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Revised simplex method: An alternative solver to the out-of-kilter method for water balance model decision support system

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Water resource decision support systems are used to support the management of water resource planning networks and catchments. For years, the out-of-kilter algorithm has been the only solver used in the Water Resource Yield Model. The out-of-kilter algorithm can solve a wide range of network problems, including transportation, assignment, shortest path, and caterer problems. However, the Water Balance Model decision support system introduced the revised simplex algorithm, which was used for comparison. The revised simplex algorithm is well suited to large, sparse problems and is widely regarded as fast and robust in practice. This research seeks to revive, explore, and improve the understanding of the decision support system by evaluating the revised simplex algorithm as an alternative to the out-of-kilter algorithm for solving water resource catchment networks. The findings indicate that the out-of-kilter algorithm focuses solely on supply costs, whereas the revised simplex algorithm considers both supply and sink nodes, resulting in improved yield and reduced supply costs. Furthermore, the revised simplex algorithm prioritises satisfying demand from the lowest-cost available sources and avoids using the pumping channel if water can be sourced from elsewhere.

Keywords: out-of-kilter algorithm, revised simplex algorithm, Water Balance Model Decision Support System, Water Resource Yield Model, water resource catchment management planning

INTRODUCTION

Water resource decision support systems (DSS) have been used to improve the management of water resource planning networks and catchments. The development of the Water Balance Model (WBM) DSS, along with an alternative algorithm for existing catchments across South Africa, the Southern African Development Community, and potentially globally, offers an effective solution to current technological limitations. Additionally, the Water Resource Yield Model (WRYM) DSS is used to calculate the volume of water that should be stored or made available to satisfy a specified demand with a designated level of reliability for water catchment systems (Brown *et al* 2015, Coleman 2007, Coleman & Pitman 2008, de Jager *et al* 2004, Jeleni *et al* 2011, Nkwonta *et al* 2017, Nyland & de Jager 2023). The WRYM DSS has a long history of development and application in water resource catchment studies, building on the WBM DSS, and has been improved to reflect the latest advancements of the Industrial Revolution. This is the same technology that has led to the creation of DSSs such as WRYM DSS and many others used in the water resources industry. The revolution of the 1760s was called the First Industrial Revolution, followed by the Second Industrial Revolution around the 1840s. The 1950s introduced the Third Industrial Revolution to the world and the beginning of the WRYM DSS. This third phase served as a stepping stone to the Fourth Industrial Revolution or Industry 4.0, also known as the 4IR.

The characteristics of technology integration from the Third to the Fourth Industrial Revolutions are blurring the lines between the physical, digital, and biological



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realms, presenting both new possibilities and challenges (Datta 2023, Lee *et al* 2018, Sarni *et al* 2018, Schwab 2018, Xu *et al* 2018). The 4IR has significantly impacted the development of WBM DSS technology and policy, as key factors such as artificial intelligence, cloud computing, and the Internet of Things are driving the current team's progress. Among the many innovations from the 4IR, cloud technology offers connectivity, big data, and remarkable computing power. Additionally, advanced engineering provides options for 3D viewing, printing, or renewable energy.

In programming, languages are used to write, analyse, and express algorithms. These algorithms serve as blueprints specifying how a computer should perform specific tasks. Programming languages act as the tools for creating these instructions in a format that computers can understand and execute. Different algorithms are implemented across various water resource DSSs for different purposes, including Lagrangian, out-of-kilter, Monte Carlo, linear programming, interior point methods, and genetic algorithms. The WBM DSS uses the out-of-kilter algorithm, adopted from the WRYM DSS, and includes an alternative revised simplex algorithm, while MODSIM employs the Lagrangian algorithm and incorporates the simplex algorithm. The Water Evaluation and Planning tool uses the primal-dual simplex method, and Mike Basin implements the genetic algorithm (De Carvalho *et al* 2009, Comley 1996, Doulgeris *et al* 2015, Fulkerson 1961, Graham & Butts 2005, Labadie 2006, Nel & Haarhoff 1996, Nyland & de Jager 2023, Vanderbei 2020).

The WBM DSS serves as a long-term planning tool, enabling users to analyse water resources using custom scenarios. Its primary aim is to balance water supply and demand using a water balance method within any selected catchment or group of catchments (Galántai 2021, Nelder & Mead 1965, Nyland & de Jager 2023, Syifa *et al* 2023).

A common method for addressing these issues is the out-of-kilter algorithm, specifically designed to solve the minimum-cost flow problem. It works alongside the primal-dual algorithm, relying on linear programming principles. The main benefit of the out-of-kilter algorithm is its ability to find a circulation within the network that maintains water flow at each reservoir, ensuring that the flow in each channel or river stays within specified maximum and minimum limits while minimising the total cost across the network (Bailes 1989, Brown 2023a, Brown 2023b, Dantzig 1990, Nelder & Mead 1965, Protsenko 2023, Syifa *et al* 2023, Tejera-Martínez *et al* 2020, Vanderbei 2014).

During the 1970s, two main algorithms for linear network optimisation gained popularity: the simplex method and its variants, and the primal-dual method, along with the out-of-kilter algorithm which is closely related. The 1980s saw the emergence of additional techniques, such as the relaxation and auction algorithms. The relaxation algorithm resembles unconstrained nonlinear optimisation and often outperforms both the simplex and primal methods in many linear programming tasks. Conversely, the auction algorithm utilises principles of complementary slackness and scaling.

The complexity of a water resource network, combined with computational analysis and the use of the out-of-kilter algorithm, made it a more practical algorithm for WRYM DSS compared to the traditional linear programming algorithm (Dantzig 1990, Fulkerson 1961, Nelder & Mead 1965).

As an alternative to the out-of-kilter algorithm, the WBM DSS offers a second option: the revised simplex algorithm, which is based on the simplex algorithm. The simplex algorithm is based on the idea of gradually moving towards one of the nodes or points on the catchment network diagram (Dantzig 1990, Nyland & de Jager 2023). When water flows from one node to another, it encounters the boundary defined by the linear programming constraints. At this boundary, each node represents an extreme point where the simplex algorithm adjusts the non-basic variables flowing to the next node until the optimal solution is reached (Dantzig 1990, Galántai 2021, Nelder & Mead 1965, Sánchez 2006).

When solving a linear programming problem with a digital computer, the simplex algorithm requires storing the entire table in memory. In contrast, the revised simplex algorithm is more memory-efficient because it only computes and stores data needed for the current step to test or improve the solution. The revised simplex algorithm aims to avoid the need to compute the complete dictionary after each pivot. Instead, it focuses on tracking specific information, such as the current basis, and uses matrix formulas to calculate the necessary parts of the dictionary (Arsham 1997, Chivers & Sleightholme 2006, Dantzig 1990, Nelder & Mead 1965, Tejera-Martínez *et al* 2020). There are two options for the revised simplex algorithm (Al-Mhanna *et al* 2023, Arsham 1997, Dantzig 1990, Galántai 2021, Nadler 1973, Nelder & Mead 1965, Syifa *et al* 2023):

1. The initial algorithm monitors only the current basis variables and solution, performing all computations with the inverse of the basis, which supplies a unique choice rule. By adding slack variables, an identity matrix is formed. This method ensures convergence and typically incurs minimal additional computational cost.
2. The second algorithm uses the standard simplex method, with a small adjustment to handle artificial variables. Instead of the usual two-phase approach, it is adapted to include the artificial variables necessary to create an identity matrix. During each cycle, the coefficients associated with the simplex variables are updated via straightforward recursion relations.

The website was written in PHP, uses an open-source relational management system (RMBMS) named My Structured Query Language (MySQL) database, and is deployed on a LAMP (Linux, Apache, MySQL, and PHP) stack. A LAMP stack is an open-source software stack used to build and deploy web-based decision support systems, with four components: the operating system (CentOS Linux version 8.0), the web server (Apache version 2.4), the database (MySQL version 8.0), and a programming language (PHP version 7.4). SQL is a programming language used to create, modify, and extract data from relational databases, as well as to control user access to the database (Hargreaves *et al* 2008, Mahachandra *et al* 2018, Martinovic *et al* 2012, Microsoft Windows 7 Administrator's Reference 2011, Windows Vista 2005, Tiensivu 2008).

Web communication has been broadly defined as the sharing of information among individuals or groups using interconnected computers and technology-based devices. Communication between the website and the WBM DSS is achieved through the hypertext transfer protocol (HTTP) call. HTTP is the foundation of data exchange on the World Wide Web, enabling seamless communication between clients (like web browsers) and servers (Bourhis *et al* 2020, Brahmia *et al* 2022, Chavarriaga *et al* 2023, Chua Hock-Chuan 2016, Dolnak & Litvik 2017, Frisbie 2023, Nyland & de Jager 2023). It functions as a request-response protocol in which DSS users submit data requests to a server, which then returns the requested

data. Users can configure each study within the model.

Data exchange enables the creation of a more efficient, optimised web-based DSS, with input data formatted in the structured JavaScript Object Notation (JSON) format. When the user accesses the uniform resource locator (URL) or web address of a webpage or file in a browser, a complete communication sequence is initiated between the browser and the server hosting the website. JSON is a lightweight data-interchange format used to transmit data in a web-based DSS, making it easy for humans to read and write (Bourhis *et al* 2020, Brahmia *et al* 2022, Chavarriaga *et al* 2023, Frisbie 2023, Nyland & de Jager 2023). The output data, referred to as export data, is formatted in JSON. The Internet Information Services (IIS) on the DSS server processes the request and forwards it to the PHP script. This script creates a COM object instance and passes the input JSON directly to it without parsing or interpreting it. The DSS receives and parses the JSON, then performs the simulation. After completing the simulation, the model organises the results into a JSON structure and returns it to the PHP script. The script then sends this data back to the website, which unpacks the simulation data from the JSON structure and stores it in the database (Bourhis *et al* 2020, Brahmia *et al* 2022).

METHODOLOGY

Research methods generally fall into two categories: quantitative and qualitative. Quantitative research focuses on numerical data and statistical analysis, whereas qualitative research emphasises words and interpretive meanings. Both are vital for gaining comprehensive knowledge. Typical quantitative techniques include experiments, numerical observations, and surveys with closed-ended questions. Conversely, qualitative methods often involve open-ended interviews, descriptive observations, and literature reviews that explore concepts and theories (Lakshman *et al* 2000, Takona 2024).

Qualitative and quantitative research methods can be used either sequentially or simultaneously within a single study. In a sequential approach, if a study produces unclear quantitative results, the same group can be engaged to provide explanations, which can then inform the development of an instrument to test those reasons. The methodology for the quantitative study is based on a positivist philosophy that aims to observe an objective reality existing independently of the observer. Quantitative data is considered reliable and reproducible, consisting of factual 'hard' data rather than subjective 'soft' data. Researchers typically use statistically validated tools, aiming for results that are objective, generalisable, and reproducible (Lakshman *et al* 2000, Takona 2024). In the case of a reservoir, this involves data related to water inflow, storage, and distribution. The reservoir serves various users with different water needs, all of which are critical. The main challenge is prioritising these needs, as DSSs analyse data to produce detailed reports and projections, unlike simpler tools that only gather data.

The WBM DSS uses advanced mathematical methods, linear programming algorithms, and programming languages to simulate the dynamics of water resource systems (Fulkerson 1961, Minty 1961, Pegram *et al* 2013). It features a sophisticated yet easy-to-use web-based graphical interface with menu-driven editors for inputting and editing data, displaying simulation results through tables and plots, and offering export options. Additional capabilities allow users to create reservoir and river system schematics by connecting components and selecting options online (Fulkerson 1961, Minty 1961, Pegram *et al* 2013).

The WBM DSS has been developed using the latest programming language technology, and its executable is written in C++ (Delphi 2020, Galanis 2018). The programming language that is used to perform the compilation is Delphi version 10.3, with the Windows Server 2019 operating system, and the Microsoft Internet Information Server (MIIS) used for deployment and to host the website requests (Microsoft Windows 7 Administrator's Reference 2011, Windows Vista 2005, Romashkov 2017, Tiensivu 2008). The hypertext preprocessor (PHP) version is installed and used on Apache on Linux to invoke the COM server, and the use of the virtual cloud server with specific central processing unit (CPU), random access memory (RAM), and hard drive space (Haris & Hasim 2019, Laaziri *et al* 2019, Sotnik *et al* 2023). PHP stands for "personal home page". The MIIS is the web server software that is flexible, secure, and manageable for hosting anything on the website (Microsoft Windows 7 Administrator's Reference 2011, Tiensivu 2008).

The WBM DSS uses the out-of-kilter algorithm along with a second alternative algorithm within the revised simplex algorithm for its online web solution. The out-of-kilter algorithm, introduced in the early 1960s by Minty and Fulkerson, was designed to solve network flow problems applicable to both physical and non-physical systems (Comley 1996, Fulkerson 1961, Minty 1961). The programming algorithm is considered a distinct linear programming method that has been effectively used to solve various water resource problems, including the Texas Water Plan in the United States of America (Sigvaldson 1976).

The revised simplex algorithm improved on the traditional simplex algorithm version by updating only the active columns at each step, which is particularly beneficial for numerical columns. Additionally, users don't need complete knowledge of all variables in a matrix to decide on the pivot or to identify the next basis. This means that knowing the current value of the objective function at each step isn't necessary, and the effort required to solve the objective mostly depends on the number of constraints rather than the number of variables (Ord 1972).

The out-of-kilter algorithm was selected instead of the simplex algorithm because it effectively distributes flows within a network to lower the overall flow costs (Dantzig 1990, Galántai 2021, Nadler 1973, Nelder & Mead 1965, Syifa *et al* 2023). Table 1 compares and summarises the two algorithms.

Given the numerous factors at play, it is unlikely that one algorithm will always outperform others in every practical situation. Instead, each algorithm performs best when suited to the right kind of problem. Consequently, the planner must understand the traits of different algorithms – such as their convergence speed, adaptability, parallelisation potential, and suitability for problem structures – to make well-informed decisions (Arsham 1997, Dantzig 1990). When dealing with complex problems, choosing the right algorithm often requires testing multiple candidates' options.

Like most water resource DSSs, WBM DSS's main goal is to address the water allocation problem by optimising an objective function to either minimise or maximise the needs of users or demands, considering supply priorities, demand site preferences, mass balances, and other constraints (Coleman & Pitman 2008, Department of Water and Sanitation 2020, Nyland & de Jager 2023). The constraint set is updated step-by-step at each time point to systematically evaluate the demand priorities and supply preferences. Each demand site, reservoir, irrigation block, mine, and wetland is assigned a

Table 1 The comparison between the out-of-kilter and revised simplex algorithms

Algorithm	Out-of-kilter	Revised simplex
Efficiency	It effectively assigns flows in a network to minimise overall flow costs.	It is memory-efficient because it computes and stores only the data needed for the current step to test or improve the solution, and it is known as the simplex algorithm with multipliers.
Objective	To identify the optimal flow within a water resource network, the selected flow set must meet certain relationships.	This approach minimises calculations per iteration by focusing solely on the coefficients needed to identify the pivot element, rather than updating the entire tableau each time. It especially saves effort when handling numerical columns, as it avoids updating all columns at every step.
Complementary slackness	The complementary slackness optimality conditions (CSOC) theorem defines the criteria for the optimality of the out-of-kilter theory and aids in categorising arcs in the network's nodes as either "out-of-kilter" or "in-kilter". Adjustments are made to out-of-kilter arcs until an optimal solution is reached, which occurs when all arcs are classified as "in-kilter".	It is a primal cost improvement method that relies heavily on price vectors and duality concepts. In particular, the complementary slackness (CS) conditions play a crucial role in that, if a variable x is feasible and together with p satisfies these CS conditions, then variable x is considered an optimal solution of the problem, and p is an optimal solution of its dual problem
Advantage/ strength	The primary strength of the out-of-kilter algorithm is its capacity to pinpoint a circulation in the network that maintains water flow at each node (reservoir). It ensures that the flow in every channel or river stays within the defined minimum and maximum limits while minimising overall network costs.	The benefit of the revised simplex algorithm over the original method is that it initially removes zero coefficients, retaining only non-zero ones. Subsequently, it stores just eight to ten significant digits for each non-zero coefficient, making most of the coefficients part of the possible matrix.
Focus	It is able to address a wider variety of network issues, such as transportation, assignment, shortest path, and caterer problems.	It effectively manages large, sparse problems and is often quick and reliable in real-world applications.

distinct priority number, from highest to lowest. Figure 1 shows the various catchment elements, each identified by a unique number. The catchment is in Mpumalanga, South Africa. Reservoirs are triangular, nodes are circles, and transfer channels are arrowed lines. Like demand priorities, supply preferences use an integer ranking system to determine which sources supply a particular demand site.

Since power generation, domestic supply, and irrigation often depend on multiple sources to satisfy their needs, an allocation mechanism is necessary to manage these multiple supply choices. The transfer for Eskom, the South African power generator, is shown in channel 509 of Figure 1. It relies on different sources, such as pumping through channel 502 and gravity channel 504 from reservoir 10 (Nooitgedacht Dam); abstraction through channel 500 from node 10 (Gemsbokhoek weir); and transfer from reservoir 26 (Vygeboom Dam) through channel 501. The transfer via channel 501 includes water from Gladdespruit weir through channel 636 to Vygeboom Dam. Additionally, the user can limit the flow through any transmission link to a maximum volume or a percentage of demand, reflecting physical limits (e.g. pipe or pump capacities), contractual limits, or preferences regarding the mixing of supplies.

A reservoir is a crucial node in the catchment network, capable of storing water over time. It is modelled using several variables tied to its physical and operational features. A reservoir is an artificial or natural

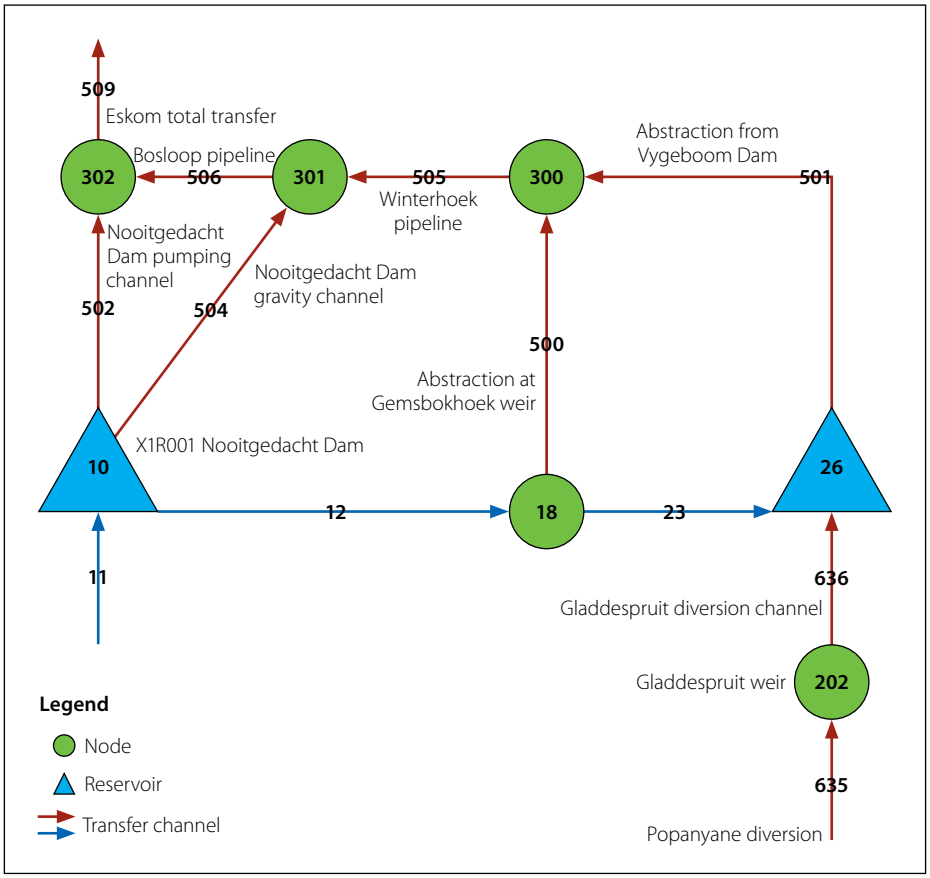


Figure 1 The water supply transfer system to Eskom channel 509

lake with a dammed outlet that stores water and regulates outflow (Manson 1998, Nagy *et al* 2002, Rugumayo & Kizza 2001, Wurbs 2005, Wurbs 2012, Wurbs *et al* 2012). The simulation of reservoir behaviour involves a straightforward calculation of the stored water volume at the end of each month. The initial storage volume at the start of the simulation is known. The storage at the end of the first month is determined by the change in storage, which equals inflows minus outflows, as shown in Equation 1.

$$\textit{Change in storage} = \textit{inflows} - \textit{outflows} \quad (1)$$

Once the DSS determines the reservoir storage at the end of the first month, the storage at the beginning of any subsequent month

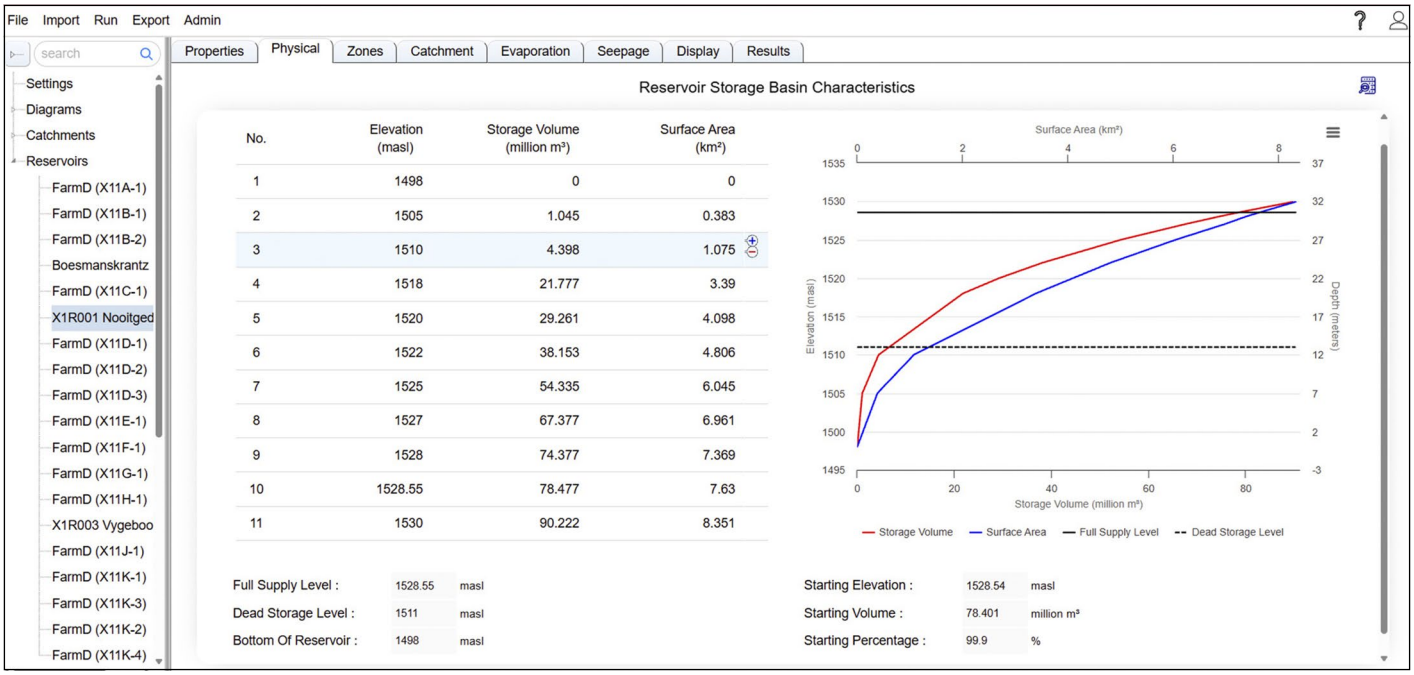


Figure 2 The Nooitgedacht Dam reservoir physical characteristics levels

is assumed to be the same as the storage at the end of the previous month (Draper *et al* 2004, Lund & Ferreira 1996, Manson 1998, Rugumayo & Kizza 2001, Wurbs 2012, Wurbs *et al* 2012). Figure 2 presents the bottom, dead, and full supply levels of the reservoir, accompanied by a table illustrating the elevation-volume-surface-area relationship and a plot of the reservoir. The example provided in Figure 2 is for Nooitgedacht Dam, located on the Komati River, 10 km away from Carolina Town in the Mpumalanga province of South Africa. The WBM DSS is flexible in terms of data editing and not reliant on a specific type of format data set to follow (Nyland & de Jager 2023).

Figure 3 shows the reservoir's catchment area, which is the land area from which water collects and flows into the reservoir. This movement of water is known as runoff. In South Africa, rainfall is the largest contributor to runoff, with snow and hail being less common. The core structure of the catchment network system is shaped by where water flows into reservoirs and other nodes within the water resources system. It is referred to as the incremental catchment for that node (Draper *et al* 2004, Hughes 1992, Lund & Ferreira 1996, Manson 1998, Nagy *et al* 2002, Rugumayo & Kizza 2001, Wurbs 2005, Wurbs 2012). The total area of each incremental catchment, measured in square kilometres, is included in the calculations. The WBM DSS uses this variable to assess natural runoff from undisturbed parts of each catchment, accounting for areas affected by irrigation, streamflow reductions, and activities such as mining.

Each designated incremental catchment within the water resource network is linked to one or more reservoir and junction node through a reference number. This number is assigned according to the DSS-defined sequence (Beard 1967, Coleman & Pitman 2008, McKenzie & Van Rooyen 1999, Nkwonta *et al* 2017, Rugumayo & Kizza 2001). Flow volumes and water use in the catchment files are allocated to the chosen nodes using

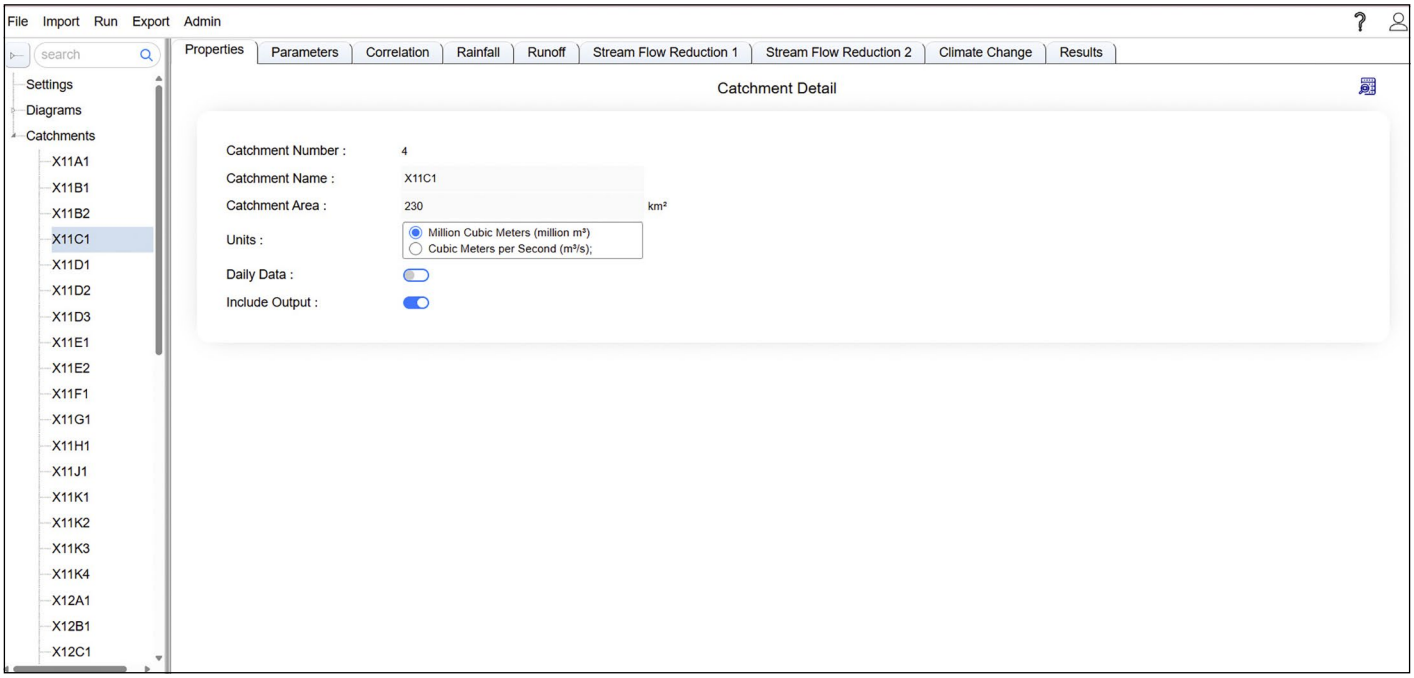


Figure 3 The WBM DSS list of catchment areas with catchment details of X11C1

scaling factors: incremental flow factor α, irrigation area factor β and afforestation area factor C. Figure 4 illustrates the different tabs that present the various values calculated using the net flow formula. The values shown are monthly, but daily values can be displayed when imported. This feature is new compared to the earlier WRYM DSS, which imported and calculated values solely from catchment hydrology data, allowing edits through the graphical user interface (GUI). The net flows entering a particular system node are then calculated using the formula in Equation 2:

$$\begin{aligned} \textit{Net flow} = \alpha \times (\textit{Incremental flow}) - \\ \beta \times (\textit{Irrigation area}) - \\ C \times (\textit{Afforestation area}) \end{aligned} \quad (2)$$

The catchment area of a reservoir is accepted as the area contributing to the full supply boundary of the reservoir.

When the reservoir drops below the full supply level, an additional intermediate area becomes visible, which isn't included in natural inflow calculations. Usually, the ratio of this full supply area to the total catchment area is very small, making its runoff impact negligible. However, there are cases where runoff from this intermediate zone can be significant and must be considered. In such cases, the rainfall-runoff coefficient helps estimate the extra runoff as a percentage of rainfall. For example, if the catchment's average runoff is 10% of annual rainfall, a coefficient of 0.1 is used (Ngoepe *et al* 2025). This coefficient is key for evaluating situations in which the reservoir's total supply area accounts for a large share of its catchment.

To verify the system's network inflows, accurately defining net runoff from incremental catchments is crucial. Model results are used to compare simulated node inflows with the corresponding net runoffs from relevant catchments. While calculating net runoffs, note that DSS simulation results are in average m^3/s, whereas time-series files show data in million m^3. Users should perform appropriate unit conversions. This process should be repeated for each inflow node, using historical model data for a specific day or month during the analysis period, as well as the overall average across the full duration. The WBM-DSS supports daily or monthly time steps, unlike the previous WRYM DSS, which was limited to monthly updates (Ngoepe *et al* 2025).

The total channel transfer allocation to Eskom is divided among various channels. The Bosloop pipeline and Nooitgedacht pumping channels are the last two, as shown

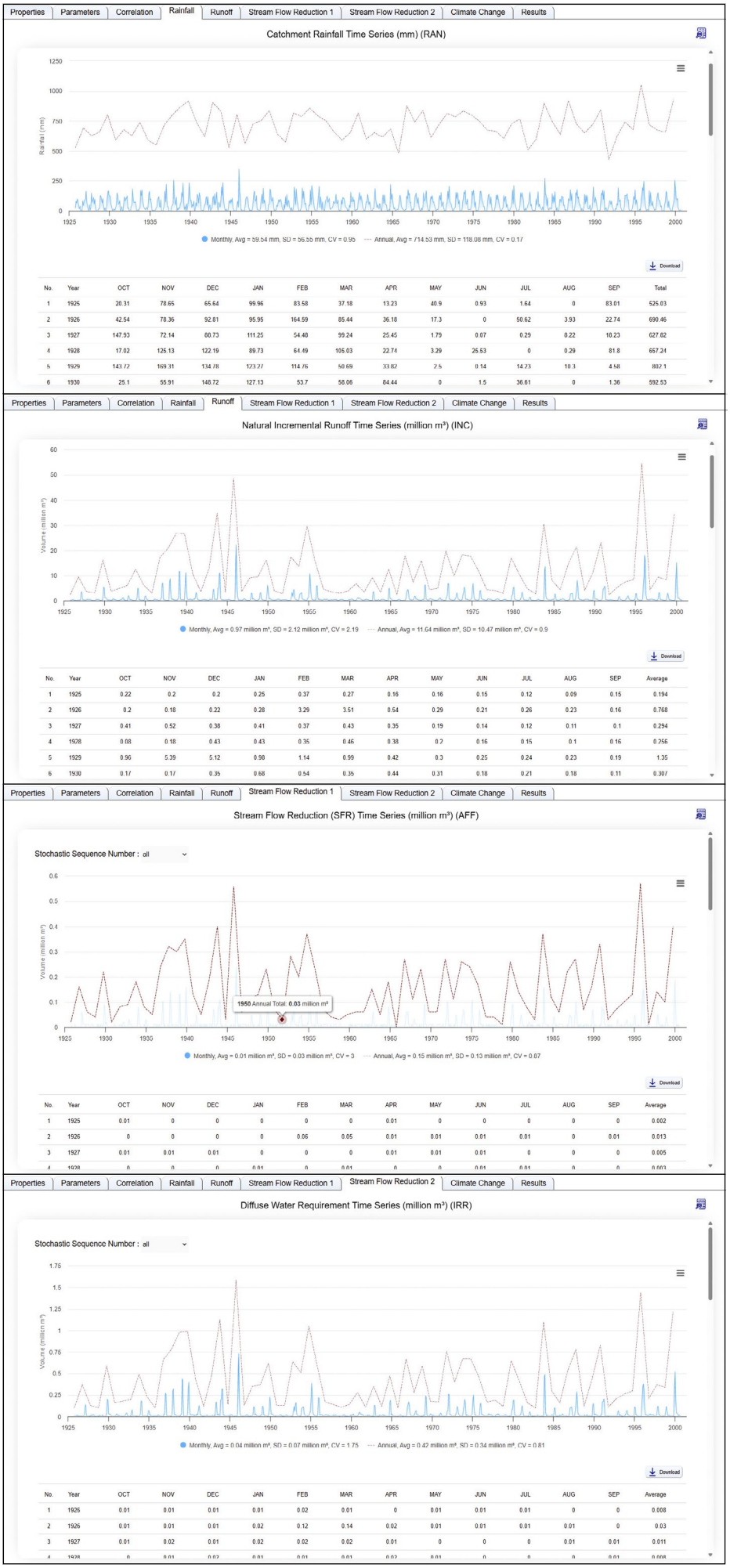


Figure 4 The WBM DSS net flow calculation values

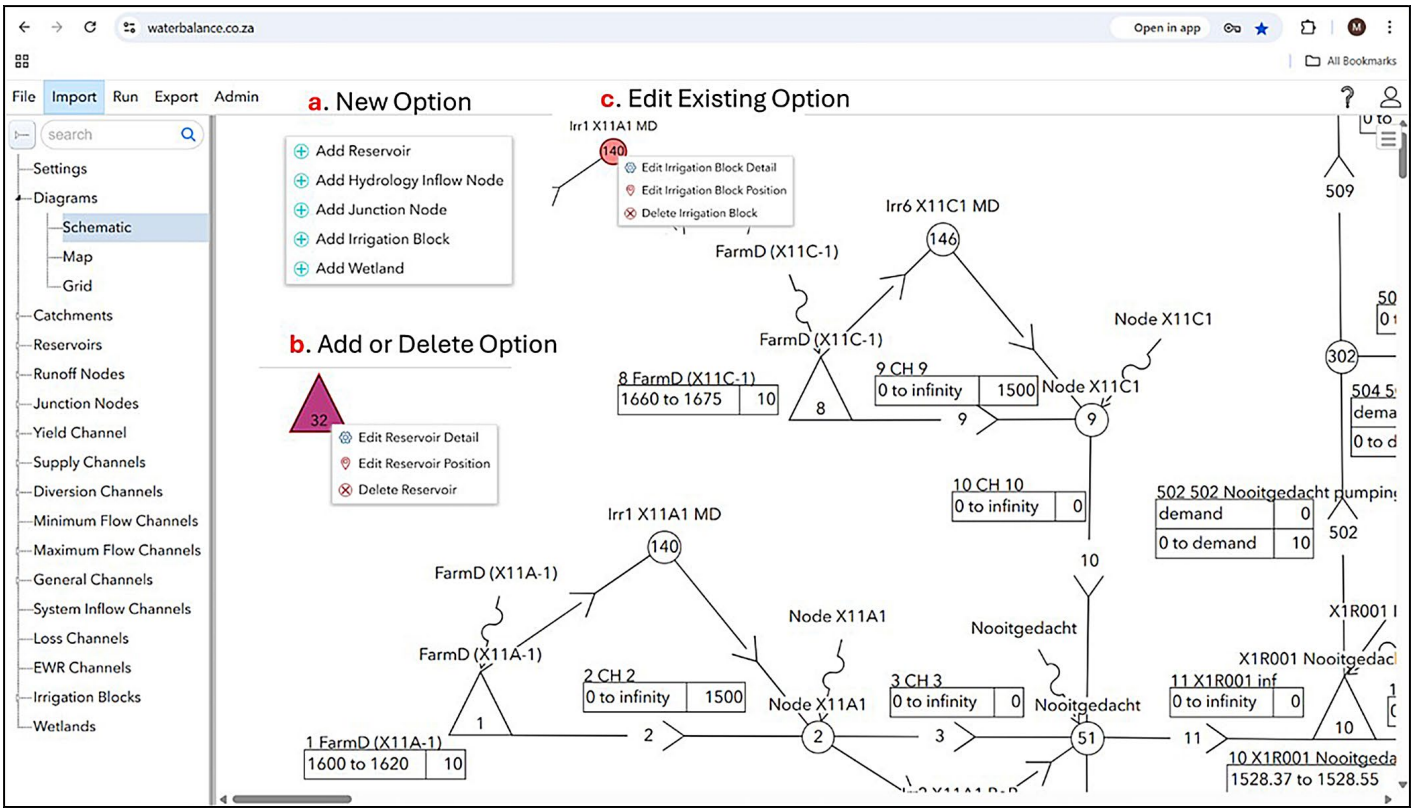


Figure 5 The WBM DSS options for editing the network element

in Figure 1. Red channels indicate transfer pathways, while blue channels denote river flows. The two algorithms allocate resources to each channel based on its maximum supply capacity. Water from Nooitgedacht Dam can be transferred either by pumping (502) or gravity (504), whereas water from Vygeboom Dam can be transferred via abstraction channel 501 including the water transfer through the Gladdespruit weir and the Popanyane pipelines. In addition to these transfers, water is occasionally pumped from the Gemsbokhoek weir to boost supply.

The input study data includes 17 files, each containing a different catchment dataset. These input files, in their original formats as used by WRYM DSS, can be imported into the web-based WBM DSS, or a new file can be created directly from the web service. The X11C natural historical streamflow data, measured in million cubic metres (Mm^3), is listed among the files in Figures 3 and 4. It covers the period from October 1925 to September 1999, totalling 84 years. The X11C is a quaternary catchment that supplies the Nooitgedacht Dam. In addition to the input data and natural historical incremental flows, generating stochastic streamflow data via stochastic analysis is crucial when using the DSS's statistical distribution option, which requires stochastic input parameter sequences.

The user can add, edit, and delete features in the web-based DSS. The user can click on the "Add Reservoir" option in (a), and option (b) appears as shown in Figure 5. Meanwhile, the DSS provides the option to delete or edit the existing element, as shown in option (c) of Figure 5.

The terminology related to water movement can be confusing for many, but it becomes easier to understand once you grasp the simple concept. The general rule is always to spend less and get a high or maximum return; water moves between nodes or noticeable points within a water resource catchment. Naturally, water flows from an upstream node to a downstream node through channels or rivers, or water is pumped from nodes with water to the node where it is needed via a pipeline. The upstream node is the source, meaning the point where water is sourced, and the downstream node, or the receiver, is the sink. Between the nodes, water is transported via the so-called arcs. Arcs are the rivers, pipelines, and transfer channels.

The objectives focus on the demand and supply of water at a specific cost, regardless of whether demand is fully or partially met. The algorithm considers different options, and the results are always available. The law of flow conservation is strictly applied, ensuring continuous observation of flow between source and sink, with net flow calculated as the water from the source minus the water reaching the sink. Solving water resource problems is straightforward when demand equals supply (Ploskas *et al* 2013). However, challenging situations arise when demand exceeds supply or vice versa. Does the algorithm handle such cases? When the water supply exceeds requirements, the optimal solution shows how to distribute the surplus. Conversely, if requirements are unmet, it indicates how unmet demand is allocated among destinations. The algorithm's flow from one node to another is based on the profit gained, which must be non-negative; otherwise, no movement occurs. Limitations of the revised simplex and out-of-kilter algorithms include that objective functions and constraints are not always linear, and there is no guarantee of finding a solution. Additionally, large-scale problems may not be solvable using linear programming because their parameters are often fixed and treated as constants.

Any linear programming problem to solve depends on the data and the problem size within a certain range. Key factors include the solution time, the quality of initial solutions, the speed of sensitivity analysis, and the available computing power.

RESULTS

Graphs provide crucial information and are often an excellent way to display data behaviour, offering visual summaries. One of their main purposes is to provide the analyst with insight into the data and showcase essential concepts when explaining results (Helsel

Table 2 Nooitgedacht Dam month-end reservoir volume (mcm) 1925 to 1962

Year	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Avg
1925	76.190	74.665	72.780	71.691	71.505	69.950	67.802	66.057	64.137	61.869	59.294	57.573	67.793
1926	55.662	54.122	52.695	51.695	67.578	78.477	78.477	77.200	75.541	74.699	72.735	70.477	67.447
1927	71.034	71.467	71.008	70.986	70.419	70.634	69.780	67.755	65.715	63.452	60.995	58.521	67.647
1928	55.881	54.526	54.359	54.187	53.669	54.086	53.440	51.610	50.039	47.982	45.547	43.922	51.604
1929	47.828	78.057	78.477	78.477	78.477	78.477	78.185	76.808	75.412	73.975	72.046	69.748	73.831
1935	62.426	59.945	57.911	56.548	57.172	56.896	55.675	55.875	55.140	53.471	51.047	48.626	55.894
1936	46.535	49.220	51.795	78.477	78.477	78.305	77.211	75.321	73.497	71.428	69.009	66.824	68.008
1937	64.655	62.636	78.477	78.477	78.477	78.095	78.477	78.211	77.454	76.148	74.594	72.523	74.852
1938	70.839	69.628	70.548	72.181	78.477	78.477	78.209	77.593	76.622	76.642	75.298	73.998	74.876
1939	73.650	78.477	78.477	78.477	78.477	77.704	77.242	76.862	78.477	78.477	76.924	76.132	77.448
1955	71.083	71.734	78.477	78.477	78.477	78.477	77.453	77.601	77.428	76.217	74.082	72.819	76.027
1956	72.598	73.105	74.071	73.636	73.816	75.141	74.735	73.001	71.692	70.353	68.577	67.577	72.359
1957	67.043	65.779	63.885	63.985	63.944	63.073	63.577	62.650	61.063	58.933	56.439	55.020	62.116
1958	53.527	53.287	53.239	53.162	53.027	51.658	49.817	47.994	46.042	43.904	41.467	39.331	48.871
1959	36.965	36.584	36.242	35.255	35.736	36.205	37.663	37.480	36.162	34.291	32.151	29.874	35.384
1960	28.139	27.961	31.362	34.363	36.596	39.733	42.336	42.710	41.989	40.590	38.585	36.918	36.774
1961	35.859	34.985	35.509	36.118	35.690	35.187	34.710	33.074	31.298	29.257	27.142	24.954	32.815
1962	23.400	41.307	59.383	58.727	56.884	55.030	53.373	51.418	50.892	50.521	48.692	46.256	49.657

& Hirsch 2002). The DSS applies the two algorithms to the data and executes. Once execution is complete, the DSS can present each algorithm's results or combine and compare the results data.

Table 2 presents the comparison results with colour coding: black, blue, and red. It shows end-of-month volume data for the Nooitgedacht Reservoir from 1925 to 1962, covering the following periods: 1925–1929, 1935–1939, 1955–1959, and 1960–1963. Blue values indicate matching outputs from both algorithms, while red values highlight discrepancies. Reservoir volume is not the primary concern regarding the algorithms, but transfer allocation and priority introduce complexities about supply order and timing. The revised simplex algorithm has not yet been modified to align with the specific data and rules, unlike the out-of-kilter algorithm. The latter employs a benefit/disbenefit mechanism to regulate water flow between nodes, typically from upstream to downstream, unless water is pumped out of a node.

The results in Table 2 are shown in a graph for each method. Focusing on the yellow highlight for March 1960, plots were extracted to illustrate the differences in the results. The choice of March was random, and any other month could have been selected to demonstrate the output differences. Figure 6 illustrates the results from the two algorithms (out-of-kilter and

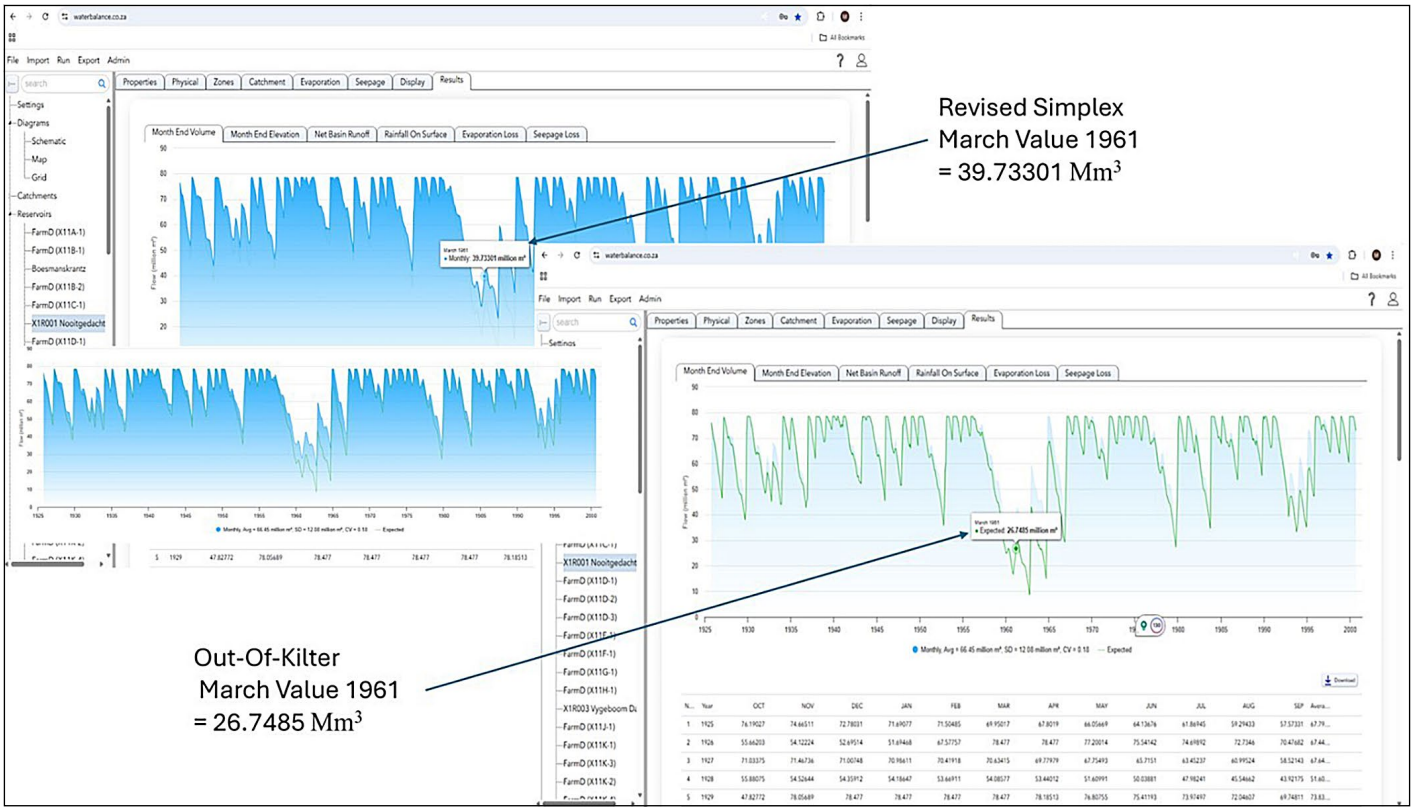


Figure 6 The reservoir results with two different algorithms. The green line is the out-of-kilter, and the blue line is the revised simplex algorithm

Table 3 Monthly reservoir volume variances for Nooitgedacht Dam: Revised simplex minus out-of-kilter algorithms, selected years (1925–1962)

Year	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep
1925	0.00	0.00	0.00	2.25	2.81	0.00	0.00	0.00	0.00	2.86	0.00	0.00
1926	0.00	0.00	4.30	5.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.51
1927	0.00	0.00	2.38	3.13	3.20	3.20	0.00	0.00	0.00	0.00	3.16	0.00
1928	0.00	0.00	4.61	5.53	5.71	5.97	5.97	5.95	5.96	0.00	5.88	5.87
1929	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1935	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.63	3.60	0.00
1936	0.00	0.00	5.47	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1937	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1938	0.00	0.00	2.01	2.69	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1939	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1955	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1956	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.83	2.85	0.00	0.00
1957	0.00	0.00	4.29	5.33	5.61	5.60	5.61	5.61	5.61	0.00	0.00	5.54
1958	5.83	6.46	7.49	8.26	8.25	8.21	0.00	8.14	8.11	8.07	8.01	0.00
1959	8.04	8.63	9.62	10.40	10.74	10.86	10.88	10.84	10.82	10.77	10.69	10.59
1960	10.68	11.24	12.17	12.96	12.99	12.98	12.98	12.95	12.93	12.89	12.79	12.72
1961	12.93	13.61	14.55	15.30	15.33	15.29	15.25	15.15	15.06	14.95	14.83	14.66
1962	14.73	15.21	16.24	17.08	17.16	17.12	17.05	16.96	17.12	17.09	16.97	16.83

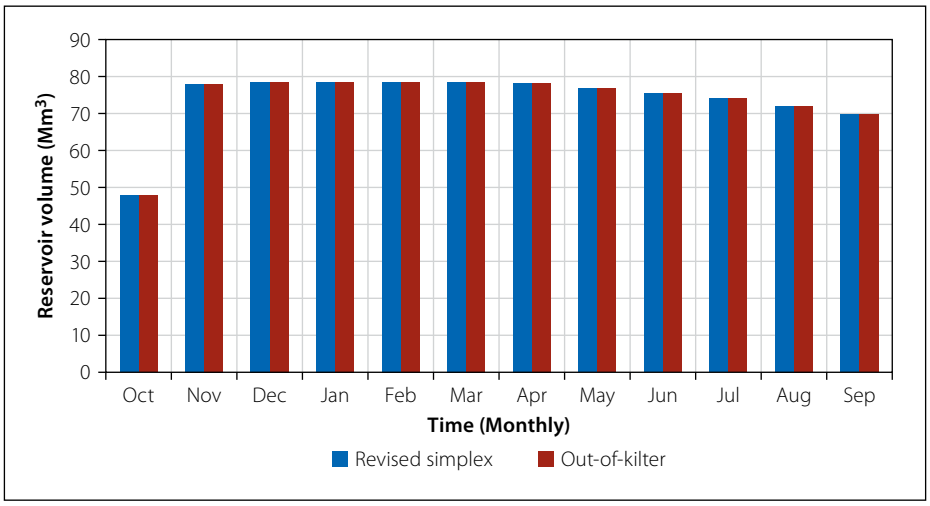


Figure 7 Reservoir volume results comparison for the revised simplex algorithms for the hydrological year 1928/9

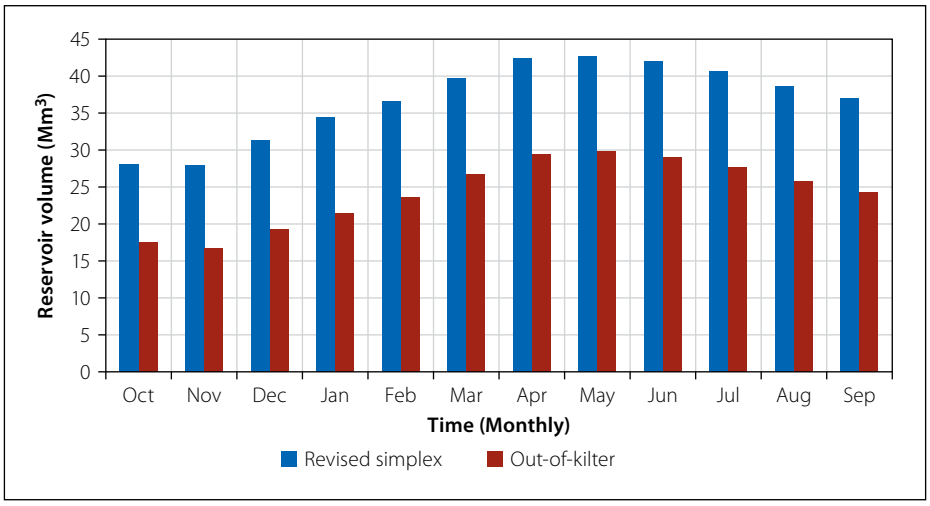


Figure 8 Reservoir volume results comparison for the revised simplex algorithms for the hydrological year 1959/60

revised simplex) for March 1961, specifically noting values 26.7485 and 39.73301 Mm^3. In some cases, the monthly values are similar, whereas in others, they differ (Ngoepe 2016, Pegram *et al* 2016). The green line graph represents the out-of-kilter algorithm, while the blue line graph displays the monthly values for the revised simplex algorithm.

The revised simplex algorithm results show that, in some months, the algorithm has a higher reservoir volume than the out-of-kilter algorithm in the same month, or, in some instances, for the entire hydrological year, i.e. the 1957/8 to 1962/63 years, as displayed in Table 3. The results in Tables 2 and 3 are supported by Figure 7, which shows similar results for both algorithms in the hydrological year 1928/29. Figure 8 shows the differences for the full hydrological year 1959/60 and Figure 9 shows fewer differences during the hydrological year 1924/5.

The results from the channels align with the reservoir outcomes shown in Figure 6. Figure 10 displays the results, with the green line representing the out-of-kilter algorithm and the blue line illustrating the revised simplex algorithm. Figure 10 provides an example of the Gladdespruit canal from the study data.

The findings indicate that the developed DSS will enable water resource planners to make efficient, strategic decisions based on comprehensive information. It relies not

only on algorithms but also on effective visualisation of catchment system networks. Moreover, the results allow users to select from a range of linear programming algorithms tailored to specific catchments. This flexibility helps users recognise that different catchments vary in size and complexity, requiring unique algorithms for decision making. Overall, this research offers a streamlined approach for sharing and utilising big data within the user community.

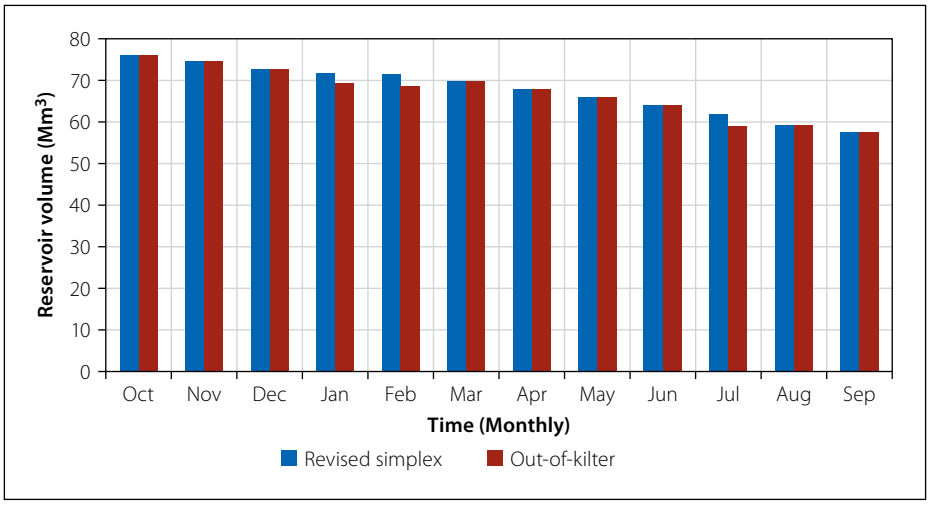


Figure 9 Reservoir volume results comparison for the revised simplex algorithms for the hydrological year 1924/5

CONCLUSIONS

The out-of-kilter algorithm focuses solely on supply costs, whereas the revised simplex algorithm considers both supply and sink nodes, resulting in higher yields and lower supply costs. The results in Table 2, from hydrological years 1957/8 to 1962/3, support the statement of high yield compared to its counter algorithm. To achieve this, proper penalty configurations in the revised simplex are essential to prevent default channel assumptions. Initially, the out-of-kilter algorithm uses Gaussian elimination to identify the feasible region, then proceeds similarly to the revised simplex approach along the network edges. Both algorithms can unexpectedly operate the network to produce a lower total cost, which might be numerically optimal but practically questionable; thus, comparing their results is recommended. The alternative algorithm aims to meet demand using the lowest-cost sources and avoids using the pumping channel when alternative sources are available. Both algorithms can unexpectedly yield results with a lower overall cost. The out-of-kilter algorithm applies distinct penalty structures for channels and reservoirs, whereas the revised simplex algorithm uses a standard, uniform penalty.

Future research should investigate adding penalties to both algorithms to see whether a different method or algorithm produces similar or alternative results. While this possibility will be explored, the outcomes could remain unchanged or vary. The DSS was tested on a water resource system that excludes certain advanced catchment components, such as mines and wetlands, which could be incorporated. It is advisable to test the product on a system that includes all these remaining elements to build confidence in its deployment. A DSS meant to guide management in policy-related decisions should be flexible enough to update with ongoing technological advancements.

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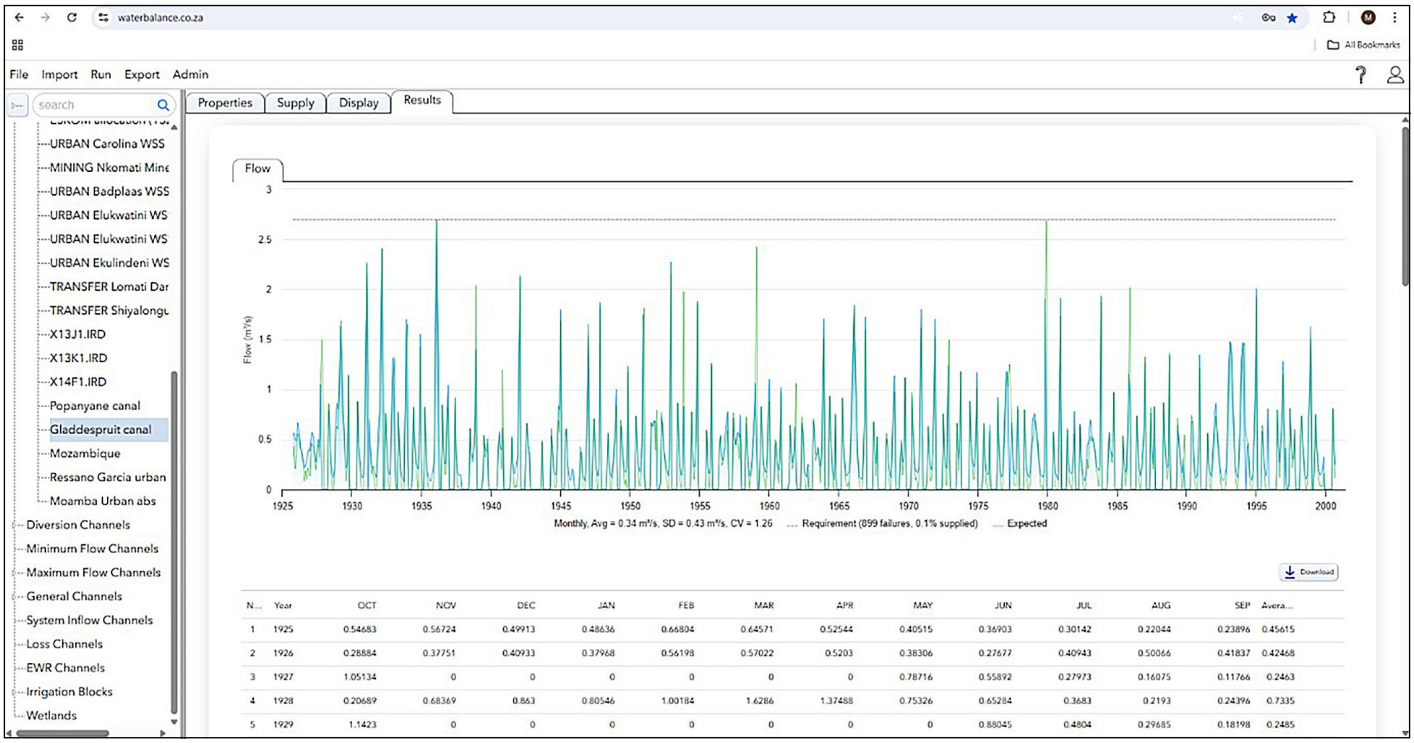


Figure 10 The channel results with two algorithms. The blue line is the out-of-kilter algorithm, and the green is the revised simplex algorithm

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