

# A comparison between the duplicate series analysis method and the heterogeneity test as methods for calculating the sampling constants, $K$ and alpha

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## Synopsis

The compilation of the sampling nomogram is an essential tool in the optimization of a sampling protocol and allows the operator to track the change in relative variance of the sampling error (the Fundamental sampling error) during steps of comminution and mass reduction in the process of recovery of the aliquot for analysis. The data required for the compilation of the nomogram are centred on the sampling constant  $K$  and the exponent, alpha ( $\alpha$ ) of the nominal fragment size. Five methods for establishing  $K$  have been identified, but two of them, the duplicate sampling analysis method and the heterogeneity test method purport to provide such constant. The step by step procedure for each of these two methods was examined using broken ores from three mines, Mponeng mine, Kloof mine, and Lily mine. Two of these mines (Mponeng and Kloof) are typical Witwatersrand-type gold mines, and the third is an Archaean-type shear related gold deposit situated in the Barberton Greenstone belt. The results suggest that the factor  $K$  derived using the heterogeneity test is very accurate, but applies only to a single size fraction in the spectrum of comminuted materials. The calculation of gold grain sizes using this method appear to be too small and do not conform with the results from detailed mineralogical studies. The factors  $K$  and alpha derived using the duplicate series analysis are appropriate for use across a wide spectrum of comminution sizes and also provide realistic gold grain sizes, comparable with the equivalent circular diameter of gold grains identified in mineralogical studies of the ores.

problem is worthy of consideration because the constants are applied in an identical manner when it comes to their use in compiling sampling nomograms and protocols. The FSE is also the absolute minimum error that manifests principally because of the constitution heterogeneity arising from the 'within fragment' variability in grade, shape, volume and density. The sampling constant  $C$  is the product of the shape factor  $f$ , the granulometry factor  $g$ , the mineralogical constant  $c$ , and the liberation factor  $\ell$ . The challenge is to produce an accurate estimation of the sampling constant  $K$ , (a simple derivative of  $C$ ), to be used in the compilation of the sampling nomogram and protocol. The protocol is used as a basis for calculating cut-off grades and ultimately determines what material is sent to the ore stockpile or the waste dump. The establishment of such protocols on an experimental basis is not common in the South African mining industry and the benefits of undertaking the somewhat lengthy and costly procedures have yet to be fully realized.

Currently there are five different methods that have been suggested for estimating values for  $K$ . The first three methods of estimation arise from the work of Lyman<sup>2</sup> who reports three different, but practical ways for calculating the sampling constant  $K$ ; these are mentioned here for the sake of completeness. In each case a detailed deportment study of ore mineralogy is essential. The first method employs the coarse fraction in the flow stream, i.e. that portion greater than 95% passing on any screen size in a mill feed. Approximately 100 of the largest fragments are collected from a composite sample, weighed, and analysed for

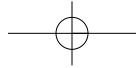
## Introduction

The sampling nomogram, a graphic which illustrates how the fundamental sampling error varies with changes in sample mass and degree of comminution, is an important guide on how to establish an optimal sampling protocol for broken ores. The compilation of such nomograms and ultimately the sampling protocol depends critically on the value of the sampling constant  $K$  that is used in the formula for the FSE determined by Pierre Gy<sup>1</sup>. The way in which this constant  $K$  is determined is the focus of this paper. It is understood that a direct and simple comparison between the methods for estimating  $K$  may not be appropriate because of the calibration techniques by which the constants are derived. Nevertheless the

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the element of interest. The rest of the material is analysed to give an average grade for the total sample. The second method uses fine-grained material such as concentrates, but requires data inputs derived from mineral liberation analyser (MLA) and Qem-Scan analyses. The sampling constant  $K$  can then be determined using estimates of the composition distribution within individual size fractions. The third method suggested by Lyman, involves estimation of an upper bound on the sampling constant using an overall grade  $\alpha_L$  and by making the assumption that any size fraction consists of only two composition classes, namely, liberated high grade particles with density  $\rho_1$ , and liberated low grade particles with density  $\rho_2$ . These three approaches are not explored any further in this paper, but provide rich grounds for further research. In addition it should be noted that a considerable advantage of the three methods suggested by Lyman<sup>2</sup>, is that none of them involves the liberation factor, nor do they require the determination of the liberation factor.

### Approaches to determining $K$

The two other methods for determining values for  $K$  examined in this paper, namely the heterogeneity test (HT) and the duplicate sampling analysis (DSA), are the most commonly and widely disseminated in the literature. Accepting all the caveats that attend its use and application, the familiar formula for the FSE derived by Gy<sup>1</sup> takes the form given in Equation [1]:

$$s_{FSE}^2 = \frac{Cd_N^3}{M_S} \quad [1]$$

FSE is therefore a function of the sampling constant  $C$ , the cube of the nominal size of the fragments ( $d_N^3$ ), and the inverse of the mass of material collected in the sample ( $M_S$ ). According to Pitard<sup>3</sup>, Gy's earlier literature defined the constant factor of constitution heterogeneity  $IH_L$  as shown in Equation [2].

$$IH_L = f \cdot g \cdot c \cdot \ell \cdot d_N^3 = C \cdot d_N^3 \quad [2]$$

Because the liberation factor is a function of  $d_N$  the constant  $C$ , the product of four factors including  $\ell$ , changes as  $d_N$  changes. For practical purposes it is now customary to express  $IH_L$  as shown in Equation [3]: with little doubt that the exponent of  $d$  is  $\alpha = 3$ , unless the liberation factor is modelled as a function of  $d$  itself.

$$IH_L = f \cdot g \cdot c \cdot \ell \cdot d_N^3 = K \cdot d_N^\alpha \quad [3]$$

Thus it is evident that estimating the liberation factor is one of the primary concerns, there being three methods by which this parameter can be estimated. Here the distinction between the liberation factor and the liberation size is emphasised. The liberation size,  $d_L$  the of the constituent of interest is defined as the size to which 95% of the material must be ground in order to completely liberate at least 85% of the constituent of interest. Convention dictates that one never liberates all the minerals. The best crush liberates only 85%–90% of the mineral.

The first is the mineralogical method, which requires knowledge of the critical content of the lot  $\alpha_L$  and the critical content of the largest fragments,  $\alpha_{max}$ <sup>4</sup>. Following microscopic

investigation of the coarsest fragments to be sampled, a few of the largest fragments, where the content of the constituent of interest is high, are isolated. These fragments are assayed to find the maximum content which is defined as  $\alpha_{max}$ .

$$\ell = \frac{\alpha_{max} - \alpha_L}{1 - \alpha_L} \quad [4]$$

This formula (Equation [4]), is used when looking at the coarsest fraction. Take about 300 kg of material and crush it all to the same size, about 1.2 cm. Separate this into 50 samples of 6 kg each. Screen out the 1 cm sized fragments and select those fragments with the highest sulphide content or ash content depending on what one is analysing for. This can be very difficult for gold ores unless there is a clear association between sulphide and gold content. Analyse the 1cm fragments for the mineral of interest. Assume the following results are obtained: average content of ore = 1.5%Cu,  $\alpha_L = 0.015$  and the average content of 1 cm fragments = 15%Cu,  $\alpha_L = 0.15$ , then the liberation factor according to Equation [4], is shown in Equation [5] as:

$$\ell = \frac{\alpha_{max} - \alpha_L}{1 - \alpha_L} = \frac{0.15 - 0.015}{1 - 0.015} = 0.15 \quad [5]$$

Pitard<sup>5</sup> favours the approach using  $\alpha_{max}$  for the determination of the liberation factor, in which case  $\alpha = 3$ .

The second method for determining the liberation factor is application of the mineral processing formula when the liberation size  $d_L$  of the constituent of interest is known. The approximate model by which to calculate  $\ell$  as a function of the nominal fragment size  $d_N$  as shown in Equation [6].

$$\ell = \sqrt{\frac{d_L}{d_N}} \quad [6]$$

This equation is only a rough model that should not be used unless there is supporting mineral processing information at hand. Assuming  $d_L \sim 100$  m it is then possible to calculate the liberation factor when fragments are at 0.5 cm in diameter.

$$\ell \approx \sqrt{\frac{d_L}{d_N}} \approx \sqrt{\frac{0.01}{0.5}} \approx 0.14 \quad [7]$$

A third method for estimating the liberation factor proposed by François-Bongarçon suggests that the factor should be calibrated according to the nature and characteristics of the material being sampled. This so-called calibration method for the liberation factor  $\ell$ , is assumed to follow the empirical model shown in Equation [8].

$$\ell = \left( \frac{d_L}{d_N} \right)^b \text{ or } \ell = \left( \frac{d_L}{d_N} \right)^{3-\alpha} \quad [8]$$

Here the exponent  $b$ , is related to the slope of the calibration line above the liberation size  $d_L$ , and varies between 0 and 3 depending on the nature of the ore. Generally  $b = 3-\alpha$ , where  $\alpha$  is the slope of the calibration curve above the liberation size of the critical components. According to François-Bongarçon<sup>6</sup> exponent  $b$  takes values close to 1.5 in most gold ores as well as in cases where it has not been possible to calibrate the exponent, and so we can write Equation [9]:

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$$\ell = \left( \frac{d_\ell}{d_N} \right)^{1.5} \quad [9]$$

According to Pitard<sup>3</sup> empirical evidence indicates that if the constituent of interest is a single mineral, the exponent  $b$  is very close to 0.5. However, as clearly shown by Gy<sup>7</sup> for the liberation of ash in coals, and demonstrated by François-Bongarçon<sup>6</sup> for liberation of gold,  $b$  may be significantly different from 0.5, especially if the constituent of interest is located in a variety of different minerals. Under such conditions, the conventional equation should be approximated as shown in Equations [10] and [11]:

$$s_{FSE}^2 = \frac{K \cdot d_N^\alpha}{M_S} \quad [10]$$

and

$$K = f \cdot g \cdot c \cdot (d_\ell)^{3-\alpha} \quad [11]$$

In this case  $K$  and  $\alpha$  are the parameters that must be quantified in the so-called duplicate series analysis (DSA) experiments described by François-Bongarçon.

Model curves of the liberation factor as a function of the exponent alpha in Equation [10] are shown for critical components with a liberation size of 20 m and 150 m in host rock with nominal fragment sizes from 300 m to 1 cm are shown in Figures 1 and 2, respectively. For gold ores with medium to fine grained mineralization (anything less than 70 m in diameter), these curves indicate that the liberation factor is low (less than 0.25) for values of alpha greater than 1.0. For fine-grained ores, <20 m, the liberation factors are consistently very small, less than 0.1 for all values of alpha greater than 1.0. For values of alpha less than 1 the liberation factor increases exponentially, even in fragments that are 1 cm or more in diameter (Figure 1).

For coarse-grained gold mineralization, where the liberation size is greater than 150 m ( $d_L > 150$  m), the liberation factor is large and increases for all values of alpha less than 2.5 (Figure 2).

An argument against the DSA approach is that components of the grouping and segregation error (GSE) variance may become overwhelming and swamp the FSE as we approach or reach the liberation size. In particular gold (or other minerals of interest) because of its malleability may not comminute as fast as the gangue<sup>8</sup>, in which case the largest particles could be larger than  $d_N$ . Arguments against the HT method is that, apart from detailed mineralogical studies and gold deportment tests, there is no model of the liberation factor curve or other appropriate information from the heterogeneity tests by which we can accurately calculate the exponent  $\alpha$  ascribed to  $d$  in Gy's formula in Equation [10]. Ideally it should be possible to model the liberation factor  $\ell$  as a function of  $d_N$ , the topsize of the fragments.

Accepting the differences that exist between  $C$  and  $K$ , the form of Equations [1] and [10] are identical except that the cube term shown in Equation [1] is replaced by a variable exponent ( $\alpha$ , alpha) in Equation [10]. In order to estimate the FSE for any given sampling conditions of the ore, the parameter  $K$  specific to that ore must be established if Equation [1] is to be used, and  $K$  and  $\alpha$  if Equation [10] is to be used. It is noted that  $s_{FSE}^2$  in Equations (1) and (10) are in fact the same thing, namely the variance of the FSE, and

that  $MS$  and  $d_N$ , have the same meaning in both equations. Typically these parameters have in the past been established using two quite different experimental procedures, which are referred to by Pitard<sup>5</sup> and others as the heterogeneity test and the duplicate series analysis, the latter being a preferred name for what is also known as the 'sampling tree experiment' which includes GSE and delayed comminution problems. The units of  $K$  differ depending on estimation method; using the heterogeneity test the units are in  $g/cm^3$ , whereas  $K$  derived using DSA, has units in  $g/cm^\alpha$ .

### Description of the duplicate series analysis

The more common and better understood method for establishing the correct values for the sampling constants  $K$  and alpha is that proposed by Francois-Bongarçon<sup>6,9</sup>. These tests have been described by Becker<sup>10</sup>, Assibey-Bonsu<sup>11</sup>, and Minnitt *et al.*<sup>12</sup>. The procedure for the duplicate Series analysis that aims at experimentally calibrating  $K$  and  $\alpha$  is simple, but tedious, and care must be taken that the rules and principles pertaining to the use of the riffle splitter are upheld throughout the repetitive procedures. Usually an amount of 40–50 kg is sufficient for this experiment, but if visible gold is present 40 kg is much too small to perform a valid test; the size of the sample should be increased substantially. Having collected ~40 kg of run-of-mine ore or mineralized reef material, the following steps apply:

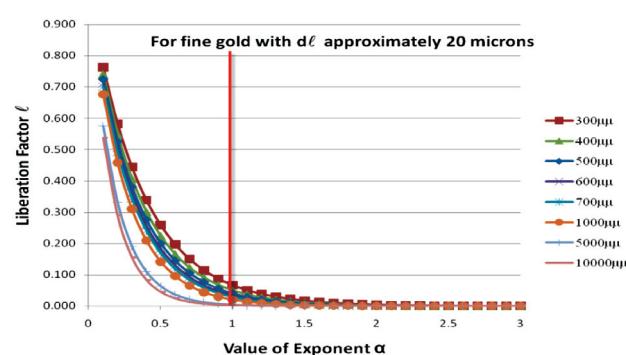


Figure 1—Model liberation curves for a range of exponent values (alpha) in host rock ranging from 300 m to 1 cm; liberation size of the critical component is 20 m

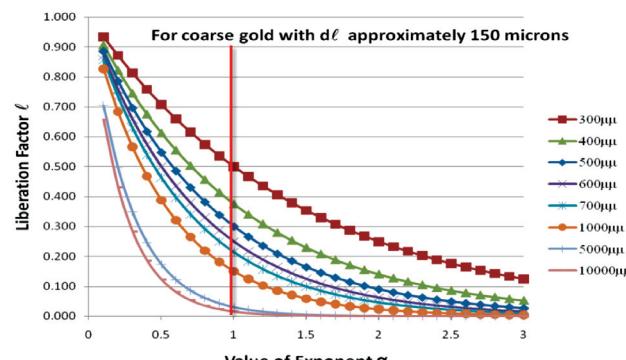


Figure 2—Model curves for liberation factor versus exponent values of alpha; liberation size  $d_L$  of critical component is 150 m in host rock ranging from 300 m to 1 cm

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- *Step 1*—reduce the lot to 95%–1.9 cm in a jaw crusher, split the lot into four sub-lots (series) each comprising ~10 kg and separate out the first quarter, Series 1 (~10 kg) at 95%–1.9 cm. The size to which the material is crushed is a matter of choice, in this case 1.9, 1.2, 0.63 and 0.1 cm, but they should provide sufficient spread across the calibration curve
- *Step 2*—crush the remaining 30 kg to 95%–1.2 cm in a jaw crusher, split out the second quarter Series 2 (~10 kg) at 95%–1.2 cm
- *Step 3*—crush the remaining 20 kg to 95%–0.63 cm in a jaw crusher and split out a third quarter Series 3 (~10 kg) at 95%–1.2 cm
- *Step 4*—crush the remaining 10 kg to 95%–0.1 cm in a ball mill or similar crusher and split out a fourth quarter Series 4 to 95%–0.1 cm
- *Step 5*—each of the four sub-lots, approximately 10 kg each, are split into 32 sub-samples using a riffle splitter with appropriately sized openings. Whereas some may consider the riffle splitter an ineffective tool by which to minimize GSE, others believe that using a rotary splitter to do the job does not preserve the essential variance that should be measured for the DSA technique. The reason for using a riffle splitter is firstly convenience, but Petersen, Dahl, and Esbensen<sup>13</sup> investigated the behaviour of sampling variance using a variety of different methods of mass reduction using different types of equipment and splitting techniques. They concluded that only devices based on riffle splitting principles (static or rotational) pass the ultimate representativity test. Two samples are selected at random from each series in order to check the calibration of the nominal top size of that series. This should result in 120 more-or-less equal mass samples that are then submitted for analysis depending on the element of interest. The authors accept that the method used to split samples for the duplicate series analysis-DSA (i.e. Steps 1 to 5 and the splitting of the respective 10 kg samples into 312 g sub-samples) will incur some error, but would be minimal due to the size of the lot being sampled. Generally it is useful to have one of the Series at the pulverized stage; this provides the error incurred at the last sub-sampling stage when the aliquot is extracted for analysis, and since it is common to all series it can simply be subtracted from each of them. This is different from the analytical variance (analytical precision is 4%), which has been accounted for, but is also subtracted from the pulverized stage variance.

### Data reduction for the duplicate series analysis method

Equation [10] is rearranged in the following manner to give Equation [12] representing a straight line equation relating the standardized variance and the top size of the fragments in the four series.

$$\ln(\sigma_R^2 \times M_S) = \alpha \ln d_N + \ln(K) \quad [12]$$

The raw unedited data for the duplicate series analysis from the assay laboratory are reported as shown in Table A1 of Appendix 1. Outliers invariably occur within these data so

the importance of identifying and eliminating such outliers is emphasized here. A typical QA/QC Excel control chart shown in Figure 3, which allows samples with biases greater than 5% to be identified, was used to identify and eliminate all outliers. Appropriate iterative adjustments are made to the data-set until all outliers have been identified and removed; the greyed-out data in Tables A1 to A6, were excluded from the calculations.

Descriptive statistics of the adjusted data-sets with outliers removed are listed in Table I.

The statistics for the Mponeng, Kloof, and Lily mines have been reduced to provide the relevant data for the duplicate series analysis, a specimen for the reduction of the data from the Mponeng mine being shown in Table II.

Data reduction for the duplicate series analysis data for the Kloof and Lily mines (Appendix Tables A2 and A3) are performed in precisely the same manner as that for the Mponeng mine as shown in Table II. The  $\ln(s^2 \times MS)$  and  $\ln(d_N)$  data for duplicate series analysis of the Mponeng, Kloof, and Lily mines are given in Table III.

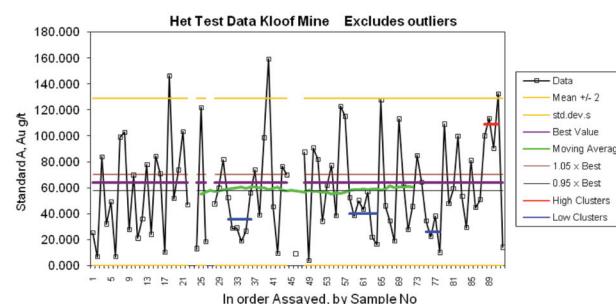


Figure 3—Control chart to test for outliers in Kloof Mine data for the heterogeneity test

Table I Descriptive statistics for gold fire assays for the four DSA series from each mine (outliers removed)				
Mponeng Mine	Series 1	Series 2	Series 3	Series 4
Size (cm)	5.00	1.50	0.50	0.15
Mean (g/t)	13.819	13.990	15.446	15.291
Variance (g/t <sup>2</sup> )	16.7231	3.1257	1.3448	0.4651
Std dev (g/t)	4.0894	1.7680	1.1597	0.6820
RSD	0.2959	0.1264	0.0751	0.0446
Average mass (g)	564.483	556.090	463.623	502.383
Kloof Mine	Series 1	Series 2	Series 3	Series 4
Size (cm)	1.900	1.270	0.670	0.200
Mean	61.48	64.62	64.75	60.41
Variance	258.8147	85.7236	37.9902	30.7804
Std dev	16.0877	9.2587	6.1636	5.5480
RSD	0.2617	0.1433	0.0952	0.0918
Average mass (g)	317.6	313.8	318.0	308.6
Lily Mine	Series 1	Series 2	Series 3	Series 4
Size (cm)	1.900	0.950	0.475	0.200
Mean	0.87	1.08	0.91	1.02
Variance	0.0567	0.0742	0.0192	0.0160
Std dev	0.2381	0.2724	0.1387	0.1263
RSD	0.2748	0.2527	0.1523	0.1239
Average mass (g)	263.47	279.76	248.44	248.52

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Table II

### Data reduction for duplicate series analysis, Mponeng Mine

Mponeng Mine	Series 1	Series 1	Series 1	Series 1
$d_{95}$ (cm)	5.00	1.50	0.50	0.15
Measured multi-stage rsd	0.2959	0.1264	0.0751	0.0446
Measured multi-stage var	0.0876	0.0160	0.0056	0.0020
Less analytical var#	0.0860	0.0144	0.0040	0.0004
Standardized var	1.1880	0.2010	0.0624	0.0060
Mass (g)	564.48	556.09	463.62	502.38
Mass assayed (g)	50.0	50.0	50.0	50.0
Single stage var	1.1880	0.2010	0.0624	0.0060
$s^2 * M_s$	670.6179	111.7934	28.9083	2.9893
Mponeng $\ln(s^2 * M_s)$	6.5082	4.7167	3.3641	1.0951
$\ln(d_{\max})$	1.6094	0.4055	-0.6931	-1.8971

#Analytical precision is 4%

Table III

### $\ln(s^2 * M_s)$ and $\ln(d_{\max})$ data Series 1–4 for Mponeng, Kloof, and Lily mines

Mponeng Mine	Series 1	Series 2	Series 3	Series 4
Size (mm)	50.0	15.0	5.0	1.5
$\ln(s^2 * M_s)$ (actual)	6.508	4.717	3.364	1.095
$\ln(d_{\max})$	1.609	0.405	-0.693	-1.897
Mponeng model $\ln(s^2 * M_s)$	6.58	4.75	3.09	1.26
Kloof Mine	Series 1	Series 2	Series 3	Series 4
Size (mm)	19.0	12.7	6.7	2.0
$\ln(s^2 * M_s)$ (actual)	7.175	5.950	5.034	4.847
$\ln(d_{\max})$	0.642	0.239	-0.400	-1.609
Kloof Model $\ln(s^2 * M_s)$	6.61	6.24	5.64	4.51
Lily Mine	Series 1	Series 2	Series 3	Series 4
Size (mm)	19.0	10.0	5.0	2.0
$\ln(s^2 * M_s)$ (actual)	2.826	2.932	1.587	1.248
$\ln(d_{\max})$	0.642	-0.051	-0.744	-1.609
Lily model $\ln(s^2 * M_s)$	3.03	2.46	1.90	1.20

The data shown in Table III including the actual data ( $\ln$  [product of error variance and sample mass]), for each mine as well as the model data for each mine, are plotted against the  $\ln$  [nominal fragment size] in the calibration curves of Figure 4.

The data shown in Table IV are derived directly from the calibration graphs shown in Figure 7. For example, the slope of the straight line for the Mponeng mine is given by the coefficient 1.52, and is equivalent to alpha,  $\alpha$  which is the exponent in Equation [10], and the exponent of 4.14 which is the intercept, gives a  $K$  value of 62.7 (Table IV).

Unlike the other two sets of data, the residual errors for the Kloof mine DSA model (Figure 7) are quite high. This could be due to several practical problems, including the outlier effects, and the non-removal of pulverized stage variances from the respective single-stage variances. Several sensitivity analyses have been conducted to examine the impact of the practical issues on the DSA results. These include removal of outliers based on a statistical analysis.

Unfortunately, the assay laboratory did not provide the analyses for pulverized stage series, before discarding the reject powders; as a result, the single-stage variances have not been removed from the respective series for Kloof mine. Based on analysis of pulverized stage variances for low grade gold ores in the literature<sup>14</sup>, it was established that the last stage (2 mm) of the Kloof series will be the most affected of the fragment sizes in this regard. The literature review indicates that the variances for this size of material could be overstated by as much as 20%, whereas the larger sizes could be affected by up to 4%. Accordingly, a reduction in variance of 20% for the smallest sized fraction (2 mm), and 4% for the other three size fractions has been considered as a possible scenario. Another scenario considers the elimination of the smallest size fraction (2 mm), because it could be materially affected by the non-removal of the pulverized stage variance. The expected range of values for alpha and  $K$  for these scenarios is shown in Table V for changing slope and intercept of the calibration curves.

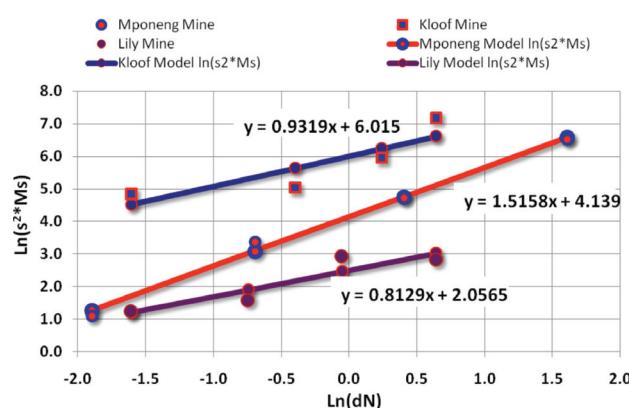


Figure 4—Calibration curve of  $\ln(s^2 * M_s)$  versus  $\ln$  (nominal fragment size) for Mponeng, Kloof, and Lily Mines to find  $K$  (intercept) and  $\alpha$  (slope)

Table IV

### Sampling Constants $K$ and $\alpha$ derived from the calibration curve (Figure 6)

	Mponeng	Kloof	Lily
Alpha $K$	1.5158 62.7	0.9319 409.4	0.8129 12.3

Table V

### Changes and adjustments to Kloof data series and associated changes in alpha and $K$

State of data	Slope, alpha	Intercept, $K$
Raw data, no outliers removed	0.505	580.59
Outliers removed#	0.93	409.37
Percentage adjustments*	1.25	293.04
Series 4 (2 mm) removed (Table III)	2.00	309.00

\*A reduction of 20% to variance of smallest fragment size and 4% to the other three fragment sizes.

#These results have been used for the compilation of Kloof nomograms

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Thus the authors are aware of the sensitivities of the calibration curves, especially for the way in which small changes in the variances at the small fragment sizes can significantly affect the value of  $K$  and alpha. At these small fragment sizes, and with a limited number of data (32 assays per series), one outlier in the series of gold assays can give rise to material changes in the variance. The impact of these parameter changes for the DSA on gold liberation sizes will be demonstrated later in this paper.

### Compilation of the DSA nomograms

Having once established estimates of the constants  $K$  and alpha, it is quite simple to substitute the parameters  $K$  and  $\alpha$  into Equation [10] and compile the nomograms for the specific mine ores from these data as shown in Figure 5.

The differences in the nature of the ores is emphasized in the nomograms of Figure 5. The ores from Mponeng are evenly sized and rounded pebble-supported conglomerates containing fairly low grade very fine-grained gold. This makes for easy sampling and well behaved (generally declining) nomograms as the sample mass is reduced. The nomogram for the Kloof ores also declines from right to left as the sample mass is reduced, but less so than for the Mponeng ore (Figure 5). The relatively high value of  $K$  suggests that these ores require considerable comminution before mass reduction in order to ensure that the nomogram lies below the danger and design thresholds that demarcate the limits of the variances to ensure representative sampling of these ores. The Kloof ores are from the matrix-supported milky white cobble facies of the high grade Ventersdorp Contact Reef conglomerates. This reef facies contains pebbles of widely differing size (1 to 8 cm) and very coarse-grained gold, with an average of 220 m<sup>15</sup> (equivalent circular diameter).

The Lily ores are quite different from those of the Mponeng and Kloof mines, both of which are developed in the auriferous conglomerates of the Witwatersrand Supergroup. Lily ores from the Barberton Mountain Land occur in a shear zone along a contact between the upper formations of the Onverwacht Group and the greywackes of the Fig Tree Group. The ores are finely bedded to laminated and are overprinted

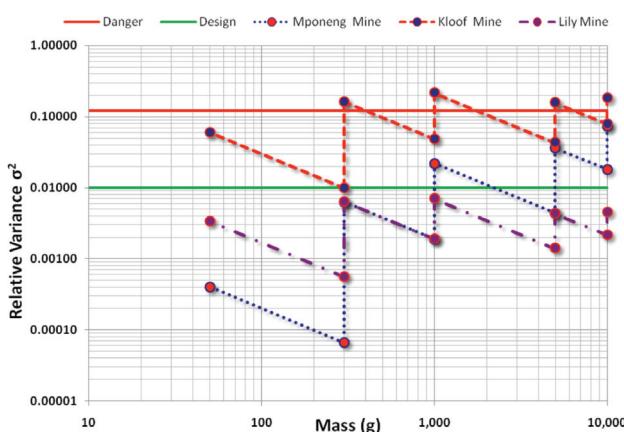


Figure 5—Nomograms for the ores from Mponeng, Kloof, and Lily mines using data derived from the ores by the duplicate series analysis method

by a strong shear fabric. The trend of the nomogram for the Lily ores (Figure 5) is almost horizontal, suggesting that it should not be difficult to obtain representative samples from these ores.

Values of the constant alpha ( $\alpha$ ) for the three mines examined in this study vary from 0.81 to 1.52 (Table IV), but detailed work in eight Anglogold-Ashanti mines in Witwatersrand-type gold mining operations produced the values for alpha and  $K$  shown in Table VI. The nomograms for these operations are shown in Figure 6, and the trend lines through the nomograms are shown in Figure 7.

Table VI  
Calibration values for  $K$  and alpha using the DSA method for South African Wits-type gold mining operations

Mine type	Operation	Reef	$K$	Alpha
Witwatersrand	Bambanani	Basal	36	0.68
Witwatersrand	Tshepong	Basal	275	0.71
Witwatersrand	Kopanang	Vaal	497	0.77
Witwatersrand	Savuka	Carbon leader/VCR	31	0.67
Witwatersrand	Joel	Beatrix	30	0.72
Witwatersrand	Elandsrand	VCR	454	1.28
Witwatersrand	Tau Lekoa	VCR	970	0.63
Witwatersrand	Great Noligwa	Vaal	2849	1.09

Source: V. Chamberlain, AnglogoldAshanti, personal communication, 2004

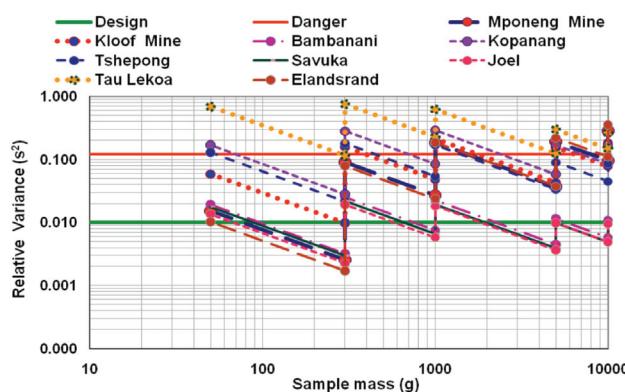


Figure 6—Nomograms for eight gold mining operations in the Witwatersrand Basin; values for  $K$  and alpha taken from Table III

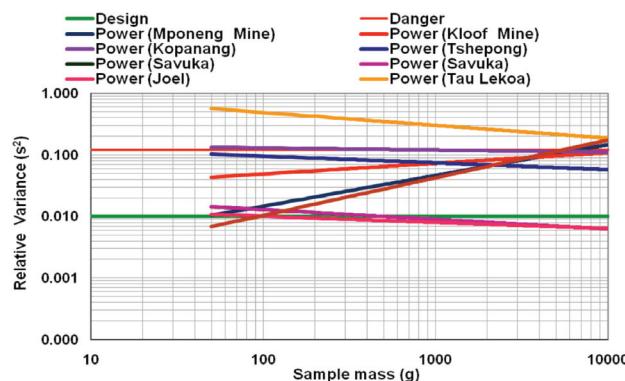


Figure 7—Trend lines (power) through nomograms shown in Figure 6 to illustrate the overall slope of the nomograms

## A comparison between the duplicate series analysis method

The trend lines of the eight nomograms from South African gold mines shown in Figure 7 suggest that quite large masses of ore have to be crushed to very fine grain sizes before mass reduction in order to remain below the design and safety lines and to protect the integrity of the sampling protocols.

### Description of the heterogeneity test

The aim of the heterogeneity test is to determine the sampling constants for different styles or types of mineralization as a basis for designing and optimizing sampling and sample preparation protocols for exploration drill hole samples, blasthole samples, conveyor belt samples, etc<sup>16</sup>. Heterogeneity testing is a critically important step in material characterization that focuses on determining the sampling constant  $K$  in the formula  $Gy$  proposed for determining the fundamental sampling error. The test is performed by controlling  $d_N$  to a size of one centimeter so that the exponent of  $d_N$  is irrelevant and the value of  $d_N^3$  is very close to one; as it turns out the value is 1.05 cm. The mass of each sample is controlled to an exact weight so that  $M_S$  is exactly known. The variance of the selected samples is also known, which leaves  $K$  as the only unknown value that can be solved for in Equation [1].

The first step should also include a mineral deportment study allowing the nature of the mineral distribution, the mineral associations and mode of occurrence, the size distribution of the mineral grains (especially the possibility of nugget formation or occurrence), and the sampling characteristics of the ores to be determined<sup>17</sup>. Heterogeneity testing is not done on a regular basis, in fact it probably needs to be done only once for a given type of mineralization over the lifetime of a project, but it is essential that the process followed is done correctly from the beginning. Results from the heterogeneity test should be congruent and support the results of the deportment study.

The following section describes a recommended test that can be used in order to achieve an accurate determination of  $IH_L$  if it is difficult to estimate the liberation factor of a constituent of interest. This is particularly true for minor constituents such as gold, molybdenum, copper, arsenic etc. The test has been proposed as a means of providing a definition of the value of  $K$ . The idea is to experimentally estimate the error variance  $s^2_{FE}$  and on this basis compile a nomogram that describes the practical sampling unit operations that allows one to optimize the sampling protocol.

- *Step 1*—collect 300 kg of ore in as large fragments as can be handled from a mine and dry overnight at 1100°C
- *Step 2*—crush the material to about 95% passing minus 1.9 cm with a jaw crusher with an opening adjusted accordingly
- *Step 3*—screen the entire lot through 1.25 cm and 0.63 cm screens; the  $d_N$  between these two screens is 1.05 cm on average as shown in the following calculation.

Top screen (cm)	Bottom screen (cm)
1.266	0.630

$$d_N = \sqrt[3]{\frac{d_{\text{upper screen size}}^3 + d_{\text{lower screen size}}^3}{2}}$$

$$= \sqrt[3]{\frac{1.266^3 + 0.63^3}{2}} = \sqrt[3]{1.1396} = 1.0445 \text{ cm}$$

- *Step 4*—weigh the -1.25 cm+0.63 cm fraction, and spread the material on a clean surface. The heterogeneity test will be performed on this fraction
- *Step 5*—from this fraction (about 50 kg), collect 100 samples. Each sample must be made of  $p$  fragments selected randomly one at a time, up to 50g. (Use 50-gram as a target, collecting whatever number of fragments it takes to do so. So later on you have  $M_S = 50$  grams in the formula). Number these samples from 1 to 100, weigh each of them, and record values for  $p$ . Equivalent mass is 5000 g
- *Step 6*—pulverize each sample directly in an enclosed ring and puck pulverizer (LM5 mill) to about 95% minus 106  $\mu\text{m}$  and assay samples 1 to 100 for gold using 50 g aliquots.

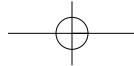
Performing the heterogeneity test on 1-cm fragments is ideal. The average size of the fragments between the 1.25-cm screen and 0.65-cm screen is 1.05 cm. 1.05 cm is very close to the ideal 1 cm, so the exponent of  $d_N$  does not play any role in the calculation of the sampling constant. Samples collected to perform the heterogeneity test should consist of relatively few fragments in order to make the analytical error negligible. Usually, the fragments are collected one by one until a weight nearest to 50 grams is obtained; it simplifies the calculations if using sample mass instead of number of fragments. Fragments collected to make each sample must be selected one by one at random, in order to destroy any distributional heterogeneity (DH) and minimizes the contribution from the grouping and segregation error.

The problem with the Heterogeneity Test (HT) is that the calibration exactly represents  $K$  for the specific ~1 cm size at which the calibration is done. For other size distributions the sampling characteristics will be different and can be determined only by undertaking the analysis at that particular fragment size, as demonstrated by Pitard (2009). The HT technique assumes a general rule based on Gy's empirical estimate of the liberation factor  $\ell$  as  $\text{SQRT}(d/d_N)$ , which was experimentally calculated, but not for low grade gold ores<sup>6</sup>. The general use of this a priori liberation factor in some cases provide meaningless results<sup>11</sup> and while it may be correct for ~1 cm size fraction it is not transferable to other size distributions.

### The difference between $C\phi$ and $K$

As mentioned above the sampling constant may take the two forms shown in Equations [1] and [2] depending on how the exponent of the nominal fragment size is specified. If the nominal size  $d$  has an exponent of 2.5 ( $d^{2.5}$ ), or  $\alpha$ , where  $\alpha = 3-x$ , ( $d^{3-x}$ ), the appropriate symbol is  $K$ . If the exponent of  $d$  is 3 ( $d^3$ ), then the appropriate symbol is  $C$ . The fundamental sampling error variance for any increment in a sampling protocol is given by Pierre Gy's<sup>1</sup> well-known formula:

$$s_{FSE}^2 = \left( \frac{1}{M_S} - \frac{1}{M_L} \right) K d_N^\alpha \quad [13]$$



## A comparison between the duplicate series analysis method

We define  $C_\phi$  as the sampling constant for a specific size-fraction  $\phi$ , i.e. for a single stage of comminution in the sampling process, identified by subscript  $\phi$ . The single stage error variance is defined as:

$$\sigma_\phi^2 = \left( \frac{1}{M_\phi} - \frac{1}{M_L} \right) C_\phi d_\phi^3 \quad [14]$$

Where variables in the equation represent the mass of sample ( $M_\phi$ ), mass of lot ( $M_L$ ), nominal fragment size ( $d_\phi$ ) and sampling constant ( $C_\phi$ ) for a specific or single stage variance in a sampling protocol. If  $M_S$  is small in relation to  $M_L$  it is possible to rearrange Equation [14] to give:

$$C_\phi = \frac{M_S}{d_\phi^3} \times \sigma_{FSE}^2 \quad [15]$$

and since

$$\sigma_{FSE}^2 = \frac{s_{ai}^2}{a_i^2} \quad [16]$$

we write:

$$C_\phi = \frac{\bar{M}_\phi}{d_\phi^3} \frac{s_{ai}^2}{a_i^2} = f_\phi g_\phi c_\phi \ell_\phi \quad [17]$$

where  $s_{ai}^2/a_i^2$  is the relative variance calculated from the experimental assay data derived from the heterogeneity test procedure. Again it is important to emphasise that  $\sigma^2$  is used for infinite populations, but in practice we use  $s^2$  because we are dealing with finite, limited amounts of material. Furthermore, provided we accept the model for the liberation factor  $\ell = \sqrt{\frac{d_\ell}{d_N}}$ , a model that is hotly contested by some, it is possible to substitute for  $\ell$  and rearrange to get a value for  $C_\phi$ :

$$C_\phi = f_\phi \times g_\phi \times c_\phi \times \sqrt{\frac{d_\ell}{d_\phi}} \quad [18]$$

$$C_\phi \sqrt{d_\phi} = f_\phi g_\phi c_\phi \sqrt{d_\ell}$$

The three factors  $f$ ,  $g$ , and  $c$  are independent of the maximum diameter of the fragments ( $d$ ), as is  $d_\ell$ , the liberation size, so that from one size fraction to another the term  $f_\phi g_\phi c_\phi \sqrt{d_\ell}$  remains reasonably constant. This invariant term which is the sampling constant  $K$ , does not change from one stage of comminution to another so that:

$$K = C_\phi \sqrt{d_\phi} \quad [19]$$

This derivation for  $K$  assumes of course that Equation [6] is valid and appropriate for all  $d_N$ .

### Data reduction for the heterogeneity test

The sample masses and assay grades compiled from the experimental work undertaken as part of the heterogeneity tests for Mponeng, Kloof and Lily mines are listed in Appendix Tables A4, A5, and A6, respectively. The relative variance ( $s_{ai}^2$ ) of the 100 assays of material specifically classified as  $d_\phi = 1.05$  cm is given by the total variance of the assays divided by the mean squared as indicated in Equation [16]. Thereafter, for each of the different sampling stages, the constant  $C_\phi$  can be derived from the above equation using the different values of  $d_N$ . The descriptive statistics for these sets of data are presented in Table VII.

### Compilation of the HT nomograms

Having calculated values for  $K$  for the Mponeng, Kloof, and Lily mines, shown in Table VII, it is possible to compile the nomograms for a specific gold grade and ore type using these data. In the case of the heterogeneity test the sampling constant  $K$  is used together with an exponent of 2.5 (Table VIII) to derive the sampling nomogram as shown in Figure 11.

In this case the exponent used in the formula for the relative variance is 2.5 rather than 3. The equation commonly used is:

$$s_{FSE}^2 = \frac{f g c \ell d^3}{M_S} \quad [20]$$

The metallurgically tested equation for the liberation factor  $\ell = \sqrt{\frac{d_\ell}{d_N}}$  is now substituted into the equation as follows:

$$\begin{aligned} s_{FSE}^2 &= \frac{f g c d_N^3}{M_S} \times \frac{d_\ell^{0.5}}{d_N^{0.5}} \\ s_{FSE}^2 &= \frac{f g c \sqrt{d_\ell} \times d_N^{2.5}}{M_S} \end{aligned} \quad [21]$$

Substituting  $K$  for  $f g c \sqrt{d_\ell}$  gives:

$$s_{FSE}^2 = \frac{K \times d_N^{2.5}}{M_S} \quad [22]$$

The data derived from using Equation [22] are shown in Table IX and are presented in the nomograms compiled from these data in Figure 11.

The three nomograms derived by plotting these data are shown in Figure 8.

The sampling constant  $K$  was the only value derived from the heterogeneity test, a simply constructed experiment that produces an answer for the value of  $K$ . There is a reasonably

Table VII  
Descriptive statistics for the raw data derived from the heterogeneity test

	Mponeng Mine	Kloof Mine	Lily Mine
Average mass	100.00	50.11	100.73
Average grade	1.66	56.35	0.71
Variance	0.1204	1071.6908	0.1714
Relative variance	0.0439	0.3375	0.3390
$C$	3.790	14.609	29.503
$K$	3.88	14.97	30.23

Table VIII  
Sampling constant  $K$  for the Mponeng, Kloof, and Lily ores determined from the heterogeneity tests

	Mponeng Mine	Kloof Mine	Lily Mine
$K$	3.88	14.97	30.23
Exponent	2.50	2.50	2.50

## A comparison between the duplicate series analysis method

Table IX Table of standardized relative variances for the Mponeng, Kloof, and Lily mines					
Mass (g)	Alpha $K$	Mponeng Mine	Kloof Mine	Lily Mine	
		2.50	2.50	2.50	
		3.88	14.97	30.23	
	Size (cm)	SRV	SRV	SRV	
10000	5.0000	0.0217095	0.083685	0.169000	
10000	2.0000	0.0021968	0.008468	0.017102	
5000	2.0000	0.0043937	0.016937	0.034203	
5000	0.5000	0.0001373	0.000529	0.001069	
1000	0.5000	0.0006865	0.002646	0.005344	
1000	0.1000	0.0000123	0.000047	0.000096	
300	0.1000	0.0000409	0.000158	0.000319	
300	0.0050	0.0000000	0.000000	0.000000	
50	0.0050	0.0000001	0.000001	0.000001	
	Total error	5.27	10.34	14.70	
	Analytical error	4.0	4.0	4.0	
	Total error	6.61	11.09	15.23	

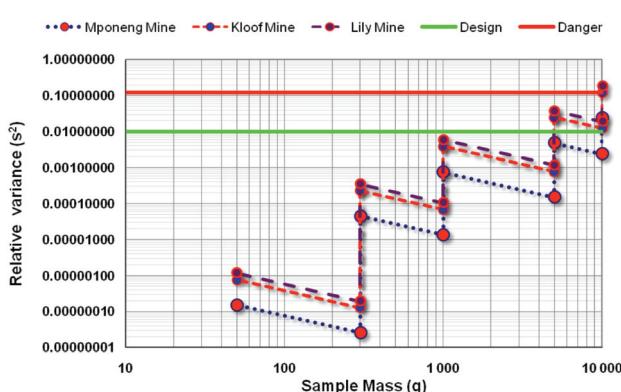


Figure 8—Sampling nomograms for the HT method for Mponeng, Kloof, and Lily mine ores

close correspondence between the nomograms for the three ore types, and the declining trend in the nomogram from right to left suggests that the ores behave in an accomodating fashion during sampling and that representative sampling should not be that difficult.

### Comparison of results from the DSA and the heterogeneity test methods

The methods described as the duplicate sampling analysis (DSA) and the heterogeneity test (HT) both attempt to derive a sampling constant  $K$  in the formula for the fundamental sampling error, by different methods. These values are not the same thing in that they have different units,  $\text{g}/\text{cm}^2$  and  $\text{g}/\text{cm}^3$ , respectively. In addition, the DSA method derives a calibrated value for the exponent, alpha that is specific to the ores under consideration; the heterogeneity test by contrast uses a constant value of 2.5 in the compilation of the nomogram, a value which Pierre Gy later rescinded<sup>18</sup>. According to Pitard<sup>8</sup>, the exponent of 2.5 may also vary depending on the model for the liberation factor that is derived from the department study. The standardized relative variances from three mines, Mponeng, Kloof, and Lily for the two different approaches to estimation of the sampling parameters  $K$  and alpha are presented for comparison in Table X. The values for  $K$  and alpha are significantly different for the DSA and the HT methods. These differences are reflected in the nature of the nomograms for the different ores and are shown together as a range of nomograms in Figure 9.

The nomograms for the three different ore types (using the same sample masses and fragment sizes throughout), are shown in Figure 9 for the sake of a visual comparison between the different methods of derivation of values for  $K$ .

In general the DSA-derived nomograms show a flat to shallow downward trend from right to left in Figure 9, whereas the HT-derived nomograms show a steep downward

Table X  
Comparative standardized relative variances for the DSA and HT methods

Mass (g)	Alpha	Mponeng Mine		Kloof Mine		Lily Mine	
		DSA	Het test	DSA	Het test	DSA	Het test
		1.52	2.50	0.930	2.50	0.81	2.50
		62.80	3.88	411.58	14.97	12.30	30.23
	Size (cm)	SRV (FE)	SRV	SRV (FE)	SRV	SRV (FE)	SRV
10000	5.00	0.071953	0.0217095	0.18343	0.083685	0.00454	0.169000
10000	2.00	0.017941	0.0021968	0.07810	0.008468	0.00215	0.017102
5000	2.00	0.035883	0.0043937	0.15620	0.016937	0.00431	0.034203
5000	0.50	0.004388	0.0001373	0.04292	0.000529	0.00140	0.001069
1000	0.50	0.021942	0.0006865	0.21458	0.002646	0.00698	0.005344
1000	0.10	0.001913	0.0000123	0.04789	0.000047	0.00189	0.000096
300	0.10	0.006378	0.0000409	0.15963	0.000158	0.00629	0.000319
300	0.01	0.000068	0.0000000	0.00979	0.000000	0.00055	0.000000
50	0.01	0.000408	0.0000001	0.05873	0.000001	0.00330	0.000001
	Total error	20.07	5.27	64.07	10.34	12.20	14.70
	Analytical error var	4.00	4.0	4.00	4.0	4.00	4.0
	Total error	20.47	6.61	64.19	11.09	12.84	15.23

SRV (FE): standardized relative variance (fundamental error)

## A comparison between the duplicate series analysis method

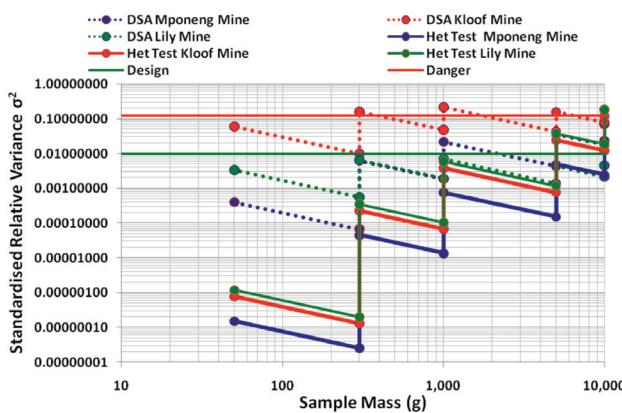


Figure 9—Comparison of all nomograms using the duplicate series analysis (DSA) and the heterogeneity test (HT) for Mponeng, Kloof, and Lily ores; maximum acceptable variance at 0.01

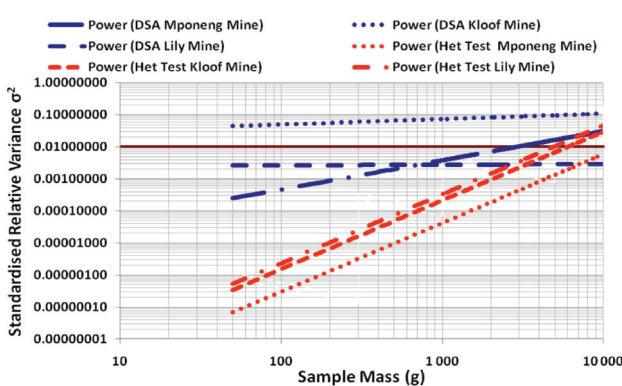


Figure 10—Trend lines (power) through the nomograms for the different methods of estimating K and alpha; heterogeneity test trend lines are shown in red, DSA trend lines are shown in blue

trend from right to left in Figure 9. A visual comparison of these differences, and ultimately the end result between the DSA and the heterogeneity test methods of estimating the sampling constant  $K$ , is shown by the trend lines for these nomograms in Figure 10.

### Comparison of liberation and gold grain sizes

Once the models have been fully specified, liberation and gold grain sizes can be calculated for the different ore types provided the data listed in Table XI are available for substitution into Equation [23], which is a simple rearrangement of Equation [11].

$$d_{\ell} = \left[ \frac{K}{c \cdot f \cdot g} \right]^{\frac{1}{3-\alpha}} \quad [23]$$

where  $d_{\ell}$  = liberation size of component of interest,  $K$  is the calibrated parameter,  $c$  = the mineralogical constant (density/average grade),  $f = 0.5$ ,  $g = 0.25$ , and  $\alpha$  the calibrated slope for DSA; for HT the assumed value is 2.5. Results for calculation of the liberation and the gold grain sizes for both the DSA and the HT methods are provided in Table XII.

The gold grain liberation sizes calculated in Table XII indicate a range of values from the Mponeng, Kloof, and Lily mines of 56 m, 1218 m, and 41 m, respectively. Detailed mineralogical studies of the Kloof ores indicate that the gold is coarse grained with an equivalent circular diameter of about 220 m<sup>15</sup>. This is an order of magnitude less than the calculated value, but the mineralogical methods to determine the gold grain sizes can only ever provide a minimum value. In general the gold grain liberation sizes for the DSA method appear to be acceptable. By contrast the gold grain sizes derived using the HT method, would be far too small, less

Table XI  
K, alpha and grade data for the Mponeng, Kloof, and Lily mines

DSA method	Mponeng Mine	Kloof Mine	Lily Mine
$K$	62.74	409.37	12.26
Alpha	1.52	0.93	0.81
Grade	14.64	62.82	0.97
HT method	Mponeng Mine	Kloof Mine	Lily Mine
$K$	3.88	14.97	30.23
Alpha	2.5	2.5	2.5
Grade	1.657	56.348	0.711

Table XII  
Summary of K and Alpha values, and liberation sizes and gold grain sizes, for both the DSA and heterogeneity test methods

DSA method	Mponeng	Kloof	Lily
Grade	14.64	62.82	0.97
$g/g(100 000)$	0.000014637	0.000062819	0.000000969
$\rho/g^*$	1093147.05	254700.64	16516129.03
$K$ (calibrated)	62.74	409.37	12.26
$f$	0.5	0.5	0.5
$g$	0.25	0.25	0.25
$c$	1093147.05	254700.64	16516129.03
Alpha calibrated	1.52	0.93	0.81
Exponent (1/(3- $\alpha$ ))	0.67	0.48	0.46
$cfg$	136643.38	31837.58	2064516.13
$d\ell$ (cm)	0.005636254	0.121823303	0.004077556
$d\ell$ (m)	56.36	1218.23	40.78
Heterogeneity test method	Mponeng	Kloof	Lily
Grade	1.657	56.348	0.711
$g/g(100000)$	0.000001657	0.000056348	0.000000711
$\rho/g^*$	9658158.91	283947.71	22503863.99
$K$ (calibrated)	3.88	14.97	30.23
$f$	0.5	0.5	0.5
$g$	0.25	0.25	0.25
$c$	9658158.91	283947.71	22503863.99
Alpha (given for HT)	2.5	2.5	2.5
Exponent (1/(3- $\alpha$ ))	2.00	2.00	2.00
$cfg$	1207269.86	35493.46	2812983.00
$d\ell$ (cm)	0.000000000	0.000000178	0.000000000
$d\ell$ (m)	0.0000001	0.0017789	0.0000012

\*Density used is that for amalgam (gold-silver alloy) ~ 16 g/cc

## A comparison between the duplicate series analysis method

than the size of the gold atom, if the exponent of 2.5 and the corresponding value of  $K$  is used in the calculations. This approach does not yield intuitively appealing gold grain liberation sizes. Thus the DSA method provides liberation and gold grain sizes that are comparable with the known mineralogical characteristics of the ores.

The sensitivity studies undertaken on the changes of variance due to the removal or adjustments of the assay values for the Kloof data using the DSA method yielded the results presented in Table V. These data together with the grade data shown in Table XIII are used to calculate a range of liberation and gold grain sizes for the respective scenarios.

Gold grain liberation sizes range from about 100 m to 2000 m (Table XIII), and are in keeping with what is known from detailed mineralogical studies<sup>15</sup>. Despite this range of grain sizes in the scenario results the DSA gold grain estimates are practically acceptable. It seems that the exponent of 2.5 in the HT model does not reflect what is known from metallurgical studies, and consequently the efficiency of predicting real sampling variances is therefore dubious.

### Conclusion

This study shows that the calibrated values for the duplicate series analysis and heterogeneity test methods are significantly different. The study further indicates that the DSA gold liberation sizes are more in keeping with the mineralogical evidence from the respective mines. It is recommended work be undertaken on similar deposits to confirm these preliminary results.

### General considerations for calibration procedures

The average trend lines for nomograms with a fixed exponent (in this case  $\alpha = 1$ ), for different values of  $K$  are shown in Figure 11, the principle effect being to shift the nomogram up or down along the vertical axis (relative variance  $\sigma^2$  FSE) of the graph.

The average trend line for nomograms for a fixed value of  $K$  (in this case  $K=100$ ), and different values of alpha as shown in Figure 12, causes a rotation of the nomogram about a common axis.

The overall changes in the trends associated with the nomograms for changes in alpha and  $K$ , simply serves to emphasize that the principal reason for differences between the DSA and heterogeneity test methods is due to the use of different values for the exponent. It is likely that a portion of the substantial difference in solutions is due to GSE which becomes very large closer to the liberation size<sup>3</sup>.

Evidence from this initial study into the different methods for calculating experimental values for  $K$  and alpha ( $\alpha$ ), indicates that the values derived using the HT may be appropriate for a specific fragment size, but that such values for  $K$  are not transferable to other fragment sizes, even within the same material. The DSA approach by contrast provides  $K$  and alpha values that are applicable to a given material across a wide spectrum of comminution sizes, and these experimental values would therefore qualify as true 'constants'. While the proponents of the different approaches to establishment of the sampling constants have good

reasons for their choice of exponent, it is clear that the broader mining industry requires clarity on an issue as important as this.

One reason that using an exponent of  $\alpha = 2.5$  does not provide a useful result is the way in which it is linked to the shape of the model curves for the liberation factor. Model curves for the liberation factor described by Equation [10] are shown in Figure 13 for a range of different liberation sizes for critical components ( $d\ell$  from 10 to 10 000 m) using an exponent of 2.5, i.e. this is the simple ratio of  $d\ell/d_N$ . For example a 1 000 m grain of the critical component is totally liberated at the point where  $d_N = 0.1$  cm. Likewise a 50 m grain of gold would be totally liberated at the point where  $d_N$

Table XIII Sensitivity data for the Kloof data using DSA and gold grain liberation sizes				
DSA method	Grade	K	Alpha	Gold grain size, $d\ell$ (m)
Raw data no outliers removed	61.78	580.59	0.51	1995
Outliers removed	62.82	409.37	0.93	1218
Percentage adjustments	62.82	293.04	1.25	686
Last point removed	62.46	309.00	2.00	97

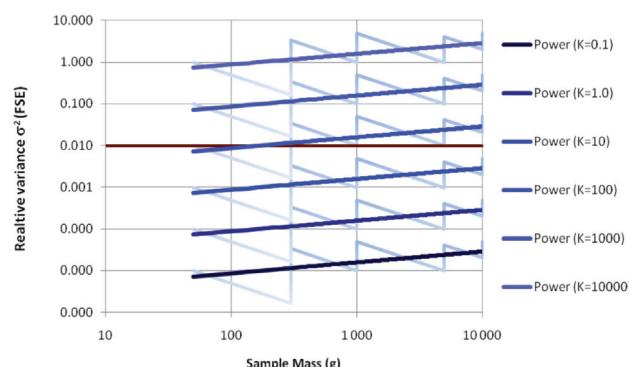


Figure 11—Nomograms shift along the Y-axis due as the sampling constant  $K$  changes

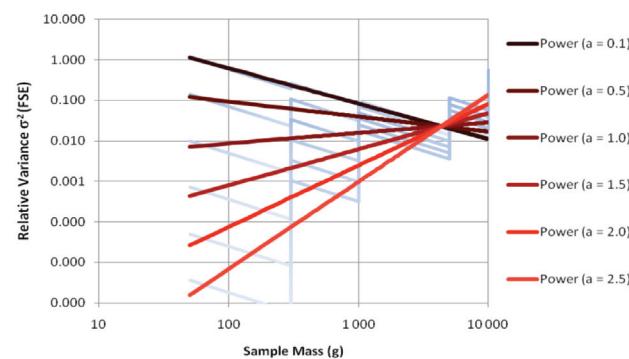


Figure 12—Nomograms rotate about a common axis as the exponent alpha changes; for the heterogeneity test the exponent is fixed at 2.5

## A comparison between the duplicate series analysis method

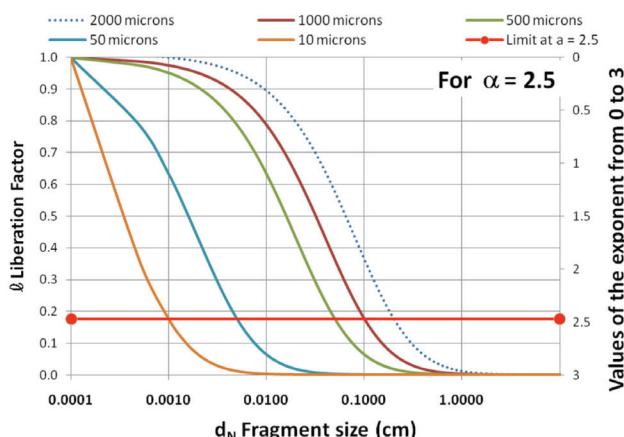


Figure 13—Model curves for the liberation factor when  $d$  ranges from 10 m to 2 000 m and the exponent is 2.5 implying the empirical liberation factor shown in Equation [6]

$= 0.005$  cm. These points lie along the horizontal line shown as the limit at  $\alpha = 2.5$ . This means that the curves above this line have no meaning and that the maximum liberation size could at most be about 0.18 (Figure 13). This restricts the calculation of the FSE to a limit where  $\ell = 0.18$ , when in fact the FSE may in truth be much higher.

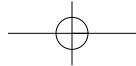
Other consideration that have emerged from this study apply principally to the DSA method and include the importance of evaluating the effect of outliers on changes in variance, especially at the smaller fragment sizes. In order to reduce the impact of outliers, it is suggested that the number of samples used to evaluate each series should be increased from 32 to 64 samples. The impact of errors arising from the extraction of the 50 g aliquot from the pulverized ores has to be corrected using the pulverized stage series. Good deportment studies of the ores provides a range of gold grain sizes, and helps to confirm and validate the sampling parameters. Finally, any method that uses series of analyses must include the basic quality assurance, quality control framework through introducing duplicates, standards, and blanks in the analytical procedure.

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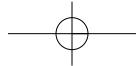
## A comparison between the duplicate series analysis method

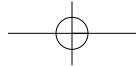
### Appendix

#### Tables A1–A6

Table A1  
Sample mass and grade data (g/t) from Mponeng Mine for DSA

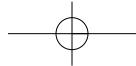
Size (cm)	Series 1		Series 2		Series 3		Series 4	
	5.00		1.50		0.50		0.15	
Sample no.	Mass (g)	Grade						
1	493.5	8.82	570.7	13.45	450.2	14.52	568.0	15.49
2	448.5	20.48	513.3	12.46	450.0	13.86	540.5	14.98
3	486.0	19.17	497.3	13.76	453.0	14.96	510.5	15.17
4	491.5	11.35	540.5	14.74	449.0	14.17	526.5	14.67
5	552.0	15.21	580.9	16.28	456.5	13.87	449.5	15.62
6	640.5	19.46	698.0	12.64	464.5	16.49	539.0	15.73
7	558.5	8.33	536.0	12.98	465.0	14.97	416.5	13.98
8	489.5	12.82	546.0	11.75	473.0	17.08	489.0	15.23
9	568.0	10.70	526.0	14.61	466.5	16.17	468.0	14.87
10	<b>532.0</b>	<b>26.71</b>	542.0	14.22	473.0	14.86	453.5	14.87
11	533.5	11.56	562.0	16.05	451.5	13.61	463.5	15.30
12	457.0	8.86	503.0	15.55	464.5	15.01	468.5	14.12
13	600.0	14.72	588.5	15.23	476.5	15.09	493.0	14.88
14	688.0	9.13	559.5	13.48	471.5	15.66	461.0	16.45
15	673.0	18.59	<b>600.0</b>	<b>19.90</b>	471.5	15.00	<b>558.0</b>	<b>13.36</b>
16	612.5	15.88	675.5	15.40	456.5	13.71	520.5	15.30
17	627.0	12.20	580.0	13.43	444.5	15.91	484.5	15.94
18	598.5	8.72	524.5	13.19	456.0	15.10	497.0	15.71
19	557.0	13.35	531.5	11.17	457.0	16.65	551.0	14.36
20	527.0	17.53	533.0	15.59	453.0	15.62	515.5	15.42
21	508.5	10.90	557.5	14.14	466.0	16.20	543.5	16.26
22	480.0	12.41	609.5	12.53	479.5	17.85	539.0	16.88
23	532.5	9.89	522.5	12.26	467.5	16.94	454.0	15.20
24	549.5	8.00	364.5	13.03	471.5	14.87	444.0	16.33
25	718.5	12.43	508.0	18.57	471.5	16.84	584.5	14.92
26	552.5	16.41	602.5	16.78	472.0	15.14	445.0	14.58
27	573.0	15.18	629.0	13.51	443.0	17.84	<b>522.0</b>	<b>13.22</b>
28	482.5	20.29	553.0	16.00	449.5	15.49	440.5	14.72
29	747.5	16.49	559.0	11.56	<b>492.5</b>	<b>11.29</b>	580.5	15.49
30	656.5	21.88	569.0	11.36	492.5	14.47	545.0	15.69
Mean		13.82		13.99		15.45		15.29
Variance		16.7231		3.1257		1.3448		0.4651
Std dev.		4.0894		1.7680		1.1597		0.6820
RSD		0.2959		0.1264		0.0751		0.0446
Average mass (g)	564.5		556.1		463.6		502.4	
Size (cm)	5.00		1.50		0.50	0.150		





## A comparison between the duplicate series analysis method

Table A2 Sample mass and grade data (g/t) from Kloof Mine for DSA											
Series 1 (1.90 cm)			Series 2 (1.27 cm)			Series 3 (0.67 cm)			Series 4 (0.20 cm)		
No.	Mass (g)	Assays g/t	No.	Mass (g)	Assays g/t	No.	Mass (g)	Assays g/t	No.	Mass (g)	Assays g/t
KL1	344.9	52.5	KL33	296.6	49.7	KL65	326.8	71.0	KL97	307.1	60.6
KL2	290.3	53.7	KL34	304.7	50.2	KL66	329.3	54.7	<b>KL98</b>	<b>308.0</b>	<b>20.3</b>
KL3	365.9	58.6	KL35	339.9	79.2	KL67	325.9	60.2	KL99	305.1	75.0
KL4	287.0	46.5	KL36	317.7	50.1	KL68	334.0	60.7	KL100	310.8	57.1
KL5	340.2	42.2	KL37	298.3	60.7	KL69	320.4	67.2	KL101	293.6	60.2
KL6	287.3	54.9	KL38	311.7	68.9	KL70	313.8	65.7	KL102	293.1	63.6
KL7	314.1	59.7	KL39	298.0	63.2	KL71	331.4	72.0	KL103	305.1	65.6
KL8	324.7	40.5	KL40	293.3	73.1	KL72	313.2	65.0	KL104	298.7	55.8
KL9	274.0	37.7	KL41	308.3	65.2	KL73	314.3	60.6	KL105	311.2	56.2
<b>KL10</b>	<b>358.3</b>	<b>5.0</b>	KL42	298.2	63.5	KL74	316.9	56.7	KL106	308.6	58.7
KL11	323.2	68.5	KL43	302.3	77.1	KL75	317.4	70.0	KL107	305.9	56.7
KL12	286.7	91.9	KL44	319.4	65.2	KL76	308.8	57.4	KL108	302.8	60.7
KL13	329.2	76.9	KL45	326.5	69.4	KL77	311.1	61.2	KL109	309.9	60.5
KL14	274.1	60.8	KL46	313.6	73.4	KL78	305.7	59.0	KL110	306.1	74.8
KL15	286.0	80.9	KL47	338.7	76.0	KL79	317.1	60.5	KL111	301.7	58.2
KL16	262.7	39.9	KL48	307.2	53.9	KL80	311.5	63.7	KL112	293.7	70.4
KL17	335.7	78.6	KL49	315.9	50.9	KL81	313.3	66.7	KL113	314.9	62.4
KL18	308.9	67.9	KL50	317.5	76.8	KL82	314.5	66.5	KL114	314.0	62.4
KL19	322.6	41.7	KL51	315.3	64.2	KL83	314.9	66.8	KL115	312.0	57.9
KL20	322.6	72.2	KL52	327.4	72.3	KL84	314.8	62.9	KL116	309.4	68.7
KL21	340.0	51.0	KL53	325.1	82.4	KL85	322.0	52.1	KL117	318.1	61.9
KL22	302.8	59.8	KL54	303.2	60.4	KL86	316.9	74.6	KL118	318.0	56.2
KL23	353.0	61.1	KL55	320.4	61.4	KL87	326.9	75.6	KL119	318.0	59.8
KL24	340.9	74.9	KL56	301.7	57.6	KL88	328.4	65.0	KL120	311.0	52.4
KL25	292.4	43.3	KL57	327.5	48.9	KL89	317.7	66.0	KL121	320.1	58.1
KL26	285.6	73.9	KL58	291.2	58.3	KL90	317.2	64.5	KL122	316.5	56.2
<b>KL27</b>	<b>318.8</b>	<b>23.7</b>	KL59	336.8	57.4	KL91	311.7	68.9	KL123	314.2	60.0
KL28	321.1	50.9	KL60	319.3	66.4	KL92	318.6	64.5	<b>KL124</b>	<b>311.3</b>	<b>78.7</b>
KL29	399.5	83.3	KL61	320.4	64.6	KL93	324.9	79.1	KL125	313.3	53.7
KL30	347.2	92.9	KL62	312.7	62.7	KL94	304.3	57.3	KL126	313.2	57.7
KL31	328.5	82.7	KL63	309.3	69.0	<b>KL95</b>	<b>318.1</b>	<b>47.9</b>	KL127	310.3	56.6
KL32	295.9	45.1	KL64	322.8	75.8	KL96	314.4	71.3	KL128	299.0	54.3
Mean		61.48			64.62			64.75			60.41
Variance		258.8147			85.7236			37.9902			30.7804
Std dev.		16.0877			9.2587			6.1636			5.5480
RSD		0.2617			0.1433			0.0952			0.0918
Average mass (g)		317.6			313.8			318.0			308.6
Size (cm)		1.90			1.27			0.67			0.20

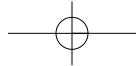


## A comparison between the duplicate series analysis method

Table A3

Sample mass and grade data (g/t) from Lily Mine for DSA

Series 1 (1.90 cm)			Series 2 (0.95 cm)			Series 3 (0.475 cm)			Series 4 (0.20 cm)		
No	Mass (g)	Assay g/t	No	Mass (g)	Assay g/t	No	Mass (g)	Assay g/t	No	Mass (g)	Assay g/t
ST01	249.6	0.92	ST33	307.8	1.12	ST65	251.6	0.85	ST097	237.3	1.21
ST02	279.1	0.82	ST34	311.5	1.04	ST66	247.1	1.15	ST098	242.5	1.04
ST03	284.0	1.62	ST35	298.2	1.22	ST67	248.3	0.96	ST099	247.1	0.92
ST04	280.4	0.45	ST36	295.8	1.20	ST68	255.1	0.94	ST100	250.4	1.16
ST05	238.0	0.60	ST37	266.7	1.18	ST69	226.3	0.79	ST101	272.6	0.97
ST06	243.3	0.83	ST38	292.0	1.49	ST70	226.4	0.85	ST102	269.6	1.07
ST07	258.5	0.93	ST39	253.2	1.22	ST71	205.8	0.85	ST103	260.1	1.20
ST08	268.4	1.06	ST40	253.7	0.98	ST72	202.0	0.73	ST104	271.2	1.06
ST09	284.8	0.79	ST41	310.8	0.90	ST73	254.1	1.07	ST105	272.1	0.87
ST10	222.7	0.42	ST42	314.4	0.90	ST74	254.9	0.81	ST106	277.2	1.23
ST11	231.7	0.72	ST43	343.8	1.73	ST75	249.3	0.86	ST107	245.7	1.12
ST12	253.9	0.71	ST44	300.7	0.83	ST76	243.5	1.03	ST108	256.7	1.01
ST13	281.9	0.70	ST45	318.5	1.69	ST77	291.8	1.14	ST109	281.5	0.93
ST14	260.4	1.17	ST46	333.6	0.78	ST78	286.0	1.00	ST110	284.7	1.04
ST15	283.6	0.68	ST47	346.3	0.87	ST79	293.2	0.95	ST111	268.3	0.89
ST16	279.8	1.44	ST48	253.4	0.95	ST80	288.9	0.02	ST112	278.3	1.10
ST17	278.3	0.88	ST49	257.8	1.49	ST81	222.3	0.72	ST113	189.5	0.84
ST18	258.6	0.97	ST50	262.3	1.48	ST82	219.1	1.02	ST114	194.6	1.22
ST19	281.5	1.00	ST51	249.6	0.78	ST083	208.1	0.99	ST115	182.0	1.03
ST20	262.8	0.82	ST52	263.8	0.95	ST084	221.8	0.99	ST116	192.4	1.32
ST21	264.3	1.65	ST53	269.6	0.80	ST085	241.3	0.32	ST117	196.5	0.86
ST22	253.7	0.77	ST54	284.7	1.56	ST086	239.6	0.93	ST118	198.6	0.82
ST23	262.0	0.73	ST55	262.2	1.03	ST087	239.9	1.07	ST119	198.4	1.15
ST24	279.2	0.69	ST56	258.0	1.36	ST088	241.3	0.80	ST120	209.0	2.47
ST25	267.7	0.75	ST57	239.2	0.62	ST089	239.4	0.79	ST121	258.0	0.87
ST26	245.6	1.11	ST58	235.5	1.16	ST090	242.3	0.96	ST122	270.1	0.87
ST27	263.2	0.85	ST59	253.3	0.73	ST091	224.4	0.99	ST123	258.9	0.86
ST28	254.3	0.67	ST60	274.2	1.35	ST092	221.9	0.73	ST124	270.5	1.06
ST29	267.4	1.36	ST61	261.9	0.77	ST093	305.1	0.88	ST125	273.4	1.09
ST30	255.3	1.34	ST62	269.1	0.97	ST094	301.4	0.92	ST126	287.5	1.18
ST31	263.6	0.89	ST63	251.6	0.97	ST095	272.9	1.05	ST127	274.9	0.97
ST32	273.3	0.93	ST64	259.1	1.47	ST096	285.0	0.50	ST128	283.1	0.95
Mean	263.47	0.8667		279.76	1.0780		248.44	0.9107		248.52	1.0197
Variance		0.0567			0.0742			0.0192			0.0160
Std dev.		0.2381			0.2724			0.1387			0.1263
RSD		0.2748			0.2527			0.1523			0.1239
Size (cm)		1.90			0.95			0.475			0.20

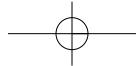


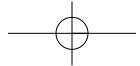
## A comparison between the duplicate series analysis method

Table A4

Sample mass and grade (g/t) for heterogeneity test from Mponeng gold mine

Mass (g)	Grade	Mass x grade	IHL	No.	Mass (g)	Grade	Mass x grade	IHL	No.
100.05	1.27	127.064	0.047	1	<b>100.02</b>	<b>1.21</b>	<b>121.024</b>	<b>0.065</b>	<b>55</b>
100.02	2.05	205.041	0.069	2	99.97	1.5	149.955	0.006	56
100.01	2.13	213.021	0.097	3	99.97	1.27	126.962	0.047	57
100.03	1.56	156.047	0.002	4	99.94	1.49	148.911	0.007	58
99.99	1.23	122.988	0.059	5	100.04	1.47	147.059	0.009	59
99.96	1.75	174.930	0.006	6	100.01	1.45	145.015	0.011	60
100.03	1.8	180.054	0.012	7	99.96	1.58	157.937	0.001	61
100.05	2.33	233.117	0.190	8	99.99	1.89	188.981	0.027	62
99.95	1.56	155.922	0.002	9	100.04	2.25	225.090	0.149	63
99.97	1.62	161.951	0.000	10	100	1.78	178.000	0.009	64
100	2.03	203.000	0.063	11	<b>99.95</b>	<b>1.35</b>	<b>134.933</b>	<b>0.028</b>	<b>65</b>
99.95	1.66	165.917	0.001	12	<b>100.02</b>	<b>1.46</b>	<b>146.029</b>	<b>0.010</b>	<b>66</b>
100.05	1.79	179.090	0.011	13	<b>100.05</b>	<b>1.23</b>	<b>123.062</b>	<b>0.059</b>	<b>67</b>
99.98	1.49	148.970	0.007	14	<b>99.96</b>	<b>1.34</b>	<b>133.946</b>	<b>0.030</b>	<b>68</b>
99.96	1.08	107.957	0.112	15	99.96	2.29	228.908	0.169	69
100.02	1.88	188.038	0.025	16	100.07	1.95	195.137	0.041	70
100.05	1.69	169.085	0.002	17	100.06	2.42	242.145	0.241	71
99.95	1.47	146.927	0.009	18	100.03	1.18	118.035	0.075	72
100.03	1.48	148.044	0.008	19	99.95	1.39	138.931	0.021	73
100	1.73	173.000	0.004	20	100.01	1.49	149.015	0.007	74
99.99	1.9	189.981	0.029	21	99.99	1.42	141.986	0.016	75
100.05	1.48	148.074	0.008	22	100.01	1.33	133.013	0.033	76
99.96	2.16	215.914	0.109	23	99.99	1.74	173.983	0.005	77
100	1.98	198.000	0.048	24	99.98	2.32	231.954	0.184	78
99.98	1.63	162.967	0.000	25	100.05	1.74	174.087	0.005	79
99.96	2.36	235.906	0.206	26	100.05	1.4	140.070	0.019	80
99.99	1.65	164.984	0.000	27	99.97	2.01	200.940	0.057	81
100.05	2.05	205.103	0.069	28	100.03	1.26	126.038	0.050	82
100.02	1.87	187.037	0.023	29	100.04	1.22	122.049	0.062	83
100.05	1.39	139.070	0.021	30	100.06	2.06	206.124	0.072	84
100.06	1.27	127.076	0.047	31	100.01	2.03	203.020	0.063	85
100.01	2.34	234.023	0.195	32	99.97	1.63	162.951	0.000	86
99.98	1.21	120.976	0.065	33	99.95	1.72	171.914	0.004	87
100.05	1.78	178.089	0.009	34	100.01	1.41	141.014	0.017	88
99.99	1.33	132.987	0.033	35	100.05	1.81	181.091	0.013	89
100.06	1.77	177.106	0.008	36	100	1.26	126.000	0.050	90
100.04	1.14	114.046	0.089	37	100.06	1.87	187.112	0.023	91
99.94	1.79	178.893	0.011	38	99.95	1.33	132.934	0.033	92
99.95	0.99	98.951	0.152	39	100.03	1.7	170.051	0.002	93
99.95	1.35	134.933	0.028	40	100	1.43	143.000	0.014	94
100.06	1.59	159.095	0.000	41	100.02	2.08	208.042	0.079	95
100.03	1.45	145.044	0.011	42	99.97	1.45	144.957	0.011	96
100.02	1.19	119.024	0.071	43	100.01	1.18	118.012	0.075	97
100.01	1.93	193.019	0.036	44	99.96	1.24	123.950	0.056	98
99.98	1.5	149.970	0.006	45	100.01	2.2	220.022	0.126	99
99.96	1.99	198.920	0.051	46	100.04	2.07	207.083	0.076	100
99.95	1.07	106.947	0.116	47	100.004	1.6233	100.01	50	
99.97	1.69	168.949	0.002	48	10000.4	162.33	IHL =	4.04	
99.95	1.49	148.926	0.007	49			Average mass =	100.00	
100.05	1.21	121.061	0.065	50			Average grade =	1.66	
100.02	1.95	195.039	0.041	51			Variance =	0.1204	
<b>100.06</b>	<b>1.35</b>	<b>135.081</b>	<b>0.028</b>	<b>52</b>			Relative variance =	0.0439	
99.96	0.69	68.972	0.330	53			C =	3.790	
100.04	1.29	129.052	0.042	54			K =	3.88	

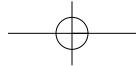




## A comparison between the duplicate series analysis method

Table A5  
Sample Mass and Grade (g/t) for heterogeneity test from Kloof gold mine

Mass (g)	Grade	Mass x grade	IHL	No.	Mass (g)	Grade	Mass x grade	IHL	No.
49.86	25.38	1265.511	0.364	1	49.76	38.20	1901.081	0.163	55
49.96	6.71	335.106	0.800	2	49.96	122.79	6134.917	0.833	56
49.96	83.94	4193.874	0.095	3	49.96	115.03	5747.379	0.627	57
50.56	32.13	1624.859	0.251	4	49.96	52.35	2615.591	0.034	58
50.56	49.19	2487.466	0.055	5	50.16	38.36	1924.318	0.162	59
49.96	6.89	344.029	0.795	6	49.96	50.46	2521.244	0.045	60
50.36	99.08	4990.216	0.298	7	49.96	43.39	2168.086	0.104	61
50.36	102.75	5174.887	0.364	8	50.36	56.72	2856.674	0.014	62
49.96	27.71	1384.316	0.322	9	49.76	21.89	1089.510	0.431	63
50.06	69.82	3495.295	0.008	10	50.06	16.31	816.409	0.556	64
50.26	21.18	1064.678	0.450	11	49.76	127.96	6368.005	0.982	65
50.06	35.77	1790.815	0.196	12	50.46	46.06	2324.563	0.080	66
50.46	77.99	3935.550	0.047	13	50.26	34.31	1724.739	0.217	67
50.06	24.15	1209.288	0.388	14	50.16	18.69	937.330	0.503	68
49.86	84.38	4207.376	0.099	15	50.06	113.07	5660.636	0.581	69
49.96	70.74	3534.695	0.011	16	50.26	63.44	3188.612	0.000	70
49.76	10.41	518.211	0.697	17	50.06	27.91	1397.120	0.319	71
<b>49.86</b>	<b>146.24</b>	<b>7291.927</b>	<b>1.629</b>	<b>18</b>	49.96	45.62	2279.257	0.083	72
50.36	51.92	2615.086	0.037	19	50.06	84.89	4250.014	0.104	73
50.26	73.65	3701.971	0.022	20	50.06	64.35	3221.408	0.000	74
49.76	103.40	5145.580	0.372	21	49.76	34.32	1707.946	0.215	75
49.76	47.02	2340.142	0.071	22	50.06	22.27	1114.992	0.426	76
<b>50.36</b>	<b>215.62</b>	<b>10859.315</b>	<b>5.602</b>	<b>23</b>	49.96	38.17	1906.938	0.164	77
50.16	13.11	657.795	0.634	24	50.46	10.17	513.096	0.713	78
50.26	121.73	6118.795	0.808	25	50.26	109.07	5482.227	0.492	79
50.16	18.44	925.035	0.508	26	49.86	47.76	2381.554	0.065	80
<b>50.26</b>	<b>216.44</b>	<b>10879.433</b>	<b>5.652</b>	<b>27</b>	49.96	59.23	2959.351	0.006	81
49.86	47.55	2370.903	0.067	28	49.96	99.88	4990.210	0.309	82
50.26	59.81	3006.389	0.005	29	50.26	53.33	2680.565	0.029	83
50.36	81.72	4115.590	0.075	30	50.06	29.50	1476.837	0.292	84
49.96	52.40	2618.350	0.033	31	50.06	81.09	4059.910	0.070	85
49.86	29.02	1447.107	0.299	32	50.16	44.95	2254.800	0.090	86
50.06	29.15	1459.219	0.298	33	49.96	50.90	2543.163	0.043	87
50.06	18.87	944.799	0.498	34	50.26	100.04	5028.426	0.314	88
50.36	26.67	1343.216	0.343	35	50.36	113.44	5713.413	0.593	89
50.46	55.73	2812.305	0.017	36	50.26	90.42	4544.964	0.168	90
49.86	73.75	3677.533	0.022	37	49.86	132.27	6595.492	1.122	91
49.96	38.94	1945.550	0.154	38	50.16	14.16	710.347	0.608	92
50.16	98.58	4945.263	0.288	39	50.16	74.28	3726.094	0.025	93
<b>49.76</b>	<b>159.23</b>	<b>7924.130</b>	<b>2.181</b>	<b>40</b>	49.96	76.85	3839.907	0.039	94
49.86	45.23	2255.537	0.087	41	50.06	34.23	1713.716	0.217	95
49.76	9.56	475.792	0.719	42	50.16	32.51	1631.087	0.244	96
49.86	76.39	3808.955	0.036	43	49.86	44.35	2211.322	0.095	97
50.26	69.99	3517.753	0.008	44	50.36	111.28	5604.510	0.542	98
<b>49.76</b>	<b>171.66</b>	<b>8542.576</b>	<b>2.789</b>	<b>45</b>	50.26	60.00	3015.858	0.004	99
50.06	9.15	458.073	0.735	46	49.86	33.32	1661.489	0.230	100
<b>50.26</b>	<b>209.80</b>	<b>10545.230</b>	<b>5.170</b>	<b>47</b>	50.09	64.16	50.10		
50.06	87.55	4383.045	0.133	48	5008.63	6415.73	IHL =	31.70	
49.76	3.94	196.176	0.875	49			Average mass =	50.11	
50.46	90.66	4575.306	0.172	50			Average grade =	56.35	
50.06	81.97	4103.706	0.077	51			Variance	1071.69	
50.06	34.10	1706.988	0.219	52			Relative Variance	0.3375	
50.26	61.88	3110.230	0.001	53			C	14.609	
50.16	77.07	3866.319	0.041	54			K	14.97	



## A comparison between the duplicate series analysis method

Table A6

Sample mass and grade (g/t) for heterogeneity test from Lily gold mine

Mass (g)	Grade	Mass x grade	IHL	No.	Mass (g)	Grade	Mass x grade	IHL	No.
101.40	0.37	37.518	0.329	1	100.60	0.46	46.276	0.218	55
100.30	1.20	120.360	0.152	2	<b>101.40</b>	<b>2.94</b>	<b>298.116</b>	<b>5.826</b>	<b>56</b>
101.50	0.34	34.510	0.370	3	100.70	1.66	167.162	0.852	57
97.30	1.00	97.300	0.024	4	100.80	1.13	113.904	0.096	58
<b>102.00</b>	<b>1.88</b>	<b>191.760</b>	<b>1.405</b>	<b>5</b>	100.40	0.41	41.164	0.275	59
100.60	0.67	67.402	0.050	6	<b>100.80</b>	<b>2.44</b>	<b>245.952</b>	<b>3.339</b>	<b>60</b>
100.60	0.31	31.186	0.410	7	101.40	1.13	114.582	0.096	61
100.70	0.75	75.525	0.017	8	101.40	0.33	33.462	0.384	62
100.10	0.23	23.023	0.535	9	100.80	0.53	53.424	0.149	63
102.40	0.47	48.128	0.211	10	101.00	0.37	37.370	0.327	64
100.80	1.53	154.224	0.597	11	100.40	0.40	40.160	0.287	65
100.70	0.35	35.245	0.353	12	100.40	0.41	41.164	0.275	66
102.10	0.52	53.092	0.160	13	100.30	0.29	29.087	0.439	67
102.00	0.42	42.840	0.267	14	100.40	0.61	61.244	0.086	68
100.60	0.63	63.378	0.073	15	100.80	0.51	51.408	0.168	69
100.20	0.55	55.110	0.131	16	101.20	1.22	123.464	0.172	70
100.60	0.47	47.282	0.207	17	100.80	0.99	99.792	0.022	71
100.00	0.44	44.000	0.239	18	101.40	1.60	162.240	0.733	72
100.00	0.42	42.000	0.262	19	<b>101.00</b>	<b>2.12</b>	<b>214.120</b>	<b>2.125</b>	<b>73</b>
100.10	0.40	40.040	0.286	20	100.60	0.72	72.432	0.027	74
<b>99.80</b>	<b>2.87</b>	<b>286.426</b>	<b>5.354</b>	<b>21</b>	101.80	0.61	62.098	0.087	75
102.00	0.54	55.080	0.142	22	100.50	1.57	157.785	0.669	76
102.00	0.67	68.340	0.051	23	102.23	0.18	18.401	0.636	77
100.50	0.59	59.295	0.100	24	100.40	0.53	53.212	0.148	78
100.60	1.01	101.606	0.029	25	102.00	0.76	77.520	0.014	79
101.30	0.85	86.105	0.000	26	100.80	1.12	112.896	0.089	80
99.60	1.32	131.472	0.277	27	100.60	0.45	45.270	0.229	81
<b>100.60</b>	<b>2.83</b>	<b>284.698</b>	<b>5.184</b>	<b>28</b>	100.60	1.16	116.696	0.118	82
100.60	0.53	53.318	0.149	29	100.25	0.47	47.118	0.206	83
100.50	0.74	74.370	0.020	30	100.37	0.64	64.237	0.067	84
101.20	0.55	55.660	0.132	31	100.72	0.45	45.324	0.229	85
100.40	0.43	43.172	0.251	32	101.44	1.08	109.555	0.064	86
100.90	0.96	96.864	0.013	33	100.59	0.40	40.236	0.288	87
100.60	1.75	176.050	1.054	34	100.29	0.98	98.284	0.018	88
100.70	0.55	55.385	0.132	35	100.36	0.22	22.079	0.553	89
100.80	1.78	179.424	1.129	36	100.42	0.45	45.189	0.228	90
<b>100.50</b>	<b>2.20</b>	<b>221.100</b>	<b>2.392</b>	<b>37</b>	100.78	0.52	52.406	0.158	91
100.80	0.52	52.416	0.158	38	100.67	0.90	90.603	0.002	92
<b>100.90</b>	<b>2.33</b>	<b>235.097</b>	<b>2.892</b>	<b>39</b>	99.81	0.69	68.869	0.040	93
100.70	0.57	57.399	0.115	40	100.25	0.58	58.145	0.107	94
100.80	0.89	89.712	0.001	41	100.81	0.20	20.162	0.591	95
100.80	0.39	39.312	0.301	42	100.20	0.59	59.118	0.100	96
100.50	0.60	60.300	0.093	43	100.73	2.20	221.606	2.398	97
100.80	0.75	75.600	0.017	44	100.40	0.18	18.072	0.624	98
101.20	0.65	65.780	0.061	45	100.02	0.34	34.007	0.365	99
100.90	0.27	27.243	0.473	46	100.24	1.17	117.281	0.126	100
101.58	0.42	42.664	0.266	47	100.75	0.86	100.75		
<b>100.30</b>	<b>2.01</b>	<b>201.603</b>	<b>1.757</b>	<b>48</b>	<b>10074.66</b>	<b>86.32</b>	IHL =	28.70	
100.50	1.22	122.610	0.170	49			Average mass =	100.73	
100.90	0.72	72.648	0.028	50			Average grade =	0.69	
100.80	0.61	61.488	0.086	51			Variance	0.15	
100.60	0.71	71.426	0.031	52			Relative variance	0.31	
100.10	0.29	29.029	0.438	53			C	26.77	
102.00	1.54	157.080	0.622	54			K	27.44	

