Evaporation from a Water Surface Exposed to the Natural Environment

Detlev G. Kröger[1]

The problem of evaporation from a heated horizontal water surface that is exposed to the natural environment is analysed. An approximate equation is deduced that predicts the turbulent convection mass transfer or evaporation rate for cases where the water temperature is higher than that of the ambient air.

NOMENCLATURE

c	Molar concentration of species, mol/m^3
C_f	Friction coefficient
c_p	Specific heat, J/kgK
D	Diffusion coefficient, m^2/s
g	Gravitational acceleration, m/s^2
k	Thermal conductivity, W/mK
h	Heat transfer coefficient, W/m^{2}K
p	Pressure, N/m^2
q	Heat flux, W/m^2
T	Temperature °C or K
t	Time, s
v	Speed, m/s
z	Coordinate

Dimensionless numbers

Le	Lewis number, $k/(\rho c_p D)$
Pr	Prandtl number, $\mu c_p/k$
Ra	Rayleigh number, $g\delta^3_{Du}(\rho_{avi} - \rho_{avo})\rho_{av}c_p/(k\mu)$
Sc	Schmidt number, $\mu/(\rho D)$

Greek letters

α	Thermal diffusivity, $k/(\rho c_p)$, m^2/s
δ	Concentration or partial density layer thickness, m
μ	Dynamic viscosity, kg/ms
ρ	Density, kg/m^3
ϕ	Relative humidity

Subscripts

a	Air
av	Air-vapour mixture
avi	Air-vapour mixture at initial condition
avo	Air-vapour mixture at z = 0
c	Concentration
D	Mass diffusion
i	Initial condition
m	Uniform mass flux
o	At z = 0
q	Uniform heat flux
T	Temperature
t	Time
u	Unstable condition
v	Vapour
w	Water or wind

[1]Department of Mechanical Engineering, University of Stellenbosch, Private Bag X1, Matieland, 7602, Stellenbosch

Introduction

At the time of writing exactly 200 years ago, i.e. 1802, Dalton's classical paper was published entitled "Experimental essays on the constitution of mixed gases; on the force of steam or vapour from water and other liquids in different temperatures, both in a Torricellian vacuum and in air; on evaporation and on the expansion of gas by heat". In this paper Dalton stated that the rate of evaporation from a water surface is proportional to the difference in vapour pressure at the surface of the water and that in the surrounding air, and furthermore that the wind speed affects this proportionality. Subsequently numerous researchers investigated the problem of evaporation on the basis of Dalton's model. A recent critical and comprehensive review of many of the equations employed to predict evaporation rates from water surfaces is presented by Sartori[2]. It follows from these publications that there was essentially no further more detailed physical modelling of the process of evaporation of water from a horizontal surface into the natural environment subsequent to Dalton's publication. Much uncertainty exists and significant discrepancies occur between empirical equations that predict rates of evaporation under different conditions.

Analysis

In the following analysis an approximate equation is deduced that predicts the convective mass transfer or evaporation rate from a horizontal water surface exposed to the natural environment. Initially the transfer rate due to natural convection only is deduced and the equation is then extended to make provision for windy (forced convection) conditions.

Consider a stationary semi-infinite fluid (binary mixture consisting of air and water vapour) in which the concentration or partial density ρ_{vi} of the species of interest (water vapour) is initially uniform. Beginning with the time t = 0, the partial vapour density at the z = 0 boundary or surface is maintained at a greater level ρ_{vo} as shown in figure 1(a)

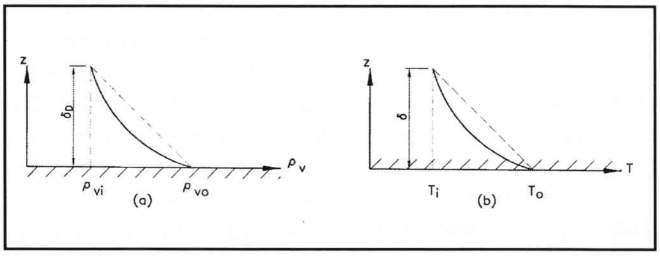

Figure 1: Partial density or temperature distribution in semi-infinite medium

Water vapour will diffuse into the semi-infinite medium to form a concentration boundary layer, the thickness of which increases with time.

The mathematical equation of time dependent diffusion in a binary mixture, expressed in terms of the molar concentration c is as follows:

$$D\frac{\partial^2 c}{\partial x^2}=\frac{\partial c}{\partial t} \tag{1}$$

The diffusion flux is driven solely by the concentration gradient strictly in an isothermal and isobaric medium. Nevertheless, equation (1) is a good approximation in the present non-isothermal system, where concentration gradients are usually superimposed on temperature gradients. For the present problem the concentration c can be replaced by the partial water vapour density ρ_v such that equation (1) becomes

$$D\frac{\partial^2 \rho_v}{\partial z^2}=\frac{\partial \rho_v}{\partial t} \tag{2}$$

Since changes in Kelvin temperature are relatively small the diffusion coefficient can be assumed to be constant.

This equation is analogous to the time-dependent equation for conduction into a semi-infinite body i.e.

$$\alpha\frac{\partial^2 T}{\partial z^2}=\frac{\partial T}{\partial t} \tag{3}$$

If the temperature of a semi-infinite solid is initially uniform at T_i and a sudden increase in temperature to T_o occurs at z = 0 as shown in figure 1(b), Schneider[3] shows that the temperature gradient at z = 0 is given by

$$\frac{\partial T}{\partial z}=(T_i-T_o)/(\pi\alpha t)^{1/2} \tag{4}$$

The corresponding heat flux is

$$q_T=-k\frac{\partial T}{\partial z}=\frac{k(T_o-T_i)}{(\pi\alpha t)^{1/2}} \tag{5}$$

An effective heat transfer coefficient can be expressed in terms of this heat flux i.e.

$$h_{Tt}=q_T/(T_o-T_i)=k/(\pi\alpha t)^{1/2} \tag{6}$$

Similarly, by solving equation (3) for the case where the semi-infinite solid at an initial uniform temperature T_i is suddenly exposed to a constant surface heat flux q_q, the latter can, according to Holman[4], be expressed in terms of an effective surface temperature T_{oq} as

$$q_q=k(T_{oq}-T_i)/\left[2(\alpha t/\pi)^{1/2}\right] \tag{7}$$

The corresponding effective heat transfer coefficient is defined as

$$h_{qt}=q_q/(T_{oq}-T_i)=k/\left[2(\alpha t/\pi)^{1/2}\right] \tag{8}$$

It follows from equations (6) and (8) that for the same temperature difference i.e. for

$$\begin{aligned}&(T_{oq}-T_i)=(T_o-T_i)\\&h_{qt}/h_{Tt}=\pi/2=h_q/h_T\end{aligned} \tag{9}$$

Due to the analogy between mass and heat transfer the solution of equation (2) gives the following relations corresponding to equations (4) to (9) respectively:

If the initial partial density at z = 0 is suddenly increased to ρ_{vo}

$$\frac{\partial \rho_v}{\partial z}=(\rho_{vi}-\rho_{vo})/(\pi Dt)^{1/2} \tag{10}$$

The corresponding vapour mass flux is

$$m_v=-D\frac{\partial \rho}{\partial z}=(\rho_{vo}-\rho_{vi})\,[D/(\pi t)]^{1/2} \tag{11}$$

An effective mass transfer coefficient can be expressed in terms of this mass flux i.e.

$$h_{Dt}=m_v/(\rho_{vo}-\rho_{vi})=[D/(\pi t)]^{1/2} \tag{12}$$

If vapour is generated uniformly at a rate m_{vm} at z = 0, this mass flux can be expressed in terms of an effective partial vapour density ρ_{vom} to give analogous to equation (7)

$$\begin{aligned}m_{vm}&=D(\rho_{vom}-\rho_{vi})/\left[2(Dt/\pi)^{1/2}\right]\\&=(\rho_{vom}-\rho_{vi})(\pi D/t)^{1/2}/2\end{aligned} \tag{13}$$

The corresponding effective mass transfer coefficient is defined as

$$h_{Dmt}=m_{vm}/(\rho_{vom}-\rho_{vi})=(\pi D/t)^{1/2}/2 \tag{14}$$

It follows from equations (12) and (14) that for the same effective difference in partial density i.e. for

$$\begin{aligned}&(\rho_{vom}-\rho_{vi})=(\rho_{vo}-\rho_{vi})\\&h_{Dmt}/h_{Dt}=\pi/2=h_{Dm}/h_D\end{aligned} \tag{15}$$

These latter equations are applicable in the region of early developing concentration distribution in a semi-infinite region of air exposed to a water or wet surface. According to Merker[5] for a Rayleigh number Ra ≥ 1101, unstable conditions prevail with the result that water vapour is transported upwards away from the wetted surface by means of "thermals" as shown in figure 2.

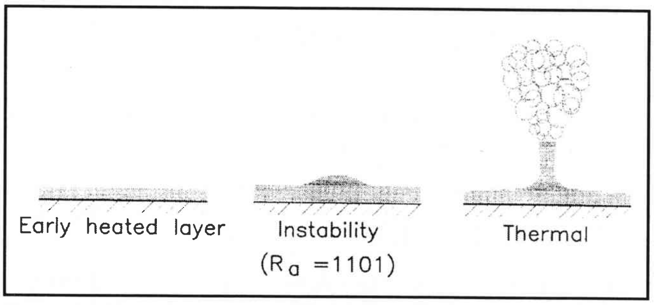

Figure 2: Flow development on surface

The generation of such thermals is periodic in time, and both spatial frequency and rate of production are found to increase with an increase in heating rate.

For an analysis of the initial developing concentration of vapour partial density distribution near the suddenly wetted surface at z = 0, consider figure 1(a).

The approximate magnitude of the curvature of the partial density profile is the same as the change in slope $d\rho v/\partial z$ across the relatively small concentration layer thickness or height δ_D i.e.

$$\frac{\partial^2 \rho_v}{\partial z^2} \approx \frac{(\partial \rho_v / \partial z)_{z=\delta_D} - (\partial \rho_v / \partial z)_{z=0}}{\delta_D - 0} \qquad (16)$$

Figure 1(a) suggests the following partial density gradient scales:

$$(\partial \rho_v / \partial z)_{z=\delta_D} = 0,$$

$$(\partial \rho_v / \partial z)_{z=0} \approx (\rho_{vi} - \rho_{vo}) / \delta_D$$

Substitute these gradients into equation (16) and find

$$\partial^2 \rho_v / \partial z^2 \approx -(\rho_{vi} - \rho_{vo}) / \delta_D^2 \qquad (17)$$

The approximate magnitude of the term on the right-hand side of equation (2) can be deduced by arguing that the average partial density of the δ_D-thick region increases from the initial value ρ_{vi} by a value of $(\rho_{vo} - \rho_{vi})/2$ during the time interval of length t.

$$\partial \rho_v / \partial t = (\rho_{vo} - \rho_{vi})/(2t) \qquad (18)$$

According to equations (2), (17) and (18) find

$$-(\rho_{vi} - \rho_{vo})/\delta_D^2 \approx (\rho_{vo} - \rho_{vi})/(2Dt)$$

or (19)

$$\delta_D = (2Dt)^{1/2}$$

The concentration layer becomes unstable when

$$Ra = g\delta_{Du}^3 (\rho_{avi} - \rho_{avo})\rho_{av} c_p / (k\mu) = 1101 \qquad (20)$$

where

$$\rho_{av} = (\rho_{avo} + \rho_{avi})/2$$

At this condition

$$\delta_{Du} = 10.33[k\mu / \{g(\rho_{avi} - \rho_{avo})\rho_{av} c_p\}]^{1/3} \qquad (21)$$

From equations (19) and (21) find

$$t_u = 53.31[k\mu / \{g(\rho_{avi} - \rho_{avo})\rho_{av} c_p\}]^{2/3} / D \qquad (22)$$

Substitute equation (22) into equation (12) to find

$$h_{Dt}[k\mu / \{g(\rho_{avi} - \rho_{avo})\rho_{av} c_p\}]^{1/3} / D = 0.0773 \qquad (23)$$

The average mass transfer coefficient during the period t is found by integrating equation (12) i.e.

$$h_D = 2[D/(\pi t_u)]^{1/2} = 2h_{Dt}$$

or upon substitution of equation (23)

$$h_D[k\mu / \{g(\rho_{avi} - \rho_{avo})\rho_{av} c_p\}]^{1/3} / D = 0.155 \qquad (24)$$

If the surface generates vapour at a uniform rate it follows from equation (15) that

$$h_{Dm} = \pi h_D / 2$$

or (25)

$$h_{Dm}[k\mu / \{g(\rho_{avi} - \rho_{avom})\rho_{av} c_p\}]^{1/3} / D = 0.243$$

It is stressed that these equations are only applicable to the first phase of the heat transfer process and do not include the second phase during which thermals exist. No simple analytical approach is possible during this latter phase, although the mean mass transfer coefficient during the breakdown of the concentration layer will probably not differ much from the first phase. This would mean that the mean mass transfer coefficient over the cycle of conduction layer growth and breakdown is of approximately the same value as that obtained during the first phase of the cycle.

By following a procedure similar to the above, the analogous problem of heat transfer during natural convection above a heated horizontal surface for a constant surface temperature of T_o can be analysed to find according to Kröger[8]

$$h_T[\mu T / \{g(T_o - T_i) c_p k^2 \rho^2\}]^{1/3}$$

$$= h_T[k\mu / \{g(\rho_i - \rho_o) c_p \rho\}]^{1/3} / k = 0.155 \qquad (26)$$

and for the case of uniform heat flux q

$$h_q\left[\mu T/\left\{g\left(T_{oq}-T_i\right)c_p k^2\rho^2\right\}\right]^{1/3} = h_q\left[k\mu/\left\{g\left(\rho_i-\rho_o\right)c_p\rho\right\}\right]^{1/3}/k = 0.243 \quad (27)$$

where

$$\rho = \left(\rho_i + \rho_o\right)/2$$

Note the similarity between equations (24) (25) and (26) (27) respectively. These equations are applicable to natural convection mass and heat transfer respectively.

During windy periods (forced convection) the transfer coefficients generally increase with increasing wind speed. According to the Reynolds-Colburn analogy and the analogy between mass and heat transfer, the following relations exist (see Holman[4])

$$\frac{h_w \, Pr^{2/3}}{\rho c_p v_w} = \frac{C_f}{2} = \frac{h_{Dw} Sc^{2/3}}{v_w}$$

or (28)

$$h_{Dw} = h_w/\left(\rho c_p Le^{2/3}\right)$$

For air water-vapour mixtures Sc » Pr and

$$Le = k/\left(\rho c_p D\right) \approx 1$$

such that (29)

$$h_{Dw} \approx h_w/\left(\rho c_p\right) = C_f v_w/\left(2\,Pr^{2/3}\right)$$

For Le = 1 find $D = k/(\rho c_p)$. Substitute this into equation (24), to find together with equation (29) the rate of evaporation from a horizontal wetted surface at a uniform partial vapour pressure ρ_{vo}.

$$\begin{aligned} m_{vo} &= \left[h_D + h_{Dw}\right]\left(\rho_{vo}-\rho_{vi}\right) \\ &= \left[0.155\left\{g\left(\rho_{avi}-\rho_{avo}\right)k^2/\left(\rho_{av}c_p\mu\right)\right\} + C_f v_w/\left\{2\left(\mu c_p/k\right)^{2/3}\right\}\right]\left(\rho_{vo}-\rho_{vi}\right) \\ &= \left[0.155\left\{g\left(\rho_{avi}-\rho_{avo}\right)\mu/\rho_{av}^2\right\}^{1/3} + C_f v_w/2\right]\left(\rho_{vo}-\rho_{vi}\right)/Pr^{2/3} \end{aligned} \quad (30)$$

If the vapour is generated uniformly at z = 0 find the rate of evaporation by adding equations (25) and (29).

$$m_{vom} = \left[0.243\left\{g\left(\rho_{avi}-\rho_{avom}\right)\mu/\rho_{av}^2\right\}^{1/3} + C_f v_w/2\right]\left(\rho_{vo}-\rho_{vi}\right)/Pr^{2/3} \quad (31)$$

The dimensionless mass transfer coefficient corresponding to equation (31) is

$$\frac{h_{Dm}\left(Pr\,\rho_{av}\right)^{2/3}}{\left[g\left(\rho_{avi}-\rho_{avo}\right)\mu\right]^{1/3}} = 0.243 + \frac{C_f v_w\left(Pr\,\rho_{av}\right)^{2/3}}{2\left[g\left(\rho_{avi}-\rho_{avo}\right)\mu\right]^{1/3}} \quad (32)$$

The effective friction coefficient C_f depends on wind velocity, surface roughness and heat and mass transfer rates, and must be determined experimentally.

Sartori[2] lists many empirical correlations that predict the rate of evaporation. The values given by these equations may differ significantly over a range as shown by the shaded area in figure 3. In figure 3 the rate of evaporation is shown as a function of wetted surface temperature T_o and ambient air temperatures $T_i = T_o - 5$, a relative humidity of $\phi = 45$ per cent and a wind speed of $v_w = 3$ m/s at about 1m above the wetted surface. Sartori[2] recommends the equations proposed by the WMO[6] and McMillan[7] as shown in figure 3. Equation (31) is also shown in figure 3 for an ambient pressure of $p = 10^5$ N/m^2 and different values of the effective friction coefficient C_f.

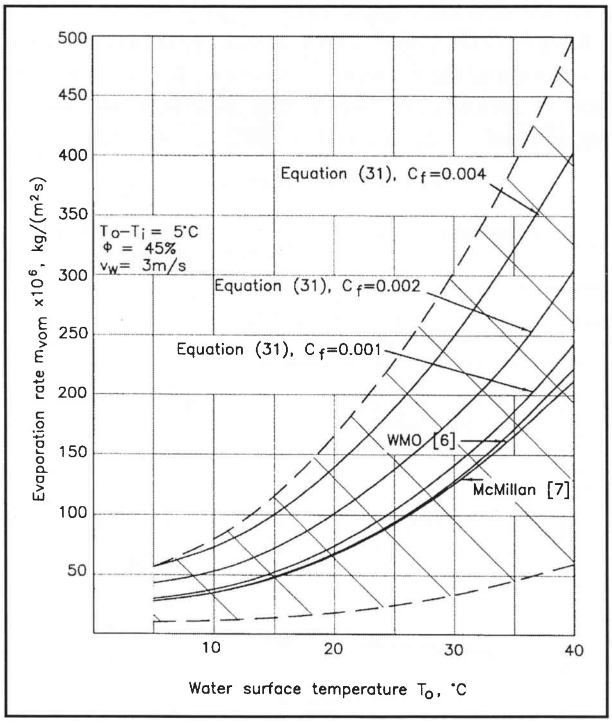

Figure 3: Rate of evaporation

Conclusion

Many horizontal natural water or wetted surfaces having a relatively low thermal conductivity and exposed to solar radiation, transfer both mass and heat to the natural environment.

Under these conditions the rate of evaporation at the wetted surface will be determined primarily by the heat flux due to solar radiation and should thus be evaluated according to equation (31) for cases where $\rho_{avi} > \rho_{avo}$. Although equation (31) is an approximation it is based on a sound theoretical approach. In this equation the effective friction coefficient C_f must be determined experimentally for a wind velocity measured at a particular height near the wetted surface.

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