# Optimizing Transmission Rate in BSC Model-Based Middleton Class-A Noise Impaired Channel with Transceiver Side Information

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***Abstract*— This paper introduces a Binary Symmetric Channel (BSC) model, which leads to a simple and practical representation for channels corrupted by the Middleton Class-A impulse noise, such as Powerline communication channels. The proposed BSC channel is based on a variable transition probability according to a Poisson distribution. The average transition probability, which provides insight into the error rate, as well as the capacity bounds for this channel are analyzed in all cases of informed and non-informed transmitter and/or receiver. Particularly, an achievable transmission rate is determined in case of informed transmitter by proposing a suboptimum solution that involves employing a variable transmitted energy, proportional to the known noise variance. Analytical formulation of the achievable rate and simulation results in case of a binary Frequency Shift Keying communication system allow to conclude that the channel state information at the receiver side is helpful when the transmitter is non-informed and uses a fixed transmitted energy. However, it no longer contributes to significant performance enhancement when the transmitter is informed.**

***Index Terms*— BSC, Middleton Class-A noise, transition probability, Poisson distribution, transmission rate.**



## I. Introduction

Various transmission mediums are being adversely affected by the impulse noise. This type of noise refers to sudden, brief, and high-amplitude disturbances that significantly degrade the quality of transmission [1-4].

The impulse noise is a main source of errors in data reception across several digital communication systems, including both wireless and wired links, such as power line communications systems [1-2, 5-6]. Typical research topics in this area include channel measurements, channel characterization and noise modeling.

In the realm of binary communication, the classical Binary Symmetric Channel (BSC) is widely recognized in the literature as a fundamental concept in information theory [11-12]. It is a simple mathematical model, where errors may occur with a fixed transition probability, based on the assumption of a channel affected by only the standard Additive White Gaussian Noise (AWGN), see Fig. 1.

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In this work, we present an adaptation of the BSC model specifically designed to characterize channels affected by Middleton Class-A impulse noise. Our aim is to establish an easily applicable and practical mathematical model for such scenarios while providing a robust framework within information theory for applying Middleton Class-A noise, which is broadly adopted across various research domains, particularly in PLC communications [5-7]. Additionally, we delve into the analytical formulation of how transmitter/receiver channel state information impacts capacity, along with proposing a suboptimum solution to maximize achievable transmission rates. This study is of major importance for further investigating how transmitter awareness of the channel state can contribute to improved and optimized performance through adaptive power and coding strategies, mainly in communication environments characterized by impulse noise [13-16]. Extending classical capacity results for memoryless channels with transmitter-side Channel State Information (CSI) to impulse-noise-affected channels fills gaps in the literature and provides valuable contributions to both theory and practical system design.

The paper is organized as follows. In section II, a review of the classical BSC channel, as well as the Middleton Class-A impulse noise characteristics are given. Section III introduces the proposed modification of the BSC model to include impulse noise. We also detail the analytical formulation of the capacity expressions in both cases of non-informed, and informed transmitter/receiver, while suggesting a suboptimum solution to maximize the achievable transmission rate in case of an informed transmitter. Section IV is dedicated to simulation results and a discussion to validate the analytical study.

## II. Review of the classical BSC channel and Middleton Class-A impulse noise

### A. BSC Channel overview

The Binary Symmetric Channel model represents a distinct case of the discrete memoryless channel with two input (binary) symbols $X \in \{0,1\}$ and two binary outputs $Y \in \{0,1\}$. Notably, this channel exhibits symmetry, meaning that the likelihood of receiving a 1 when 0 is transmitted equals the likelihood of receiving a 0 when 1 is transmitted. This symmetry is governed by a fixed transition probability, denoted by $p$, which characterizes the conditional probability of error, or in other words, the likelihood of a bit flipping [11-12]. Fig. 1 gives an illustration of a standard BSC channel.

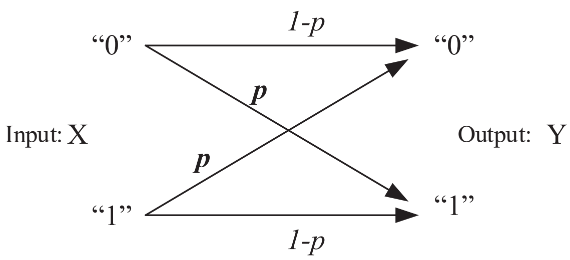


Fig. 1. BSC Channel illustration [11]

The probability $p$, which represents the error probability, can be easily evaluated for diverse transmission systems, when the transmitted signal is corrupted by Additive White Gaussian Noise, [11]. In case of a coherent binary FSK communication system with a match filter and hard decision at the receiver, it is expressed in (1) using the Q-function.

$$p = Q\left(\sqrt{\frac{E_b}{2\times\sigma_g^2}}\right), \tag{1}$$

where $E_b$ is the signal energy per bit and $\sigma_g^2$ is the AWGN noise variance, respectively. The $Q$ function is defined as $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp(-t^2/2)\mathrm{dt}$.

The BSC channel capacity $C$, which is an intrinsic property of the channel, is computed as a function of the binary entropy function, $H(p)$,

$$C = 1 - H(p), \tag{2}$$

where $H(x) = -x\log_2(x) - (1-x)log_2(1-x)$ [11]. In case of a 2-FSK system, the capacity can then be expressed as:

$$C = 1 - H(Q\left(\sqrt{\frac{E_b}{2\times\sigma_g^2}}\right)). \tag{3}$$

### B. Middleton Class-A impulse noise characteristics

Several signal transmission environments are suffering from the presence of impulse noise, in addition to AWGN, which leads to transmission issues. In particular, impulse noise has long been considered the most challenging constraint in Powerline Communication (PLC), affecting both broadband and narrowband communication [1, 5-6]. This kind of noise is characterized by a random occurrence, duration and power spectral density, the intensity of which can be destructive for communication system performance [5].

The Middleton Class-A model is a commonly used tool to describe impulse noise [2, 7-8]. It is a memoryless model in which the number of impulses in a symbol period is represented by the random variable $K$, where $K$ is Poisson distributed with parameter $A$. The probability of observing $k$ impulses is given by:

$$P(K = k) = P_k = \frac{A^k e^{-A}}{k!}. \tag{4}$$

The impulse noise amplitudes are Gaussian distributed with mean zero and variance $\frac{\sigma_I^2}{A}$. Using the Middleton Class-A model, impulse noise effects on signal transmission can be interpreted as follows. In addition to the background noise, each transmitted symbol is affected independently by $k$ impulses each of them Gaussian distributed with a variance equal to $\frac{\sigma_I^2}{A}$. For a coherent detection, the noise sample after correlation and integration over a symbol period is the integration of the Gaussian noise and the $k$ impulses with a total variance $\sigma_k^2 = \frac{k\sigma_I^2}{A} + \sigma_g^2$, where $\sigma_I^2$ and $\sigma_g^2$ are the average variance of the impulse - and the background noise, respectively, see also [11]. As $k$ is a realization of a Poisson random variable with parameter $A$, the resulting average noise variance is equal to $\sigma_I^2 + \sigma_g^2$. Note that the transitions probabilities are independent from each other (Poisson property). We also refer to $k$ as the channel state, which varies from one time period to the next. Note that $A$ is equivalent to the average number of impulses per transmitted symbol [2, 7]. We assume in our calculations that the value of $A$ is fixed and known to receiver and transmitter.

## III. Proposed BSC Model for Middleton Class-A noise

### A. Channel model description

The motivation of this work is to define a simple channel model for a binary communication system when the transmission medium is characterized by the presence of impulse noise according to the Middleton Class-A model.

As discussed in the previous section, each symbol transmission is corrupted by Gaussian background noise with variance $\sigma_g^2$, in addition to $k$ Gaussian impulses, each with variance $\frac{\sigma_I^2}{A}$, during every symbol period. The random variable $K$ generates $k$ impulses using the Poisson distribution with parameter $A$. Based on this observation, a variant of the conventional BSC can be defined using a variable transition probability [17]. For every transmission, the transition probability depends on the particular value of $k$, generated by the random variable K. Fig. 2 depicts the diagram of the proposed BSC channel.

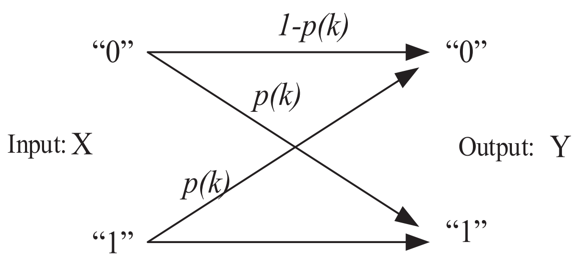


Fig. 2. Proposed BSC Channel for Middleton Class-A noise environment

The channel can be considered as AWGN with a discrete output, while the input message remains binary with equal probabilities for '0' or '1'. It is also memoryless since the output depends only on the current state and not on previous states, consistent with the Middleton Class-A model. The transition probability *p(k)* is determined solely on the channel state, defined by the parameter $k$.

Using the same binary FSK system as in section (II.A), the transition probability *p(k)* is expressed as given in (5).

$$p(k) = Q\left(\sqrt{\frac{E_b}{2\times\left(\sigma_g^2+\frac{k\sigma_I^2}{A}\right)}}\right). \tag{5}$$

The average transition probability defines the Bit Error Rate (*BER*) for the proposed channel, and it is formulated as:

$$\overline{p(k)} = BER = \sum_{k=0}^{\infty} P_k \times Q\left(\sqrt{\frac{E_b}{2\times\left(\sigma_g^2+\frac{k\sigma_I^2}{A}\right)}}\right), \tag{6}$$

where $P_k$ is given by (4).

### B. Proposed BSC channel capacity for non-informed transmitter

For a binary symmetric channel, the capacity achieving distribution is given by equal probable binary input symbols and symmetric transition probabilities, see (2). We are interested in the capacities denoted $C(-,-)$, $C(-,+)$, $C(+,-)$ and $C(+,+)$ as the capacities for the four possible different situations of non-informed or informed transmitter and receiver about the value of $k$ for every transmitted symbol, respectively.

We start by examining the channel capacity when the transmitter has no knowledge about the channel noise, denoted by $C(-,*)$ , which reflects the maximum achievable transmission rate under these conditions.

When the receiver is informed about the value of $k$, and considering that our BSC channel is memoryless, the capacity can be calculated as the infinite sum of capacities at each time period (or channel state) as given [12, 18]:

$$\begin{aligned} C(-,+) &= P_0 \times C(k=0) + P_1 \times C(k=1) + \cdots \\ &= 1 - P_0 \times H(Y|X,k=0) + P_1 \times H(Y|X,k=1) + \cdots \\ &= 1 - \textstyle\sum_{k=0}^{\infty} P_k \times H\big(p(k)\big), \end{aligned} \tag{7}$$

where $H(Y|X,k)$ is the output entropy given the input $X$ and $k$, and $Y$ is the binary output of the channel.

When the receiver has no information about the value $k$, the entropy $H(Y|X)$ equals the entropy of the output $Y$ given the input $X$, which is determined by the entropy of the average transition probability. The capacity expression can thus be formulated as [12, 18-19]

$$C(-,-) = 1 - H\big(\overline{p(k)}\big). \tag{8}$$

Using the entropy convexity, we conclude that $C(-,+) \geq C(-,-)$, [11].

An interesting question revolves around the potential impact on the channel capacity when having knowledge of the noise variance at the transmitter side [13-16].

While this assumption may not be realistic in general, especially given our study's focus on a communication system without incorporating a channel coding stage, it provides a useful starting point for further investigation. Previous theoretical studies have shown that significant improvements in channel capacity are achievable when the transmitter has CSI, as it enables adaptive power and coding strategies, offering insights into optimal power allocation and coding schemes. For that matter, modern systems with transmitter-side CSI in real-word scenarios, such as wireless systems, employ adaptive coding and modulation to adjust transmissions based on channel conditions. However, these studies are limited to cases involving fading or additive white Gaussian noise (AWGN) channels [13-16, 20]. The application of capacity results for memoryless channels with CSI at the transmitter to specific noisy environments, such as impulse noise in PLC, is not extensively explored [21-23].

This ongoing study is particularly relevant for systems utilizing channel coding blocks to enhance transmission performance in environments exhibiting time-varying impulse noise characteristics.

### C. Capacity of the proposed BSC for an informed transmitter

The approach of using the channel state information at the transmitter side to enhance the capacity of the transmission channel in impulsive noise environment has been studied in [22-23]. We are interested in the improvement of the capacity when the transmitter is informed about the value of $k$. The input to the communication channel is still binary with equal input probability**.** At the modulator, a variation in the transmitted symbol energy is introduced as a function of the channel state. Note that the output $Y$ is still binary and the channel remains symmetric because the transition probability is the same for both input symbols.

The mathematical formulation of this problem is given in (9), in which the vector $\{E_b^k\}$ denotes the transmitted symbol energy used for a particular value of $k$. Hence, we have to find the energy distribution that maximizes $C(+,*)$ under the condition that the average energy is less than or equal to $E_b$, i.e.

$$\{E_b^k\} = \underset{0<\sum_{k=0}^{\infty} P_k \times E_b^k \leq E_b}{\operatorname{argmax}} \{C(+,*)\}, \tag{9}$$

under the constraint

$$\sum_{k=0}^{\infty} P_k \times E_b^k \leq E_b. \tag{10}$$

The constraint (10) is used to extend formulas (7) and (8) to the cases of informed and non-informed receiver:

The problem is to find the vector $\{E_b^k\}, k = 0, 1, \cdots$, under the constraint (10) which maximizes [22]

$$C(+,+) \;= 1 - \min_{\{E_b^k\}} \left\{\sum_{k=0}^{\infty} P_k \times H\left(p\left(k, E_b^k\right)\right)\right\}, \tag{11}$$

and

$$C(+,-) \;= 1 - \min_{\{E_b^k\}} \left\{H\big(\overline{p(k, E_b^k)}\big)\right\}, \tag{12}$$

where:

$$p\left(k, E_b^k\right) = Q\left(\sqrt{\frac{E_b^k}{2\times\left(\sigma_g^2+\frac{k\sigma_I^2}{A}\right)}}\right). \quad (13)$$

The maximization problem is difficult to solve in general. An equivalent problem, called water-filling, is considered in calculating the capacity for fading channels [12].

One possible solution is to define the variable transmitted energy as follows:

$$\begin{cases} E_b^0 = \alpha\frac{E_b}{e^{-A}} & k = 0 \\ E_b^k = (1-\alpha)\frac{kE_b}{A} & k \geq 1 \end{cases}, \quad (14)$$

with $0 \leq \alpha \leq 1$.

Expression (14) satisfies the condition $\sum_{k=0}^{\infty} P_k \times E_b^k = \alpha E_b + (1-\alpha)\sum_{k=1}^{\infty} P_k \times \frac{kE_b}{A} = E_b$.

By taking into account the definition (14) of the transmitted energy, the mathematical formulation of equations (11) and (12) yields the achievable transmission rate expressions (15) and (16), representing a suboptimum solution.

$$R_\alpha(+,+) = 1 - \min_\alpha \sum_{k=0}^{\infty} P_k \times H\left(Q\left(\sqrt{\frac{E_b^k}{2\left(\sigma_g^2+\frac{k\sigma_I^2}{A}\right)}}\right)\right), \quad (15)$$

and

$$R_\alpha(+,-) = 1 - \min_\alpha H\left\{\sum_{k=0}^{\infty} P_k \times Q\left(\sqrt{\frac{E_b^k}{2\left(\sigma_g^2+\frac{k\sigma_I^2}{A}\right)}}\right)\right\}. \quad (16)$$

In the next section, these rates are evaluated for $\alpha = \alpha_0 = 0.01$.

## IV. Simulation Results & Discussion

In the following, the impulse - and Gaussian noise variances are set equal to $\sigma_I^2 = 7.28 \times 10^{-4}$ Watt, $\sigma_g^2 = 7.28 \times 10^{-7}$ Watt, which match the average values deduced from measurements in [6]. In addition, we set the energy per bit $E_b = 7.28 \times 10^{-3}$ Joules to achieve a signal to impulse noise ratio $10 \times log_{10}(E_b/\sigma_I^2) = 10$ dB.

Our goal is to obtain high achievable data rates for a constant value of $\alpha = \alpha_0$. We therefore consider separately the cases of small and large values of the parameter $A$, for any value of $\alpha$.

### *A. Optimizing the achievable transmission rate across α variations*

- **Small *A*:**

In Fig. 3, we evaluate (15) for $0.001 < A < 1$, for different values of $\alpha$ for informed transmitter and - receiver.

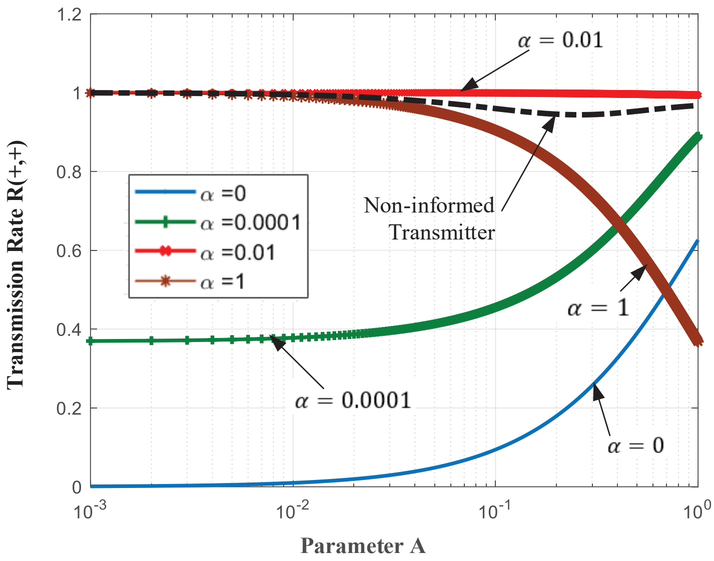


Fig. 3. Transmission Rate for informed transmitter and - receiver versus small *A*

For A = 0, the channel will behave as an AWGN channel with variance $\sigma_g^2$ and thus for $\alpha = 1$ the channel capacity is given by

$$C_{A=0}(+,+) = C_{A=0}(+,-) = 1 - H\left(Q\left(\sqrt{\frac{E_b}{2\sigma_g^2}}\right)\right), \quad (17)$$

and for $\alpha = 0$, $R(+,+)_{A=0,\ \alpha=0} = 0$ .

The transmission rate in case of an informed transmitter and the specific case of $A = 0$ and $\alpha = \alpha_0$ is given by

$$R(+,+)_{A=0} = R(+,-)_{A=0} = 1 - H\left(Q\left(\sqrt{\frac{\alpha_0 E_b}{2\sigma_g^2}}\right)\right). \quad (18)$$

From Fig. 3, we conclude that for $\alpha = \alpha_0 = 10^{-2}$, and $E_b/\sigma_G^2 = 10^{+4}$, the transmission rate $\lim_{A\to 0}\{R(+,+)\}$ is close to the capacity $C_{A=0}(+,+)$.

<u>**Note:**</u> In case of non-informed transmitter and small values of $A$, the capacity is calculated using (7) and (8) as follows:

$$\begin{aligned}\lim_{A\to 0} C(-,+) &= \lim_{A\to 0}\Big[1 - e^{-A} \times H\left(Q\left(\sqrt{\frac{E_b}{2\sigma_g^2}}\right)\right) \\ &\quad - Ae^{-A} H\left(Q\left(\sqrt{\frac{E_b}{2(2\sigma_g^2+\sigma_I^2/A)}}\right)\right) - \ldots\Big] \\ &= 1 - H\left(Q\left(\sqrt{\frac{E_b}{2\sigma_g^2}}\right)\right), \quad (19)\end{aligned}$$

and

$$\lim_{A\to 0} C(-,-) = 1 - H\left(\lim_{A\to 0}\{\overline{p(k)}\}\right) = 1 - H\left(Q\left(\sqrt{\frac{E_b}{2\sigma_g^2}}\right)\right). \quad (20)$$

- **For large $A$:**

We observe that the average value of $\{\frac{k\sigma_I^2}{A}\} = \sigma_I^2$ and the variance of $\{\frac{k\sigma_I^2}{A}\}$ is given by $\frac{\sigma_I^4}{A}$, which approaches zero when $A$ is large compared to $\sigma_I^4$. In this case, the receiver will observe an AWGN noise with variance $(\sigma_g^2 + \sigma_I^2)$. Similarly, the average value of $\{\frac{kE_b}{A}\} = E_b$ and the variance of $\{\frac{kE_b}{A}\} = \frac{E_b{}^2}{A}$.
Since the asymptotic variance of the noise at the receiver is equal to $(\sigma_g^2 + \sigma_I^2)$, information at the receiver about k becomes irrelevant and the limiting capacity for the informed transmitter is given by

$$\lim_{A\to\infty,} C(+,*) = 1 - \mathrm{H}\left(Q\left(\sqrt{\frac{E_b}{2(\sigma_g^2+\sigma_I^2)}}\right)\right). \quad (21)$$

We also investigate the variations of $R_\alpha(+,+)$, (15), for 1 < A < 100 and different values of $\alpha$. The results are illustrated in Fig. 4. From Fig. 4, we conclude that again, for $\alpha = \alpha_0 = 10^{-2}$, the transmission rate $\lim_{A\to\infty}\{R(+,+)\}$ is close to $\lim_{A\to\infty,} C(+,*)$.

The average transition probability in case of an informed transmitter and the specific case of $A \to \infty$ *and* $\alpha = \alpha_0$ is given by

$$\lim_{A\to\infty} \overline{p(k)} =$$

$$= \lim_{A\to\infty} \left[ e^{-A} Q(0) + \sum_{k=1}^{\infty} P_k \times Q\left(\sqrt{\frac{(1-\alpha_o)\frac{kE_b}{A}}{2\left(\sigma_g^2+\frac{k\sigma_I^2}{A}\right)}}\right)\right]$$

$$= \lim_{A\to\infty} \left[ e^{-A} Q(0) + (1\text{-}e^{-A})\, Q\left(\sqrt{\frac{(1-\alpha_o)\, E_b}{2(\sigma_g^2+\sigma_I^2)}}\right)\right]$$

$$= \mathrm{Q}\left(\sqrt{\frac{(1-\alpha_o)\, E_b}{2(\sigma_g^2+\sigma_I^2)}}\right). \quad (22)$$

The corresponding achievable rate for A⟶∞ is equal to

$$\lim_{A\to\infty,\,\alpha_0} R(+,*) = 1 - H\left[Q\left(\sqrt{\frac{(1-\alpha_o)\, E_b}{2(\sigma_g^2+\sigma_I^2)}}\right)\right]. \quad (23)$$

Hence, $\lim_{A\to\infty,\,\alpha_0} R(+,*) < \lim_{A\to\infty} C(-,*)$. Even though the difference is negligible, choosing a fixed $\alpha_o$ for the informed transmitter case reduces the transmission rate for extremely large values of $A$.

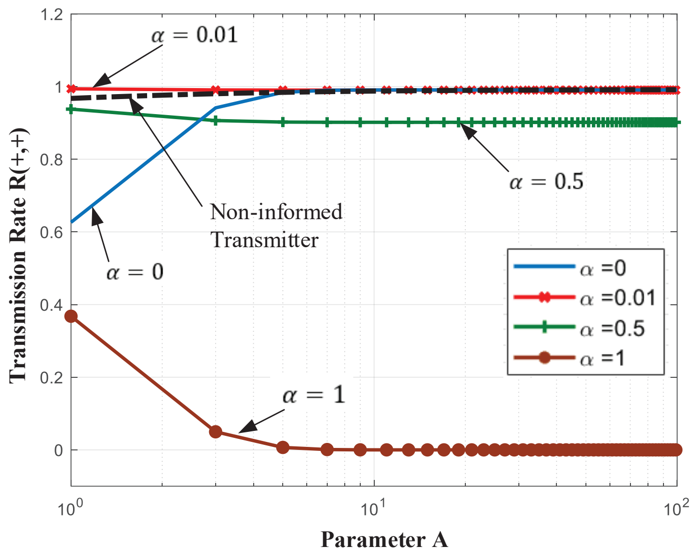


Fig. 4. Transmission Rate R(+,+) for 1 < A < 100

Based on the findings presented in Fig. 3 and Fig. 4, we continue with the value $\alpha = \alpha_o = 0.01$, which gives a good result for the whole range of interest of $0 \le \mathrm{A} \le 100$.
**Note1:** It is worth noting that for $\alpha = \alpha_o = 0$,

$$\lim_{A\to\infty,\ \alpha=0} R(+,*) = 1 - H\left(Q\left(\sqrt{\frac{E_b}{2\times(\sigma_g^2+\sigma_I^2)}}\right)\right), \quad (24)$$

and for $\alpha = \alpha_o = 1$, $\lim_{A\to\infty,\ \alpha=1} R(+,*) \to 0$.
**Note2:** The limiting capacity for the non-informed transmitter for large $A$ is also equal to

$$\lim_{A\to\infty} C(-,+) = \lim_{A\to\infty} C(-,-) = 1 - \mathrm{H}\left(Q\left(\sqrt{\frac{E_b}{2(\sigma_g^2+\sigma_I^2)}}\right)\right). \quad (25)$$

Next, we examine the $\overline{p(k)}$ and the achievable rates for values of 0 < A < ∞, and $\alpha = \alpha_0$.

### B. *Transition probability evaluation*

We reformulate the expression for the $\overline{p(k)}$, using (14) as:

$$\overline{p(k)} = e^{-A} Q\left(\sqrt{\frac{\alpha_0\, E_b/e^{-A}}{2\sigma_g^2}}\right) + $$
$$+ \sum_{k=1}^{\infty} P_k \times Q\left(\sqrt{\frac{(1-\alpha_o)\frac{kE_b}{A}}{2\times\left(\sigma_g^2+\frac{k\sigma_I^2}{A}\right)}}\right). \quad (26)$$

Fig. 5 depicts the $\overline{p(k)}$ variations with respect to the parameter $A$. The $\overline{p(k)}$ curve for fixed $\mathrm{E_b^k} = \mathrm{E_b}$ for k $\ge 0$ is added for comparison purposes.

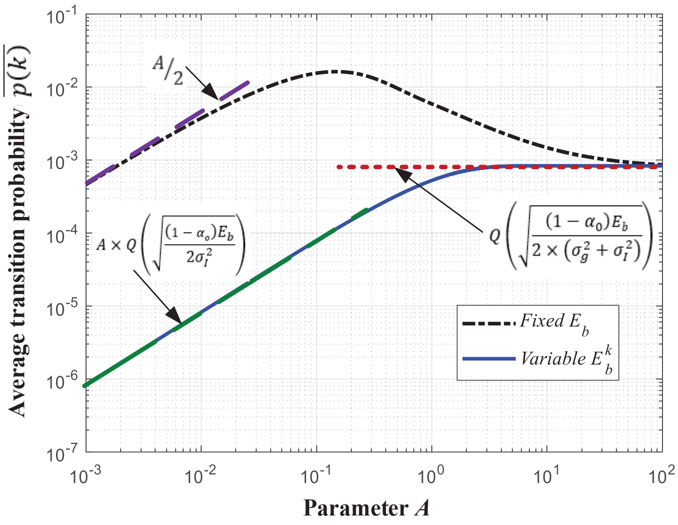


Fig. 5. $\overline{p(k)}$ for variable transmitted energy, as a function of *A*.

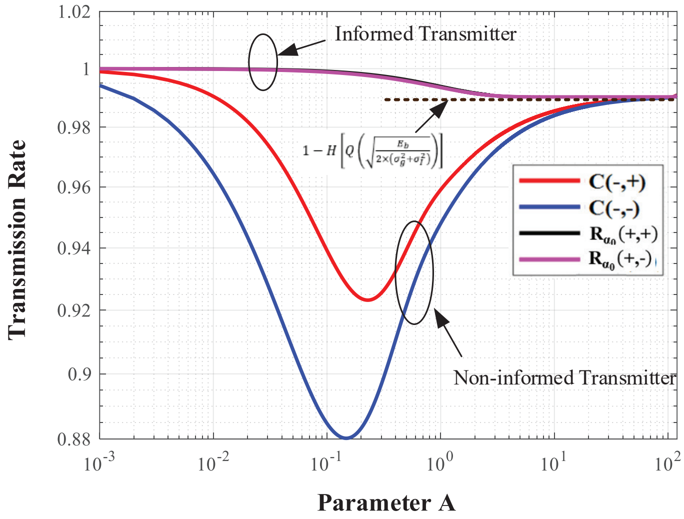


Fig. 6. Achievable transmission rates as a function of *A*.

Fig. 5 shows that the average transition probability $\overline{p(k)}$, for an informed transmitter using the suboptimum solution from (14), is significantly lower than that for a non-informed transmitter with fixed transmitted energy. Ranging between $10^{-6}$ and $10^{-3}$, this transition probability reflects the low error rate of the described BSC channel, and it results in a higher achievable channel rate.

For sufficiently large *A*, the asymptotic $\overline{p(k)}$ value depicted in Fig. 5 concurs with the earlier-discussed outcomes in section IV.A, expression (22).

For *A* approaching the value 0, we observe that the slope is dominated by $A \times Q\left(\sqrt{\frac{(1-\alpha_o)\,E_b}{2\sigma_I^2}}\right)$. This is a result of the asymptotic behavior of $\overline{p(k)}$, as described in equation (27), where

$$\lim_{A\to 0}\left\{\overline{p(k)}\right\} =$$

$$= \lim_{A\to 0}\left\{P_0 \times Q\left(\sqrt{\frac{\alpha_o \frac{E_b}{e^{-A}}}{2\sigma_g^2}}\right) + P_1 \times Q\left(\sqrt{\frac{(1-\alpha_o)\frac{E_b}{A}}{2\times\left(\sigma_g^2+\frac{\sigma_I^2}{A}\right)}}\right) + \cdots\right\}$$

$$= \lim_{A\to 0}\left\{e^{-A} Q\left(\left(\sqrt{\frac{\alpha_o \frac{E_b}{e^{-A}}}{2\sigma_g^2}}\right)\right) + A e^{-A} Q\left(\sqrt{\frac{(1-\alpha_o)^{E_b}/_A}{2\left(\sigma_g^2+\frac{\sigma_I^2}{A}\right)}}\right) + \ldots\right\}$$

$$= Q\left(\sqrt{\frac{\alpha_o\, E_b}{2\sigma_g^2}}\right) + \lim_{A\to 0}\left\{A \times Q\left(\sqrt{\frac{(1-\alpha_o)E_b}{2\sigma_I^2}}\right)\right\}. \quad (27)$$

## C. *Transmission rate evaluation*

Based on the results for the $\overline{p(k)}$ and expressions (7), (8), (15) and (16), we evaluate the transmission rate of the proposed BSC channel for the different cases of informed/non-informed transmitter and/or receiver. In Fig. 6, we illustrate the resulting transmission rate curves, versus the parameter *A*.

The results of Fig. 6 confirm that the transmission rate of the studied BSC channel in a Middleton Class-A impulse noise environment significantly improves with an informed transmitter compared to the non-informed one. Its values varying from 0.99 to 1 exceed those of both curves corresponding to non-informed transmitter, even when the receiver is informed, and this is regardless of the value of the parameter *A*.

Thus, we conclude that when the transmitter is not informed about the impulse noise variance, i.e. the value of *k*, the capacity of the channel clearly improves with the knowledge at the receiver. In fact, for *A=0.2*, the capacity value goes from 0.88 for non-informed receiver to 0.92 when the receiver is informed. This difference results in a SNR gain of about 1.2 dB of using the information about the channel at the receiver. Also, Fig. 6 indicates that the capacity for the non-informed receiver at *A=0.01* equals that of informed receiver at *A=0.05*. This implies that the same performance is achieved by an informed receiver with an average transition probability $\overline{p(k)} = 10^{-2}$ as by a non-informed receiver with $\overline{p(k)} = 4.10^{-3}$ (see Fig. 5). Consequently, the same SNR gain of approximately 1.2 dB is observed with the receiver's knowledge.

However, in case of an informed transmitter, the transmission rate of the channel approaches its maximum value equal to 1 when $A \leq 0.1$. This implies a gain of approximately 4.6 dB in SNR compared to a non-informed transmitter and an informed receiver. Additionally, the curves for both scenarios with an informed transmitter, regardless of the receiver's state, are close together. Indeed, as the transmitter adjusts the energy of its modulated signal in accordance with the noise variance level, the $\overline{p(k)}$ value remains very low. Consequently, the receiver's knowledge of the channel state no longer contributes to the capacity enhancement, as its value is already near its maximum.

## V. CONCLUSION

In this paper, we propose a form of a binary symmetric channel model adequate for channels affected by both Gaussian and impulse noise, as described by the Middleton Class-A model. We investigate the influence of the transmitter and receiver knowledge of the channel state information on communication performances in terms of transmission rate/capacity.

The suggested BSC model operates as a channel with an infinite number of states, each characterized by its proper transition probability derived from the analytical formula of the Middleton Class-A noise variance. This approach effectively models channels corrupted by impulse noise, commonly found in transmission mediums, such as powerline and industrial environments.

The interpretation of the mathematical expression of the Middleton Class-A noise effects on transmission reveals that each transmitted symbol is subject to noise with a statistically varying variance, following a Poisson distribution. This is the inspiration behind determining the transition probability of the new BSC channel by a Poisson distribution with characteristic parameter $A$.

The study of the average transition probability and the capacity expressions in case of a non-informed transmitter about the channel state, shows that the information about the impulse noise variance at the receiver side can help to improve the capacity of the channel. In this case, advanced coding techniques and mitigation strategies based on noise estimation at the receiver can be employed to enhance the overall system performance.

In order to further improve the transmission rates of the studied channel, the knowledge of the noise variance at the transmitter side can be explored. This is inspired by previous research results which deal with the possibility of knowing the channel state by the transmitter, although this can be not practical in real systems. By channel state, we refer the exact value of the noise variance at each symbol transmission (the value of $k$). We give a solution based on using a variable transmitted energy in direct proportion to the noise variance.

Simulations of a binary FSK system operating on the proposed BSC model have been accomplished. It was shown that when the transmitter is informed about the channel state, the average transition probability, which is indicative of the Bit Error Rate (*BER*), is maintained low and almost constant regardless of the value of the parameter $A$. Moreover, the side information at the receiver no longer aids in further performance improvement of the system that achieves nearly its maximum transmission rates.

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