THE SOLAR CHIMNEY AIR STANDARD THERMODYNAMIC CYCLE

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The paper presents an air standard thermodynamic analysis for the solar chimney cycle. A solar chimney is a power plant in which air, heated under a glass-decked solar collector, escapes through a power generating turbine at the base of a chimney. Simple equations were derived for the cycle efficiency, power per unit mass flow and available turbine pressure drop, assuming ideal air as process gas and ideal reversible processes. The main conclusions were the following: the cycle efficiency is directly proportional to the chimney height; unlike the gas turbine cycle, there is no optimal pressure ratio (chimney height) for maximum specific power; the power per unit mass flow developed by the plant is directly proportional to the cycle efficiency and the temperature rise in the solar collector; the pressure drop across the turbine is proportional to the chimney height and the collector temperature rise. A design condition where the chimney exit temperature equals the collector inlet temperature was also proposed for initial calculations. The specific power of the plant is then approximately proportional to the square of the cycle efficiency. The analysis enables one to do initial sizing of a plant for a given required power.

Nomenclature

c compression temperature ratio
c_p specific heat [J/kgK]
D chimney inside diameter [m]
g gravitational acceleration [m/s²]
h enthalpy [J/kg]
K pressure drop coefficient
m mass flow [kg/s]
n polytropic exponent
P power [W]
p pressure [Pa]
r pressure ratio
T absolute temperature [K]
t cycle temperature ratio

V velocity [m/s]
z altitude [m]
Δ change in value
γ specific heat ratio
η cycle efficiency
ρ density [kg/m³]

Subscripts

c chimney base
d design
opt optimum
t turbine
te turbine exit
1 gas turbine cycle inlet
2 combustor (collector) inlet
3 turbine inlet
4 gas turbine (solar chimney) exit
23 between 2 and 3

Superscripts

$*$ normalized value
$'$ idealised value

Introduction

A solar chimney power plant consists of a large area covered by glass or translucent plastic sheets to collect solar energy. A central, high chimney, containing a power generating turbine at its base, allows the hot air to escape. Haaf et al.[1] and Haaf[2] discussed the operation of a prototype solar chimney plant.

The thermodynamic processes occurring in a solar chimney could be presented as a thermodynamic cycle. Air standard cycles are simplified thermodynamic cycles that are useful in the understanding of the real cycles that they simulate. They are so named because their working fluid is assumed to be air behaving as perfect gas. Their analytical nature serves to point out the main relationships between variables, and the most important trends in the way they affect each other. The results obtained from air standard analyses are intended to render limiting (not realistic) values of variables such as efficiency and specific power. Extended analyses including pressure losses, component efficiencies, such as the solar

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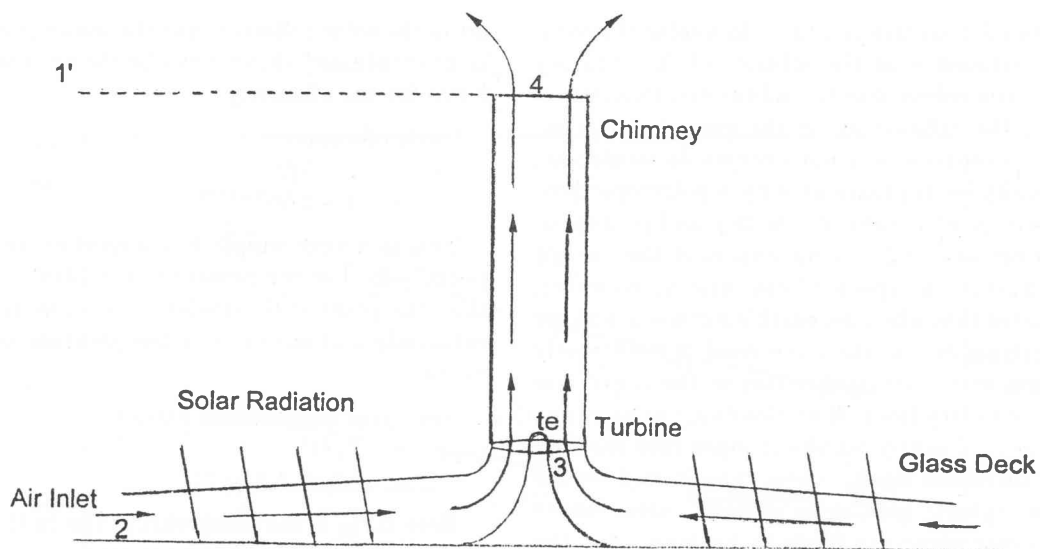†Department of Mechanical Engineering, University of Stellenbosch

Figure 1 Schematic layout of solar chimney power plant

collector efficiency, and variable air properties, are beyond the scope of an air standard analysis and will be published elsewhere.

Many investigators, for example Wilson,[3] Cohen *et al.*,[4] Oates[5] and Archer & Saarlas,[6] have found it instructive to develop air standard cycles for gas turbine and other thermodynamic cycles. Some aspects of the thermodynamics of the solar chimney cycle have been investigated, for example, by Schlaich[7] and Stephan *et al.*[8] but a complete air standard cycle analysis for the solar chimney could not be found.

Objectives of the study

The objectives of this report are as follows:

- To develop a solar chimney air standard cycle

- To use the models to determine the existence of optima if any

- To determine the efficiency potential of the cycle

- To determine the power generating potential of the cycle

- To determine the available turbine pressure drop of the cycle

- To investigate in principle what cycle modifications are necessary and feasible

Assumptions for air standard analyses

The usual assumptions for air standard cycles are as follows:

- The working fluid is dry air, considered as a constant composition perfect gas throughout, with constant specific heat

- The mass flow through the system is constant

- Compression and expansion processes are adiabatic and reversible (isentropic)

- The change in kinetic energy of the air between inlet and exit of each component is negligible

- There are no stagnation pressure drops in the combustor or connecting ducts and passages

- Inlet and exit atmospheric conditions are identical

- The only heat flow is the net heat flow into the air in the system

The solar chimney as a gas turbine

The first observation is that the solar chimney cycle is fundamentally a gas turbine (Joule or Brayton) cycle. Let us consider the gas turbine air standard cycle. The four processes are isentropic compression, constant pressure heat addition, isentropic expansion and constant pressure heat removal.

In a standard gas turbine cycle the inlet and exhaust environmental conditions are assumed to be identical, and the only thermodynamic function of the surrounding air is to complete the cycle by cooling the exhaust

air back to the inlet air temperature. In a solar chimney, however, the exhaust is at the altitude of the chimney top, and the atmosphere has the additional function of recompressing the exhaust air to the ground inlet conditions. This compression is not necessarily isentropic, but can generally be approximated by a polytropic process of the form $p/\rho^n = $ constant, with p and ρ equal to pressure and density, and n some exponent that is not necessarily equal to the specific heat ratio γ. However, Lenschow[9] states that when the earth's surface is warmer than the overlying air or when the wind is moderately strong, efficient vertical mixing occurs in the convective atmospheric boundary layer. If no clouds are present this mixing results in a nearly adiabatic lapse rate through most of this turbulent layer. These may indeed be the prevalent atmospheric conditions in desert areas where the solar chimney plant are likely to be built. For the present analysis we shall assume an adiabatic lapse rate.

We shall first look at the simulation of the solar chimney cycle by a standard gas turbine air cycle where the chimney is eliminated and an isentropic compressor driven by the turbine is added. To better simulate the environmental conditions of the solar chimney plant we assume that the gas turbine plant is situated at the physically impractical altitude of the chimney top. The pressure ratio is chosen to be equal to the ratio of ground level atmospheric pressure to atmospheric pressure at the solar chimney top. The gas turbine then has the same cycle pressure ratio as the solar chimney plant it simulates. Such a real gas turbine solar concept would have other major practical disadvantages, such as the collector pressure being higher than local atmospheric (at chimney top altitude), and the huge amount of power to be transmitted between turbine and compressor. The low temperature solar gas turbine cycle will however serve as a useful benchmark for the solar chimney cycle.

Air standard analysis of solar powered gas turbine cycle

The T-s diagram in Figure 2 shows the cycle and the numbering used for the conditions between the processes in the cycle. The analysis below follows Cohen et al.[4] and Archer & Saarlas.[6] The definitions below are used to simplify the writing of the equations:

Cycle pressure ratio:
$$r = p_2/p_1 = p_3/p_4$$

Compression temperature ratio:
$$
\begin{aligned}
c &= T_2/T_1 \\
&= r^{(\gamma-1)/\gamma}
\end{aligned}
$$

The cycle efficiency is defined as the turbine output power divided by the thermal power transferred to the air in the solar collector. For the assumptions and conditions mentioned above it can be shown that the following holds for the efficiency:

Cycle efficiency:
$$
\begin{aligned}
\eta &= 1 - 1/c \\
&= 1 - 1/r^{(\gamma-1)/\gamma}
\end{aligned}
$$

This is a very simple but important relationship, especially for low temperature ratio ideal cycles. It shows that the plant cycle efficiency is a function of pressure ratio only and not of cycle temperature ratio, t, defined below.

The cycle temperature ratio is:
$$
\begin{aligned}
t &= T_3/T_1 \\
&= (T_2 + \Delta T_{23})/T_1
\end{aligned}
$$

Here ΔT_{23} is the temperature rise in the collector, T_2 is the compressor exit temperature.

As shown by Cohen et al.[4] the specific power of an ideal gas turbine, as normalised with T_1 in terms of t and c is:
$$
\begin{aligned}
P_1* &= P_{\text{shaft}}/(m\,c_p\,T) \\
&= \{(t/c) - 1\}(c - 1)
\end{aligned}
$$

The equation above implies that the specific power depends on pressure ratio as well as collector temperature rise (or power input per unit mass flow). It is clear that the specific power increases with plant temperature ratio (or collector temperature rise), but the effect of compression ratio cannot be seen just by inspection. The significance of the specific power is that the size and capital costs of a plant are determined largely by the mass flow through it. For any given value of t the relationship above can be differentiated with respect to c and equated to zero. The result is:
$$c_{opt} = \sqrt{t}$$

The value of the optimum specific power is then:
$$
\begin{aligned}
P_1*_{opt} &= \{(t/c_{opt}) - 1\}(c_{opt} - 1) \\
&= (t/\sqrt{t} - 1)(\sqrt{t} - 1) \\
&= (\sqrt{t} - 1)^2
\end{aligned}
$$

As pointed out by Cohen et al.[4] this optimum occurs when the turbine and compressor exit temperatures are equal, i.e. when $T_2 = T_4$. The optimum specific power can also be expressed in terms of the optimal cycle pressure ratio:
$$P_1*_{opt} = (c_{opt} - 1)^2$$

The general implication of the two equations for optimal power output is then that when an optimum combination of cycle temperature and pressure ratio is chosen, the specific power can be expressed as a function of one of them alone. Combinations off the optimal will progressively reduce the specific power output compared to the optimum as they deviate from it.

For a low temperature gas turbine air standard cycle simulating a solar chimney cycle the most important conclusions are:

- Cycle efficiency depends on cycle pressure ratio only

- For each cycle temperature ratio there is an optimal pressure ratio for maximum power per unit mass flow

Air standard analysis of solar chimney cycle

The analysis of the solar chimney cycle proceeds largely as for the gas turbine cycle except for the following:

- The compression process does not occur in the system, but in the environment, as the air descends. We have pointed out that isentropic compression may be a realistic process in desert conditions. However, we shall not enforce that condition strictly during the analysis.

- Any reference in the analysis to the compression inlet temperature T1 will be avoided, because this is a variable temperature determined by the point where the constant pressure cooling line from point 4 intersects the polytropic (not isentropic) compression line to point 2. See Figure 3. Use will be made however of the point 1', the intersection between the constant pressure line through point 4 and the isentropic line through point 2.

A convenient reference temperature in the cycle is the collector inlet temperature, T_2, since it is the ground level environmental temperature.

As before, the cycle efficiency is:

η = (Shaft power out) / (Solar power in)

The solar power input is:

$P_{23} = m\, c_p\, (T_3 - T_2)$

The shaft power out is:

$P_{\text{shaft}} = m\, c_p\, (T_3 - T_4) - m\, c_p\, (T_2 - T_1')$

Why the second term, $m\, c_p\, (T_2 - T_1')$ must be subtracted for the solar chimney cycle is explained in the following paragraphs.

The power that could in principle be extracted from the flow by an ideal turbine as it expands from the collector exit pressure, p_3 to the chimney exit pressure, p_4, is $m\, c_p\, (T_3 - T_4)$, but not all of it is available as shaft power.
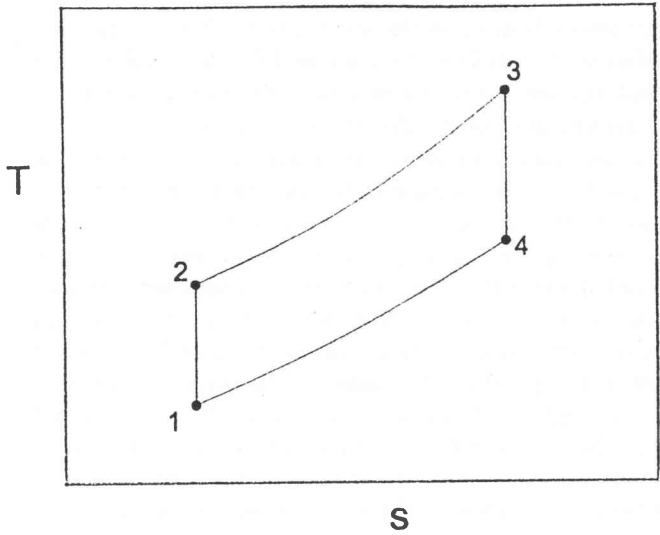

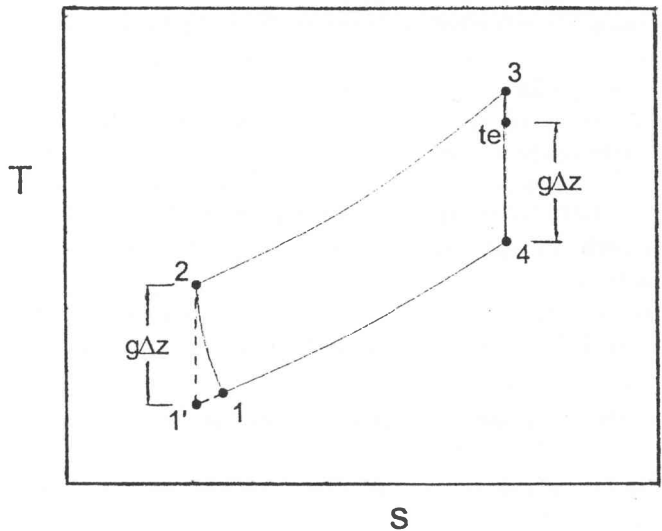Figure 2 Temperature-entropy diagram for gas turbine air standard cycle

Figure 3 Temperature-entropy diagram for solar chimney air standard cycle

Consider the section of the chimney between the turbine exit and the chimney exit. The power required to lift the air in the chimney is $mg\Delta z$. There is no heat transfer or shaft work in this section of the chimney. Energy conservation then determines the enthalpy change in the chimney to be $\Delta h = c_p(T_4 - T_{te}) = -g\,\Delta z$.

The energy exchange is isentropic, following from the assumption of zero friction and heat transfer. The magnitude of Δh may also be evaluated by considering the air as it descends again in the atmosphere after having been cooled to T_1'. If heat transfer and frictional effects are small, the process is reversed and the enthalpy increases by the amount $\Delta h = g\,\Delta z = c_p(T_2 - T_1')$.

In the argument above we made some assumptions that will now be discussed. To construct the thermodynamic cycle one has to follow the flow from the chimney exit through the atmosphere where we assume that it is cooled down at constant pressure to the same entropy value as at the collector inlet. An isentropic compression between this point, at temperature T_1' and the collector inlet at temperature T_2 completes the cycle. If the process through which the air goes between the chimney exit and collector inlet follows some other path, determined by the atmospheric model, the idealized cycle may still be used for developing the analysis, as long as the imaginary temperature T_1' is factored out during the analysis. This may be done because state point 1' describes a condition external to the solar chimney, and is merely a construct used to develop the air standard analysis.

Although the air standard gas turbine and solar chimney cycle T-s diagrams are similar, there are some differences. In the gas turbine cycle, energy is transferred from the turbine to the compressor as shaft work, that is, energy is extracted from the cycle by the turbine and reintroduced by the compressor. In the solar chimney cycle there is an internal exchange between enthalpy and geopotential energy in the chimney, and the process is reversed in the external environment. In spite of the differences the gas turbine air standard cycle analysis can be applied directly to the solar chimney cycle since similar enthalpy and entropy changes occur in the two air standard cycles.

In the solar chimney cycle the pressure ratio is defined in terms of the chimney exit and collector exit pressures:
$$r = p_3/p_4$$
and the compression temperature ratio is:
$$c = r^{(\gamma-1)/\gamma}$$

As for the gas turbine cycle the cycle efficiency is then:
$$\eta = 1 - 1/c$$
$$= (T_2 - T_1')/T_2$$
Since:
$$c_p(T_2 - T_1') = g\,\Delta z$$
$$\eta = g\,\Delta z/(c_p T_2)$$

The cycle efficiency of an ideal solar chimney plant is directly proportional to the chimney height, and inversely proportional to the collector inlet temperature. Schlaich[7] uses the definition above for the efficiency of the chimney only, and defines a plant efficiency that includes the solar collector heat losses.

The cycle temperature ratio is defined as with T_1', as discussed above:
$$t = T_3/T_1\prime$$

For the solar chimney cycle the specific power is normalized with respect to the collector inlet temperature, T_2:
$$P_2* = P/(m\,c_p T_2)$$
$$= m\,c_p\,(T_3 - T_4) - m\,c_p\,(T_2 - T_1')/(m\,c_p T_2)$$
$$= (T_3 - T_4)/T_2 - (T_2 - T_1')/T_2$$
$$= (t/c)(1 - 1/c) - (1 - 1/c)$$
$$, = (t/c - 1)(1 - 1/c)$$

It is tempting to differentiate this equation with respect to c for fixed t as for the gas turbine cycle, but for the solar chimney t does not remain constant when c changes, because T_1' decreases with altitude as chimney height (and c) increases. In other words t is a function of c:
$$t = T_3/T_1\prime$$
$$= (T_2 + \Delta T_{23})\,c/T_2$$
$$= c\,(1 + \Delta T_{23}/T_2)$$
So:
$$P_2* = (\Delta T_{23}/T_2)\,(1 - 1/c)$$

If ΔT_{23} and T_2 are fixed, P_2* will be maximized by maximizing c (i.e. chimney height). There is then no optimum chimney height that maximizes specific power. Similarly, there is no optimum collector temperature rise that maximizes specific power for a fixed chimney height. Any increase in collector temperature rise will increase the specific power.

The specific power can be written in terms of η as follows:
$$P_2* = (\Delta T_{23}/T_2)\,\eta$$

The specific power can be written in terms of Δz as follows:
$$P_2* = (\Delta T_{23}/T_2)\,(g\,\Delta z/c_p T_2)$$

The power output of an ideal solar chimney plant is then:
$$P = m\,c_p T_2 P_2*$$
$$= m\,g\,\Delta z\,\Delta T_{23}/T_2$$

The equation above shows that the specific power is proportional to the chimney height and the collector temperature rise, and inversely proportional to the collector inlet temperature. The required specific power can be obtained by any suitable combination of Δz or ΔT_{23}, or in other words by any suitable combination of chimney height and collector area. The choice will ultimately be determined by the relative costs of chimney and collector. Very high chimneys will be expensive, and may promote condensation in the chimney, and very large collectors will be expensive, and inefficient due to high radiation losses.

Table 1 gives the power per unit mass flow for various combinations of chimney height and collector temperature rise.

Table 1 Power per unit mass flow [kW/(tonne/s] for $T_2 = 303.2$ K

Δz [m] $\Downarrow$ ΔT_{23} [K] $\Rightarrow$	5	10	15	20
500	81	162	243	323
1000	162	323	485	647
1500	243	485	728	970
2000	323	647	970	1294

The available pressure drop across the turbine may now be calculated. We define the available temperature and pressure ratio across the turbine as follows, where subscript "te" denotes turbine exit:

$t_t = T_3/T_{te}$

and:

$r_t = p_3/p_{te}$

The plant output power may be expressed as follows:

$$
\begin{aligned}
P &= m\,c_p\,(T_3 - T_{te}) \\
&= m\,c_p T_3\,(1 - 1/t_t) = m\,g\,\Delta z\,\Delta T_{23}/T_2 \\
1/t_t &= 1 - (g\,\Delta z/c_p\,T_2)\,\Delta T_{23}/\,(T_2 + \Delta T_{23}) \\
&= 1 - \eta\,(\Delta T_{23}/T_2)\,/\,\{1 + (\Delta T_{23}/T_2)\} \\
1/r_t &= \left\{ \frac{[1 + (1 - \eta)\,(\Delta T_{23}/T_2)]}{[1 + (\Delta T_{23}/T_2)]} \right\}^{\gamma(\gamma - 1)} \\
\Delta p_t/p_3 &= (1 - 1/r_t) \\
&= 1 - \left\{ \frac{[1 + (1 - \eta)\,(\Delta T_{23}/T_2)]}{[1 + (\Delta T_{23}/T_2)]} \right\}^{\gamma(\gamma - 1)} \\
&= 1 - \{1 - \eta/\,[(1 + (T_2/\Delta T_{23}))]\}^{\gamma(\gamma - 1)}
\end{aligned}
$$

The equation above was used to draw up Table 2.

Table 2 Available turbine pressure drop [Pa] for $T_2 = 303.2$ K and $p_3 = p_2 = 90$ kPa

Δz [m] $\Downarrow$ ΔT_{23} [K] $\Rightarrow$	5	10	15	20
500	82	162	239	313
1000	164	323	477	626
1500	246	485	715	937
2000	329	646	952	1248

For incompressible flow we may assume that $P = Q_3 \Delta p_t$ where Q_3 is the volume flow m/ρ_3 at the turbine inlet. The available turbine pressure drop Δp_t is then:

$$
\begin{aligned}
\Delta p_t &= P\,\rho_3/m \\
&= (g\,\Delta z\,\Delta T_{23}/T_2)\,\rho_3 \\
&= (g\,\Delta z\,\Delta T_{23}/T_2)\,p_3/\,\{R\,(T_2 + \Delta T_{23})\} \\
&= \{p_3\eta\gamma/\,(\gamma - 1)\}\,/\,(1 + T_2/\Delta T_{23})
\end{aligned}
$$

When the available turbine pressure drop is calculated with this equation for $\Delta z = 2000$ m and $\Delta T_{23} = 20$ K, then $\Delta p_t = 1255$ Pa, which corresponds to within 0.5 % with the value in the table above. The incompressible approximation, which can also be derived as the first term binomial expansion of the compressible equation, is accurate enough for the present simple analysis.

Bearing in mind that $p_2 = p_3$ the available turbine pressure drop may also simply be written in terms of atmospheric ground level density ρ_2 as:

$\Delta p_t = \rho_2\,g\,\Delta z\,\Delta T_{23}/T_3$

Schlaich[7] gives a similar equation. This is as far as we can go when the system is considered to have no pressure losses.

Practical application of air standard analysis

It is useful in the application of the theory to make a few simple additional assumptions. Schlaich[7] points out that the maximum power that can be generated occurs when two thirds of the drop in total pressure in the system is across the turbine. It can easily be shown that this applies to any system with a fixed available pressure difference and a loss proportional to flow squared. So only two-thirds of the pressure drop is available to drive the turbine, and the power extraction is reduced by the same factor. The fact that a third of the pressure losses are associated with system losses may be used to calculate the optimum chimney draught velocity. Since the velocity in the chimney increases as the density decreases due to expansion across the turbine and with altitude in the chimney, we have to choose a reference station. As reference position we choose the velocity at the chimney base, before the turbine. The chimney exit loss referred to this position will be larger than 1.0, because of the chimney exit velocity is larger than the chimney base velocity.

$$
\begin{aligned}
K\,\rho_3\,V_c{}^2/2 &= \Delta p_t/3 \\
K\,V_c{}^2/2 &= (\Delta p_t/\rho_3)\,/3 = (1/3)\,P/m \\
V_c &= [2/\,(3K)]^{0.5}\,(P/m)^{0.5}
\end{aligned}
$$

To allow for all the effects, we assume a constant system loss coefficient referred to the chimney base velocity of $K = 2.0$.

Table 3 Maximum power chimney draught velocity [m/s] for $K = 2.0$ and $T_2 = 303.2$ K

Δz [m] $\Downarrow$ ΔT_{23} [K] $\Rightarrow$	5	10	15	20
500	5.2	7.3	9.0	10.4
1000	7.3	10.4	12.7	14.7
1500	9.0	12.7	15.6	18.0
2000	10.4	14.7	18.0	20.8

Table 3 shows that for the chosen conditions the chimney base velocity for maximum power can be given approximately by:

$V_c = (\Delta T_{23}\,\Delta z/1000)^{0.5}$

The mass flow required for a 250 MW plant is calculated as follows below. (In this analysis we assume

the turbine efficiency to be 100%, but with an 80% turbine efficiency the output would be 200 MW.) The construction of a large plant of 200 MW is currently being considered.

$$m_{250\text{MW}} = 250 \times 10^6 / \{(2/3)\, P/m\}$$

Table 4 Mass flow [tonne/s] for 250 MW plant for $T_2 = 303.2$ K

Δz [m] $\Downarrow$ ΔT_{23} [K] $\Rightarrow$	5	10	15	20
500	4636	2318	1545	1159
1000	2318	1159	773	580
1500	1545	773	515	386
2000	1159	580	386	290

The appropriate chimney diameter is calculated as follows:

$$\begin{aligned} m &= \rho A_c V_c \\ &= [p_2 / \{R(T_2 + \Delta T_{23})\}](\pi/4)\, D^2 V \\ D^2 &= 4mR(T_2 + \Delta T_{23}) / (\pi p_2 V) \end{aligned}$$

Table 5 Chimney diameter [m] for 250 MW and $K = 2.0$ and $T_2 = 303.2$ K

Δz [m] $\Downarrow$ ΔT_{23} [K] $\Rightarrow$	5	10	15	20
500	1058	634	471	383
1000	629	377	280	228
1500	464	278	207	168
2000	374	224	167	135

Tables 4 and 5 show that a 250 MW plant (with a 100% efficient turbine) designed for a temperature rise of 20 K and a chimney height, requires a chimney diameter of about 170 m.

A possible design condition

One of the disadvantages of the analysis so far is that it does not lead to any preferential solar collector temperature rise for a given chimney height or vice versa. A possible preferential combination of parameters is the one implied by $c = \sqrt{t}$. In the case of the solar chimney it is not an optimal combination in the sense that it maximizes the specific power. We propose to choose this combination as a basis for initial design. In recognition of this fact, the subscript "d" (for design) will be used instead of "opt".

The choice of a design condition with $cd = \sqrt{t}$ implies the following:

- Neither the collector size nor the chimney height will be extreme

- The chimney exit temperature, T_4, is equal to the solar collector inlet temperature, T_2

- The condition above means that such a cycle will not benefit from the introduction of a heat exchanger, using exit air to heat entry air

- Condensation in the chimney will be unlikely even if the air is saturated at the collector inlet, as the chimney exit condensation temperature will be only a few degrees below the collector inlet temperature due to the decrease in pressure in the chimney

We shall now derive the dependence of collector temperature rise on chimney height for the design conditions:

$$\begin{aligned} g\,\Delta z &= c_p(T_2 - T_1') \\ g\,\Delta z / c_p &= (c-1)T_1' \end{aligned}$$

But:

$$\Delta T_{23} = (t - c)T_1'$$

and:

$$\begin{aligned} (\Delta T_{23})_d &= (c^2 - c)T_1' \\ &= \{(c-1)T_1'\}c \\ &= (g\,\Delta z / c_p)T_2 / T_1 \\ &= (g\,\Delta z / c_p)T_2 / (T_2 - g\,\Delta z / c_p) \\ &= T_2 / \{c_p T_2 / (g\,\Delta z) - 1\} \\ (\Delta T_{23})_d / T_2 &= 1 / \{c_p T_2 / (g\,\Delta z) - 1\} \end{aligned}$$

The equation above gives the design value of collector temperature rise for any chosen chimney height.

Since $g = 9.81 \approx 10$ m/s^2 and $c_p = 1005 \approx 1000$ J/kgK, the design temperature rise in the collector in degrees K is about $1/100$ times the chimney height in metres. It is quite likely that the design temperature rise of about 15 K for a 1500 m high chimney will be exceeded in practice.

Table 6 Design value of collector temperature rise versus chimney height for $T_2 = 303.2$ K

Δz [m]	500	1000	1500	2000
$(\Delta T_{23})_d$ [K]	5.0	10.1	15.4	20.9

If the collector temperature rise is fixed, the design value of chimney height can be found by inverting the equation given above:

$$(\Delta z)_d = (c_p T_2 / g) / (1 + T_2 / \Delta T_{23})$$

Table 7 reflects the dependence of design chimney height on collector temperature rise.

Table 7 Design value of chimney height versus collector temperature rise for $T_2 = 303.2$ K

ΔT_{23} [K]	5	10	15	20
Δz_d [m]	504	992	1464	1922

The implications of the chosen design condition on the specific power will be derived below.

$$\begin{aligned} P_2* &= (t/c)(1 - 1/c) - (1 - 1/c) \\ &= \{(t/c) - 1\}(1 - 1/c) \\ P_2*_d &= (c_d{}^2 / c_d - 1)(1 - 1/c_d) \\ &= c_d(1 - 1/c_d{}^2) \\ &= \eta_d{}^2 / (1 - \eta_d) \\ &\approx \eta_d{}^2 \end{aligned}$$

The equation above draws the attention again to the extreme importance of cycle efficiency and chimney height in determining the plant power per unit mass flow.

The power per unit mass flow is then:

$$P/m = c_p T_2 \eta_d^2 / (1 - \eta_d)$$

and the mass flow per unit power is:

$$m/P = (1 - \eta_d) / (c_p T_2 \eta_d^2)$$

For a 250 MW plant and a turbine efficiency of 100 % the required mass flow is:

$$m_{250} = (250 \times 10^6)(1 - \eta_d) / (c_p T_2 \eta_d^2)$$

Example values are given in Table 8.

Table 8 Various design variables for $T_2 = 303.2$ K and $K = 2.0$ and $\eta_t = 0.80$

Δz [m]	500	1000	1500	2000
$(\Delta T_{23})_d$ [K]	5.0	10.1	15.4	20.9
η_d (%)	1.61	3.22	4.83	6.44
P/m [kW/(tonne/s)]	80.2	326	747	1350
m [tonne/s] / 200 MW	4673	1149	502	278
Chimney velocity [m/s]	5.2	10.4	15.8	21.2
Chimney diameter [m]	1060	375	203	131

Table 8 shows that a 250 MW plant with $T_4 = T_2$ would require a chimney diameter of about 200 m.

Conclusions

An air standard analysis has been done for a solar chimney cycle. The main conclusions are listed below. The conclusions should be read in the context of the assumptions listed at the beginning.

- The cycle efficiency is $\eta = g \Delta z / c_p T_2$. It is proportional to chimney height and inversely proportional to collector inlet temperature. Once the chimney is built, the cycle efficiency is virtually cast in concrete.

- There is no optimum combination of cycle pressure ratio and cycle temperature ratio for maximum specific power as in gas turbine cycles. It means that there is no thermodynamic optimum combination of collector size and chimney height.

- The power developed by the plant is $P = m g \Delta z \Delta T_{23} / T_2$. It is proportional to the mass flow, chimney height and collector temperature rise, and inversely proportional to collector inlet temperature.

- The available pressure drop across the turbine is $\Delta p_t = \rho_3 g \Delta z \Delta T_{23} / T_2$. It is proportional to the turbine inlet density, to chimney height and collector temperature rise and inversely proportional to collector inlet temperature.

- By assuming a value of a pressure loss coefficient, $K = 2$, and that the maximum power that can be extracted occurs when two-thirds of the pressure drop is across the turbine, the required chimney velocity can be derived as $V_c = \sqrt{(1/3)} \sqrt{(P/m)}$.

- An initial design condition where the chimney exit temperature equals the collector inlet temperature is proposed for initial calculations. These conditions enable one to do initial sizing of a plant for a given required power.

- For the proposed design cycle a heat exchanger between chimney exit and collector inlet would not improve specific power (apart from being impractical).

- For the design cycle the maximum power per unit mass flow is approximately proportional to cycle efficiency squared. This implies that although the input energy costs nothing, the cycle efficiency exerts a strong influence through its coupling to specific power and plant capital costs.

- The air standard cycle analysis seems worth while but should be extended with the incorporation of losses. A more realistic model may modify some of the conclusions based on the air standard cycle.

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