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Knowledge construction of trigonometric equations of Grade 11 learners: A constructivist perspective

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Authors:
Njabulo H. Dube[1,2]
Hlamulo W. Mbhiza[1]

Affiliations:
[1]Department of Mathematics Education, School of Teacher Education, University of South Africa, Pretoria, South Africa

[2]Department of Mathematics and Computer Science Education, Faculty of Humanities, University of KwaZulu-Natal, Durban, South Africa

Corresponding author:
Njabulo H. Dube,
dubenh@unisa.ac.za

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The way learners define mathematics shapes their perceptions of its nature as a subject and its relevance to their lives. In South Africa, many learners have limited exposure to meaningful mathematical experiences, which contributes to low achievement levels. This study investigates how 17 Grade 11 learners understood and solved trigonometric equations. Grounded in constructivist learning theory, the research explores how learners build mathematical knowledge through problem-solving. A qualitative approach was used, involving purposive sampling of learners from an afternoon mathematics programme. Data were collected through worksheets and semi-structured interviews. According to the Curriculum and Assessment Policy Statement (CAPS) curriculum, Grade 11 learners are expected to deepen their understanding of trigonometric equations based on foundational knowledge from Grade 10. However, findings revealed that while learners attempted to construct knowledge they encountered significant challenges, particularly in applying algebraic rules.

Contribution: The study concludes that reinforcing basic algebraic concepts and using varied teaching strategies can enhance learners' comprehension of trigonometric equations.

Keywords: Trigonometric equations; learners; construction learning theory; knowledge construction.

Introduction

Trigonometric concepts are a fundamental part of the high school mathematics curriculum in South Africa, with applications extending beyond pure mathematics into practical fields such as engineering, construction, and physics (Arhin & Hokor, 2021; Ferede et al., 2026). Despite its importance, trigonometry often presents significant learning challenges due to its abstract nature and the fragmented conceptual understanding learners tend to develop – especially regarding trigonometric functions and their complexities (Maphutha et al., 2023; Ngu & Phan, 2020). A solid grasp of trigonometry is essential for learners aiming to pursue careers in science and engineering, as well as for success in advanced mathematics tasks (Ngcobo et al., 2019). However, foundational weaknesses in basic mathematical operations – such as working with integers, fractions, and proportional reasoning – can hinder learners' ability to engage effectively with trigonometric problems (Makhubele, 2021; Suryawati & Hasriani, 2019; Wahyuningrum et al., 2017).

Trigonometry was formally introduced into the South African mathematics curriculum in 1994 and remains a key component of mathematics education. According to the Curriculum and Assessment Policy Statement (CAPS), trigonometry accounts for a significant portion of the Grade 11 Mathematics Paper 2 – 50 out of 150 marks – highlighting its importance (Department of Basic Education [DBE], 2011). The CAPS document outlines a clear progression of trigonometric content from Grade 10 to Grade 12, with Grade 11 serving as a critical bridge to more advanced concepts taught in Grade 12 (Mukuka & Tatira, 2025; Spangenberg, 2021). The consistent weighting of trigonometry – 40% or more in Grades 10 and 12, and 50% or more in Grade 11 – underscores its central role in the curriculum and its impact on learners' academic performance.

Despite its prominence, many learners struggle with trigonometry due to gaps in their foundational understanding developed in earlier grades (Sahlu et al., 2025). These challenges must be explored to support learners who wish to pursue mathematics at the tertiary level. Addressing these issues requires a deeper understanding of how learners construct knowledge of, and the ability to solve trigonometric equations, as well as reliance on their prior mathematical knowledge.

This study is guided by the following research questions:

- *How do Grade 11 learners understand and solve trigonometric equations in mathematics?*
- *What prior mathematical knowledge do Grade 11 learners rely on when solving trigonometric equations?*

By investigating these questions, the study aims to provide insights into learners' cognitive processes and inform teaching strategies that can enhance their understanding and problem-solving abilities in trigonometry.

Literature review

Trigonometry is consistently identified as one of the most challenging and abstract areas of high school mathematics (Dhungana et al., 2023). Learners often perceive it as overly complex and struggle to determine which form – triangle, circle, or analytic trigonometry – is appropriate in different problem contexts (Nanmumpuni & Retnawati, 2021). This confusion is frequently linked to a lack of conceptual grounding and the informal use of algebraic notation (Sebsibe & Feza, 2020). Several studies have explored the nature of learners' difficulties in trigonometry. Orhun (2015) found that learners often make fundamental errors due to weak foundational knowledge. Similarly, Usman and Hussaini (2017) identified common errors – such as comprehension, transformation, and process skill errors – among high school learners solving trigonometric problems. These findings highlight a recurring issue: learners are not only struggling with procedural fluency but also with conceptual understanding. To address the abstract nature of trigonometry, Büttner and Erath (2023) advocate for the integration of visual representations – such as graphs and diagrams – to help learners connect symbolic, numerical, and graphical forms. However, despite these recommendations, many classrooms still rely heavily on symbolic manipulation, with limited use of visual tools to support understanding.

In the South African context, systemic challenges further compound these learning difficulties. The Trends in International Mathematics and Science Study (TIMSS) has consistently shown that South African learners perform below the international benchmark of 400 points, reflecting a lack of basic mathematical proficiency (Mullis et al., 2020). Nationally, the Grade 12 mathematics pass rate remains low (DBE, 2020), and studies have pointed to inadequate teacher content knowledge as a contributing factor (Mosvold, 2022; Ngcobo et al., 2019; Pournara et al., 2015; Stols, 2015). Brijlall and Maharaj (2015) argue that ineffective teaching strategies and shallow pedagogical content knowledge hinder learners' progress. Ndlovu (2017) emphasises the need for pre-service teachers to develop deep subject knowledge before entering the classroom. Internationally, similar concerns persist. For example, Rohimah and Prabawonto (2019) found that Indonesian learners struggle with factoring trigonometric quadratics, interpreting problem forms, and applying appropriate solution strategies. In South Africa, Chikiwa (2015) observed that the symbolic language used in trigonometry can be a barrier to understanding, especially in multilingual classrooms, underscoring the need for teachers to explicitly unpack mathematical terminology.

While existing literature has extensively documented the types of errors learners make and the general challenges they face in trigonometry, there is limited research focusing specifically on how learners *construct* knowledge of trigonometric equations – particularly within the South African Grade 11 context. Most studies emphasise error analysis or teaching strategies, but few explore the cognitive processes learners engage in when solving trigonometric equations. This study addresses that gap by investigating how Grade 11 learners construct knowledge of trigonometric equations and what prior mathematical knowledge they draw upon in the process. By focusing on learners' reasoning and problem-solving strategies, the study aims to provide deeper insights into their conceptual development. These insights can inform more targeted teaching approaches that build on learners' existing knowledge while addressing specific misconceptions and gaps.

Theoretical framing

This study is grounded in constructivist learning theory, drawing primarily from the work of Jean Piaget and Lev Vygotsky. Constructivism posits that learners actively construct knowledge through experiences, reflection, and interaction with their environment. This theoretical lens is particularly relevant for understanding how Grade 11 learners engage with and make sense of trigonometric equations. According to Piaget's cognitive constructivism (1936), learners build mental models through the processes of assimilation (integrating new information into existing frameworks) and accommodation (modifying existing frameworks to incorporate new information). In this study, these processes are operationalised by examining how learners use their prior mathematical knowledge – such as algebraic manipulation and understanding of functions – when solving trigonometric equations. The study explores how learners adjust or expand their conceptual frameworks as they encounter new trigonometric identities and problem types.

Contextual factors, such as learners' backgrounds, prior experiences, and classroom environments, also play a critical role in shaping knowledge construction (Mbhiza, 2021). This study considers these factors by investigating how learners' previous exposure to algebra and trigonometry in Grade 10 influences their current understanding and problem-solving strategies in Grade 11. Social constructivism, influenced by Vygotsky (1962, 1978), adds another dimension by emphasising the role of social interaction and cultural tools in learning. Central to Vygotsky's theory is the concept of mediation, where learners acquire cognitive tools – such as language, symbols, and problem-solving strategies – through guided interaction with more knowledgeable others (Tzuriel, 2021). In this study, mediation is observed through teacher scaffolding, peer collaboration, and classroom discourse, which support learners in navigating the complexities of trigonometric equations.

A key construct from Vygotsky's theory, the Zone of Proximal Development (ZPD), is particularly relevant.

The ZPD refers to the gap between what a learner can do independently and what they can achieve with guidance. This study operationalises the ZPD by identifying the types of support learners require – whether from teachers, peers, or teaching materials – to move from basic procedural manipulation to deeper conceptual understanding of trigonometric equations. Furthermore, constructivist learning strategies – such as reflection, experimentation, and discussion – are central to this study's design. Learners are encouraged to engage with trigonometric identities, test their understanding through problem-solving, and articulate their reasoning during semi-structured interviews. These activities reflect the constructivist emphasis on active learning and meaning-making (Resnick & Glaser, 2016; Saleem et al., 2021). The constructivist learning theory provides a comprehensive framework for examining how learners build and refine their understanding of trigonometric equations. It informs the study's focus on:

- The role of prior knowledge in shaping new learning (Piaget, 1936).
- The importance of social interaction and scaffolding (Vygotsky, 1978).
- The cognitive strategies learners use to internalise and apply mathematical concepts.

By applying this framework, the current study uncovers not only what learners know, but how they come to know it – highlighting both the cognitive and social dimensions of learning trigonometry. The following section presents the methodological approach for the study.

Research methods and design

This study employed a qualitative research approach, underpinned by a phenomenological design, to explore the lived experiences of Grade 11 learners as they engaged with trigonometric equations. Phenomenology, as described by McLeod (2024), seeks to understand the meaning of lived experiences from the perspective of individuals, aiming to uncover the essence of how people perceive and make sense of specific phenomena. In this context, the study focused on how learners experience, understand, and navigate the learning of trigonometric equations in a real classroom setting. The research was conducted over a one-month period during an afternoon mathematics support programme at a South African high school. From a Grade 11 mathematics class of 67 learners, 17 participants were purposively selected based on their regular attendance in the afternoon programme and their willingness to participate, as indicated by signed consent forms. In the South African education system, learners begin selecting subject streams in Grade 10, with Mathematics being a core subject for those pursuing the science stream. The selected learners were engaged in a series of activities designed to elicit their understanding of trigonometric equations, including diagnostic testing, classroom observations, and interviews. Table 1 presents the research methods we employed in the current study.

As depicted in Table 1, data collection involved multiple methods to ensure a rich and comprehensive understanding of learners' experiences. Learners completed a diagnostic test focused on trigonometric equations, which was analysed to identify patterns of understanding, misconceptions, and procedural errors.

Their written responses were closely examined to gain insight into their reasoning and problem-solving strategies. In addition, semi-structured and informal interviews were conducted to allow learners to articulate their thought processes, reflect on their learning experiences, and discuss the challenges they encountered.

Informal discussions during the afternoon sessions further enriched the data by capturing spontaneous insights and clarifications. This phenomenological study provided a detailed exploration of learners' experiences with trigonometric equations, using a qualitative approach to capture the depth and complexity of their understanding. The combination of diagnostic testing, observations, and interviews allowed for a holistic view of how learners construct mathematical knowledge, offering valuable insights for improving teaching practices in trigonometry.

Data analysis

The data collected in this study were analysed using a thematic approach, guided by the principles of constructivist learning theory. The analysis focused on three key areas: how learners constructed knowledge of trigonometric equations; the role of their prior mathematical understanding; and the influence of teaching support on their learning processes. Drawing on the cognitive and social dimensions of constructivism as proposed by Piaget and Vygotsky, the analysis explored how learners assimilated and accommodated new trigonometric concepts, and how social interaction and scaffolding contributed to their conceptual development. Vygotsky's concept of the ZPD was particularly useful in identifying the gap between what learners could achieve independently and what they could accomplish with guidance.

TABLE 1: Data collection methods.

Data collection method	Description
Diagnostic tests	Learners completed a trigonometric diagnostic test designed to reveal their conceptual grasp and procedural fluency.
Observation of test scripts	Researchers analysed learners' written responses to identify patterns of understanding and common errors.
Semi-structured and informal interviews	These interviews provided opportunities for learners to reflect on their problem-solving processes, articulate their reasoning, and discuss challenges they encountered.
Classroom interaction and discussions	Informal discussions with learners during the afternoon sessions offered additional insights into their thinking and learning strategies.

The analysis process began with the administration of a diagnostic test consisting of seven questions designed to assess learners' understanding of trigonometric equations. Seventeen Grade 11 learners completed the test individually under controlled conditions, with no interaction permitted during the session. Their written responses were collected and reviewed to identify patterns of understanding, misconceptions, and problem-solving strategies.

Following the test, each learner participated in an individual session where they were asked to reflect on their responses and explain their reasoning for each question. These sessions provided valuable insights into learners' thought processes, assumptions, and conceptual challenges. In addition to these formal interviews, informal discussions during the afternoon programme further enriched the data.

All interviews were recorded, transcribed, and subjected to a coding process to identify recurring themes.

Thematic analysis was then applied to interpret the data in relation to the study's research questions. Each test question was treated as a separate item (numbered 1 to 6), and learners' responses were analysed accordingly.

Where a learner did not attempt a question, this was noted as 'not an attempt'.

This analytical process allowed for a deeper understanding of how learners constructed knowledge in the context of secondary-level trigonometry. The examples presented in the findings illustrate the ways in which learners engaged with the content, revealing both their conceptual progress and the challenges they encountered. Through this approach, the study offers meaningful insights into the cognitive and social dimensions of learning trigonometric equations. The next section presents the data and findings for the study.

Ethical considerations

Ethical clearance to conduct this study was obtained from the Humanities and Social Sciences Research Ethics Committee of the University of KwaZulu-Natal (No. HSSREC/00003000/2021).

Results

Determining the quadrant where the function has a solution

Item 1 was designed at a Level 1 in terms of the cognitive levels specified in the CAPS document, as it required recall of facts: *Given tan θ = –2.5, in which quadrant would the function have a solution? Explain.*

All 17 learners gave correct responses to Item 1. Learners were considered to have made all the necessary constructions for Item 1 if they could correctly identify the quadrants where the given function had a solution. In this item, all learners correctly identified the quadrant and provided a corresponding explanation.

However, not all of them provided an accurate explanation, but they were able to justify the quadrant where the given function has a solution. Item 1 was designed to explore whether learners had constructed the concept of the sign needed to determine the quadrant in which a solution lies. The sign (+ or –) serves as a visual cue to trigger recall of which quadrant the function solution lies in, as learners understand which trigonometric ratio is positive and negative in each quadrant.

The findings showed that all 17 learners had constructed knowledge of the concept of the relationship between the sign of the function and the quadrant in which the solution lies. Figure 1 provides an example of the responses given by the learners for this item.

The response indicates that Shozi knew that the solution belongs in the second and fourth quadrants. Her explanation indicates that she understood the sign (–) indicates where the given function will have a solution; however, her explanation also shows that she confused the meaning of the sign (–) to imply that the answer would be negative. While Shozi was able to identify the correct quadrant when the given function had a solution, the meaning of the sign (–) in this context was misunderstood. During the interview, the following transpired:

Researcher: In this question, you were given *tan θ* = –2.5. The question asks in which quadrant the function would have a solution and explain. Therefore, how did you solve this question?

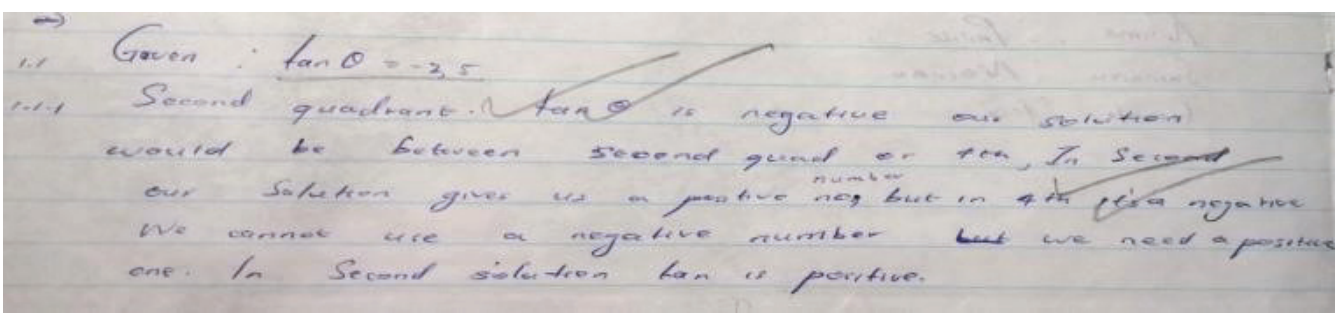

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FIGURE 1: Shozi's written response to Item 1.

Shozi: Firstly, I said that *tan* $\theta = -2.5$. Okay. I checked the sign of *tan*, and then determined which quadrants *tan* would be negative, finding that it would be the second and fourth quadrants.

Researcher: In your explanation in the first question, you also mentioned that *tan* is positive in the second quadrant because it yields a positive solution. Do you want to explain how?

Shozi: Sir … *eish* … I made a mistake in my explanation. Because the reference angle is negative, the solution will be in quadrants 2 and 4.

Researcher: Is there any other method you could have used?

Shozi: I would have used a calculator if I wanted to know exactly what the reference angle is and wanted to find the exact value of *tan* in quadrants 2 and 4.

Shozi's responses throughout the interview demonstrated that she comprehended the idea of trigonometric ratios and the quadrants in which each trigonometric ratio is positive and negative, and she was able to rectify the error made in her written response about the solution being negative. However, the error was noted that Shozi didn't attach the angle when she answered the first question. She kept on referring to *tan* instead of *tan* θ. In the same category, Zulu also demonstrated an understanding of the meaning of the signs (±) in determining the quadrant where the function would have a solution, as shown below in his written response (Figure 2).

In the interview, the researcher asked Zulu to explain his response:

Researcher: In 1.1, you were given *tan* $\theta = -2.5$. The question says, 'In which quadrant would the function have a solution', and you said the second and fourth quadrants. Why?

Zulu: I said that the function will have a solution in quadrants 2 and 4 because *tan* is negative in these quadrants, as I was given the negative value of y. According to the question, it states, 'Which quadrant would the function have a solution?' This is why I said the second and fourth quadrants – because we are given the value of y as negative. *Uyabona* [you see], Sir, in the Cartesian plane, *tan* is negative in the second and fourth quadrant *kodwa ke* [but] where the y value is negative is in the fourth quadrant, if I was plotting the coordinates.

Researcher: Then, tell me, while we are still discussing this question, is there any other method you could have used to solve it?

Zulu: Yes, sir.

Researcher: How?

Zulu: If I wanted the exact solutions, I would have calculated the reference angle at the end, I would say 180° reference angle, then finds the answer and 360 reference angle. Therefore, that will tell which quadrant my answer belongs to.

During the interview, Zulu also demonstrated that he understood and constructed knowledge of the value –2.5 as the *y*-coordinate and related his explanation to plotting coordinates, which showed that he understood the concept in relation to the function. While both Shozi and Zulu's responses showed that the external cue (–) triggered their identification of the quadrant where the solution lay – which indicated which indicated that they constructed knowledge of the concept – Zulu's explanation during the interviews showed that he had interiorised most of the understanding of the concept, as he was able to relate the position where the solution lay to a graphical representation in the Cartesian plane. For Item 1, the findings showed that most learners were able to construct knowledge of the given concept. It was also noted that Zulu made the same errors while explaining, as he also said *tan* instead of *tan* θ. This common error was noted among most learners who do not attach the word *tan* to the angle.

Determining the value of the angle

While in Item 1, learners were asked to identify the quadrant where the solution lay, in Item 2, they were required to show their understanding of the meaning of the reference angle in determining the value of the function: *Given tan* $\theta = -2.5$*, is the value of* $180 < \theta$ *or* $\theta > 180$*?*

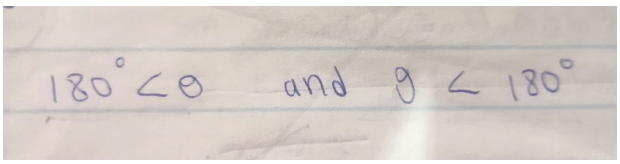

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FIGURE 3: Makwanza's written response to Item 2.

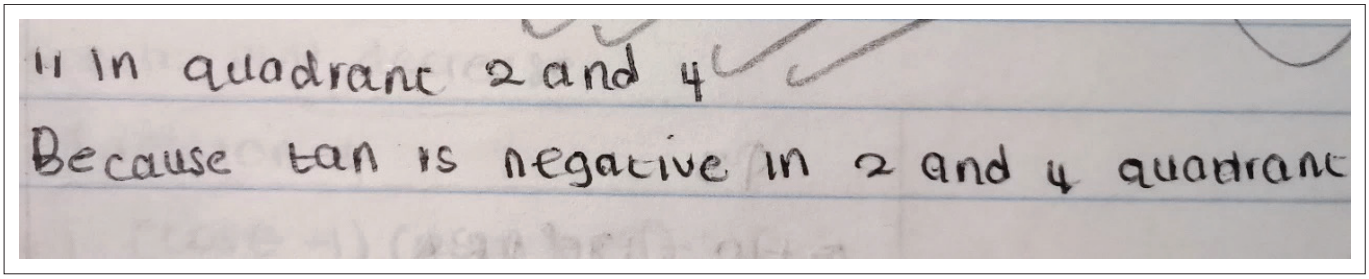

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FIGURE 2: Zulu's written response to Item 1.

In contrast to Item 1, while learners were able to identify where the solution lay, they struggled to predict the value of *tan* $\theta = -2.5$. Only six learners made the necessary constructions. However, nine attempted to write the question, but no construction was made, and two learners did not attempt to write the item. Those who had made the connection of where the solution lay, and the value of the function, were deemed to have encapsulated the construction. Figure 3 shows the response for Makwanza.

In the interview the researcher asked her to describe how she approached the problem:

Researcher: I see you chose two answers. Explain how you solved the question.

Makwanza: I wrote two answers to this question because in the second quadrant θ is less than 180°, Because I will subtract the reference angle, and in the fourth quadrant θ is greater than 180°, because it will be 360° subtract the reference angle.

Researcher: Ok. So, can you explain the connection between the quadrant where the solution lies and the value of the angle?

Makwanza: *Mhhh … angazi* [*I do not know*], *Sir*.

While Makwanza could predict the value without first attempting to perform calculations, she could not make the connection between the position where the solution lay and the value of θ; she could only explain the procedural process that, since *tan* is positive in the second quadrant, the value will be less than 180°. Due to subtracting the reference angle, it is greater in the fourth quadrant. Although she was able to predict the value and explain the procedural parts, she could not make a connection between the concepts. This supports the claim by Roberts et al. (2022) that learners' responses are not necessarily indicative of a conceptual understanding.

Determining whether the trigonometric equation is defined or not

Item 3 was designed to provide insight into whether the learners understood whether the trigonometric equation was defined or not. This item also assessed learners' recall of the rules to solve related questions. Recall questions may involve facts, definitions, terms, or basic instructions, as well as running a quick algorithm or using a formula (Singh et al., 2022). In this item, for the learner to be classified as making the necessary constructions, they must state whether theta is defined or not and provide the correct explanation or calculations: *Given tan* $\theta = -2.5$*, would you say this trigonometric equation is defined or not? Explain.*

The results showed that 8 out of 17 learners displayed an understanding of the question: out of 15 learners who said the trigonometric equation is defined, eight learners provided a correct justification. The remainder failed to justify their answers. This is illustrated in Ntaka's response (Figure 4).

All of these eight learners explained why they were saying it was defined by performing the calculation and solving for *tan* $\theta = -2.5$. Their failure to explain why the trigonometric equation was defined suggests that they had not understood what it means when the trigonometric equation is defined. The knowledge construction was evident in Ntaka's response during the interview, as shown below:

Researcher: When you were answering this question you said that a trigonometric equation is defined. Why are you saying it is defined?

Ntaka: I said it was defined.

Researcher: Why?

Ntaka: Because when I calculate the value of θ, I get the reference angle subtracted from 180 degrees and 360 degrees, and get the answer. So, it is defined.

Researcher: What do we mean when we say the trigonometric equation is defined?

Ntaka: It means the answer is known. If you punch the values in the calculator and get an error, it means it does not have an answer, so it is undefined, *njenga masidvider ngo* 0 [*as we are dividing by 0*].

Ntaka indicated that she understood the trigonometric equation to be defined as the one that calculates and provides the answer.

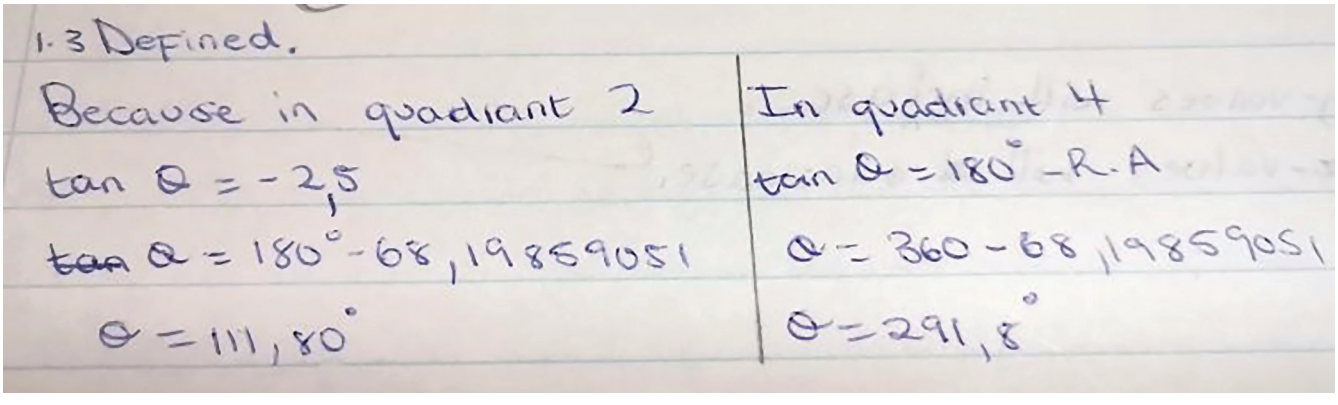

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FIGURE 4: Ntaka's written response to Item 3.

Analysis of learners' responses to Item 4 and Item 5 (equations of the form $a\text{cosec}x$; $\sec x$; $\cot x \pm b = 0$ where $a \geq 1$; $b < 0$ or $b > 0$)

Items were designed to investigate learners' knowledge constructions for solving reciprocals and assess their problem-solving skills. Faulkner et al. (2023) state that to solve a problem efficiently, one must acquire new information and select relevant information to apply in solving the given problem. To solve Item 4 and Item 5, knowledge of algebra and reciprocals was key.

Item 4 and Item 5 were grouped because they test the same knowledge and are reciprocals of sinθ and cosθ, which are continuous functions, and the question was taken from a Grade 10 exam: *Solve for x: 2cosecx + 3 = 0 and 4secx – 12 = 0.*

For Item 4, 16 learners attempted to solve the problem, but only 11 were categorised as making the necessary constructions. Five attempted but failed to solve the problem, suggesting that they had not constructed an understanding of how to solve trigonometric equations involving reciprocals. A larger number of learners (n = 14) were able to solve the problem in Item 5.

Learners in Grade 11 demonstrated a mathematical understanding of the concept of the reciprocals of *cosecθ* and secθ. The responses indicate that learners identified the coefficient and that they needed to divide by it on both sides, they understood the inverse of (Equation 1):

$$\sec x = 1/\cos x \text{ and } \text{cosec}x = 1/\sin x \quad \text{[Eqn 1]}$$

Learners were found to be able to make the necessary constructions. In the extract shown in Figure 5, Zulu performed the required procedures to answer Item 4; Makwanza also carried out the correct procedures to determine the answer for Item 5. Some of the learners did not use the correct notation to indicate that they were calculating an angle; however, while they performed the procedure, it was evident that they did so without any meaning-making: $cos\theta^{-1}$. They did not include the angle, but indicated in the final answer that they were determining the value of the angle. This suggests that the learner views a procedure as an isolated fact.

In that case, it suggests that they are viewing the procedure as an externally directed transformation, similar to solving any other algebraic equation, where the power of the first variable determines the number of solutions. In trigonometric functions, the power is viewed as 1, thus the learner cannot think of the equation having two solutions. In addition, the meaning of the notation is not yet internalised.

Analysis of learners' responses to Item 6

Item 6 was designed to provide insight into learners' constructions when solving a challenging trigonometric equation. An understanding of algebra and trigonometric ratios was required to solve this problem. Özreçberoğlu and Çağanağa (2018) argue that understanding a problem is as important as solving it to developing an understanding of the meaning of mathematics.

Item 6 required learners to solve a challenging trigonometric equation that involved problem-solving skills: *Solve for x: 5sinx – 3 = 2sinx.*

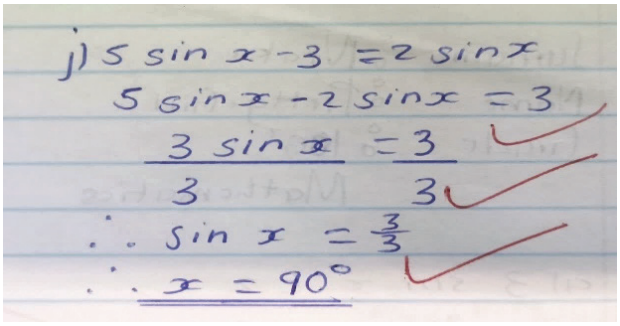

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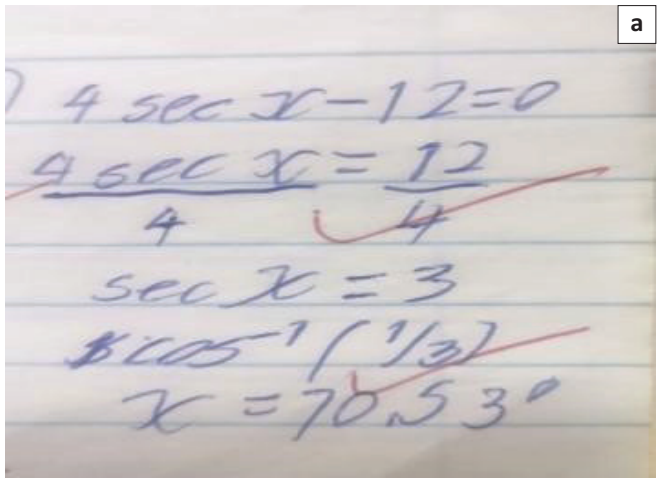
FIGURE 6: Ntaka's response to Item 6.



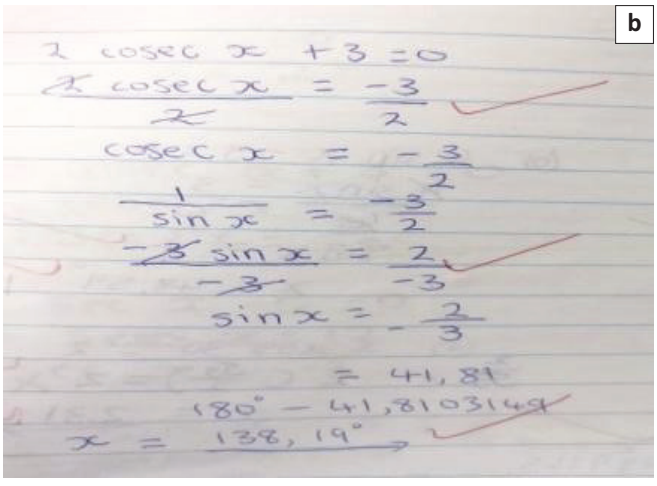


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FIGURE 5: a: Zulu's response for Item 4, and b: Makwanza's response for Item 5.

Item 6 was attempted and solved by 14 of the 17 learners. Only 13 learners demonstrated an understanding of this item. Only one learner gave an incorrect answer; they showed no evidence of understanding the problem. Three learners did not attempt to answer Item 6.

Learners' written responses revealed that most learners understood the question and knowledge construction took place. Figure 6 shows Ntaka's written response to Item 6.

Ntaka demonstrated that she knew the procedure to solve the problem and had constructed a meaningful solution, as evidenced by her answer being given in degrees. In terms of construction theory, the understanding of the structure of the trigonometric equation was accommodated. Ntaka's response indicated she was able to identify the like terms as she transposed $2\sin x$ to the left, subtracted and ended up with $3\sin x$. She continued to divide both sides by 3 and solve for the angle. She jumped the second last step as she divided both sides by 3, which is equal to 1. However, in some cases, most learners demonstrated that they constructed knowledge, with minor errors.

Discussion

The findings of this study offer a comprehensive view of how Grade 11 learners construct knowledge of trigonometric equations, revealing both strengths and gaps in their conceptual understanding. The analysis was guided by constructivist learning theory, particularly Piaget's cognitive constructivism and Vygotsky's social constructivism, which emphasise the active role of learners in building knowledge through experience, reflection, and interaction. In Item 1, all learners correctly identified the quadrants in which the function has a solution, indicating that they had constructed the concept of sign and its relationship to quadrant placement.

This reflects successful assimilation of prior knowledge about trigonometric ratios and their signs in the Cartesian plane. However, while learners like Shozi and Zulu demonstrated procedural accuracy, their explanations revealed varying levels of conceptual depth. Shozi initially misinterpreted the negative sign but corrected her understanding during the interview, illustrating the role of scaffolding in the ZPD. Zulu's explanation went further, connecting the sign to coordinate plotting, suggesting a more internalised understanding and accommodation of the concept.

Item 2 required learners to determine the value of the angle based on quadrant placement. Unlike Item 1, this task exposed a gap in learners' ability to connect procedural steps with conceptual reasoning. Although some learners, like Makwanza, could predict whether the angle was greater or less than 180°, they struggled to explain the relationship between the quadrant and the angle value. This supports Roberts et al. (2022), who argue that correct answers do not necessarily reflect conceptual understanding. Learners may apply procedures without grasping the underlying principles, indicating a need for deeper meaning-making in teaching.

Item 3 explored whether learners understood the concept of a function being defined. While 15 learners stated that the function was defined, only eight provided correct justifications. Ntaka's response exemplifies this partial understanding: she equated a defined function with the ability to compute a value, referencing calculator output and the absence of errors. Her explanation suggests that learners often rely on procedural cues rather than conceptual definitions, reinforcing the importance of explicit teaching in foundational concepts.

Item 4 and Item 5 assessed learners' ability to solve trigonometric equations involving reciprocal functions. These items required knowledge of algebra and the reciprocal identities of sine and cosine. Most learners demonstrated procedural competence, with 11 learners successfully solving Item 4, and 14 learners successfully solving Item 5. Learners like Zulu and Makwanza correctly applied inverse operations and identified reciprocal relationships, indicating that knowledge construction had occurred. However, some learners failed to use correct notation or understand the implications of trigonometric function structure, treating the equations as algebraic rather than trigonometric. This suggests that, while procedural fluency was present, conceptual internalisation of trigonometric notation and function behaviour was still developing. In relation to this, Hurrell (2021) notes that while procedural knowledge is important, performing procedures without understanding the underlying concepts can lead to peculiar and unreasonable solutions.

Item 6 presented a more complex trigonometric equation requiring problem-solving skills. Fourteen learners attempted the item, and 13 learners demonstrated understanding. Ntaka's response showed clear procedural steps and a meaningful solution, indicating successful accommodation of the structure of the equation. Her ability to identify like terms, transpose expressions, and solve for the angle reflects a deeper level of construction.

However, minor errors in notation and skipped steps among some learners suggest that, while the overall understanding was strong, attention to detail and full internalisation of procedures remain areas for improvement.

Across all items, the findings highlight the importance of integrating multiple teaching approaches to support learners' conceptual development. Learners benefit from opportunities to explain their reasoning, receive feedback, and engage with visual and symbolic representations. The study reinforces the need for teachers to scaffold learning, address misconceptions, and connect algebraic and trigonometric concepts to promote deeper understanding.

As JoJo (2015) and Faulkner et al. (2023) suggest, effective problem-solving requires both procedural skill and conceptual insight, both of which must be cultivated through intentional, learner-centred teaching.

Conclusion

This study explored the knowledge construction of trigonometric equations among Grade 11 learners, revealing both promising developments and persistent challenges in their conceptual understanding. The findings showed that, while learners were generally able to identify the correct quadrants for trigonometric functions based on sign, their explanations often lacked conceptual clarity. This suggests that, although procedural knowledge had been assimilated, deeper meaning-making and internalisation of concepts were still developing. Learners demonstrated varying levels of success across different types of trigonometric problems.

In simpler tasks, such as identifying the quadrant in which a function is defined, most learners performed well.

However, when asked to determine angle values or explain whether a theta is defined, many relied on procedural recall rather than conceptual reasoning. This gap between knowing how to perform a task and understanding why it works highlights the need for teaching approaches that prioritise conceptual development alongside procedural fluency. Figure 7 shows a flowchart derived from the research findings that will assist teachers in helping students understand the concept of a trigonometric equation.

The study also found that learners were more confident and accurate when solving equations involving reciprocal functions and more complex expressions, although some treated these problems as purely algebraic.

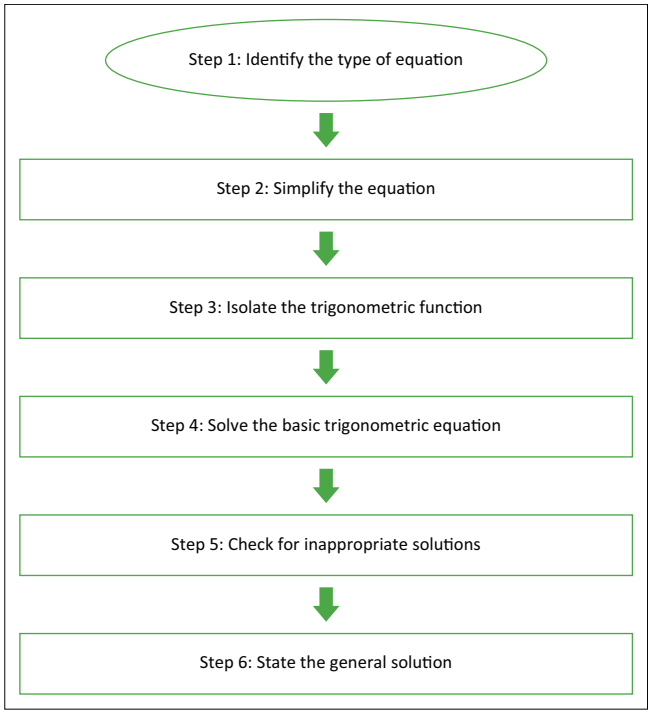

FIGURE 7: Flowchart model for solving trigonometric equations.

This tendency suggests that learners may not fully grasp the unique properties of trigonometric functions, reinforcing the importance of explicitly linking algebraic and trigonometric reasoning in teaching. To support learners in constructing a more robust understanding of trigonometric equations, teaching should incorporate a variety of teaching strategies. Visual representations, real-life applications, and interactive tools can help learners connect abstract concepts to concrete experiences. Encouraging learners to explain their reasoning – both in writing and through discussion – can reveal misconceptions and promote metacognitive awareness.

These explanations also provide teachers with valuable insights into learners' thinking, allowing for more targeted support.

Scaffolded teaching is essential, particularly when introducing complex problems. Teachers should guide learners through problem-solving processes, gradually reducing support as learners gain confidence and independence. Peer collaboration can also be a powerful tool, as learners often benefit from discussing their strategies and challenges with classmates. Grouping learners by ability and encouraging cooperative learning can create a supportive environment where knowledge is co-constructed. Furthermore, the integration of algebra and trigonometry should be a deliberate focus in the curriculum. Given the foundational role of algebra in solving trigonometric equations, reinforcing these connections can help learners transfer their knowledge more effectively. Diagnostic assessments can be used regularly to monitor learners' progress and adjust teaching to meet their evolving needs.

In conclusion, while learners in this study demonstrated encouraging signs of knowledge construction in trigonometry, there remains a need for teaching practices that foster deeper conceptual understanding. By creating learning environments that support exploration, explanation, and reflection, teachers can help learners build the confidence and competence needed to succeed in trigonometry and beyond.

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adapted for publication as a journal article. The original thesis is publicly available at: https://researchspace.ukzn.ac.za/items/09a55934-df4f-4f85-81c3-6b603af2add7.

This article is based on data from a larger study. One other article was published from the same thesis. A related article focusing on an analysis of learners' mental constructions in learning and solving trigonometric equations has been published in *Multidisciplinary Science Journal*, Volume 7(3), e2025120 (https://10.31893/multiscience.2025120). The present article addresses a distinct research question, focusing on how learners understand and solve trigonometric equations in mathematics and prior mathematical knowledge of Grade 11 learners rely on when they are solving trigonometric equations.

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CRediT authorship contribution

Njabulo H. Dube: Conceptualisation, Formal analysis, Investigation, Methodology, Project administration, Writing – original draft, Writing – review & editing. Hlamulo W. Mbhiza: Formal analysis, Methodology, Supervision, Writing – review & editing. Both authors reviewed the article, contributed to the discussion of results, approved the final version for submission and publication, and take responsibility for the integrity of its findings.

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