

Convection in Concentric Annular Regions for Turbulent Flow of Liquid Water

Jaco Dirker¹ and Josua P. Meyer²

The geometric shape of a passage's cross-section has an effect on its convective heat transfer capabilities. For concentric annuli, the diameter ratio of the annular space plays an important role. The purpose of this investigation was to find a correlation that will accurately predict heat transfer coefficients at the inner wall of smooth concentric annuli for turbulent flow of water. Experiments were conducted with a number of annular diameter ratios. The Wilson plot method was used to determine annular heat transfer coefficients from which a correlation was developed for annular convective heat transfer. The deduced correlation predicted Nusselt numbers accurately within 3% of measured values for annular diameter ratios between 1.7 and 3.2 and a Reynolds numbers range, based on the hydraulic diameter, of 4 000 to 30 000.

Nomenclature

a	annular diameter ratio [D_2/D_1]
c_p	specific heat, J/kgK
C_i	inner tube convective heat transfer correlation coefficient - Wilson plot
C_o	annulus convective heat transfer correlation coefficient - Wilson plot
D_1	diameter of outer wall of inner tube, m
D_2	diameter of inner wall of outer tube, m
D_h	hydraulic diameter of annulus [$D_2 - D_1$], m
D_i	inner diameter of inner tube, m
h	convective heat transfer coefficient, W/m ² K
k	thermal conductivity, W/mK
L	length of heat exchanger, m
Nu_i	inner tube Nusselt number [$h_i D_i/k_i$]
Nu_o	annular Nusselt number [$h_o D_h/k_o$]
P	exponent of Reynolds number in Wilson plot function
Pr	Prandtl number [$c_p \mu/k$]
Re_i	inner tube Reynolds number [$\rho_i u_i D_i/\mu_i$]
Re_o	annular Reynolds number [$\rho_o u_o D_h/\mu_o$]
U	overall heat transfer coefficient, W/m ² K
u	axial velocity, m/s
μ	viscosity, Ns/m ²
ρ	density, kg/m ³

Subscripts

D_h	based on the hydraulic diameter of the annulus
i	inner tube side
o	annulus side
w	wall

¹ Department of Mechanical Engineering, Rand Afrikaans University
² Fellow, South African Institute of Mechanical Engineering. Department of Mechanical and Aeronautical Engineering, University of Pretoria, Pretoria, 0002, South Africa, Tel: 27 12 420 2590, Fax: 27 12 362 5124, Email: jmeyer@up.ac.za

Introduction

Since the early nineteen hundreds, many researchers have investigated heat transfer in annuli, particularly in order to find correlations that can describe the Nusselt number and convective heat transfer for a wide range of flow conditions and annular diameter ratios. Figure 1 shows a schematic representation of a basic tube-in-tube element. Most of the proposed equations for calculating annular Nusselt numbers are functions of the annular diameter ratio, the Reynolds number and the Prandtl number. Most correspond with the Dittus-Boelter type form.

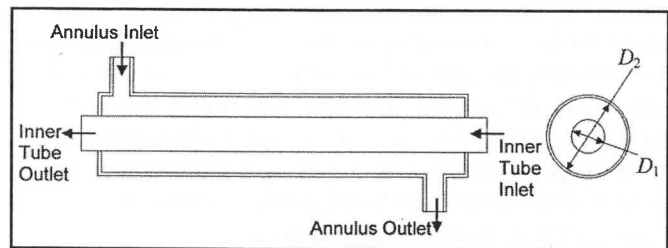


Figure 1 Schematic representation of a basic tube-in-tube heat exchanger element, not to scale

Figure 2 indicates a comparison of Nusselt number predictions of some of the equations cited in literature for a case with an annular ratio of 2 and a Prandtl number of 3.36. Reynolds numbers were based on the hydraulic diameter, D_h .

Even though both the Dittus-Boelter and Gnielinski equations were derived for smooth tubes, they are in some cases used for the determination of annular Nusselt numbers.

All correlations predict an almost linear increase in the Nusselt number with an increase in the Reynolds number. Compared to the other predictions, the equation by Foust and Christian overpredicts the Nusselt number by approximately a factor of three.

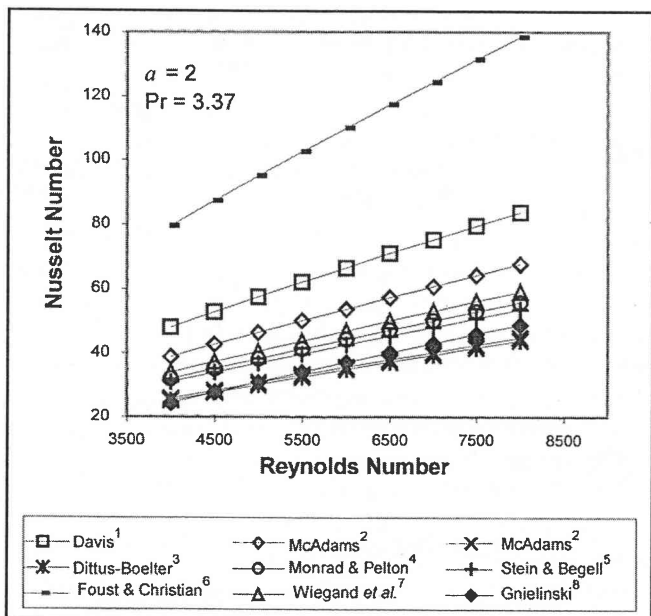


Figure 2 The predictions of various Nusselt number correlations in terms of the Reynolds number for $a = 2$

When the predictions of Foust and Christian are omitted, a difference in predicted values of $\pm 20\%$ relative to the average predicted value is found to exist. The same was found at other annular diameter ratios and flow conditions.

No literature was found that indicates the existence of an accurate heat transfer correlation for concentric annuli. It was thus the purpose of this investigation to deduce a correlation with which accurate predictions could be made of average Nusselt numbers at the inner annular wall under turbulent flow conditions of water.

Experimental facility

A schematic representation of the test facility is shown in Figure 3.

A number of concentric tube-in-tube heat exchangers, each with a different annular diameter ratio, were used during the experimental investigation. Each heat exchanger had an effective heat transfer length of about 6 m and was operated in a counterflow arrangement with hot water in the inner tube and cold water in the annulus. The heat exchangers were constructed from hard drawn refrigeration copper tubing and were operated in a horizontal configuration. Dimensions of the different heat exchangers are listed in Table 1. Standard commercial refrigeration tube sizes were used.

The inner tubes were kept in concentric positions by employing sets of either three or four radial supporting metal pins along the length of each heat exchanger. The pins had a diameter of 0.6 mm and were at an offset of either 90° or 120° at each supporting position. The pins were soldered to the outer tube while only supporting the inner tube. The pins were not soldered to the inner tube in order to minimise the effect of possible thermal conduction from the hot inner tube to the metal pins, which might have acted as fins.

By using a symmetrical configuration, possible unbalanced flow patterns were minimised. Supporting sets were placed at distances of between 0.75 m and 0.667 m apart along the length of each heat exchanger depending on the cross-sectional area

Heat Exchanger	D_i	D_1	D_2	a	L
	(mm)	(mm)	(mm)		(m)
1	5.30	6.35	11.15	1.76	6.170
2	5.30	6.35	14.10	2.22	6.095
3	5.30	6.35	17.30	2.72	6.160
4	5.30	6.35	20.30	3.20	6.170
5	5.30	6.35	32.00	5.04	6.240
6	17.30	19.05	32.00	1.68	6.200

Table 1 Physical dimensions of heat exchangers

of the annulus. The size and position of the supporting pins were carefully calculated to minimise possible sagging of the inner tube. The supporting structures occupied at most 6.5% of the cross sectional area of the smallest annulus.

Temperature measurements were facilitated by means of K-type thermocouples fixed on the outside surfaces of entry and exit regions of the heat exchangers. Temperature errors were usually less than 0.1 K. Measuring points were sufficiently insulated from the ambient. Temperature data were captured with the aid of a data logger.

Volumetric flow rates were measured by using semi-rotary circular piston-type displacement flowmeters with a measuring accuracy greater than 98%. The flowmeters were installed at the exits of both the hot and cold-water tubes of the heat exchangers. By allowing a distance of at least 1 m from the outlets, the chaotic flow patterns generated at the exit regions were decreased before entering the flowmeters. This ensured more accurate flow rate measurements. The flowmeters were installed at the exit regions as the high pressures at the inlet regions might influence the operation of the flowmeters negatively.

Hot water supplied by an on-site hot-water storage tank (1000 l), fitted with an electric resistance water heater, was pumped through the inner tube by means of a positive displacement pump and then returned to the storage tank.

The hot-water flow rates were controlled with a hand-operated bypass system.

Cold water was supplied from a cold-water storage tank (1000 l) connected to a chiller and pumped through the annulus by means of two series connected centrifugal pumps to ensure high flow rates before being returned to the storage tank.

Experimental procedure

Experimental tests were performed at a wide range of flow rate combinations between the inner tube and annulus. As a correlation for the annulus was being deduced, the flow rates in the annulus were of more importance and a bigger spectrum was covered whilst ensuring a turbulent flow regime.

Hot and cold water had entry temperatures in the vicinity of 50°C and 10°C respectively while exit temperatures varied depending on volumetric flow rates.

Experiments were started with the inner

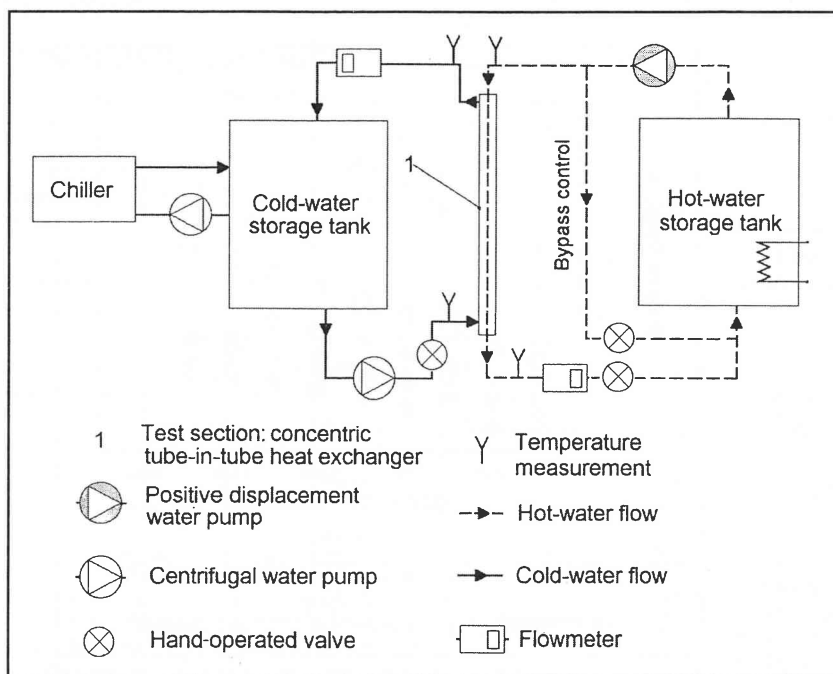


Figure 3 Schematic representation of experimental facility

Heat Exchanger	a	Number of Tests Performed	Number of Data Points Used for Analyses	Re _{o,Dh}		Energy Balance Errors (%)	
				Minimum	Maximum	Median	Standard Deviance
1	1.76	50	50	8 065	24 117	0.731	0.330
2	2.22	50	50	6 422	31 484	0.049	0.407
3	2.68	30	30	6 066	34 887	0.029	0.613
4	3.2	50	50	4 562	28 093	0.612	0.473
5	5.04	30	21	3 457	18 538	0.463	0.558
6	1.68	50	50	2 631	14 425	1.159	0.739

Table 2 Information on experimental data sets and energy balance errors

tube flow rate constant for a number of tests while the annular flow rate was altered through a spectrum. Thereafter a new inner tube flow rate was used and the procedure repeated. After sufficient time was allowed for steady state conditions to be established, inlet and outlet temperatures of the inner tube and annulus were recorded by means of the data logger.

Before moving on to a next test, it was ensured that the energy balance error was at a satisfactory low level. If the error was too high, a longer time period was given for the system to reach steady state conditions. A high level of accuracy in the experimental data was maintained. More than 90% of all data points exhibited an energy balance error of less than 1% between the inner tube and annular heat transfer rates. A Reynolds number range, based on the hydraulic diameter, of 2 600 to 35 000 was covered in experiments performed on eight heat exchangers. Information on the experimental data and data sets used for analysis purposes is given in Table 2.

Processing of data

The internal and annular Nusselt numbers can be correlated by means of equations (1) and (2) respectively:

$$Nu_i = \frac{h_i D_i}{k_i} = C_i Re_i^{0.8} Pr_i^{\frac{1}{3}} \left(\frac{\mu}{\mu_w} \right)_i^{0.14} \tag{1}$$

$$Nu_o = \frac{h_o D_h}{k_o} = C_o Re_{o,D_h}^P Pr_o^{\frac{1}{3}} \left(\frac{\mu}{\mu_w} \right)_o^{0.14} \tag{2}$$

P, *C_i* and *C_o* are added to account for geometric influences. For the inner tube the exponent of the Reynolds number was kept at 0.8 as proposed in literature⁹. The modified Wilson plot method developed by Briggs and Young⁹ was used to determine these values for the different annular diameter ratios.

Obtained values are listed in Table 3 accompanied by information on the error of the resulting correlations (equation 2) on the data in terms of the annular overall heat transfer coefficient *U_o*.

More than 95% of all data points were predicted within a 3% accuracy range by the Wilson plot correlations for the different heat exchangers. All Wilson plot correlations exhibited a median error of less than or in close proximity to 1%. Standard deviances for error values were less than 2%.

Derivation of correlation

As was expected, both *P* and *C_o* (equation 2) showed a dependence on the annular diameter ratio. Figures 4 and 5 illustrate the general trends of *P* and *C_o* in terms of the diameter ratio. The value of *P* exhibited a downward trend when the annular diameter ratio was increased. On the other hand, the value of *C_o* had an upward trend for an increasing annular diameter ratio.

From the available experimental data, the exact behaviour of the values of *P* and *C_o* could not be described for annular ratios greater than 3.2. More experimental data covering additional annular diameter ratios in this region are necessary. Unfortunately commercial sized tubes, which could produce more ratios between 3 and 5, are not readily available making the investigation process more difficult.

For annular diameter ratios of less than 3.2 the trend of *P* and *C_o* is much more consistent and can be described with greater confidence. After evaluating different curve fits, mathematical relations (3) and (4) were obtained in this region in order to predict the values of *P* and *C_o* in terms of the diameter ratio. The solid lines in Figures 4 and 5 show how these relations approximate the Wilson plot variables of *P* and *C_o* respectively.

Other mathematical relations such as linear approximations were also considered, but due to the loss in accuracy, relations

Heat Exchanger	a	Wilson Plot Variables		U _o Prediction Error (%)	
		<i>P</i>	<i>C_o</i>	Median	Standard Deviation
1	1.76	0.8973	0.0085	0.895	1.188
2	2.22	0.8319	0.0171	0.991	1.429
3	2.68	0.8533	0.0152	0.775	1.059
4	3.2	0.8351	0.0194	1.028	1.920
5	5.04	0.7175	0.0637	0.158	1.447
6	1.68	0.9256	0.0068	0.305	1.116

Table 3 Annular Wilson plot correlation results obtained for the eight heat exchangers

(3) and (4) were chosen.

By substituting relations (3) and (4) into equation (2) a correlation for the prediction of the Nusselt numbers is produced.

$$P = 1.013e^{-0.067a} \quad (3)$$

$$C_o = \frac{0.003a^{1.86}}{0.063a^3 - 0.674a^2 + 2.225a - 1.157} \quad (4)$$

The validity of the resulting correlation for the prediction of Nusselt numbers were tested with experimental data from all heat exchangers having an annular diameter ratio of less than 3.2. All predictions were within 3% of experimentally obtained values. This is demonstrated in Figure 6.

The correlation obtained has been compared to other correlations in Figure 2 over a range of Reynolds numbers for a diameter ratio of $a = 2$. The present correlation is also compared to others in Figure 7 for a Reynolds number of 20 000 and for diameter ratios of 1.25 to 3.5. This trend was found to be true

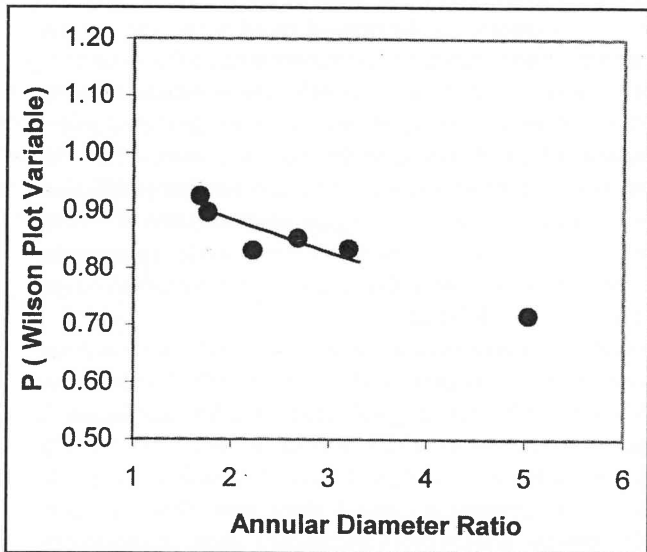


Figure 4 P values obtained from Wilson plot analyses

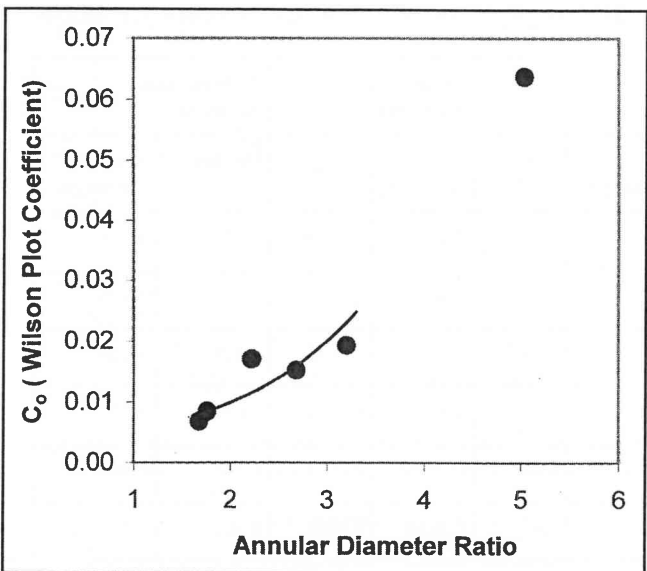


Figure 5 C_o values obtained from Wilson plot analyses

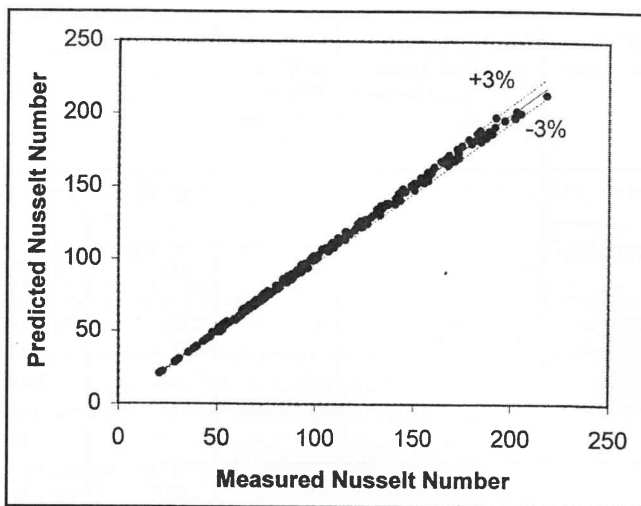


Figure 6 Comparison between predicted Nusselt numbers and measured Nusselt numbers.

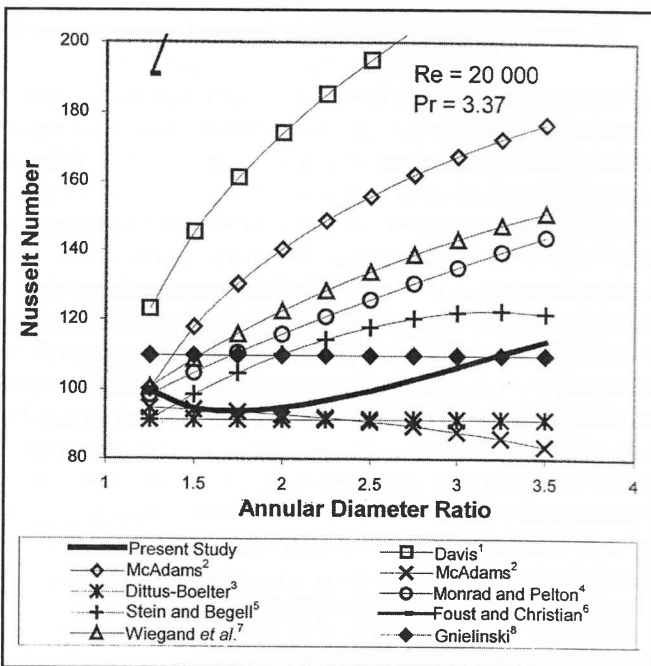


Figure 7 Comparison between the derived correlations and correlations by other authors for a wide range in annular diameter ratios.

for a side range of Reynolds numbers and Prandtl numbers.

For small annular diameter ratios, up to about 2.5, the predictions correspond well with the correlations by Dittus and Boelter³, and McAdams². In the region of an annular ratio of 3.5, a close agreement exists with the correlations of Stein and Begell⁵ and Gnielinski⁸.

Conclusion

As was expected, it was found that the convective heat transfer correlation for an annulus is dependent on the annular diameter ratios. A correlation was deduced from experimental results that predicts Nusselt numbers accurately within 3% from the measured values for diameter ratios between 1.7 and 3.2 and a Reynolds numbers range of 4 000 to 30 000.

References

1. Davis ES, *Heat transfer and pressure drop in annuli*, Transactions of ASME, 1943, Oct, pp 755-760
2. McAdams WH, *Heat Transmissions 3rd ed.*, pp 241-244,

1954

3. Dittus FW, Boelter LMK, University of California, Berkeley, Publications on Engineering, 1930, 2, pp 443
4. Monrad CC, Pelton JF, Heat transfer by convection in annular spaces, American Institute of Chemical Engineers, 1942, 38, pp 593-611
5. Stein RP, Begell W, Heat transfer to water in turbulent flow in internally heated annuli. American Institute of Chemical Engineers Journal, 1958, 4(2), June pp 127- 131
6. Foust AS, Christian GA, Non-boiling heat transfer coefficients in annuli, American Institute of Chemical Engineers, 1940, 36, pp 541-554
7. Wiegand JH, McMillen EL, Larson RE, Discussion on: Annular heat transfer coefficients for turbulent flow., American Institute of Chemical Engineers, 1945, 41, pp 147-153
8. Incropera FP, DeWitt DP, Introduction to heat transfer 3rd ed., pp 413, 1996
9. Briggs DE, Young EH, Modified Wilson plot technique for obtaining heat transfer correlations for shell and tube heat exchangers, Chemical Engineering Progress Symposium, 1969, 65, pp 35-45