of neutrons depends on the reactor power level, the Xenon concentration and the absorption cross-section. We will neglect the direct production of Xenon by fission and any time dependence of cross-sections.

Governing Equations

Implicit integration of the governing differential equations for the point kinetics, decay heat and Iodine and Xenon decay leads to the following set of simultaneous equations:

Point Kinetics:

$$\left(\frac{\Lambda}{\Delta t} - \alpha(\rho - \beta)\right)P_n - \alpha \sum_{i=1}^6 \lambda_i C_i = \frac{\Lambda}{\Delta t} P_n^o + \alpha Q_{ex} + (1 - \alpha) \left(\begin{matrix} (\rho^o - \beta)P_n^o \\ + \sum_{i=1}^6 \lambda_i C_i^o + Q_{ex}^o \end{matrix} \right) \quad (1)$$

and

$$-\alpha\beta_i P_n + \left(\frac{1}{\Delta t} + \alpha\lambda_i\right)C_i = \frac{1}{\Delta t} C_i^o + (1 - \alpha)(\beta_i P_n^o - \lambda_i C_i^o) \text{ for } i = 1 \text{ to } 6 \quad (2)$$

Decay Heat:

$$-\alpha\beta_k P_n + \left(\frac{1}{\Delta t} + \alpha\lambda_k\right)P_k = \frac{1}{\Delta t} P_k^o + (1 - \alpha)(\beta_k P_n^o - \lambda_k P_k^o) \text{ for } k = 1 \text{ to } 3 \quad (3)$$

Iodine and Xenon Decay:

$$-\alpha\lambda_J P_n + \left(\frac{1}{\Delta t} + \alpha\lambda_J\right)J = \frac{1}{\Delta t} J^o + (1 - \alpha)\lambda_J(P_n^o - J^o) \quad (4)$$

and

$$\alpha\sigma_X \varphi_0 X P_n - \alpha(\lambda_X + \sigma_X \varphi_0)J + \left(\frac{1}{\Delta t} + \alpha\lambda_X\right)X = \frac{1}{\Delta t} X^o + (1 - \alpha) \left(\begin{matrix} (\lambda_X + \sigma_X \varphi_0)J^o \\ -(\lambda_X + \sigma_X \varphi_0 P_n^o)X^o \end{matrix} \right) \quad (5)$$

In these equations Δt represents the size of the discrete integration time step and α a weighting factor. The superscript α refers to the values at the previous time step.

Total Heat Generation:

The total heat generation in the core P_{tot} can be calculated from

$$P_{tot} = P_{nom} \frac{(P_n + P_{decay})}{\left(1 + \sum_{k=1}^3 \beta_{kk}\right)} \quad (6)$$

with

$$P_{decay} = \sum_{k=1}^3 \lambda_{kk} P_k \quad (7)$$

and P_{nom} the design nominal power output of the reactor.

Reactivity feedback equations

The total reactivity can be calculated from

$$\rho = \rho_f + \rho_m + \rho_x + \rho_{ex} \quad (8)$$

The first three terms on the right-hand side are dependent on the fuel and moderator temperatures. The value of ρ_{ex} is determined by the insertion depth of the control rods. The relations used for the PBMR reactor were derived by Scherer et.al.⁴ and are as follows:

$$\rho_f = \Gamma_{f1}(T_f - T_{f0}) + \Gamma_{f2} \ln\left(\frac{T_f}{T_{f0}}\right) - \Gamma_{f3} \left(\frac{1}{T_f} - \frac{1}{T_{f0}}\right) \quad (9)$$

$$\rho_m = \Gamma_{m1}(T_m - T_{m0}) + \Gamma_{m2}(T_m - T_{m0})^2 + \Gamma_{m3}(T_m - T_{m0})^3 \quad (10)$$

and

$$\rho_x = \Gamma_{x1} + \Gamma_{x2}(T_m - T_{m0})(X - 1) \quad (11)$$

The reference moderator (T_{m0}) and fuel (T_{f0}) temperatures are determined by the detail design of the fuel and the reactor.

Sphere Heat Conduction Model

The sphere heat conduction model is based on a finite difference solution of the transient one-dimensional spherical heat conduction equation⁵. Each sphere is divided into a number of discrete 'onion ring' shaped control volumes each represented by a single node. Half control volumes represent the inner- and outermost layers. Implicit integration of the governing differential equation for each node leads to a set of equations that must be solved simultaneously for each node in the representative sphere.

The node on the surface of the sphere represents the surface temperature of all the spheres in that section of the reactor that is exposed to the coolant. From the viewpoint of the coolant, the spheres will therefore have the same effect as a constant surface temperature heat exchanger with a total area equal to the sum of the surface areas of all the spheres in that layer. The convection heat transfer can therefore be simulated quite easily with the aid of the effectiveness-NTU method⁶.

For the calculation of the surface heat transfer coefficient the correlation put forward by Kugeler & Schulten⁷ is employed which states that the Nusselt number is given by

$$Nu = 1.27 \frac{Pr^{0.33}}{\epsilon^{1.18}} Re^{0.36} + 0.033 \frac{Pr^{0.5}}{\epsilon^{1.07}} Re^{0.86} \quad (12)$$

Re is the Reynolds number and Pr the Prandtl number which are based on the pebble outer diameter. The void fraction ϵ is taken as 0.39. The correlation is valid for core outer diameter to pebble diameter ratios greater or equal to 20, core height to pebble diameter ratios greater or equal to four, $100 \leq Re \leq 10^5$ and $0.36 \leq \epsilon \leq 0.42$.

Fluid Flow Model

The fluid flow model is based on the differential equations for the conservation mass, momentum and energy for compressible gasses⁸.

Governing Equations

Implicit integration of the governing differential equations for a flow element leads to equations (13) through (15) which are solved simultaneously for all the nodes and elements in the complete reactor network with the aid of the so-called 'Pressure Correction Implicit Method' (PCIM) that is described elsewhere

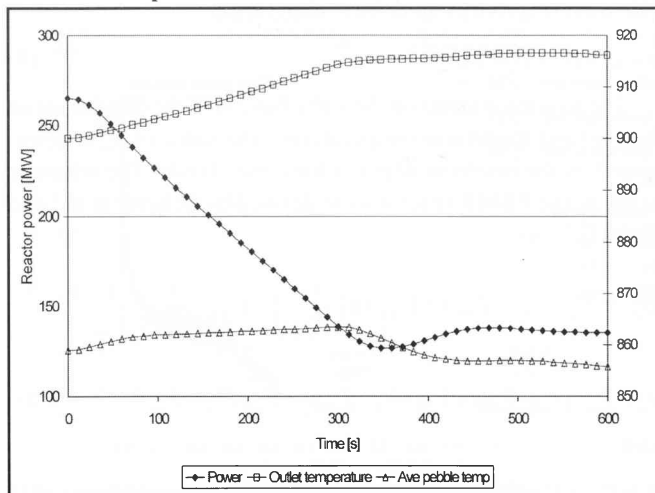


Fig 4: Results of mass flow transient simulation

by Greyvenstein⁸.

Conservation of Mass

$$\bar{\rho} - \bar{\rho}^o = \frac{\Delta t}{V} \left(\alpha \left(\sum m_i - \sum m_e \right) + (1 - \alpha) \left(\sum m_i^o - \sum m_e^o \right) \right) \quad (13)$$

Conservation of Momentum

$$\bar{\rho} \bar{V} - \bar{\rho}^o \bar{V}^o = - \frac{\Delta t}{L} \left(\alpha \left(\frac{\bar{p}}{\bar{\rho}^o} (p_{oe} - p_{oi}) + \bar{\rho} g (z_e - z_i) + (T_{oe} - T_{oi}) \frac{1}{T_o} \frac{\bar{m}^2}{2 \bar{\rho} A^2} + \left(\frac{fL}{D} + \sum K \right) \frac{|\bar{m}| \bar{m}}{2 \bar{\rho} A^2} \right) + (1 - \alpha) \left(\frac{\bar{p}^o}{\bar{\rho}^o} (p_{oe}^o - p_{oi}^o) + \bar{\rho}^o g (z_e - z_i) + (T_{oe}^o - T_{oi}^o) \frac{1}{T_o^o} \frac{(\bar{m}^o)^2}{2 \bar{\rho}^o A^2} + \left(\frac{f^o L}{D} + \sum K^o \right) \frac{|\bar{m}^o| \bar{m}^o}{2 \bar{\rho}^o A^2} \right) \right) \quad (14)$$

Conservation of Energy

$$(\bar{\rho} c_p \bar{T}_o - \bar{p}) - (\bar{\rho}^o c_p \bar{T}_o^o - \bar{p}^o) = - \frac{\Delta t}{V} \left(\alpha \left(\sum (m_e c_p T_{oe}) - \sum (m_i c_p T_{oi}) + \sum (m_e g z_e) - \sum (m_i g z_i) - Q \right) + (1 - \alpha) \left(\sum (m_e^o c_p T_{oe}^o) - \sum (m_i^o c_p T_{oi}^o) + \sum (m_e^o g z_e) - \sum (m_i^o g z_i) - Q^o \right) \right) \quad (15)$$

Friction Factor

For the pebble bed the friction factor (f) is calculated with the aid of the correlation also put forward by Kugeler & Schulten⁷ namely

$$f = \psi \frac{1 - \epsilon}{\epsilon^3} \quad (16)$$

with

$$\psi = \frac{320}{\left(\frac{Re}{1 - \epsilon} \right)} + \frac{6}{\left(\frac{Re}{1 - \epsilon} \right)^{0.1}} \quad (17)$$

The correlation is valid for core outer diameter to pebble diameter ratios greater or equal to 20, core height to pebble diameter ratios greater or equal to four, $1 \leq Re / (1 - \epsilon) \leq 10^5$ and $0.36 \leq \epsilon \leq 0.42$.

Results

Grid and timestep independence

The evaluation of grid dependence was done by comparing steady-state solutions based on simulations employing different numbers of layers in the reactor as well as different numbers of material layers in each of the representative pebbles. For all of these cases the resultant gas outlet conditions are exactly the same since it is independent of the grid size. However, different values of pebble temperatures and reactivity resulted.

An analysis of the results showed that 10 layers in each pebble is sufficient. Furthermore, for 20 layers or more along the length of the reactor, the average pebble temperature is within 0.3 % of that obtained with 500 layers or more. The external reactivity is within 3.8 % of that obtained with 500 layers or more. From this it can be concluded that 20 layers along the length of the reactor is sufficient.

In order to investigate the time step size dependence, a reactivity transient was investigated. The transient entails a ramp release of 500 mNile at a rate of 10 mNile per second for different simulation time step sizes, varying between 5 seconds and 0.005 seconds.

A detailed analysis of the results show that for a time step of 0.5 seconds or smaller, the maximum error is less than 1 % when compared to the results obtained with a 0.005 second time step. For a time step of 0.1 second or smaller the maximum error is less than 0.2 %.

Similar results were obtained for the average pebble temperatures. Therefore, based on both the neutronic power and temperature results, it can be concluded that an acceptable degree of time step independence is obtained for any time step size smaller than 0.5 seconds.

Sample calculation 1: Reactivity transient

The first sample calculation entails a ramp release of 500 mNile [0.5 %] at a rate of 10 mNile [0.01 %] per second over a time span of 50 seconds. Fig 3. shows the results obtained for the reactor power output, coolant outlet temperature and average pebble temperature.

The figure shows that the reactivity release results in an increase in power followed by a sharp increase in the pebble temperature. This in turn results in a decrease in power output, due to the negative temperature gradient characteristics of the fuel. After 300 seconds the gas outlet temperature is still rising in response to the continuing power and pebble temperature rise, which is less noticeable on the graph.

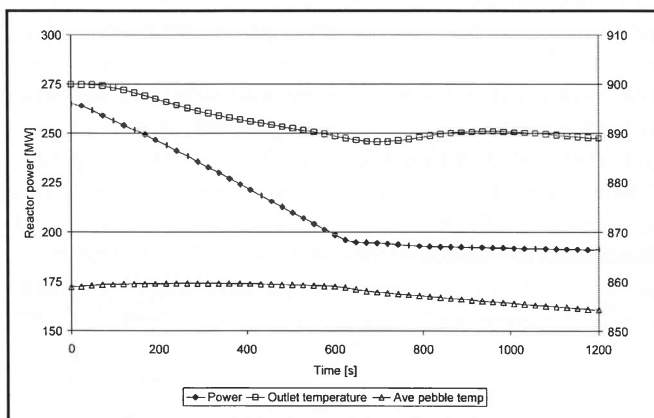


Fig 5: Results of gas inlet temperature transient

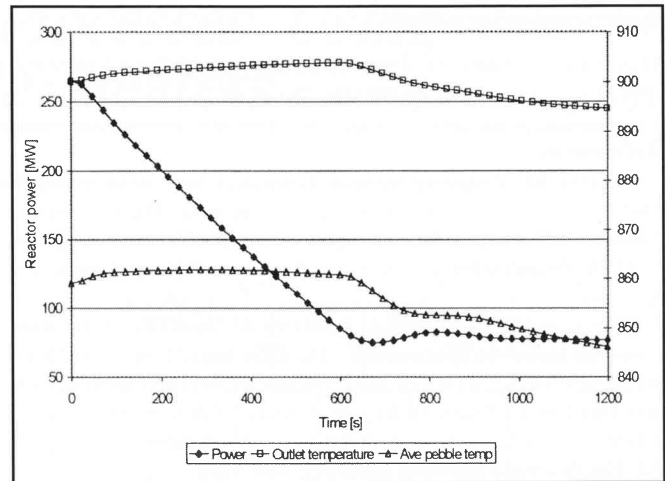


Fig 6: Results of simultaneous temperature, pressure and mass flow transient

Sample calculation 2: Mass flow transient

The second sample calculation entails a ramp reduction in mass flow rate to 50 % of the initial value over a time span of 300 seconds. Fig 4. shows that the decrease in coolant flow initially results in an increase in the pebble temperature, which in turn results in a decrease in the power output. Following this, the pebble temperature starts to decrease again. However, despite the decrease in pebble temperature, the power continues to drop. This is due to an increase in the Xenon concentration with Xenon acting as a neutron poison.

Sample calculation 3: Gas inlet temperature transient

The third sample calculation entails a ramp increase of the inlet gas temperature to 100°C higher than the initial value over a time span of 600 seconds. Fig 5. shows the same phenomena as in Sample 2.

Sample calculation 4: Simultaneous inlet temperature, pressure and mass flow transient

This sample entails a simultaneous ramp increase in the inlet gas temperature to 100°C higher than the initial value, a ramp reduction in the inlet pressure by 45 bar and a ramp reduction in mass flow rate to 40 % of the initial value over a time span of 600 seconds. Fig 6. shows the same phenomena as in Samples 2 and 3, including the Xenon effect.

Conclusions

This paper described the integrated one-dimensional simulation model for the pebble bed nuclear reactor contained in Flownet. The simulation consists of three distinct models namely the point kinetics and decay heat generation model, the sphere heat conduction model and the fluid flow model. These three models are solved simultaneously at each time step to obtain the integrated transient response of the reactor.

Results obtained with the new model showed that grid independence is obtained with at least 10 layers in each representative fuel sphere and 20 layers along the height of the reactor. An adequate degree of time step independence is obtained for time step sizes smaller than 0.5 seconds.

Sample calculations have shown that the correct qualitative results are obtained. However, detailed comparative studies must still be undertaken with more comprehensive software models such as TINTE in order to properly quantify the accuracy of the results obtained with the new model.

Acknowledgments

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