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# The views and experiences of Grade 3 teachers in developing learners' algebraic representations

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The global concern over low mathematical achievement consistently challenges educational ministries. Poor development of learners' algebraic representations contributes to poor mathematical achievement from the Foundation Phase classes (Grades 1–3). In short, in teaching algebraic representation in Grade 3, success is measured by the extent to which the learners can reason, handle abstractions, manipulate symbols and finally be able to communicate mathematical ideas. This article explores the views and experiences of Grade 3 teachers in developing learners' algebraic representations. The concrete-representational-abstract (CRA) approaches underpin this study as they guide teachers in using concrete objects, drawings, and symbols to build learners' algebraic representations. A single case study research design using a qualitative approach was employed to explore the phenomenon under study. A homogenous purposive sampling was used to select six Grade 3 teachers from three schools in Limpopo province, South Africa. Semi-structured interviews, interpretivist document analysis and non-participant observations were used to collect data. Thematic narrative analysis was used to interpret the data to explore the views and experiences of Grade 3 teachers in developing learners' algebraic representations. The findings indicate that teachers use concrete manipulatives and indigenous games to develop Grade 3 learners' algebraic representations. However, the lesson plans and observations revealed that Grade 3 teachers face challenges teaching abstract number symbols, as learners rely more on concrete objects.

**Contribution:** Considering these findings, the novelty of this article contributes to teachers' knowledge on expanding the CRA approach with scaffolding approaches to teach abstract number symbols to develop Grade 3 learners' algebraic representations. As a result, this research addresses Africanisation, decolonisation, and pedagogical transformation in the alarmingly poor results of mathematics.

**Keywords:** algebraic representations; concrete-representational-abstract; Grade 3 teachers; indigenous games; mathematics

## Introduction

Over the years, poor performance in mathematics has been a problem for many learners. The focus on teaching and learning algebra is premised on the vision of the Department of Basic Education (DBE, 2011), which argued that there is a need for Grade 3 learners to exit the Foundation Phase with high mathematical knowledge, including algebraic skills. By emphasising algebraic skills, the intention is to equip learners with significant abilities to identify patterns, engage with functions, and reason algebraically (Demonty et al., 2018). Despite this idea being noble and desirable, poor performance in mathematics persists, hence the need for research focused on improving performance in algebra from the early grades, and thus the need and significance of this article.

The global concern over low mathematics achievements among learners has led developing nations to take part in programmes such as Trends in International Mathematics and Science Study (TIMSS) and the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ), aimed at improving the pass rate of the subject (DBE, 2023; Mabena et al., 2021). Over the years, poor performance in mathematics has been a problem for many learners. The Foundation Phase is not exempt from poor mathematical literacy and comprehension challenges. The literature has shown that an inadequate understanding of algebra contributes to low overall achievement in mathematics in lower grades. Zapatera and Quevedo (2021) state that learners in secondary education have difficulty learning algebra because of the conventional methods of teaching mathematics in primary education and further state that while some scholars have linked these challenges to cognitive development constraints or the abstraction of algebra,

for many others, these challenges are a result of inadequate mathematics teaching that places an excessive computational emphasis on algebra in primary school.

Curriculum policies indicate that the purpose of teaching algebraic representation in the lower grades is to lay the foundation for the learners to accept and realise algebra as generalised mathematics. For example, the exit-level learning outcomes in the Curriculum and Assessment Policy Statement (CAPS) for mathematics in the Foundation and Intermediate Phases include learners' ability to recognise, describe, and extend patterns; use symbolic language to represent number relationships; and apply properties of operations to solve problems (DBE, 2011). Sun et al. (2023) agree that the development of algebraic thinking should start in primary school and continue throughout mathematics education. Algebraic representations are essential because they allow learners to connect these representations to the real world (Du Plessis, 2018). By explaining each arithmetic step using the properties of operations, learners can understand how algebraic thinking can help them solve everyday problems (Du Plessis, 2018). Extending on the views of Du Plessis (2018), the authors contend that Grade 3 learners need algebraic representation skills to solve real-life problems.

One important method for teaching algebra to Grade 3 learners is to start with regular problem-solving lessons that involve writing and solving problems step by step (Aktürk, 2023). One of the expectations from the CAPS mathematics curriculum is that Grade 3 learners need to be able to identify, describe, and extend number patterns; solve problems; and begin to represent number sentences using algebraic representations (DBE, 2011). Mason (2017) agrees that these expectations encourage early algebraic thinking and develop confidence in symbolic representations. A learner can express numbers with symbols, use scenarios or illustrate using actual situations. Since these techniques demonstrate a high degree of effectiveness in teaching algebraic representation to first graders, they should be promoted in the classroom (Ainsworth & Scheiter, 2021). Learners who overlook the algebraic nature of mathematics fail to acquire the essential algebraic skills of generalisation, expression and reasoning (Zapatera & Quevedo, 2021). The learners evaluate the patterns and relations presented in number sequences and geometric properties at an early stage of learning (Levin & Walkoe, 2022). As a result, there is a need to explore the views and experiences of Grade 3 teachers in developing learners' algebraic representations.

While mathematics has various components, our article investigates the challenges of teaching and learning algebra to and by young learners. The focus on the teaching and learning of algebra is premised on the vision of the DBE (2011), which posits that there is a need for Grade 3 learners to exit the Foundation Phase with high mathematical knowledge, including algebraic skills and representations. Through the emphasis on algebra, it is hoped that learners will develop patterns, functions, and algebraic thinking skills (DBE, 2011).

## Research question

What are the views and experiences of Grade 3 teachers in developing learners' algebraic representations?

## Research aim

This article explored the views and experiences of Grade 3 teachers in developing learners' algebraic representations. This was done through a case study in qualitative research design using interviews, observations, and interpretivist document analysis with six Grade 3 teachers from selected schools. Data were analysed thematically to identify patterns in their pedagogical approaches and challenges. In the South African curriculum, teaching algebraic representation in Grade 3 forms part of broader skills of pattern, functions and algebra (DBE, 2011). However, algebra is not explicitly labelled as such in the Foundation Phase curriculum, which results in inconsistent classroom practices.

## Literature review

Teaching algebra contributes to improving performance in mathematics results. The authors reviewed various studies globally from Pramesti and Retnawati (2019), Ying et al. (2020), Stephens et al. (2021) and Bråting (2023) that have been conducted on mathematics and, more specifically, on algebra. The study of Pramesti and Retnawati explored the challenges of teaching and learning algebra in Indonesia. They used a phenomenological research design within a qualitative approach. Their findings show that understanding algebraic problems and variables is still challenging. Drawing from Pramesti and Retnawati, Mathaba et al. (2024), Moru and Mathunya (2022), and Stemele and Jina Asvat (2024), it is clear that learners make many errors, contributing to misunderstanding algebraic problems and the variables in learning algebra. To address this problem, the authors argued that the higher-order thinking skills (HOTS) learning method can teach algebraic skills.

Another study by Ying et al. (2020) focused on learners' difficulties in solving algebraic problems. A quantitative research approach guided the selection of a descriptive case study. Among other findings, Ying et al.'s study shows that learners struggle with visual-spatial skills in solving algebraic problems. By understanding the findings of Ying et al. the authors have determined that there is a need to develop learners' visual-spatial skills from the early grades by engaging them in mathematical games like Hopscotch (Grover et al., 2022; Van der Walt, 2022). We distinguish our research from others by emphasising the importance of developing algebraic ability for young learners through games that enhance mathematical outcomes.

A study by Stephens et al. (2021) investigated learners' understanding of algebra following a three-year early algebra intervention. The learners started the intervention in Grade 3 and were assessed through written activities in Grade 6. Their findings show that controlled learners performed well in algebraic understanding (Stephens et al., 2021). Guided by

Lederman et al. (2021) and Hornburg et al. (2022), the authors note that there should be a follow-up to check learners' understanding throughout their academic years. Bråting's (2023) study, which explored the delivery of algebra content in Swedish schools, assumed that there was a link between arithmetic and algebra. That study revealed that most algebra topics are taught to young learners through number sense skills. This was absent in the Swedish curriculum. However, Bråting recommended implementing algebraic thinking skills in all mathematics content areas in early grades as number sense skills. Through the studies of Bråting, Adamuz-Povedano et al. (2021), and Piriya et al. (2019), the authors maintain that there is a strong relationship between number sense and algebra skills. Thus, learners could be taught these mathematical concepts through a similar pedagogical approach.

While there is growing empirical research on algebra, current literature remains scarce on teachers' experiences facilitating learners' development of algebraic representations in resource-constrained schools. Much of the existing research focused on higher grades, focusing on learners' achievement in mathematics and teachers' perceptions of curriculum demands. This article, therefore, addresses a critical gap by exploring Grade 3 teachers' views and experiences in implementing algebraic representation, particularly in settings where professional support and pedagogical resources are limited. By centring teachers' voices, the study contributes to a more nuanced understanding of the realities they face and the strategies they use to cultivate early algebraic thinking. Developing algebraic representations in the early years of schooling is increasingly recognised as essential for building foundational mathematical thinking. Reviewing the literature from Wettergren (2022), the authors agree that engaging young learners in algebraic thinking needs to begin in the primary grades through meaningful patterns and symbol use. However, early grade teachers often struggle with conceptualising what early algebra entails and how to integrate it meaningfully into the curriculum. Jung et al. (2024) emphasise that teachers are central in bridging concrete mathematics understanding with abstract algebraic reasoning. Yet, their teaching practices are often shaped by a lack of professional development and rigid curriculum guidelines. Furthermore, Kirk et al. (2023) highlight that mathematics classrooms reflect tension between curriculum coverage and teachers defaulting to encouraging representational understanding.

## Theoretical framework

The views and experiences of Grade 3 teachers in developing learners' algebraic representations are grounded in the CRA approach. Its relevance in this article emphasises three stagesof learning algebraic representation: concrete, representational, and abstract (Flores et al., 2020). Teachers can use concrete objects to represent algebraic concepts. At the same time, representation involves using pictures or drawings to represent those objects, and abstract symbols and numbers to represent mathematical ideas. The CRA approach provides a robust understanding of how teachers can use this approach to help Grade 3 learners gradually build their knowledge of algebraic representations.

### Using physical objects in developing algebraic representation

Teachers can use the elements of physical representation in teaching and learning mathematics (Mainali, 2021). Khan and Khan (2021) state that using physical objects in developing algebraic representation for young learners is an effective strategy for helping them transition from concrete to abstract mathematical thinking. In addition, Quane (2024) asserts that manipulatives provide a tangible way for young learners to understand symbolic mathematics, particularly when transitioning to algebraic thinking. As a result, the teachers can introduce algebraic concepts by using everyday objects like blocks, counters or beads to represent unknowns in mathematics. This indicates that in mental mathematics activities, teachers can introduce teaching and learning algebraic representations by using physical objects with Grade 3 learners.

### Using drawings to represent objects

Donovan and Fyfe (2022) indicate that this stage allows children to transition from the concrete manipulation of physical objects to more abstract forms of mathematical thinking. Wilkie (2024) agrees that drawings in algebraic representation are a critical tool because they maintain a visual and intuitive connection to the real world in higher grades. Singh and Azman (2023) maintain that drawings allow learners to create simplified representations of objects and relationships in mathematical situations, enhancing their understanding of algebraic concepts like variables, unknowns and operations. However, Başkan Takaoğlu (2024) shows that some learners may confuse drawings with the objects they represent, leading to misunderstandings. Thus, teachers should explicitly guide learners in using drawings as representations, not literal depictions.

### The use of symbols and numbers to represent mathematical ideas

A symbol or set of symbols, characters, diagrams, objects, images, or graphs is called a representation, and it can be used to help teach and understand mathematics (Mainali, 2021). It is common to underestimate or ignore learners' shift from interacting only with symbols and abstracting from concrete manipulatives. As a result, teachers need to introduce a smooth shift between materials and algebraic symbols (Sun et al., 2023). Using diverse modes of representation would improve the teaching and learning of mathematics, making representation a crucial component of mathematics education.

# Research methods and design

Due to the experiential and constructivist nature of the CRA approach, this study adopted an interpretivist paradigm. To understand the views and experiences of Grade 3 teachers in developing learners' algebraic representations through CRA requires exploring their subjective experiences and meaning-

making processes within specific classroom contexts. The interpretivist paradigm is relevant as it recognises that reality is not objective and fixed, but shaped by individuals' cultural, social, and historical backgrounds (Paudel, 2024). Rotella (2023) supports this by stating that interpretive researchers must acknowledge how participants' realities are influenced by their lived experiences. Thus, this article used the interpretivist lens to explore how learners and teachers interpret and respond to the CRA approach in teaching and learning algebraic representations.

The interpretivist paradigm emphasises qualitative research (Muzari et al., 2022). It provides researchers with thick and rich data collected from a social setting. Dawadi et al. (2021) affirm that an interpretive case study research design can be applied in qualitative studies, particularly within interpretive paradigms. To contextualise the readers, an interpretive case study research design is a qualitative research approach grounded in the interpretivist paradigm. It seeks to explore and investigate how people understand their experiences within a specific real-life context (Cleland et al. 2021). In this study, the interpretive case study enabled an in-depth exploration of Grade 3 teachers' pedagogical views, experiences and contextual understandings in developing learners' algebraic representations. Six Grade 3 teachers from three schools were selected through homogeneous purposive sampling based on their experiences of teaching mathematics (Akkaş & Meydan, 2024). Two Grade 3 teachers per school participated in this research. The article acknowledged potential bias arising from having only two Grade 3 teachers per school participate. This limited sample may not fully represent the diversity of teaching practices or perspectives within each school, which could influence the generalisability of the findings (Hays & McKibben, 2021). To mitigate this bias, data triangulation (Eds. Bentalha & Alla, 2024), through semi-structured interviews, interpretivist document analysis and non-participant observations, was used to corroborate teachers' insights, views and experiences and minimise reliance on single perspectives. The teachers taught Grade 3 mathematics in three public primary schools in Limpopo, South Africa. Table 1 illustrates the participants' professional knowledge and qualifications for teaching mathematics in the Foundation Phase. These public schools were mandated to use the CAPS as their framework, and they also depended on the DBE for concrete manipulatives for teaching mathematics in Grade 3.

**TABLE 1:** Pseudonyms, qualifications, and professional experience of the participants.

| Pseudonyms | Highest qualification | Professional experience in teaching mathematics in Grade 3 (years) |
|---|---|---|
| S1T1 | Bachelor of Education, Honours | 10 |
| S1T2 | National Postgraduate Diploma in Foundation Phase | 5 |
| S2T3 | Bachelor of Arts, Postgraduate Education Certificate | 4 |
| S2T4 | Bachelor of Education | 16 |
| S3T5 | Bachelor of Education in Natural Sciences, Honours | 3 |
| S4T6 | Advanced Senior Certificate in Education, mathematics | 36 |

Guided by interpretive case study design, data were collected through semi-structured interviews, interpretivist document analysis and non-participant observation. This selection of data collection instruments was supported by Nord (2022), who states that researchers must use tools creatively to obtain authentic and rich data when conducting qualitative research. Kallio et al. (2016) define a semi-structured interview as a flexible and guided conversation between the interviewer and interviewee. The role of the interviewer is to ask open-ended questions prepared from an interview guide and probing questions based on the interviewee's responses (Kallio et al., 2016). For this research, an interview schedule with open-ended questions designed from the research question was used during the conversations to ensure that the authors (interviewers) asked relevant questions to the teachers (interviewees) with probing questions to understand the underlying issues related to teaching and learning algebra in Grade 3.

Most importantly, the interview schedule was used to ensure internal validity (Khan & MacEachen, 2022). One-on-one interviews were conducted to solicit teachers' views and experiences in developing learners' algebraic representations for Grade 3 learners. During the conversations, an audio recorder was used to capture the questions and responses from the participants. There were at least two sessions of the interview with each participant, the first interview session helped to build rapport and trust (Hershkowitz et al., 2021), allowing the participants to be comfortable in sharing their experiences while the second session enabled deeper exploration on the issue and clarification of earlier responses while enhancing the richness of the data in qualitative research (Lim, 2025).

Khatri and Karki (2022) explain interpretivism as interpretivist document analysis, a systematic procedure for interpreting printed and electronic documents. Like other qualitative research methods, interpretivist document analysis requires examining and interpreting data to elicit meaning, gain understanding, and develop empirical knowledge (Khatri & Karki, 2022). Morgan (2022) confirms that interpretivist document analysis is usually used to support triangulation, allowing researchers to confirm or challenge findings from interviews or observations. For this study, interpretivist document analysis was used, where six teachers' lesson plans (documents) were requested to corroborate their responses during the interviews and interpret how they planned their mathematics lessons for Grade 3. Morgan advised that qualitative researchers need to design an interpretivist document analysis tool to systematically review, interpret, and make meaning from existing documents related to the research topic. As such, an interpretivist document analysis tool was developed with several questions to gather text-form data from the lesson plans. This process was done to avoid bias introduced by direct interaction, just like the interviews.

Byrne (2021) asserts that non-participant observation involves the researcher taking on the role of an outsider or passive observer, without interacting with participants or

influencing the observed situation. Similarly, Bentalha and Alla (Eds. 2024) affirm that methodological triangulation in qualitative research increases the credibility of the findings. The authors observed six Grade 3 teachers teaching mathematics without interacting with them. Therefore, non-participant observations were used to triangulate the data from the interviews, lesson plans and how the teachers taught algebraic representations in their classrooms. Consequently, during the presentation and discussions of findings, data from the interviews, lesson plans and observations were triangulated. The observations were conducted with each teacher during the mathematics lesson for 45 minutes. Only one session of the observations was conducted to reduce interference with teaching and learning activities and capture real-time and authentic teaching and learning activities (Li, 2023). Hüsrevoğlu and Atici (2024) discuss that an observation schedule is a pre-planned tool used to guide and structure the data collection process during observations. Moreover, it assists the researcher in systematically focusing on key behaviours, interactions, events, or practices relevant to the research questions. Following the perspective of Hüsrevoğlu and Atici, an observation schedule was used to answer the research questions and explore the teachers' experiences in developing Grade 3 learners' algebraic representation skills.

Each participant was given a consent form for perusal and signing (if agreeable). Participants were also informed of the exit clause, which allowed them to leave at any stage of the research process if they felt unsafe or uneasy without being penalised. Audio recording (with consent) and verbatim transcriptions were executed according to the interview schedule to guarantee the reliability of the information gleaned from the interviews. The verbatim transcriptions were repeatedly read and checked against the audio recordings to identify and correct anomalies (Knott et al., 2022). The authors ensured that the interview schedule and observation sheet were in order before collecting data, and also used member-checking as a further tool to verify the gathered information in the transcriptions. Lahman et al. (2023) confirm that using pseudonyms such as false names or numerical codes is a valid approach to protect the identities of schools and participants in qualitative research. Six teachers participated in this study and, for anonymity purposes, they were assigned numerical identifiers: the codes School 1 Teacher 1 (S1T1), School 1 Teacher 2 (S1T2), School 2 Teacher 3 (S2T3), School 2 Teacher 4 (S2T4), School 3 Teacher 5 (S3T5) and School 3 Teacher 6 (S3T6) are used in the presentation and discussion of findings to illustrate specific insights, while maintaining participant anonymity. Refer to Table 1 to see the participants' pseudonyms, qualifications, and professional experiences.

## Data analysis

A thematic narrative analysis was used to interpret teachers' views and experiences of developing Grade 3 learners' algebraic representation skills. De Fina (2021) asserts that thematic narrative analysis offers insights into cultural, social, and personal aspects of views and experiences by examining how stories are conveyed and what they reveal about the storytellers and their perspectives. In the context of this article, the authors explored the similarities and differences of how individual Grade 3 teachers (participants) narrated their views on and experiences of developing learners' algebraic representations. The role of the authors was to identify recurring patterns and themes within narratives and interpret them.

Before the data analysis, a professional transcriber transcribed the recorded interview data into text. In line with qualitative research practices (Couceiro, 2024), an initial phase of data familiarisation was undertaken, which involved organising the transcripts, repeatedly reading through the data, and making preliminary notes and reflections to gain a holistic sense of the participants' responses and contextual meanings. This phase was crucial for identifying emerging patterns and preparing the data for coding. Only after this immersion process was the formal thematic narrative analysis initiated, during which codes were generated and clustered into categories aligned with the study's research questions. This sequential and layered approach ensured the analysis was grounded in the data and reflected participants' lived experiences. Consequently, three data sets, semi-structured interviews, interpretivist document analysis and non-participant observation, were examined using thematic narrative analysis. Naeem et al. (2023) explain that thematic narrative analysis involves breaking data into manageable codes and categories to uncover trends, patterns and emerging themes. As a result, data were analysed manually on a computer, and codes were created from the keywords in the research questions. An independent coder assisted in co-coding the same data to ensure data analysis's transparency, authenticity, consistency and reliability (Friedman et al., 2024). The codes were classified and subsequently grouped to establish the themes. Since interpretive authors use multiple views, perceptions and voices to analyse data, the authors interpreted the data gathered from Grade 3 teachers from three data sets. To ensure rigour and trustworthiness in the findings, the authors employed a data triangulation method aligned with the interpretive paradigm (Eds. Bentalha & Alla, 2024) to capture multiple perspectives and deepen understanding of Grade 3 teachers' views. Furthermore, the data were interpreted through the lens of the reviewed literature and theoretical framework.

## Ethical considerations

Ethical clearance was obtained from the University of South Africa College of Education Ethics Review Committee on 14 October 2020. The ethics approval number is 2020/10/14/64019209/07/AM. The participants and parents signed consent letters agreeing to share their views and experiences of teaching mathematics. Furthermore, Grade 3 learners signed the assent form to agree to be part of the study with the permission of their parents or guardians. To safeguard the anonymity and confidentiality of participants, pseudonyms were used to record and present the names of participants.

# Results

The data from the semi-structured interviews disclosed that participants S1T1 and S2T3 notice improvements in learners' ability to develop early algebraic representations when concrete manipulatives and indigenous games are intentionally integrated into mathematics lessons. In addition, these participants incorporate indigenous knowledge into their teaching strategies. In the context of this study, guided by Bihari (2023), indigenous knowledge refers to the localised, traditional knowledge systems developed by indigenous peoples or local communities through generations of lived experience, cultural practices, and interaction with their natural environment. The teachers are encouraged by DBE (2011) to incorporate indigenous games in teaching and learning mathematics in Grade 3. However, the analysis of the lesson plans revealed that planned activities were based on curriculum policies with a lack of flexibility and enrichment of adaptation of algebraic representation. To triangulate this, the observations indicate that teachers experienced challenges such as time constraints, curriculum pressures, and learners' unfamiliarity with the content that integrates traditional practices. This suggests that learners may not readily relate to or recognise the mathematical value of these culturally embedded practices. Teachers emphasised the importance of hands-on learning, cultural relevance and scaffolding in making algebraic thinking more accessible to learners.

## The use of concrete manipulatives

Concrete manipulatives reflect participants' practical strategies and pedagogical reasoning behind supporting learners' transition from concrete to abstract thinking. During the interviews, the participants articulated how well manipulatives bridged the gap between abstract algebraic representations and concrete experiences. S1T1 emphasised the role of teaching aids and visual representations in fostering algebraic understanding:

> Some learners take time to understand algebra until a teacher uses manipulatives to align with the examples. We use teaching resources and manipulatives to instil algebraic understanding … We also use visual representations such as diagrams, pictures and wall charts to align with mathematical concepts. (S1T1)

Similarly, S2T3 highlighted the role of hands-on experiences in making abstract mathematical concepts accessible:

> Young learners take time to understand mathematics. Playing educational games and using concrete objects helps. (S2T3)

S3T4 elaborated on using everyday objects such as stones, sticks, and bottle caps to demonstrate algebraic patterns, grouping and equations. This approach helps learners visualise algebra as something connected to their daily lives:

> One strategy that works well for me is using manipulatives. I bring everyday objects into the classroom like stones, sticks or bottle caps and use them to demonstrate patterns, grouping and simple equations. This helps them see algebra as something connected to their daily lives rather than just numbers on a board. (S3T4)

S2T4's strategy mirrors the tenets of the CRA approach, even if it did not specifically mention them. The slow progression from tangible items to visual representations and abstract symbols ensures a richer conceptual comprehension of algebra.

## Integration of indigenous knowledge

The results from semi-structured interviews revealed that participants integrate indigenous games such as Back-to-Nought, Pebbles, *Kgathi* [skipping ropes], Touch, and *Tsheretshere* [Hopscotch] to develop Grade 3 learners' mathematical skills. However, many learners are unfamiliar with these culturally rooted play activities, challenging their integration of indigenous knowledge systems (IKS) into algebraic representations. S1T2 and S2T3 highlighted the significance of indigenous games in reinforcing basic mathematical operations:

> We can use *Diketo*, and Touch [*indigenous games*] to teach addition, subtraction, division and multiplication. (S1T2)

> Teachers use different methodologies and practical indigenous games like *Diketo* to teach counting forward and backwards. (S2T3)

S2T4, who has over six years' teaching experience, emphasised the importance of incorporating cultural practices into mathematics instruction, particularly in rural and disadvantaged schools:

> I draw from indigenous games like stone-throwing [*Diketo*] and jumping rope [*Kgathi*] to make algebraic representations relatable to my learners. These traditional games are rich in patterns, sequences and logical thinking, which help learners connect algebraic concepts to their everyday experiences. (S2T4)

By linking algebraic representations to indigenous games, S2T4 expressed that learners become more engaged and develop a deeper appreciation for mathematical structures embedded in their cultural heritage. However, the challenge remains that many learners, especially those in township areas, are unfamiliar with these traditional practices, making it necessary for teachers to introduce the cultural elements before applying them to mathematical teaching and learning activities. Bihari (2023) explains that traditional practices are the cultural knowledge, customs, values, beliefs, and indigenous knowledge within a particular community. In the context of teaching mathematics, the beadwork or mat weaving uses geometric and numeric patterns. However, a challenge identified by S2T4 was the difficulty of making indigenous knowledge relatable to all learners, particularly those from urban and township backgrounds who may not be familiar with traditional practices:

> Another challenge is that some learners might not be as familiar with certain traditional practices as they once were, especially those growing up in township areas. For example, a learner might not fully understand the significance of stone-throwing or traditional games, making it harder to use those cultural references to teach algebraic patterns. (S2T4)

Despite this challenge, S2T4 viewed the integration of cultural practices as an opportunity to foster a deeper understanding of algebraic thinking:

> Teaching young learners, especially those from rural areas, often requires creativity and sensitivity to their cultural context. It is about connecting abstract mathematical concepts to things they know, like indigenous games they play at home, such as stone-throwing or jumping rope, which I use to teach basic algebraic representations. (S2T4)

## Time constraints to meet the curriculum standards

One of the significant challenges raised by participants was the pressure to adhere to the CAPS curriculum, which requires covering a set amount of content within a limited timeframe. This can hinder efforts to provide individualised support to learners who struggle with algebraic concepts. It was during the interviews that S2T4 expressed concerns about the fast-paced nature of the curriculum:

> Another challenge is the pressure to meet curriculum standards. The curriculum moves quickly, and sometimes, there is not enough time to spend on each concept before moving on. I often worry that some learners might be left behind, especially those who need extra time and practice to understand algebraic representations. (S2T4)

S1T1 echoed this concern, emphasising the need for instructional time to be flexible:

> Learners take time to understand concepts like algebraic thinking. It is a challenge for us teachers because we must follow the curriculum policies that show instructional time for each content area. (S1T1)

Despite these constraints, participants acknowledged that patience and differentiated teaching strategies are necessary for supporting learners with diverse mathematical abilities. S1T1 further emphasised:

> It needs one to be patient to teach Grade 3 learners. Learners are different, and we have learners with mathematical barriers. As such, teachers need to use different pedagogical practices to accommodate learners with different mathematical abilities. (S1T1)

Another challenge highlighted by participants was the limited time available to teach algebraic representations effectively. The abstract nature of algebraic thinking demands significant scaffolding, yet curriculum constraints often force teachers to move on before learners fully grasp the concepts. S2T4 explained the difficulty of helping learners transition from concrete thinking to abstract reasoning:

> Another challenge is the abstract nature of algebraic thinking itself. At this age, many children are still very concrete thinkers. They are used to counting actual objects like stones or sticks, so introducing the idea of symbols, patterns and variables can be confusing. I must spend much time helping them transition from thinking about math in purely tangible terms to understanding that numbers and symbols can represent relationships and unknowns. (S2T4)

Additionally, S1T2 highlighted the difficulty of integrating indigenous games into lessons due to learners' unfamiliarity with these activities:

> A challenge we face is that these Gen Z children do not know much about indigenous games, and when you attempt to use them, it is like you are teaching them something new. Then it takes time to deliver the lesson. (S1T2)

The analysis of teachers' lesson plans and observations indicates that using concrete manipulatives and indigenous games significantly enhanced Grade 3 learners' algebraic representations. However, the classroom observations revealed that using symbols and numbers to teach abstract algebraic concepts remains challenging. During the observation of S1T1's mathematics lesson, the teacher introduced the concept of patterns using real objects such as bottle tops and coloured sticks. Learners could initially identify and extend repeating patterns when working with the tangible materials. However, as the lesson progressed and the task required learners to represent the pattern numerically, several learners struggled to transition from the concrete stage to the abstract representation. It was observed that some learners hesitated or relied heavily on the physical manipulatives to make sense of the number sequences, indicating difficulty in recognising numerical patterns without concrete support.

In S1T2's classroom, some learners could explain why geometric patterns work, but others still needed repeated manipulatives to understand the concept. It was observed that there was a limited time to reinforce conceptual understanding before moving to the next topic. S1T3 used a traditional game, Pebbles, to develop Grade 3 learners' algebraic reasoning. Even though some learners were more engaged with the game, they found it challenging to transfer the patterns into written numbers. According to the observation schedule, S2T4 incorporated a skipping rope activity to teach number patterns through rhythmic movement and repetition. This embodied learning approach engaged learners physically, and most learners could identify and verbalise the pattern (e.g., counting in twos or fives) while participating in the game. However, when the activity shifted to a written task, where learners had to represent the number pattern on paper, many struggled to connect the movement-based experience and abstract number symbols. The shift from a physical game to numerical symbols required more scaffolding.

S3T5 used flash cards with word problems to teach symbolic representation. The mathematical equations are referred to as number sentences in the Foundation Phase. S3T5 presents a problem: Sipho has $x$ marbles. He gives away three marbles. How many does he have left? S3T5 introduced a number sentence using $x - 3 = ?$ Some learners were confused about using $x$ to represent an unknown number. Learners needed repeated practice with concrete materials before independently solving number sentences. Observation data from S3T6's classroom indicated that learners were highly engaged when the teacher used a storytelling strategy to introduce the idea of an unknown quantity (variable). The learners followed the story attentively and responded actively to questions embedded in the narrative. However, when prompted to apply the concept beyond the story, learners struggled to transfer their understanding. This suggests more

scaffolding activities during the lesson implementation before transitioning to abstract representations.

# Discussion

This study used the CRA approach to explore Grade 3 teachers' views and experiences in developing learners' algebraic representations. The findings indicate that teachers integrated concrete manipulatives and visual representations to support learners' gradual transition into algebraic thinking. Additionally, some teachers incorporated indigenous games as an alternative approach to engaging learners despite challenges related to curriculum constraints and learners' unfamiliarity with traditional practices. The teachers experienced challenges teaching abstract concepts using symbols and numbers to Grade 3 learners.

## Theme 1: The use of concrete manipulatives

The use of physical objects aligns with the concrete stage of the CRA approach, which helps young learners visualise and manipulate mathematical concepts before transitioning to symbolic representations. Participants emphasised that hands-on learning fosters engagement and builds conceptual understanding of algebraic structures. Teachers used manipulatives such as blocks, counters, beads, stones and bottle caps to introduce algebraic concepts. These objects allowed learners to physically group and rearrange elements, reinforcing patterns and number relationships (S1T1, S2T3, S2T4). The findings resonate with Mainali (2021) and Khan and Khan (2021), who argue that manipulatives bridge concrete experiences to abstract thinking. Similarly, Quane (2024) supports that hands-on learning enhances symbolic mathematics, making algebra more accessible to young learners. Teachers expressed that learners struggled with abstract mathematical thinking, and manipulatives were a stepping stone to strengthen problem-solving skills and engagement.

Transitioning from physical manipulatives to drawings marks the representational stage of the CRA approach, where learners begin to conceptualise algebraic ideas through pictorial representations. Teachers noted that drawings were an intermediary step (S1T1, S2T4), where learners visualise mathematical relationships using simple sketches before moving to abstract symbols. Donovan and Fyfe (2022) highlight that drawings provide a critical bridge between physical experience and abstract reasoning, while Wilkie (2024) argues that visual representations support long-term algebraic understanding. However, Başkan Takaoğlu (2024) warns that some learners may confuse drawings with the objects they represent, leading to misconceptions. Thus, explicit guidance is needed to ensure learners understand drawings as representations, not literal depictions.

## Theme 2: Symbols and numbers to represent algebraic knowledge

The final stage of the CRA approach requires learners to interpret mathematical symbols to generalise patterns and relationships. Participants reported that learners struggled to transition from pictorial representations to symbolic notation, leading to misconceptions about algebraic variables (S2T4). Sun et al. (2023) emphasise that symbols can drastically alter how learners perceive algebraic concepts, often making them detached from real-world applications. The study aligns with Pramesti and Retnawati (2019), who found that Indonesian learners struggled with algebra due to weak problem-analysis skills. The findings suggest that learners often struggled to apply algebraic concepts without manipulatives (S1T1) or to generalise variables beyond narrative contexts (S3T6). These challenges highlight gaps in abstract reasoning and conceptual transfer, which align with cognitive processes typically developed through HOTS. Therefore, while HOTS was not an explicit focus of the study, the data suggest that incorporating HOTS may support a more profound understanding of algebraic representations. Similarly, Ying et al. (2020) noted that learners face visual-spatial difficulties in algebra, highlighting the importance of early exposure to pattern recognition and spatial reasoning through games like Hopscotch. The findings further align with Stephens et al. (2021), who found that early algebra interventions from Grade 3 positively impacted learners' algebraic reasoning in later grades.

The study highlights that some Grade 3 teachers integrated indigenous games to enhance algebraic understanding. These activities align with CRA's concrete and representational stages by incorporating familiar, culturally relevant practices to make learning relatable. Participants identified indigenous games such as Pebbles, *Kgathi* [skipping rope], Touch, and *Tsheretshere* [Hopscotch] as valuable in teaching addition, subtraction, division, multiplication, and pattern recognition (S1T2, S2T3, S2T4). S2T4, with over six years' experience, explained how cultural practices bridged the gap between abstract mathematical concepts and real-world experiences, making algebraic ideas more accessible and engaging. These findings contrast with Swedish studies (Bråting, 2023), arguing that arithmetic and algebra should be integrated into early mathematics education rather than treated as separate disciplines.

## Theme 3: The challenges of teaching abstract concepts using symbols and numbers

Despite the innovative strategies discussed above, teachers face significant challenges in developing Grade 3 learners' algebraic representation skills. Teaching abstract algebraic concepts using symbols and numbers remains challenging, particularly for Grade 3 learners. The CRA framework highlights the importance of a gradual progression from concrete to abstract thinking, yet research and teacher experiences indicate that learners often struggle with this transition. One of the key challenges in teaching abstract algebraic concepts is helping learners transition from tangible, hands-on experiences to abstract symbols and numbers (Sun et al., 2023). The CRA approach suggests that learners should engage with physical objects before moving to symbolic representation, but some struggle with this

transition (Mainali, 2021). According to the study by Pramesti and Retnawati (2019), learners often make errors in algebraic problem-solving because they do not fully understand the meaning of symbols and variables. This highlights the need for gradual scaffolding to ensure learners develop a deep conceptual understanding.

Research by Ying et al. (2020) found that learners struggled with algebraic problem-solving due to weak visual-spatial skills, which are essential for understanding abstract notation. Teachers often focused on arithmetic operations without emphasising algebraic reasoning, which can result in difficulties when learners are introduced to algebra later in their education. Stephens et al. (2021) suggest that early exposure to algebraic thinking is critical, but the challenge remains in finding developmentally appropriate ways to introduce symbols and numbers. Başkan Takaoğlu (2024) warns that learners may confuse symbols with real objects, leading to misunderstandings. Without sufficient exposure to concrete and representational stages, learners may perceive algebraic symbols as meaningless rather than as representations of relationships. This aligns with the CRA framework, which stresses the importance of gradually building understanding before expecting learners to work with abstract symbols.

The CAPS curriculum (DBE, 2011) requires teachers to introduce algebraic representations in the Foundation Phase, but the time allocated for each concept is often insufficient. Teachers in the study expressed concerns about having to move quickly through topics, which may leave some learners behind. S2T4 noted that learners required repeated exposure and practice with symbols, yet the pressure to cover the curriculum made it difficult to provide adequate reinforcement. Participants expressed concerns that the rigid CAPS curriculum did not allow enough time to scaffold algebraic understanding appropriately (S1T1 and S2T4). Teachers felt pressured to cover topics quickly, leaving little room for differentiated instruction or additional practice for struggling learners. S2T4 noted that algebra is inherently abstract, making it challenging for young learners to grasp without extended hands-on experiences. While indigenous games and cultural knowledge can be valuable tools for introducing algebraic concepts, teachers reported challenges in integrating these activities with abstract representations. S1T2 and S2T4 indicated that many learners were unfamiliar with indigenous games, making it difficult to use them as a foundation for abstract thinking. This suggests a need for more culturally relevant teaching strategies that bridge concrete experiences with symbolic representation.

# Conclusion and recommendations

This study explored the views and experiences of Grade 3 teachers in developing learners' algebraic representations. The findings highlight that teachers predominantly used concrete manipulatives and indigenous games to support learners' understanding of algebraic concepts. However, lesson plans showed few instances where learners were expected to move beyond manipulatives to using symbols. At the same time, the classroom observations revealed the challenges in transitioning learners from concrete objects to abstract number symbols, as many relied on tangible representations during teaching and learning. Given these findings, this research suggests that teachers could progressively shift from direct modelling with concrete objects to guided practice with semi-abstract representations before introducing abstract number symbols. In addition, the authors recommend that integrating familiar cultural and indigenous games with gradual abstraction can help learners develop algebraic reasoning in a meaningful way. This article expands teachers' pedagogical knowledge by advocating for an enhanced CRA approach integrated with scaffolding strategies. By incorporating structured support mechanisms, teachers can more effectively bridge the gap between concrete and abstract mathematical thinking, enhancing a deeper understanding of algebraic representations among Grade 3 learners.


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The authors declare that they have no financial or personal relationships that may have inappropriately influenced them in writing this article.

## Authors' contributions

M.A.S. and T.A.L.P. equally contributed towards the conceptualisation and writing of this research article.

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## Data availability

The data that support the findings of this study are openly available from the corresponding author, M.A.S., upon reasonable request.

## Disclaimer

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