A new mathematical programming model for production schedule optimization in underground mining operations

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Synopsis

Mixed integer programming (MIP) has been used for optimizing production schedules of mines since the 1960s and is recognized as having significant potential for optimizing production scheduling problems for both surface and underground mining. The major problem in long-term production scheduling for underground orebodies generally relate to the large number of variables needed to formulate a MIP model, which makes it too complex to solve. As the number of variables in the model increase, solution times are known to increase at an exponential rate. In many instances the more extensive use of MIP models has been limited due to excessive solution times.

This paper reviews production schedule optimization studies for underground mining operations. It also presents a classical MIP model for optimized production scheduling of a sublevel stoping operation and proposes a new model formulation to significantly reduce solution times without altering results while maintaining all constraints. A case study is summarized investigating solution times as five stopes are added incrementally to an initial ten stope operation, working up to a fifty stope operation. It shows substantial improvement in the solution time required when using the new formulation technique. This increased efficiency in the solution time of the MIP model allows it to solve much larger underground mine scheduling problems within a reasonable time frame with the potential to substantially increase the net present value (NPV) of these projects. Finally, results from the two models are also compared to that of a manually generated schedule which show the clear advantages of mathematical programming in obtaining optimal solutions.

Keywords

Underground mine optimization, mixed integer programming, long-term scheduling, mathematical programming application.

Introduction

Mining companies face the challenge of scheduling production in their mines in a way that is economically optimal. The scheduling process should provide the company with profit maximization, a high level of equipment utilization, and high quality products in each time period according to demand requirements. Numerous authors have advanced the ability of solving large surface mine scheduling problems over the last fifty years, which has resulted in the development of a number of open pit optimization packages. The underground mine scheduling problem, however, has become more prominent of late and has thus received more attention. Research into this area will become more important as shallow deposits, which are amenable to open pit mining, are increasingly exploited and eventually diminished.

The lack of available software programs to aid the underground production scheduling process has meant that this is still largely carried out manually, often involving extensive and complex spreadsheets. This is no doubt a very time-consuming and tiresome process and whereas a feasible solution may be reached, there is very little chance of obtaining the optimal solution. As such, mathematical programming techniques, which form the basis for many open pit optimization and scheduling packages, can be investigated for simular applications into the underground mine environment.

Mixed integer programming (MIP) is well recognized within industry circles as being able to model and thus find the optimal solution to large, complex, and highly constrained problems. MIP is a combination of linear programming (LP) and integer programming (IP). An LP programming model consists of a linear objective function and a set of linear constraints, without loss of generality, of the following form:

$$\textit{Maximize or (Minimize) } Z = c_1x_1 + c_2x_2 + c_3x_3 + \ldots + c_nx_n$$

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subject to:

$$a_{11}x_1 + a_{12}x_2 + \ldots\ldots a_{1n}x_n \le b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots\ldots a_{2n}x_n \le b_2$$
$$\vdots \quad \vdots \quad \vdots \quad \vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots a_{mn}x_n \le b_m$$

and a set of non-negativity restrictions: $x_1, x_2, \ldots\ldots, x_n \ge 0$

In this model, Z represents the objective function value which could be maximization of profit or minimization of cost, x_j are decision variables, whose values determined by the model, a_{ij} and c_j are constants whose values are dictated by the nature of the problem and b_i is the right-hand side constant value. In certain problems, the decision variables must assume integer values. When this restriction is added to the system, the model is called an integer programming (IP) model. In a given model, if some variable values are allowed to be continuous while others must assume integer values; the resulting model is a mixed integer program (MIP). At present, employing classical MIP models for underground production scheduling requires significant computational effort, resulting in its true potential use being hindered due to excessive solution times. MIP model solution times depend mainly on the number of variables as well as the number of constraints in the model. The solution time often increases exponentially as the number of variables increase. The worst case scenario in finding the optimal solution involves examining each candidate solution. Therefore solving large problems may become exponentially more complex with increasing problem size. Fortunately in practice all nodes are usually not examined because some branches are fathomed early (Topal 2003).

Trout (1995) formulates and attempts to solve a mixed integer programming multi-period production scheduling model for a sublevel stoping copper ore operation located in Mt Isa, Australia. The objective is to schedule stopes for production over a two-year period at four weekly time intervals for the purpose of maximizing net present value. The data set containing 55 stopes within the 1100 orebody had to comply with numerous production, resource, and timing targets and constraints. Although the model produced a solution that improved NPV by 23% over what was realized in practice, the solution time was interrupted prior to proof of optimality at 209 hours with the reported integer solution being obtained after 1.6 hours. As such, the model was not implemented at the mine as it also lacked a number of additional important features.

Nehring (2006) continues on from Trout's (1995) model by formulating an additional constraint function to limit multiple fillmass exposures which is proposed with (Topal 2006) without violating other constraints. This made the model more applicable to the sublevel stoping method and was tested on a conceptual nine stope operation. A manual schedule was generated on the basis of selecting the next available highest cash flow stope while adhering to all constraints. This manual schedule was then compared to the model result, which showed the model increased NPV by 0.66%. The model was implemented and solved using Lindo Software's Excel add-on called What's Best. It contained 774 binary variables, 315 linear variables, 1917 constraints and took 4 h 45 mins 8 s to solve (Nehring and Topal, 2007).

Little (2007) continues on from Nehring's (2006) revised model by addressing various strategies to reduce the number of variables present in the model and thereby reduce solution time. The proposed theories by (Topal 2007) relating to natural sequence and natural commencement allow void and backfill activities to be defined as a function of the first extraction variable and both the extraction and backfill commencement variables to be eliminated. The new formulation was tested on the conceptual nine stope operation using the same software on a similar computer producing the same production schedule and NPV with an 80% and 92% reduction in the number of variables and solution time respectively (Little *et al.* 2008).

Topal (2003) uses MIP to optimize production schedules at a large scale sublevel caving iron ore operation with the objective of minimizing deviation from targeted production of each ore type. The implementation of machine placements, which were representative of LHD capacities, each contained a number of production blocks with an associated ore quantity of a certain quality for each period while in production. Scheduling machine placements as opposed to individual production blocks thus significantly increased the efficiency of the model. The extensive use of early and late start times further reduced the number of variables. The final model was implemented over 36 monthly periods and comprised 1 440 variables. Written using AMPL code, it was solved in CPLEX on a Sun Ultra 10 machine with 256 MB RAM in less than 100 seconds. Final results indicate an improved deviation from targeted production of 6% down from 10–20% compared to the manual schedule with no constraint violation. The model was ultimately implemented into Kiruna Mine's mainstream scheduling process (Topal, 2008).

Carlyle and Eaves (2001) use an integer programming model to plan a production schedule for a sub-level stoping operation at Stillwater Mining Company. The model provided near-optimal solutions, for a 10-quarter planning period, to maximize revenue from mining platinum and palladium. However, the authors did not describe any special techniques to expedite solution time or a description of the model in their publication.

Production planning of a deposit containing eleven poly-metallic zones with different geological characteristics and thus requiring different mining methods is carried out by McIssac (2005) using MIP with the objective of maximizing cash flow. Production planning took place over four years using three monthly periods with a minimum daily production capacity set at 500 tonnes. The program is built into Microsoft Excel and solved with Fontline's Xpress Solver. A total of 1 200 variables were required taking 30 minutes to solve. No mention is made if a feasible solution was reached that could guarantee optimality. It is not stated how many of these variables are integer variables and there is no mention of a previously generated manual production schedule with which to compare results.

This paper seeks to further build and develop the concepts of natural sequence and natural commencement as presented by Little *et al.* (2008) for the purpose of reducing the number of variables and thus solution time for generating and solving MIP production scheduling models specifically for sublevel stoping operations. As such this paper will implement these concepts on a much larger scale to a more realistic operating scenario while also extensively implementing predefined production data to again further reduce the number of variables. Furthermore, the proposed model also allows one to control the grade profile of the schedule more realistically by generating new monthly ore production and grade data formulations. This allows greater control over monthly ore grades being fed into the process plant, which would otherwise not have been possible. Two models are presented that highlight the difference that can be achieved by fully utilizing the natural sequence of production phases inherent in the sublevel stoping method. The first model, referred to as the 'classical model', is representative of past mathematical scheduling practices which contain a number of inefficiencies and thus require significantly longer solution times. The second model, referred to as the 'new model', seeks to take the concepts of natural sequence and natural commencement further. To appreciate the affect of this on the number of variables within the model and its impact on solution time, both models will be applied to an identical fifty-stope operation, starting with an initial ten stopes which is then incrementally increased by five stopes under the same parameters, constraints, and objective. Finally, results from the two models are also compared to that of a manually generated schedule, which shows the clear advantages of mathematical programming in obtaining optimal solutions.

General overview of sublevel stoping operation

The sublevel stoping method (also referred to as open stoping, longhole stoping or blasthole stoping) is one of a number of bulk underground mining methods. The main differentiating factor between sublevel stoping and the other methods, including sublevel caving, and block caving is that no caving takes places. For this reason voids that are created as a result of extraction are required to be backfilled. Once consolidated, this fillmass provides the support and confinement to continue mining surrounding stopes. As such, the method generally requires a competent ore and stable host rock needing minimal support. This technique is mostly suited to steeply dipping orebodies where the dip of the footwall exceeds the angle of repose of the broken ore to allow it to freely gravitate to the base of the orebody for collection at the drawpoints (Lawrence, 1998). Figure 1 shows a general layout for the method and what a typical stope within that layout would look like.

The typical mining cycle for each stope follows a number of sequential production phases, as illustrated in Figure 2. Once the main access has been developed to an orebody or a cluster of stopes, an individual stope will generally enter into production beginning with the internal development phase. Internal development for each stope may include excavation of cross-cuts and cut-offs on each of its sublevels including the extraction level. This will be followed by the production drilling phase where the entire stope is drilled out in preparation for the loading and blasting of explosives to break the ore material. The production phase will then begin with initial firing of the drawpoints as well as the winze across all sublevels. The winze is used to create an initial void into which cut-off material is able to then be fired into. Once this material is drawn out by LHD from the stope drawpoints and larger voids become available, each subsequent production blast is able to become larger, breaking up more and more ore material. As a result ore production from each stope will generally follow a common profile which can be used in predefining production over subsequent months once production is initiated. Once all ore contained within the stope has been extracted, all access drives into the now empty stope are sealed before filling of the stope begins. After filling is completed the new fillmass will generally require some period of time to fully dry out and consolidate.

There are three main scheduling constraints inherent in sublevel stoping, which mainly relate to geotechnical conditions. First, it is crucial once production starts on a stope to progress production through all phases quiete rapidly. The reason for this is not to leave the stope open for too long and thus limiting its exposure to intense geotechnical stresses. This decreases the risk of failure, which could potentially propagate through to surrounding stopes and infrastructure. Secondly, all forms of simultaneous adjacent stope production must be avoided in order to prevent excessively large unsupported voids. Thirdly, once a stope moves through the production phase and onto backfilling to ultimately become a fillmass, remaining adjacent stope production must be scheduled so as to prevent simultaneous multiple exposure of the fillmass. This is because the fillmass is generally weak and unable to transfer stresses. For this reason, once production ceases on a stope, it does not simply leave the data-set due to these ongoing interactions which may remain many years after completion of the initial stope. It should also be noted that the objective function for both models seek to maximize net present value (NPV). All activities in this case being scheduled have an associated cash flow. The NPV calculation determined in the optimization process therefore considers only those activities placed in the model for evaluation.

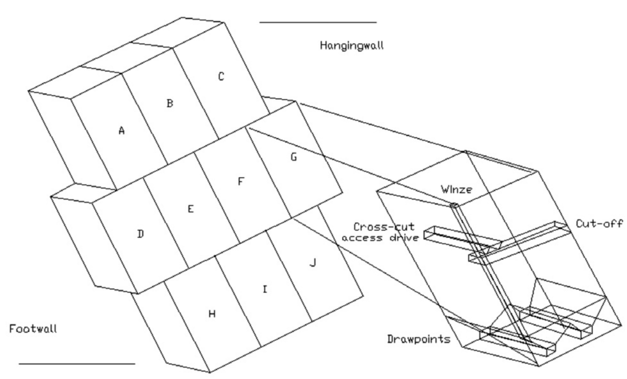

Figure 1—General stope layout for sublevel stoping method

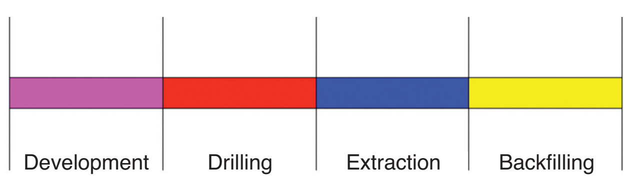

Figure 2—Typical production phases for a stope

It should also be noted that the evaluation of constraint costs or shadow prices is not the focus of either the classical or the proposed scheduling model. However, an NPV comparison between scenarios is possible by rerunning the model after each constraint adjustment.

Classical MIP model formulation

The classical model assigns a binary decision variable to signify the beginning of each phase of stope production including development, drilling, production and backfilling for each stope. Once commencement is initiated, predefined data relating to development metres, production, drilling metres, ore tonnage extracted and the backfill tonnage placed for each subsequent period thereafter is recognized and implemented. The NPV calculation determined in the optimization process considers only those activities presented to the model for evaluation. Any other activities that affect revenues in any other way need to be considered outside the model if these are not specified and presented for evaluation. No consideration has yet been given to taxation or depreciation. All subscript notation, sets, parameters, and decision variables used in the construction of the classical MIP model with descriptions are presented in Appendix A.

Proposed new MIP model formulation

In recognizing that each phase of stope production inherently follows the next without any significant time delay, this model takes the concepts of natural sequence and natural commencement further by assigning just a single binary decision variable to signify the start of development which is naturally followed by production drilling, extraction and backfilling. Predefined production data then ensure that subsequent drilling, production, and backfilling activity for each stope are recognized and implemented. It is evident that representing all four stope production phases with a single variable, as opposed to defining a separate variable for each phase, creates significant efficiencies. All subscript notation, sets, parameters, and decision variables used in the construction of the proposed new model MIP model with descriptions are presented in Appendix B.

Implementation of proposed model on a fifty-stope operation

For the purpose of comparison, implementation of both the classical and the newly developed model will take place on a small conceptual underground sublevel stoping operation. Whereas conceptual in nature, production data, parameters, and constraints are reflective of other underground operations of a similar scale and thus justify its use for testing and validating purposes.

The operation itself utilizes the conventional sublevel stoping method (as discussed and illustrated earlier) to extract copper ore from one north-south striking lens dipping at 75 to 80 degrees, reaching depths of 800 metres below surface. Current remaining ore reserves total 7.6 Mt grading 2.8% Cu for 0.2 Mt Cu, which will be exhausted at a rate of 50 000 t/month or 0.6 Mt/annum ore over the operation's remaining 12-year mine life. Once extracted from stope draw-points via LHD, all ore is channelled directly to an underground crusher station. All production ore to the crusher is handled by LHD unit with no trucking required and no rehandling required. Once crushed, ore is then hoisted to the surface via the haulage shaft. A plan view of the operation showing all stopes is provided in Figure 3.

As shown, the 50-stope data-set comprises eight stopes (indicated in green) having already completed all phases of production to become a fully consolidated fillmass. Production drilling (indicated by red) is currently taking place in one stope with another two in the extraction phase (indicated by blue), and another stope currently completing the backfill process (indicated by yellow). This leaves 38 stopes available for production. The ore tonnage and grade for these stopes as well as those currently in production are presented in Table I.

In keeping with the natural sequence of activities required to bring each stope into production, the expected length of internal development, production drilling, ore extraction tonnages, and backfill requirements, together with the anticipated time frames to complete each of these activities have been entirely predefined. Table II shows an example of the production profiles of ore tonnage and copper grade expected to be extracted from each of the first ten stopes over each month once their extraction phase is initiated.

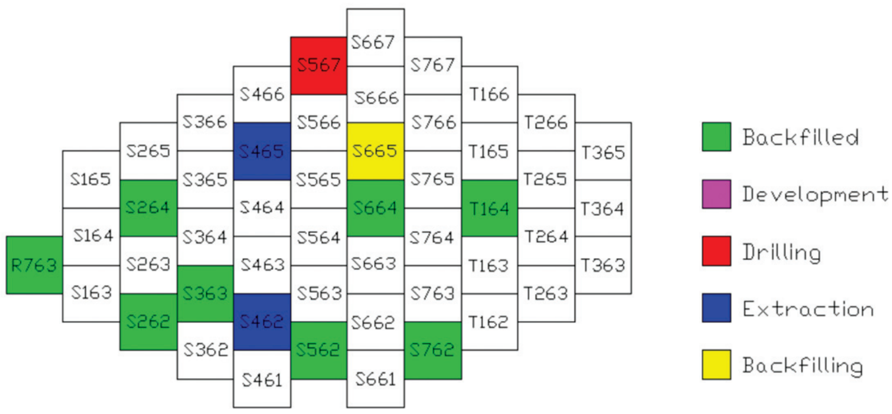

Figure 3—Plan view of a 50-stope operation

Table I

Ore tonnage and grade for fifty-stopes example

Stope	Tonnes (t)	Grade (% Cu)	Stope	Tonnes (t)	Grade (% Cu)	Stope	Tonnes (t)	Grade (% Cu)
R763			S465	19 000	2.9	S764	191 000	2.6
S163	187 000	2.8	S466	191 000	2.8	S765	179 000	2.8
S164	176 000	3.0	S562			S766	185 000	2.7
S165	218 000	2.7	S563	200 000	2.6	S767	182 000	2.9
S262			S564	217 000	2.7	T162	192 000	2.8
S263	201 000	2.9	S565	180 000	3.0	T163	175 000	2.9
S264			S566	179 000	2.9	T164		
S265	178 000	3.2	S567	211 000	2.8	T165	178 000	2.8
S362	189 000	2.6	S661	198 000	2.7	T166	197 000	3.0
S363			S662	185 000	2.8	T263	188 000	2.9
S364	183 000	2.8	S663	182 000	2.7	T264	209 000	2.8
S365	185 000	2.7	S664			T265	188 000	2.6
S366	206 000	2.6	S665			T266	196 000	2.8
S461	196 000	2.9	S666	184 000	2.8	T363	173 000	3.0
S462	148 000	2.8	S667	179 000	2.8	T364	204 000	2.7
S463	186 000	2.7	S762			T365	186 000	2.8
S464	174 000	3.0	S763	187 000	2.9			

Table II

Expected monthly ore/grade production rates for first ten stopes

Stope	Month 1		Month 2		Month 3		Month 4		Month 5		Month 6	
	Tonnes	Grade	Tonnes	Grade	Tonnes	Grade	Tonnes	Grade	Tonnes	Grade	Tonnes	Grade
R763												
S163	21 000	2.8	38 000	2.9	43 000	2.7	49 000	2.7	36 000	2.9		
S164	19 000	3.2	36 000	3.0	44 000	3.1	39 000	2.9	38 000	3.0		
S165	17 000	2.5	37 000	2.6	44 000	2.8	48 000	2.7	45 000	2.6	27 000	2.7
S262												
S263	15 000	2.7	26 000	3.0	40 000	2.7	47 000	2.9	42 000	3.1	31 000	2.9
S264												
S265	22 000	3.3	34 000	3.0	38 000	3.3	45 000	3.1	39 000	3.2		
S362	20 000	2.8	35 000	2.9	48 000	2.2	47 000	2.6	39 000	2.5		
S363												

A full breakdown of data for the extraction profiles of the remaining stopes as well as the advancement metres, drilling metres, and backfill placement profiles for the internal development, production drilling, and backfilling phases of each stope can be provided upon request. It should be noted that internal development in this case refers to all development specific to bringing a particular stope into production. It therefore does not include development activities that are to the benefit of more than one stope. For the purposes of this demonstration it will be assumed that these development activities have been completed. All development activities are given as a length in metres and are stated as a standard equivalent primary horizontal development length. For example, where a length of rehabilitation is required, this is converted into an equivalent primary horizontal length. Similarly, production drilling lengths are also stated as a standard equivalent hole diameter length. It is recognized that whereas internal development activities would result in the production of some ore, it will be assumed that no ore is produced from internal development activities. All periods are quoted in months. Planning engineers for this 50-stope operation also endeavour to meet mill feed head grade requirements of 2.8% Cu at a deviation of ±15%.

Fleet capacities place an important constraint on an operation that must also be considered. Jumbo and bolting rig fleets at the 50-stope operation limit equivalent primary horizontal development activities to 100 metres per month. Similarly, the production drill rig fleet restricts monthly production drilling activities to 20 000 metres of standard hole diameter. Backfill availability for each month is 90 000t. Monthly primary ore production in this case is limited by the haulage shaft, which has a monthly capacity of 60 000t. After taking into consideration the bucket capacity of each LHD unit as well as all tramming distances, this capacity is expected to be comfortably met by the mine's LHD fleet. All sequencing restrictions relating to adjacency and single backfill exposure limits inherent in sublevel stope mining, as discussed previously, also need to be rigorously applied.

All fixed and variable development, drilling, production, and backfilling costs associated with each stope have also been predefined. Variable costs, which are normally quoted in $/unit, are therefore incurred throughout the operation of each activity. Fixed costs, however, are incurred at the start of each phase as a one-off cost. Using an average copper price of $4 000/t and an average recovery factor of 90%, revenues for each stope are calculated. Undiscounted values for each stope are then found by subtracting all costs from the revenue. For the purposes of this demonstration, each stope is made available for the start of the internal development phase from the very first period. To ensure compliance with the natural progression of activities inherent sublevel stoping, all stopes currently in production must remain in production through to completion.

For scheduling purposes, a 10% per annum discount rate is applied. This figure is reflective of the rate currently used by smaller mining companies seeking to start up new operations. A change in economic circumstances resulting in a change in the discount rate would warrant a rerun of the model as this is likely to affect the optimal schedule and resulting NPV.

Implementation of the MIP model

The construction of both the classical and new mixed integer programming (MIP) scheduling models took place with all parameters and constraints implemented. Scheduling was carried out over 48 monthly periods. Both MIP scheduling models were written using a mathematical programming language (AMPL) code and solved using the solver package CPLEX 10.3 (ILOG™) on the same computer. A comparison of characteristics and solution times is contained in Table III for each scheduling model. This table details the implementation of the initial ten stopes through to the final fifty stopes at each five stope interval. Stopes were added according to their alphabetical and then numerical order.

It should be noted that from 30 stopes onwards both models were deemed to have solved the problem once a 5% gap solution was reached. This was to avoid excessive run times as these results can be regarded as optimal. As shown in Table III, solution times for the first 10–15 stopes are similar for both models. These times start diverging when 20 stopes are considered, which records a solution time of 104 and 65 seconds for the classical model and the new model respectively to produce the same objective value. As more stopes are added, the solution time for the classical model rises rapidly while the new model experiences moderate rises in comparison. By the full 50-stope implementation a solution time of 231 063 and 8 416 seconds are recorded for the classical and new model respectively to produce the same objective value. Due to a scheduling horizon of over 48 monthly periods scheduling results are unable to be effectively presented; however, they can be provided on request.

For comparative purposes a manual schedule was carried out on the full fifty-stope operation under the same operational and sequencing conditions. The manual approach initially selects the next available highest cash flow stope. The basis for this process involved ranking each stope in order of highest undiscounted cash flow. As would normally occur at the mine site this process was then expanded to consider alternative scheduling configurations, which did not necessarily aim to produce from the highest cash flow stope as soon as possible in the view that this may present opportunities to mine other high value stopes, which would otherwise have been deemed unavailable, while the highest cash flow stope was in production. The results of the manual schedule in comparison to both mathematical programming schedules are presented in Table IV.

As shown the resulting manually generated schedule returned an NPV of $115.8 m. Both mathematical programming models produced a superior result, increasing NPV by $6.3 m to $122.1 m, thus representing a 5.44% increase over the manual result. Furthermore, an equivalent of a full working day was spent undertaking the manual evaluation process.

Discussion and conclusion

It is evident that solution times for the mathematical models are directly linked to the number of variables associated with each model. By taking the approach of fully utilizing the concepts of natural commencement and natural sequence and thus defining production by a single variable, the new model consistently achieved a significant reduction in the total number of variables over the classical model. Over a full 50-stope data-set this significant reduction in variables translated into a significant reduction in solution time from 231 063 seconds for the classical model to just 8 416 seconds for the new model.

A significant reduction in model complexity was also achieved by using these concepts. Combining phases that naturally carried on from one another translated into a reduction in the number of variables, which in turn also translated into a reduction in the number of constraints that were required to reflect the mining process. The replacement of the four separate phase variables with a single variable also eliminated the need for vast amounts of precedence information that had to be put to the model in order to keep

Table III

Mathematical model comparisons for both models

	Classical model	New model
10 Stopes	Variables: 1 217 Constraints: 65 854 Objective ($m): 31.8 Solution time (s): 3	Variables: 288 Constraints: 44 225 Objective ($m): 31.8 Solution Ttime (s): 2
15 Stopes	Variables: 1 981 Constraints: 93 786 Objective ($m): 57.2 Solution time (s): 9	Variables: 456 Constraints: 67 072 Objective ($m): 57.2 Solution time (s): 10
20 Stopes	Variables: 2 230 Constraints: 107 721 Objective ($m): 70.1 Solution time (s): 104	Variables: 572 Constraints: 88 210 Objective ($m): 70.1 Solution time (s): 65
25 Stopes	Variables: 3 946 Constraints: 131 103 Objective ($m): 85.9 Solution time (s): 411	Variables: 711 Constraints: 116 489 Objective ($m): 85.9 Solution time (s): 268
30 Stopes	Variables: 5 072 Constraints: 167 306 Objective ($m): 91.8 Solution time (s): 1 474	Variables: 836 Constraints: 137 848 Objective ($m): 91.8 Solution time (s): 426
35 Stopes	Variables: 5 848 Constraints: 248 838 Objective time (s): 3 318	Variables: 994 Constraints: 164 525 Solution time (s): 665
40 Stopes	Variables: 7 084 Constraints: 385 481 Objective ($m): 108.2 Solution time (s): 18 749	Variables: 1 220 Constraints: 207 689 Objective ($m): 108.2 Solution time (s): 644
45 Stopes	Variables: 8 362 Constraints: 527 059 Objective ($m): 115.5 Solution time (s): 134 854	Variables: 1,402 Constraints: 237,610 Objective ($m): 115.5 Solution time (s): 1,617
50 Stopes	Variables: 9 749 Constraints: 749 523 Objective ($m): 122.1 Solution time (s): 231,063	Variables: 1 627 Constraints: 275 084 Objective ($m): 122.1 Solution time (s): 8 416a

Table IV

Fifty stope scheduling process comparison

Classical model	New model	Manual process
Objective ($m): 122.1 Solution time (s): 231 063	Objective ($m): 122.1 Solution time (s): 8 416	Objective ($m): 115.8 Solution time (s): 28 800

production progressing according to the appropriate sequence. This in turn also significantly reduced the number of constraints required by the new model.

Any further addition of stopes to the data-set would again increase solution times for both models; however, the new model would be much better placed to handle this due to its already faster solution time for 50 stopes of 8 416 seconds compared to 231 063 seconds for the classical model. This new model not only allows schedulers to obtain optimal solutions rapidly from stope data-sets such as the one used in this case study, which would have previously been virtually unachievable, it also now allows larger stope data-sets to be incorporated into the scheduling process.

Results obtained by the manual scheduling process of $115.8 m fell well short of the optimal solution of $122.1 m. This again supports the fact that even if feasible results are able to be obtained from a manually generated schedule, there is no guarantee that these results are anywhere near optimal. From a manual perspective, the further addition of stopes to the scheduling process would have increased the complexity of the problem, adding to the amount of time required to obtain an inevitable sub-optimal result. Only well formulated mathematical programming models with the additional feature of utilizing the natural sequence of activities to reduce the overall number of variables, and in turn solution time, can provide optimal solutions to complex underground scheduling problems within a reasonable and practical time frame.

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Appendix A

Classical MIP model formulation sublevel stopping operation

The formulation follows:

Subscript notation

The model is defined in general terms using the following subscript notation.

t	schedule period: $t = 1, 2, 3.... T$.
s	internal stope development activity identification: $s = 1, 2, 3.... S$.
d	stope production drilling activity identification: $d = 1, 2, 3.... D$.
e	stope extraction activity identification: $e = 1, 2, 3.... E$
f	stope backfilling activity identification: $f = 1, 2, 3.... F$
m	metal type: $m = a, b, c.... M$.
b	backfill type: $b = a, b, c.... B$.

Sets

$adjdd_s$	Set of all internal stope development activities that are adjacent to and share a boundary with internal stope development activity s.
$adjddr_s$	Set of all stope production drilling activities that are adjacent to and share a boundary with internal stope development activity s.
$adjde_s$	Set of all stope extraction activities that are adjacent to and share a boundary with internal stope development activity s.
$adjdf_s$	Set of all stope backfilling activities that are adjacent to and share a boundary with internal stope development activity s.
$adjdrd_d$	Set of all internal stope development activities that are adjacent to and share a boundary with stope production drilling activity d.
$adjdrdr_d$	Set of all stope production drilling activities that are adjacent to and share a boundary with stope production drilling activity d.
$adjdre_d$	Set of all stope extraction activities that are adjacent to and share a boundary with stope production drilling activity d.

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$adjdrf_d$	Set of all stope backfilling activities that are adjacent to and share a boundary with stope production drilling activity d.
$adjed_e$	Set of all internal stope development activities that are adjacent to and share a boundary with stope extraction activity e.
$adjedr_e$	Set of all stope production drilling activities that are adjacent to and share a boundary with stope extraction activity e.
$adjee_e$	Set of all stope extraction activities that are adjacent to and share a boundary with stope extraction activity e.
$adjef_e$	Set of all stope backfilling activities that are adjacent to and share a boundary with stope extraction activity e.
$adjfd_f$	Set of all internal stope development activities that are adjacent to and share a boundary with stope backfilling activity f.
$adjfdr_f$	Set of all stope production drilling activities that are adjacent to and share a boundary with stope backfilling activity f.
$adjfe_f$	Set of all stope extraction activities that are adjacent to and share a boundary with stope backfilling activity f.
$adjff_f$	Set of all stope backfilling activities that are adjacent to and share a boundary with stope backfilling activity f.
pd_f	Pair (2) of internal stope development activities that are adjacent to and share a boundary with fillmass f.
pdr_f	Pair (2) of stope production drilling activities that are adjacent to and share a boundary with fillmass f.
pe_f	Pair (2) of stope extraction activities that are adjacent to and share a boundary with fillmass f.
pf_f	Pair (2) of stope backfilling activities that are adjacent to and share a boundary with fillmass f.
bs	Set of all internal stope development activities that are adjacent to and share a boundary with each existing fillmass.
bd	Set of all stope production drilling activities that are adjacent to and share a boundary with each existing fillmass.
be	Set of all stope extraction activities that are adjacent to and share a boundary with each existing fillmass.
bf	Set of all stope backfilling activities that are adjacent to and share a boundary with each existing fillmass.
tpb_t	Set of periods that include all periods up to the current period t.

Parameters

r_e	Extraction reserve for each stope extraction activity e.
sc_t	Shaft/LHD/truck fleet capacity for each period t.
$devearly_s$	Earliest start time for internal stope development activity s.
$devlate_s$	Latest start time for internal stope development activity s.
$drearly_d$	Earliest start time for stope production drilling activity d.
$drlate_d$	Latest start time for stope production drilling activity d.
$extearly_e$	Earliest start time for stope extraction activity e.
$extlate_e$	Latest start time for stope extraction activity e.
$fillearly_f$	Earliest start time for stope backfilling activity f.
$filllate_f$	Latest start time for stope backfilling activity f.
n_t	Present value discount factor applied to period t.
dev_s	The undiscounted cash flow generated by each internal stope development activity s.
$drill_d$	The undiscounted cash flow generated by each stope production drilling activity d.
ext_e	The undiscounted cash flow generated by each stope extraction activity e.
$fill_f$	The undiscounted cash flow generated by each stope backfilling activity f.
gu_{me}	Difference between targeted upper ore feed head grade and stope grade for each mineral type m in each stope extraction activity e.
glm_e	Difference between targeted lower ore feed head grade and stope grade for each mineral type m in each stope extraction activity e.
d_s	Equivalent primary horizontal development length for each internal stope development activity s.
dc_t	Total equivalent primary horizontal development fleet capacity for each period t.
dr_d	Equivalent production drill length for each stope production drilling activity d.
drc_d	Total equivalent production drill fleet capacity for each period t.
bb_f	Backfill requirement of type b for each stope backfilling activity f.
ba_{bf}	Total backfill availability of type b for each period t.

Decision variables

Four decision variables are used to define and quantify all phases of stope production.

w_{st}	1	if development of internal stope development activity s starts in period t,
	0	otherwise.
x_{dt}	1	if drilling of stope production drilling activity d starts in period t,
	0	otherwise.
y_{et}	1	if production of stope extraction activity e starts in period t,
	0	otherwise.
z_{ft}	1	if filling of stope backfilling activity f starts in period t,
	0	otherwise.

Objective function

Maximize:

$$\sum_{e,t} n_t \times ext_e \times y_{et} - \sum_{s,t} n_t \times dev_s \times w_{st} - \sum_{d,t} n_t \times drill_d \times x_{dt} - \sum_{f,t} n_t \times fill_f \times z_{ft}$$

subject to:

$$\sum_t w_{st} \le 1 \quad \forall s \mid devlate_s > T$$
$$\sum_t x_{dt} \le 1 \quad \forall d \mid drlate_d > T$$
$$\sum_t y_{et} \le 1 \quad \forall e \mid extlate_e > T$$
$$\sum_t z_{ft} \le 1 \quad \forall f \mid filllate_f > T \qquad [1a]$$

$$\sum_t w_{st} = 1 \quad \forall s \mid devlate_s \le T$$
$$\sum_t x_{dt} = 1 \quad \forall d \mid drlate_d \le T$$
$$\sum_t y_{et} = 1 \quad \forall e \mid extlate_e \le T$$
$$\sum_t z_{ft} = 1 \quad \forall f \mid filllate_f \le T \qquad [2a]$$

$$w_{st} = 1 \quad \forall s \mid devearly_s = devlate_s$$
$$x_{dt} = 1 \quad \forall d \mid drearly_d = drlate_d$$
$$y_{et} = 1 \quad \forall e \mid extearly_e = extearly_e$$
$$z_{ft} = 1 \quad \forall f \mid fillearly_f = filllate_f \qquad [3a]$$

$$\sum_e r_e \times y_{et} \le sc_t \quad \forall t \qquad [4a]$$

$$\sum_e gl_{me} \times r_e \times y_{et} \ge 0 \quad \forall m,t \qquad [5a]$$

$$\sum_e gu_{me} \times r_e \times y_{et} \le 0 \quad \forall m,t \qquad [6a]$$

$$\sum_s d_s \times w_{st} \le dc_t \quad \forall t \qquad [7a]$$

$$\sum_d dr_d \times x_{dt} \le drc_t \quad \forall t \qquad [8a]$$

$$\sum_f b_{bf} \times z_{ft} \le ba_{bt} \quad \forall b,t \qquad [9a]$$

$$w_{st} + w_{s't} \le 1 \quad \forall s,t \mid s' \in adjdd_s$$
$$w_{st} + x_{d't} \le 1 \quad \forall s,t \mid d' \in adjddr_s$$
$$w_{st} + y_{e't} \le 1 \quad \forall s,t \mid e' \in adjde_s$$
$$w_{st} + w_{f't} \le 1 \quad \forall s,t \mid f' \in adjdf_s$$
$$x_{dt} + w_{s't} \le 1 \quad \forall d,t \mid s' \in adjdrd_d$$
$$x_{dt} + x_{d't} \le 1 \quad \forall d,t \mid d' \in adjdrdr_d$$
$$x_{dt} + y_{e't} \le 1 \quad \forall d,t \mid e' \in adjdre_d$$
$$x_{dt} + z_{f't} \le 1 \quad \forall d,t \mid f' \in adjdrf_d \qquad [10a]$$
$$y_{et} + w_{s't} \le 1 \quad \forall e,t \mid s' \in adjed_s$$
$$y_{et} + x_{d't} \le 1 \quad \forall e,t \mid d' \in adjedr_s$$
$$y_{et} + y_{e't} \le 1 \quad \forall e,t \mid e' \in adjee_s$$
$$y_{et} + z_{f't} \le 1 \quad \forall e,t \mid f' \in adjef_s$$
$$z_{ft} + w_{s't} \le 1 \quad \forall f,t \mid s' \in adjfd_f$$
$$z_{ft} + x_{d't} \le 1 \quad \forall f,t \mid d' \in adjfdr_f$$
$$z_{ft} + y_{e't} \le 1 \quad \forall f,t \mid e' \in adjfe_f$$
$$z_{ft} + z_{f't} \le 1 \quad \forall f,t \mid f' \in adjff_f$$

$$\sum_{t' \in tpb_t} z_{ft'} + w_{s't} + x_{d't} + y_{e't} + z_{f't} \le 2$$
$$\forall f,t \mid s' \in pd_f \mid d' \in pdr_f \mid e' \in pe_f \mid f' \in pf_f \qquad [11a]$$

$$\sum_{s' \in bs} w_{s't} + \sum_{d' \in bd} x_{d't} + \sum_{e' \in be} y_{e't} + \sum_{f' \in bf} z_{f't} \le 1 \quad \forall t \qquad [12a]$$

$$w_{st'} x_{dt'} y_{et'} z_{ft} = binary\ integer \qquad [13a]$$

$$w_{st} + x_{dt} + y_{et} + z_{ft} \le 1 \quad \forall t \qquad [14a]$$

$$\sum_{t' \in tpb_t} w_{st'} - x_{dt} \ge 0 \quad \forall t$$
$$\sum_{t' \in tpb_t} x_{dt'} - y_{et} \ge 0 \quad \forall t$$
$$\sum_{t' \in tpb_t} y_{et'} - z_{ft} \ge 0 \quad \forall t \qquad [15a]$$

The objective function of this model seeks to maximize the cash flow of all activities under consideration (when taking the time value of money into account) by determining the optimal schedule within which to carry out all activities required to progress each stope through all four phases of production. Constraint [1a] ensures that commencement of all four stope phases is initiated no more than once during the time horizon if their late start date occurs beyond the maximum time horizon. Constraint [2a] requires all four stope phases to begin at some point during the time horizon if their late start date falls within the time horizon. Constraint [3a] places the stope phases that are currently in progress into the current production schedule, ensuring continuation from previous schedules. Constraint [4a] limits ore production for all stope extraction activities in any period from exceeding the shaft/LHD/truck fleet capacity. Constraint [5a] restricts the combined mill feed ore grade from all stope extraction activities in any period from exceeding a lower grade limit. Constraint [6a] restricts the combined mill feed ore grade from all stope extraction activities in any period from exceeding an upper grade limit. Constraint [7a] limits the amount of equivalent horizontal primary development taking place from all internal stope development activities in any time period from exceeding the combined development fleet capacity. Constraint [8a] ensures that the equivalent length of production drilling taking place from all stope production drilling activities in any period cannot exceed the combined production drill rig fleet capacity. Constraint [9a] ensures that the amount of each backfill type placed across all stope backfilling activities in any period cannot exceed the supply. Constraint [10a] enforces that simultaneous production between all stope phases that share a boundary does not occur. Constraint [11a] enforces ongoing fillmass stability of all stopes by limiting simultaneous exposure to just a single side. Constraint [12a] ensures ongoing fillmass stability of all existing fillmasses by limiting simultaneous exposure to just a single side. Constraint [13a] ensures variables maintain integer values. Constraint [14a] ensures that simultaneous progression of activities within each individual each stope is avoided. Constraint [15a] enforces the sequential progression of activities inherent with moving each stope through production.

Appendix B

Proposed new MIP model formulation for sublevel stoping operation

The formulation follows:

Subscript notation

The model is defined in general terms using the following subscript notation.

t	Schedule period: $t = 1, 2, 3.... T$.
s	Stope identification: $s = 1, 2, 3.... S$.
m	Metal type: $m = a, b, c.... M$.
b	Backfill type: $b = a, b, c.... B$.

Sets

adj_s	Set of all stopes that are adjacent to and share a boundary with stope s.
p_s	Pair (2) of stopes that are adjacent to and share a boundary with stope s.
$badj$	Set of all stopes that are adjacent to and share a boundary with each existing fillmass.
tpb_t	Set of time periods that include all periods up to the current period t.

Parameters

r_s	Extraction reserve for each stope s.
sc_t	Shaft/LHD/truck fleet movement capacity for each period t.
$early_s$	Earliest start time for stope s.
$late_s$	Latest start time for stope s.
n_t	Present value discount factor applied to period t.
cf_s	The undiscounted cash flow generated by each stope s.
gu_{ms}	Difference between targeted upper ore feed head grade and stope grade for each mineral type m in each stope s.
gl_{ms}	Difference between targeted lower ore feed head grade and stope grade for each mineral type m in each stope s.
d_s	Equivalent primary horizontal development length for each stope s.
dc_t	Total equivalent primary horizontal development fleet capacity for each period t.
dr_s	Equivalent production drill length for each stope s.
drc_t	Total standard production drill fleet capacity for each period t.
b_{bs}	Backfill requirement of type b for each stope s.
ba_{bt}	Total backfill availability of type b for each period t.

Decision variables

w_{st} 1 if development from activity d is scheduled for period t,
0 otherwise.

Objective function

$$\text{Maximize:} \sum_{s,t} n_t \times cf_s \times w_{st}$$

Subject to

$$\sum_t w_{st} \le 1 \qquad \forall s \mid late_s > T \qquad [1b]$$

$$\sum_t w_{st} = 1 \qquad \forall s \mid late_s \le T \qquad [2b]$$

$$w_{st} = 1 \qquad \forall s \mid early_s = late_s \qquad [3b]$$

$$\sum_s r_s \times w_{st} \le sc_t \qquad \forall t \qquad [4b]$$

$$\sum_s gl_{ms} \times r_s \times w_{st} \ge 0 \qquad \forall m,t \qquad [5b]$$

$$\sum_s gu_{ms} \times r_s \times w_{st} \le 0 \qquad \forall m,t \qquad [6b]$$

$$\sum_s d_s \times w_{st} \le dc_t \qquad \forall t \qquad [7b]$$

$$\sum_s dr_s \times w_{st} \le drc_t \qquad \forall t \qquad [8b]$$

$$\sum_s b_{bs} \times w_{st} \le ba_{bt} \qquad \forall b,t \qquad [9b]$$

$$w_{st} + w_{s't} \le 1 \qquad \forall s,t \mid s' \in adj_s \qquad [10b]$$

$$\sum_{t' \in tpb_t} w_{st'} + w_{s't} \le 2 \qquad \forall s,t \mid s' \in p_s \qquad [11b]$$

$$\sum_{s' \in badj} w_{s't} \le 1 \qquad \forall t \qquad [12b]$$

$$w_{st} = binary\ integer \qquad [13b]$$

Once again the objective function seeks to maximize cash flow of all activities under consideration (when taking the time value of money into account) by determining the optimal schedule within which to carry out these activities. The description of all constraints for the new model (Constraints [1b]–[13b]) is reflective of those described in the classical model ([1a]–[13a]).

Constraints for each stope in this case are able to be implemented with a single function, thus avoiding repetition where the variables representing all four phases in the classical model often required four separate functions to achieve the same outcome. Constraints [14a] and [15a] in the classical model, which deal solely with the sequencing of the four phases comprising each stope, means that these are no longer required. ◆