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Manifestations of computational thinking practices on preservice mathematics teachers' pen-and-paper problem-solving

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The development of computational thinking has been declared a necessity for everyone in the current digital era. Most studies dedicated to the development of computational thinking have focused on using plugged activities. However, in developing countries like South Africa, most schools struggle with resource provision which can inhibit the integration of computational thinking in all learners. This article considers an alternative approach to computational thinking in mathematics education using pen-and-paper problem-solving. Through the adoption of exploratory qualitative research with a purposeful sample of five preservice teachers, the manifestation of computational thinking in pen-and-paper problem-solving is explored. Adopting the combination of the metacognitive framework and Schoenfeld's framework for problem-solving, this study explains from the perspectives of preservice teachers how computational thinking practices manifest in pen-and-paper problem-solving. The thematic analysis conducted in this study revealed that computational thinking practices such as abstraction, decomposition and algorithmic thinking, and evaluation were observed in preservice teachers' pen-and-paper problem-solving. Two interesting findings emerged from this study: (1) there was no discernible difference between decomposition and algorithmic thinking in preservice teachers' pen-and-paper problem-solving, and (2) there was no clear evidence of pattern recognition as an important computational thinking practice in preservice teachers' pen-and-paper problem-solving.

Contribution: The first finding indicates that during pen-and-paper problem-solving, it might be difficult to differentiate between decomposition and algorithmic thinking. The second finding indicates that more studies need to be conducted, directed towards how pattern recognition manifest in pen-and-paper problem-solving. Conclusions and recommendations are made for this study.

Keywords: Computational thinking; pen-and-paper problem-solving; mathematics education; unplugged activities

Introduction

Given the fast-paced developments in technology and its applications in education, computational thinking (CT) has become pivotal in preparing Grade R–12 learners for science, technology, engineering, and mathematics (STEM) education and careers (Grover & Pea, 2013; Long & Chan, 2021), reinforcing understanding of advanced STEM concepts (Wilensky et al., 2014), and enhancing higher-order thinking skills for solving complex STEM problems (Weintrop et al., 2016). Defined as a domain-general competence (Tsai et al., 2021) essential for systematically solving problems in everyday life and across different learning domains (Tsai et al., 2021; Wing, 2006), CT has become a huge part of the research agenda in mathematics education (Mahlaba et al., 2024). One important feature of CT is that it can be developed using plugged activities (Wang et al., 2022), unplugged activities (Li et al., 2022) and a hybrid of these activities (Lin et al., 2024). Plugged CT activities are those in which students engage in solving mathematical problems using computers or other electronic devices, whereas unplugged CT activities involve problem-solving approaches that do not require computers or any form of electronic device (Caeli & Yadav, 2020; Wang et al., 2022). Research regarding the manifestation of CT practices in plugged activities indicates that these practices manifest in ways that are related to computer programming or coding (e.g., Belmar, 2022; Chen et al., 2021; Kallia et al., 2021). However, many schools in African countries do not have access to computers nor teachers who teach programming and coding in Grades R–12 who can help develop CT skills for solving mathematical problems. Given the dominant research in plugged and hybrid activities to develop CT in mathematics education,

I followed Bell et al. (2011) who initiated the idea that it is important that CT practices are developed in line with the formal curriculum. In the present study, I examined the manifestations of CT practices in preservice teachers' unplugged problem-solving, specifically through tasks that are solved using pen and paper. Pen-and-paper problem-solving in mathematics refers to the conventional practice of using writing materials to solve mathematical problems rather than digital tools, a long-established approach in mathematics education in schools, and thus provides an important contrast to plugged CT activities, exploring how CT practices can emerge in non-digital contexts. Given that mathematics learning in the South African context predominantly occurs through pen-and-paper problem-solving, focusing on this niche area provides a valuable lens for examining how CT can be developed through day-to-day non-digital teaching practices. Such an approach emphasises the potential to develop CT competencies among learners from socio-economically disadvantaged contexts where access to digital resources is limited.

Computational thinking in mathematics education

Owing to the lack of technological (or plugged) resources to support CT development in many African countries (Ausiku & Matthee, 2021; Belmar, 2022; Mahlaba et al., 2024), research agenda in mathematics education should focus on bridging the digital divide. This may include ensuring that in CT as a competency can also be developed in learners without access to technology, which may help countries like South Africa with a huge digital divide. Ensuring the development of CT in all learners is essential, as evidenced by Wang et al. (2022) who demonstrated that a CT-based intervention significantly enhanced learners' compound thinking, decomposition, algorithmic thinking, and problem-solving abilities. Furthermore, another CT-based intervention based on using bodily movements improved young learners' abilities to solve mathematics problems and understand mathematics better (Sung et al., 2017). In a quantitative study, Sun et al. (2021) found that unplugged activities are more effective in enhancing CT within mathematics education. In the pre-test, the *p*-values for experimental groups 1 and 2 were 0.843 and 0.746, indicating no statistically significant difference. However, in the post-test, both groups demonstrated statistically significant improvements, with *p*-values of 0.00 (Sun et al., 2021). A combination of a systematic review and a meta-analysis revealed 'unplugged activities are a promising instructional strategy for enhancing students' CT skills' (Chen et al., 2023a, p. 1). Despite these and other observed benefits of CT in mathematics education, teachers still struggle to develop CT in their mathematics learners due to various reasons including the digital divide (Caeli & Yadav, 2020). While Bradshaw and Milne (2022) provided a theoretical account of how CT practices manifest in pen-and-paper problem-solving within the mathematics curriculum in South Africa, the existing literature does not report studies that explore how CT practices manifest through the analysis of actual participants' problem-solving – which is the focus of this study.

Computational thinking practices

CT practices (see Table 1) are a set of cognitive strategies used to solve complex problems systematically (Wing, 2006). Solving mathematics problems using CT involves the application of various practices that reflect the distinctive ways of thinking associated with CT such as abstraction, decomposition, algorithmic thinking and more. While such practices may vary for different problems and learning domains, in mathematics education, they include abstraction, decomposition, algorithmic thinking, pattern recognition, evaluation, and generalisation (Kallia et al., 2021; Selby & Woollard, 2013).

However, since the study did not require that participants use CT strategies to solve new problems, then generalisation was a CT practice that was not applicable in the current study. In Weintrop et al. (2016), these practices were classified as related to computational problem-solving practices. Furthermore, when Lafuente Martínez et al. (2022), Li et al. (2021) and Tsai et al. (2021) designed the tools for assessing CT, they reinforced that abstraction, algorithmic thinking, evaluation, pattern recognition, and decomposition are important CT practices. Table 1 provides a summary of the definitions of these five CT practices which are the focus of this study.

Plugged activities in computational thinking

Grover and Pea (2013) refer to plugged activities as those learning activities that require the use of computers, electronic devices or digital tools to develop CT abilities in learners. Incorporating block-based programming like Scratch which involves manipulation of programming ideas can develop CT (Fagerlund et al., 2021). Such visual programming tools also support other tools such as Hopscotch and robotic coding to enhance CT (Voon et al., 2023). Mathematical

TABLE 1: Defining computational thinking practices in mathematics education.

Practice	Definition
Abstraction	Identifying essential problem elements by simplifying and hiding unnecessary details in the problem helps to highlight elements to focus on and disregard when finding solutions to problems (Weintrop et al., 2016; Wing, 2006).
Decomposition	Breaking down a problem into smaller parts, solving these parts separately, such that partial solutions can be integrated to solve whole problem (Lafuente Martínez et al., 2022).
Algorithmic thinking	Designing step-by-step instructions for solving mathematical problems (Kadijević, 2023).
Evaluation	Systematically examining a solution to determine its correctness and exploring alternative approaches, like Polya's final step of 'looking back' (Tjoe, 2019).
Pattern recognition	Identifying patterns, trends, or regularities in a problem or process, determining the structure of problems, and finding similarities with known previous information (Chen et al., 2023b).
Generalisation	'Applying CT strategies to solve new problems' (Shute et al., 2017, p. 145).

Note: Please see full reference list of this article, Mahlaba, S.C. (2025). Manifestations of computational thinking practices on preservice mathematics teachers' pen-and-paper problem-solving. *Pythagoras, 46*(1), a855. https://doi.org/10.4102/pythagoras.v46i1.855, for more information.

CT, computational thinking.

problems that require to be solved through CT with the use of computers are also considered plugged activities (Wu & Yang, 2022). Several research studies have argued that plugged activities that involve visual and robotic programming are commonly considered a dominant approach to developing CT due to their several related benefits (Lin et al., 2024). One of the reasons is that complex and abstract CT problems can be visualised to enhance their accessibility (Fagerlund et al., 2021) and that they are more effective in developing CT than unplugged activities (Elkin et al., 2016; Lin et al., 2024). Despite such advantages, there have been challenges regarding plugged CT activities such as high cost of digital tools, lack of teacher knowledge with programming and difficulty in aligning programming and coding activities with the traditional curriculum in schools (Bell et al., 2011; Liu et al., 2024; Woo & Falloon, 2025). Hence, it is necessary to investigate the possibility of developing CT using unplugged activities and whether these activities will be user friendly for teachers and can be linked to the traditional curriculum.

Unplugged activities in computational thinking

Unplugged activities in CT are activities that teach learners the principles of CT without using computers or electronic devices that require electricity (Caeli & Yadav, 2020). Instead, unplugged activities employ physical materials, hands-on tasks, bodily movements and interactive games to facilitate the development of CT in learners. Unplugged CT activities may involve bodily movements (Sung et al., 2017), gamification (Tsarava et al., 2019), and using toys, card and puzzles (Chen et al., 2023b). These unplugged activities have been praised for their low-cost approach towards helping learners develop CT skills without using computers, digital devices, or hardware that require electricity (Chen et al., 2023b). Unplugged activities play a significant role in fostering CT in educational contexts. For example, Tsarava et al. (2019) developed three life-sized board games – *Crabs & Turtles: A Series of Computational Adventures* – to offer an unplugged, gamified and a beginner-friendly approach to introducing CT to learners. Their findings indicated that learners had a positive experience engaging with the games (Tsarava et al., 2019). Another study by Busuttil and Formosa (2020, p. 573) used five unplugged activities – Logic Gates through Twister (Ward, n.d.), Building a Tower (Caldwell & Smith, 2016), Parity Magic using cards (Bell et al., 2015), Conditionals with cards (Hour of Code, n.d.), and Adding Numbers (Cliff, 2013) – to assess CS Unplugged as a pedagogical strategy. Their findings demonstrated that these unplugged activities were effective in promoting learner involvement and collaboration. There was also evidence that learners enjoy lessons based on unplugged CT activities (Lin et al., 2024). Lastly, unplugged activities may develop skills such as sequencing that can precede mathematics education and problem-solving (Nordby et al., 2022). Given such reasons, Chevalier et al. (2020) has argued that tangible materials, game-based activities, and embodied activities should be used to develop CT and problem-solving skills in learners. The importance of developing learners' ability to think computationally extends beyond addressing the digital gap in different societies; it is also linked to the understanding that computers execute mechanical processes that are designed and directed by humans (Caeli & Yadav, 2020).

Theoretical framework

Problem-solving requires proficiency in mathematical knowledge, problem-solving strategies and metacognition (Flavell, 1979; Schoenfeld, 1985). Thus, exploring CT manifestation during pen-and-paper problem-solving requires knowledge of how CT practices relate to these requirements for problem-solving.

Metacognitive framework

Metacognition refers to the awareness and regulation of one's own thinking processes and involves metacognitive knowledge, strategies and experiences and encapsulates three dimensions: metacognitive knowledge, strategies and experiences (Flavell, 1979). During problem-solving, *metacognitive knowledge* involves the relationship between the student, the problem and strategies used to solve the posed problem (Efklides & Schwartz, 2024). *Metacognitive strategies* focus on the knowledge the student has about how to use strategies to solve problems which involves planning, monitoring, and evaluating (Efklides & Schwartz, 2024; Flavell, 1979). Lastly, *metacognitive experiences* are judgements about the students' cognitive processes that provide subjective feedback that guides control decisions based on the fluency and correctness of cognitive processing (Efklides & Schwartz, 2024). These three dimensions of metacognition are intrinsically linked to CT because CT cognitive and analytical skills are linked to effective problem-solving (Hooshyar et al., 2021; Jagals & Van der Walt, 2016; Wing, 2006; Yadav et al., 2022). Yadav et al. (2022) showed the link between metacognition and CT. For example, they argue that metacognitive monitoring and assessing the solution relates to the CT practice of evaluation. These metacognitive processes have also been shown to influence effective problem-solving (Jagals & Van der Walt, 2016). Additional findings from the field of computer science (Chen et al., 2021) further support the relationship between CT and metacognition. Yadav et al. further argued that metacognitive planning is associated with algorithmic thinking. Moreover, Jagals and Van der Walt (2016) observed that 'monitoring occurred by rereading the question and concentrating more on certain sections' (p. 8), linking monitoring to abstraction. This relationship between CT and metacognition was considered as important in this study for effective problem-solving.

Metacognition plays a central role in effective problem-solving by allowing learners to plan, monitor and evaluate their strategies as they engage in complex tasks (Efklides, 2006; Flavell, 1979; Schoenfeld, 1985; 1987). For instance, in mathematics education, successful problem-solvers use metacognitive regulation to select appropriate strategies,

monitor their progress, and revise their approach when encountering difficulties (Schoenfeld, 1985). Since CT is fundamentally rooted in structured and reflective problem-solving, requiring practices such as decomposition, abstraction, and algorithm design (Grover & Pea, 2013; Wing, 2006), it inherently depends on metacognitive processes. Students can show that they are engaged in CT if they plan solutions, test and debug their algorithms, and reflect on their strategies, all of which are governed by metacognitive knowledge and regulation (Shute et al., 2017; Yadav et al., 2022). Moreover, studies have shown that metacognitive scaffolding enhances students' ability to engage with CT-based tasks by promoting self-regulation and adaptive strategy (Basu et al., 2017). Therefore, it is evident that fostering metacognitive skills is essential for cultivating CT competencies through problem-solving.

Schoenfeld's framework for mathematical problem-solving

Schoenfeld's (1985) framework proposes that mathematical problem-solving is guided by an interplay between resources, heuristics, control, and beliefs. Resources represent students' mathematical knowledge, heuristics represents students' problem-solving strategies, control is how students select and use the resources and strategies at their disposal, and beliefs are students' mathematical views (Schoenfeld, 1985) and Schoenfeld (1987) later argued that control relates to metacognition. Collectively, these components characterise effective problem-solving and inform on how pedagogy can ensure learners engage with mathematical thinking (Schoenfeld, 2023). In its essence, Schoenfeld's framework and CT are also inherently interconnected, given the central role of problem-solving in CT (Grover & Pea, 2013; Wing, 2006). Consequently, this framework proved valuable for the purposes of this study. This framework could be related to the five CT practices as strategies for solving problems. One argument posited is that using CT practices to solve mathematical problems requires knowledge of mathematics, which is related to resources in the Schoenfeld framework. In summary metacognition and the Schoenfeld framework were considered commensurable to study problem-solving. However, problem-solving cannot be limited to the concepts explained in these two frameworks. Studies have indicated other factors such as anxiety (Jiang et al., 2021) and affective dispositions (Passolunghi et al., 2019) also affect problem-solving. Anxiety can impair problem-solving by draining cognition and engagement, while affective dispositions shape motivation, persistence, and strategic openness (Johar et al., 2025).

Rationale and research question

Methods for integrating CT into mathematics classrooms is a topical area of interest for mathematics education researchers. They argue that there is a need to incorporate CT practices to train students on how to solve problems systematically in STEM education (Mahlaba et al., 2024). Various studies have dominated the integration of CT in mathematics using plugged activities with fewer studies using unplugged activities as highlighted earlier (Lin et al., 2024; Long & Chan, 2021; Wang et al., 2022). Furthermore, there are no studies known to me that investigate how CT can manifest in pen-and-paper problem-solving. Thus, research is needed to explore this gap and find out how CT practices manifest in pen-and-paper problem-solving. To investigate this phenomenon, I used think-aloud problem-solving protocols to capture preservice teachers' (PSTs) cognitive and metacognitive strategies during mathematical problem-solving tasks (Wolcott & Lobczowski, 2021) and video-stimulated recall interviews (VSRI) to provide PSTs with the opportunity to reflect on and elaborate their thought processes while viewing recording of their own problem-solving performance (Paskins et al., 2017). The present study intends to contribute to the issue of finding out ways in which CT can be developed in learners using low-cost methods that do not require plugged activities. The research question addressed in the following study is as follows: *How do CT practices manifest in PSTs' pen-and-paper problem-solving processes?*

Research methods and design

This qualitative interpretive study adopted an exploratory approach to examine how PSTs make meaning of their problem-solving experiences (Coe et al., 2025). Specifically, it investigated how CT practices manifested in PSTs' pen-and-paper problem-solving by analysing their reasoning, strategies, and reflections, rather than testing hypotheses. Five final-year Bachelor of Education (Senior and FET Phase) mathematics PSTs were purposively sampled. At this stage in their studies, they were considered to possess the necessary content knowledge and problem-solving skills to engage with the posed non-routine problems, while not having pre-existing strategies for solving them (Schoenfeld, 1992). Ethical protocols were strictly followed: institutional clearance (SEM-1-2024-046), informed consent from all participants, and gatekeeper permission was granted. To ensure anonymity, pseudonyms were used (e.g., PSTA refers to preservice teacher A). The study employed an exploratory research design. Each PST individually solved two randomly selected non-routine problems (see Figure 1), using think-aloud protocols that were video-recorded. These sessions were followed by VSRIs, aimed at exploring PSTs' reflections on their problem-solving processes. During these interviews, PSTs viewed video recordings of their problem-solving and were asked to articulate their reasoning, decisions, and strategies. This approach is particularly suited for capturing tacit aspects of problem-solving and was used here to elicit deeper insights into how CT practices manifest during pen-and-paper problem-solving tasks. The VSRIs were audio-recorded and transcribed for analysis. Combining think-aloud protocols with VSRIs enriched the data set, as this approach enabled researchers to capture both real-time cognitive processes and retrospective metacognitive reflections (Paskins et al., 2017). This dual method provided a more comprehensive understanding of PSTs' decision-making and strategy use during problem-solving.

Data analysis involved an iterative process of meticulously examining the strategies used by PSTs to solve problems, along with the justifications they provided for their problem-solving processes in an interpretative phenomenological analysis (Nizza et al., 2021). I collected data from five (n = 5) PSTs in their final year of study, who were purposively selected because they voluntary participated in a one-month training on non-routine problem-solving, which equipped them with strategies for solving problems that do not have a pre-existing strategy to solve (Schoenfeld, 1992) but the researcher did not introduce CT practices to the PSTs during this training. This knowledge and experience were considered important in solving the two problems posed in this study. Although researchers rarely link pen-and-paper problem-solving to the development of CT in mathematics education at a practical level, Bradshaw and Milne (2022) have established a theoretical connection. Building on their work, this study seeks to generate empirical insights into how CT practices manifest in PSTs' pen-and-paper problem-solving processes.

By integrating the four core CT practices (Table 1) with metacognitive and Schoenfeld's frameworks, I provide a summary of the findings of this study, including examples of quotations from the study data and how they link with CT, Schoenfeld's framework and the metacognitive framework. Although Table 1 lists six core CT practices, generalisation and pattern recognition were not observed in the participants' pen-and-paper problem-solving practices, so they were excluded in the data analysis process. My method entailed explaining the core CT practices based on what PSTs were doing when they were solving the problems after analysing the video-recording of their problem-solving performance and the justifications they uttered for their problem-solving actions. For example, by '*reading the statement and using it to construct a visual representation of the problem*' PSTs are extracting the most important information in the statement and using this information to represent the problem visually and understand it better which is an indication of the core CT practice of 'abstraction' (see Table 2). The findings from the researchers' observations of the PSTs' problem-solving actions and processes were integrated with PSTs' justifications and narratives of these actions and processes.

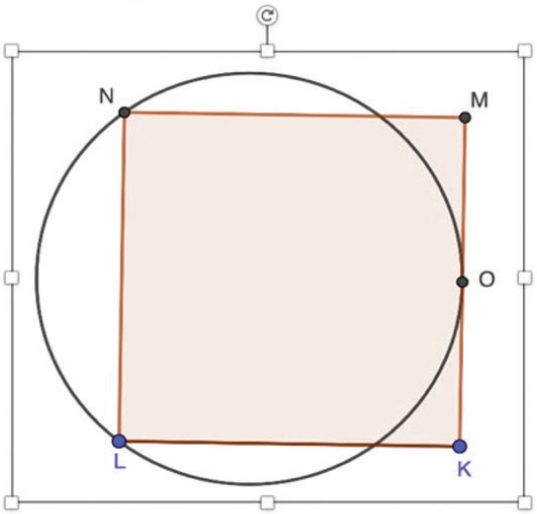
Ethical considerations

This study was conducted in accordance with ethical standards for research involving human participants. Ethical approval was obtained from the University of Johannesburg's Faculty of Education Research Ethics Committee on 14 April 2023. The study protocol was reviewed and approved as a large project with sub-projects included in the larger project under reference number SEM 1-2023-054. All participants were informed about the purpose, procedures, and voluntary nature of the study. Informed consent was obtained prior to participation. Measures were taken to ensure participant

Problem 1

The square and circle below are tangent at one point and meet at four other points, as shown.

If the side of the square is 8 units long, what's the radius of the circle?



Problem 2

A man walks $1km$ east, then $1km$ northeast, then another $1km$ east. Find the distance in kilometers between the mans' initial and final position.

FIGURE 1: Problems solved by preservice teachers.

confidentiality, anonymity, and data protection. Participants were assured that they could withdraw from the study at any time without any consequences. All data collected were used strictly for academic research purposes and stored securely in compliance with institutional guidelines.

Results and discussion

The current study intended to contribute to explaining the characteristics of CT in pen-and-paper unplugged problem-solving in PSTs' problem-solving experiences. Therefore, the problem-solving processes of the sampled PSTs provided a good window to explore how these CT practices manifested in PSTs' pen-and-paper problem-solving. Hence, the first phase of data collection involved PSTs individually solving the two problems independently through think-aloud protocols. Furthermore, based on their problem-solving videos using think-aloud protocols, PSTs were invited to participate in the second phase of data collection in a VSRI where they were asked to explain their problem-solving processes while watching their actions. Data from these two phases of the study are concurrently reported here. Due to a lack of space, I only provide a few examples to support my arguments. The reporting of the findings will anonymise participants' identities by using pseudonyms: PSTA, PSTB, PSTC, PSTD and PSTE represent the pseudonyms of the five participants of this study. Table 3 indicates the participants and the mapping of the CT skills that manifested in the pen-and-paper problem-solving.

The analysis of PSTs' pen-and-paper problem-solving revealed variation in the extent to which CT practices manifested (Table 3). Abstraction and decomposition and algorithmic thinking were the most consistently evident across participants, with four out of five PSTs engaging in abstraction and all five demonstrating some form of decomposition and algorithmic thinking. Abstraction, decomposition and algorithmic thinking are not only closely intertwined in mathematical problem-solving (Lehmann, 2025) but also represent the most common CT practices used in mathematical problem-solving (Kallia et al., 2021). Hence, it is not surprising that PSTs in this study demonstrated similar patterns, confirming that indeed CT can be meaningfully developed through both plugged and unplugged activities. This highlights the importance of incorporating pen-and-paper problem-solving into mathematics education, because it not only develops abstraction, decomposition, and algorithmic thinking but also provides accessible opportunities for developing CT without reliance on technological resources. This makes pen-and-paper problem-solving valuable in poor communities where access to digital tools is limited.

Evaluation, however, was less common, observed only in PSTB and PSTD. This pattern suggests that while PSTs were generally able to break down problems and focused on the important elements of the problem, only some extended their reasoning to critically evaluating or reflecting on their solutions. PSTD appeared to be using trial and error as means of evaluating their approaches to problem-solving, which involved trying different approaches and when the strategy did not work, they would scratch it out and use a different approach until they reached conviction in one strategy and presented it as the solution. Trial and error as a strategy is commonly associated with evaluation in CT (Cui & Ng, 2021; Lehmann, 2025). PSTB on the other hand seemed to perform their evaluation in the visual abstractions of the problem that he was producing. PSTB constructed at least three visual representations of the problem because the previous construction was deemed not suitable for solving the current problem which indicates that evaluation was conducted. From a metacognitive standpoint, this is related to 'knowledge of cognition' which means that most PSTs in this study did not intentionally reflect on their cognitive activity of problem-solving. These findings align with Schoenfeld (1992) who argued that problem-solvers can often carry out basic problem-solving steps but struggle with metacognition and evaluation. The findings

TABLE 3: Distribution of computational thinking practices across participants.

Participant	Abstraction	Decomposition and Algorithmic thinking	Evaluation
PSTA	✓	✓	-
PSTB	✓	✓	✓
PSTC	✓	✓	-
PSTD	-	✓	✓
PSTE	✓	✓	-

TABLE 2: Examples of the coding in relation to computational thinking practices, Schoenfeld's and metacognitive frameworks.

Computational thinking practice	Description	Examples	Schoenfeld's framework	Metacognitive framework
Abstraction	Focusing on important information in the problem and removing irrelevant detail to better understand and solve it.	1. Drawing the compass was important for me to find the 45°, which helped in solving the problem. 2. Reading the statement and using it to construct a visual representation of the problem. 3. Highlighting important information in the diagram such as equal angles or lines. 4. Making additional constructions to support problem-solving.	Devising a heuristic or strategy to solve the problem.	Planning the approach to solve the problem.
Decomposition and Algorithmic thinking	Breaking down a complex problem into smaller parts, solving each part, and integrating them to solve the whole problem.	1. I used sine here to get CE. 2. I then knew that DF equals 1. 3. Then, to get the distance from A to F, I added AB, BE, and EF. Each step builds on the step before reach the complete solution.	Devising and executing strategies to solve the problem.	Executing the problem-solving process carefully and checking how parts connect to the overall solution.
Evaluation (Debugging)	Recognising misunderstandings and errors and adjusting the approach to improve problem-solving accuracy.	1. Scratching a solution approach after realising it is incorrect and using a different one. 2. Using alternative visualisations to better understand the problem.	Making strategic decisions during problem-solving.	Planning and monitoring problem-solving by checking for correctness.

further highlight that evaluation is a crucial component of problem-solving, yet it remains an often-underdeveloped aspect of mathematical problem-solving (Lester, 2013). Lastly, PSTB showed evidence of all three CT aspects, whereas PSTA, PSTC, and PSTE demonstrated only two, and PSTD demonstrated decomposition and algorithmic thinking, and evaluation but not abstraction. These differences may reflect individual variation in mathematical proficiency or problem-solving approaches, although further data would be needed to confirm such associations.

Abstraction

The video observations and the VSRI indicated that PSTs used abstraction to solve problems. For example, PSTs translated the second problem from a given statement into a diagram which means they focused on the important information in the given text to construct a diagram that will help them to better understand the problem and devise a solution strategy. This is evidenced by labelling the red distances by PSTA (Figure 2) as 1 km and adding the construction of the compass to visualise the direction accurately. It emerged that drawing the compass allowed PSTs to determine the direction of the man. PSTA mentioned 'I drew the compass for me to be able to recall which direction is east and north'. PSTC stated that 'the compass is important for me to be able to draw the diagram correctly using it'. According to PSTA this direction was important in determining the turning angle of north-east as he mentioned:

> So now I know that now like they say north-east … it's more like north and east since its 90°, it's more like I cut it into half so I get this angle of 45°.

PSTB also added:

> [*B*]ecause like so in a mathematical problem you need to see the diagram, you need to see what you are working with because all these constructions I could not have done in my mind. But I could not see that it's going to be 45° there.

Previous research such as Grover and Pea (2013), Lafuente Martínez et al. (2022) and Weintrop et al. (2016) agree that focusing on the most important elements of the problem is an important element of solving problems as deciphering the text into a diagram led to the 45° angle that was important in finding the solution to the diagram. The emphasis on producing the diagram suggests that PSTA aimed to create a visual representation of the problem, reflecting the process of abstraction (Kallia et al., 2021). Furthermore, this indicates that PSTs were planning the approach to solve the problem by devising a strategy through drawing a diagram through abstraction (Schoenfeld, 1985). Correct solutions were achieved through a combination of accurate auxiliary constructions and correct interpretations of these constructions which is in line with findings from Jagals and Van der Walt (2016) who found that visualisation and metacognitive awareness are useful processes for solving mathematical problems. The significance of the importance of 'correct' abstraction during mathematical problem-solving is the case of PSTE who used 'incorrect abstractions' which led to an incorrect visualisation (Figure 3) and ultimately incorrect solution. It seems that PSTE considered the direction of the man's path 'north-west' instead of 'north-east', which may have influenced the formulation of the solution.

In the VSRI, PSTE uttered:

> [*T*]he only way that I would say is that since now they are talking about the directions, you needed to know like which direction is east which direction is north so I would have made a lot of mistakes but then I draws back to know the compass directions.

Surprisingly, even though PSTE knew that the compass was important for directions, he still could not construct a correct diagram for the man's path. According to Schoenfeld (1985), this signifies the importance of having good resources to solve problems and provides evidence that a lack of knowledge may lead to incorrect solutions. Furthermore, as per Yadav et al. (2022), PSTE did not make the additional constructions that were hidden features of the problem and were critical and required to solve the second problem. Another argument stems from PSTE's failure to apply appropriate abstractions. Despite recognising the importance of the 'compass,' the control aspect (Schoenfeld, 1985) was problematic, as he was

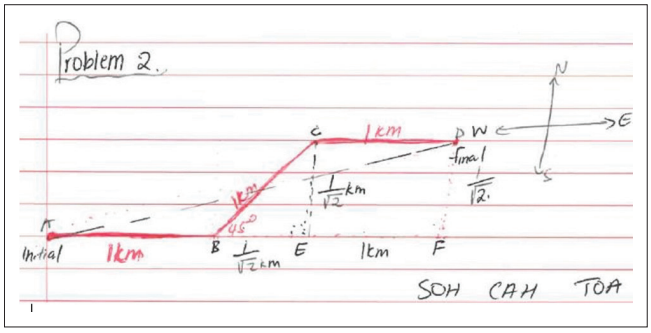


FIGURE 2: PSTA's construction of in the second problem.

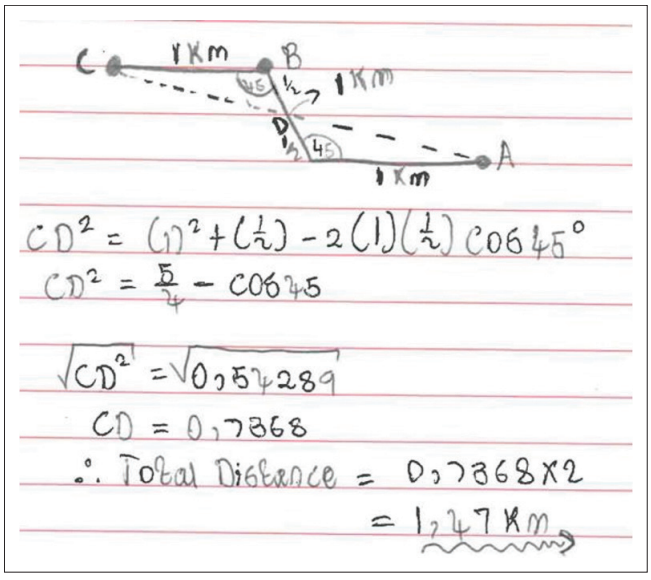


FIGURE 3: PSTE's diagram of the second problem.

unable to effectively use the knowledge and strategies available to solve the problem. Jagals and Van der Walt (2016) also highlighted the importance of metacognitive awareness in focusing on and using important information to solve mathematical problems is evident in some PSTs in this study.

Decomposition and algorithmic thinking

Evidence of PSTs solving sub-problems independently before synthesising these results contributed to enhancing PSTs' problem comprehension and facilitated the development of solutions. This is indicative of decomposition. However, the analysis revealed a striking parallel between problem decomposition and algorithmic thinking, to the extent that distinguishing between these two CT practices proved challenging. For example, if a PST decomposes a problem and solves different problems and then uses all these steps to find the final solution to the problem, it could be thought of as creating a step-by-step algorithm to solve the problem. Decomposition was exemplified by PSTA who solved the second problem by completing smaller tasks then integrated the solutions from these smaller parts to solve the larger problem. PSTA began by finding the length of CE (Figure 4) because, as he stated, 'the length of CE will help me to be able to get the length of DF there'.

Using the diagram he produced (Figure 4), PSTA applied the properties of the opposite sides of a rectangle to conclude that CE equals DF, highlighting the importance of basic mathematical knowledge or resources during problem-solving (Schoenfeld, 1985). PSTA stated:

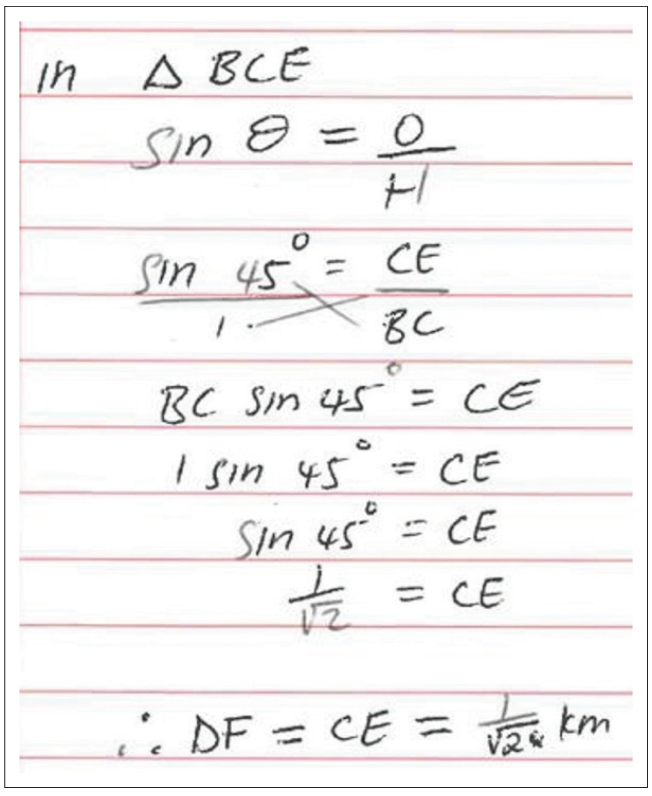


FIGURE 4: PSTA finding the length of CE.

> Yeah, since this is 90° and that is 90°, I can say now this and that they are gonna be parallel to each other, and now the distance from C to D is equal to E to F that's making this distance to be same.

This highlights the close relationship between CT practices and the metacognitive control to use such knowledge to solve mathematical problems (Yadav et al., 2022). Furthermore, it seemed that PSTA also relied on a mnemonic of 'SOH CAH TOA' (Figure 2) and stated that it was to:

> remember trig ratios, so I will use sine, then I will be able to know that its opposite over the hypotenuse, cos it's adjacent over hypotenuse, tan is opposite over adjacent. So I did this so that I will be able to recall, like the formulas for the trig ratios.

This use of the mnemonic indicates the strategies that PSTA used to make progress in this non-standard problem (Schoenfeld, 1985) but also indicates another step in the decomposition practice (Grover & Pea, 2018). The inclusion of all the information in Figure 2 is related to planning for the solution which is an important metacognitive aspect for successful problem-solving (Efklides & Schwartz, 2024; Schoenfeld, 1985). The use of the mnemonic 'SOH CAH TOA' also indicates that PSTA had already planned that trigonometry would be helpful in solving the problem. PSTA found the length of BE which then helped him to find the length of AF using the expression AF = AB + BE + EF as he mentions:

> [*T*]he reason why I wanted to get BE I knew that for me in order to get now the distance from A to F, I would be able to add these positions which is AB, BE and EF.

According to PSTA these two lengths were important for him to '*use Pythagoras theorem*' to solve the problem. Finally, PSTA integrated the solutions from these sub-problems using the Pythagorean theorem to calculate the length of AD, which indicates the length between the man's initial and final position. What is interesting with these decomposed steps of PSTA's solution is that it can be summarised in the following algorithm:

- Step 1: Find the length of segment CE.
- Step 2: Recognise that CE = DF.
- Step 3: Find the length of AD (within triangle ΔADF) using the length of DF.
- Step 4: Find BE using the cos 45°.
- Step 5: Find the sum of AB, BE and EF to obtain AF.
- Step 6: Use the Pythagorean theorem in ΔADF to determine the length of AD.

Thus, even though PSTA did not produce a step-by-step algorithm to solve the problem, the steps above can be deduced from his problem-solving processes for finding the solution explained in his decomposition of the second problem. Given the similarity between what was coded as decomposition and what could be classified as algorithmic thinking, this study found no clear distinction between the two. Both processes were demonstrated through similar methods in the PST's problem-solving approach. The findings that there is a link between decomposition and

algorithmic thinking was also evident in Lehmann (2023) who found that decomposition can be used to solve tasks that require algorithmic thinking. In a particular study, Lehmann (2024) refers to decomposition as classified decomposition as a cognitive skill for algorithmic thinking and concluded that decomposition is an important skill for designing algorithms (see also Kadijević, 2023). Unlike Lehmann (2023, 2024) who considered that decomposition is subsumed by algorithmic thinking, in this study, algorithmic thinking was thought of as less important to focus on compared to decomposition during PSTs' pen-and-paper problem-solving and when solving non-routine problems like the ones in the current study. Thus, as a strategy for solving 'non-routine' problems in mathematics, it is more important to focus on strengthening decomposition than algorithmic thinking in learners. These systematic approaches of decomposition and algorithmic thinking were observed in all PSTs and included using established mathematical procedures, such as Pythagoras' theorem, trigonometric ratios, mnemonics, and geometric principles to solve problems step by step. This further highlights the significance of both resources and heuristics (Schoenfeld, 1985) during mathematical problem-solving.

Evaluation

Evidence of evaluation was observed occurring in two different ways in the study: (1) using different visualisations of a problem, and (2) scratching attempted solutions for better ones. One example of scratching out attempted solutions and using a different approach was PSTD who, even after getting an answer, scratched out a first approach and used a different approach (Figure 5). This indicates that PSTD reflected on the solution to check its consistency and accuracy to the problem. Similar observations of scratching out solutions or attempts to solutions as an indication of evaluation was observed from other PSTs in this study. Given that most literature on CT and mathematics education is based on plugged activities, and unplugged activities have not included pen-and-paper problem-solving, the literature on whether scratching out solution is a common practice in CT-informed mathematical problem-solving is scant. However, this study determined that 'scratching out an attempt to try another attempt' is an indication of evaluation because it is informed by reflection. One study though (Lee, 2016) found that scratching out a solution can be used by students to evaluate their solutions and check their appropriateness to the problem. Thus, evaluation (or looking back, Polya, 1973) is important in problem-solving as it may 'be the most effective technique to teach children to solve problems' (Davis & McKillip, 1980, p. 91).

Another strategy used by PSTs in this study to evaluate their solutions was using different visualisations of a problem. For example, PSTB used three different visualisations for the second problem (Figure 6). PSTB stated in the interviews:

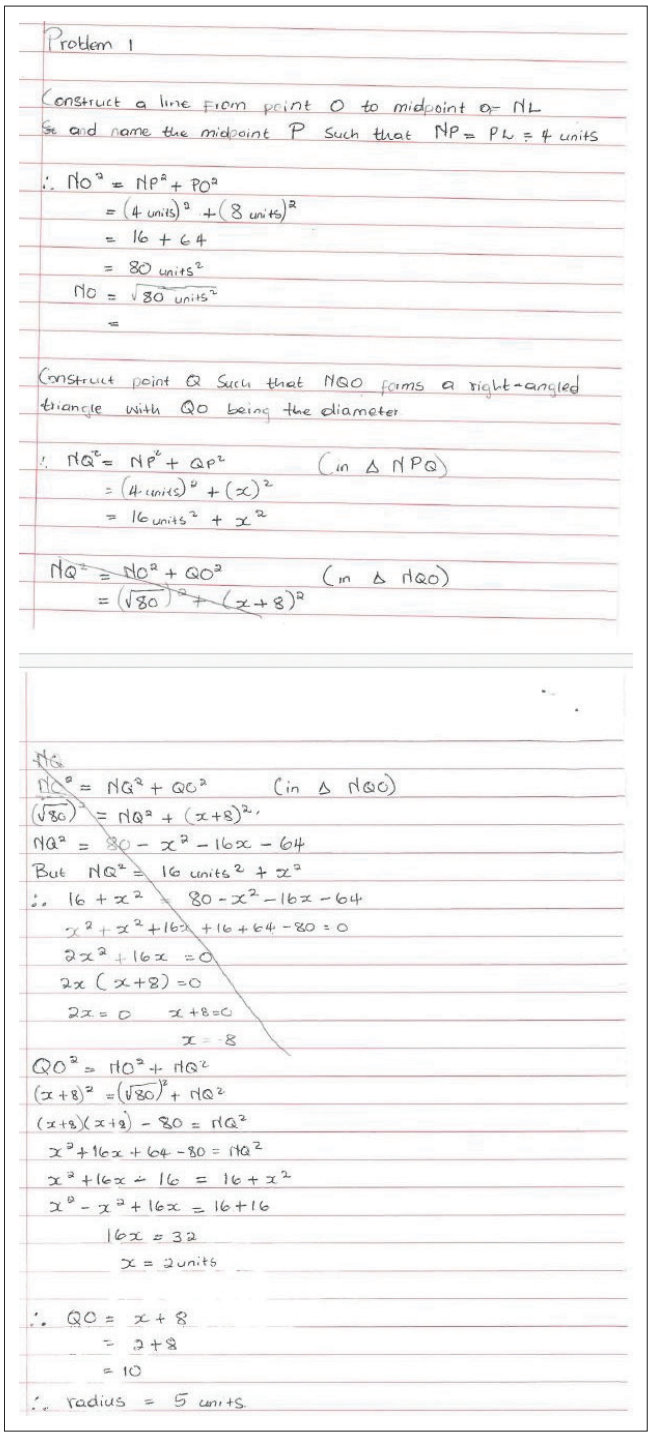


FIGURE 5: PSTD's solution to the first problem.

> [*T*]he first diagram failed; the one that succeeded at the end this one that I used. It failed because I was missing this [*pointing to 45° in C*]. Then, I thought of other ways that I can construct or create equations.

After recognising that his first approach in A did not work, PSTB constructed B because, as he said: '[*I thought*] if I can create a bigger rectangle that will have the whole diagram inside, then create and break it down into areas'. When the interviewer hinted at the fact that his approach did not work, PSTB mentioned that:

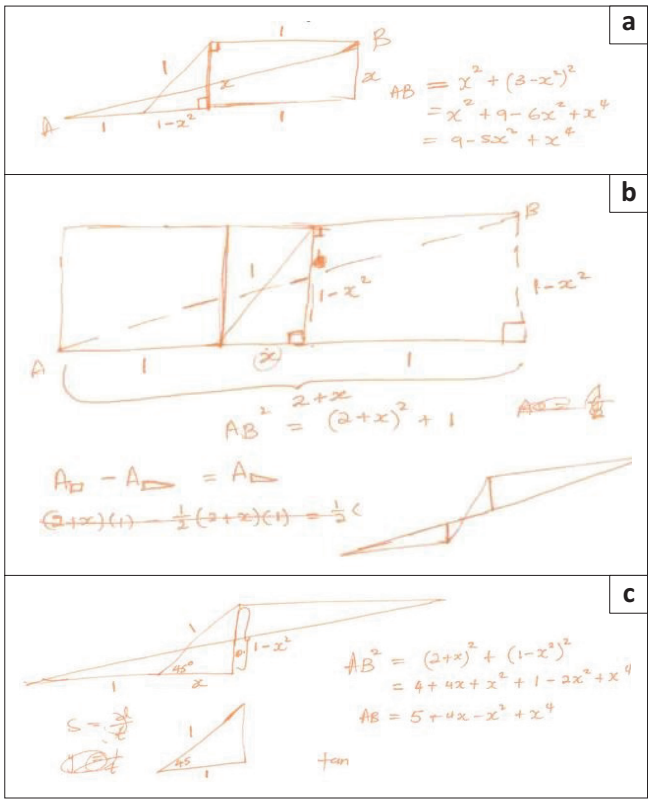


FIGURE 6: PSTB attempts to represent the second problem.

> It didn't work because even though I had this 2*x* plus *x* the other one was (1 – *x*) The areas, the area of the bigger rectangle is equal to two times the [*area of the*] triangle … because they are the same … now that would be $(1 - x^2)(2 + x) = (2 + x)(1 - x^2)$. Do you see what's happening? It cancels out, it becomes zero, it's quite true because if it becomes zero to zero it makes sense that the area of this rectangle. So, it means there was something missing in this question.

This realisation that a strategy was not working or there were gaps in the constructed diagrams highlights PSTB's ability to assess the effectiveness of his problem-solving processes and strategies. Then PSTB constructed C which included the 45° angle and that is the diagram he used to get the solution. PSTB then mentioned that:

> That's when I realised, ohh! if it's 45°, I know this is one this creates a special angle. I will have the hypotenuse am looking for, this side let's call it maybe *x* (hypotenuse). This side (*x*) now became √2. This is now obvious, now it becomes easier, I have to use Pythagoras.

As evident from PSTB's utterances and constructions, he used his evaluation skills to different visual representations of the problem (A and B) did not lead to the solution. Furthermore, through evaluation PSTB was able to diagnose the 'missing link' in his first two constructions and was later able to then find the solution.

Previous literature has revealed that the practice of evaluating solutions or solution steps is important for problem-solving and may lead to finding correct solutions (Koichu et al., 2021). The practice of evaluation during mathematical problem-solving is linked to metacognition (Kilpatrick, 2016) because each time an attempt at the solution is made, it is simultaneously checked for its appropriateness and if it is not appropriate, a different approach is designed and implemented which is important for successful problem-solving (Efklides & Schwartz, 2024; Jacobs & Paris, 1987; Schoenfeld, 1985). This is an indication of metacognitive monitoring which is an important element in problem-solving (Jagals & Van der Walt, 2016). In this case, using metacognitive monitoring, PSTB evaluated the different representations, the plan to execute a solution and the solution obtained for its appropriateness to the problem. This action was useful in finding the correct solution.

Conclusion and recommendations

The present study explored how CT practices manifested in PSTs' pen-and-paper problem-solving with a specific focus on four CT practices: abstraction, decomposition, algorithmic thinking, and evaluation. The question posed in this study was: *How do CT practices manifest in PSTs' pen-and-paper problem-solving processes?* The research findings indicated that there were several ways in which CT practices manifested in PSTs' problem-solving and the manifestation of these CT practices was closely linked to metacognition and the constructs of Schoenfeld's framework. Abstraction in this study manifested by translating textual information into diagrams, focusing on key elements of the problem, such as distances and directions. This strategy allowed them to devise solutions effectively because their accurate constructions helped them visualise and solve problems. This aligns with previous research highlighting the importance of abstraction and visualisation in mathematical problem-solving (Mudaly & Narriadoo, 2023; Rich & Yadav, 2020). There were other strategies exemplifying abstraction in PSTs' problem-solving that were based on focusing on the essential elements of the problem to find the solution. Decomposition and algorithmic thinking seemed to represent closely related processes, but decomposition was thought to be more important than algorithmic thinking. The main argument for this is that non-routine problems do not usually follow a step-by-step procedure, but they can be broken down into smaller parts, and most of the PSTs mainly spoke about 'breaking the problem down so they can understand it better' in the VSRI. Lockwood et al. (2016) and Lehmann (2024) argued that breaking down problems is a huge element of algorithmic thinking which concurs with the findings from this study. In fact, PSTA's example of breaking down the second problem into several steps and finding the solution resulted in the six-step algorithm to solve the problem. The main reason why this decomposition is more important is that in the normal curriculum for South Africa, the six-step algorithm is not as important as the decomposed solutions that results in the solution to the problem. Evaluation manifested in the reflective practices used by PSTs' pen-and-paper problem-solving. Only two

examples of evaluation manifested in this study: using different visualisations of a problem and scratching attempted solutions for better ones. These are just two examples of the many ways in which evaluation manifested from PSTs' pen-and-paper problem-solving. This reflective practice of evaluation aligns with metacognitive monitoring and reflection which is an important element of problem-solving in mathematics (Jagals & Van der Walt, 2016; Schoenfeld, 1987). The study recommends that interventions that aim to incorporate CT using pen-and-paper problem-solving should focus on abstraction, decomposition and algorithmic thinking, and evaluation. Another recommendation is that more studies should be conducted, aimed at providing a nuanced explanation of how, if ever, pattern recognition manifests in pen-and-paper problem-solving in mathematics education.

The study expanded the current CT discourse from programming-based plugged and unplugged activities to realistic school-based pen-and-paper problem-solving for incorporating CT in mathematics education. This opens a new way for learners to learn mathematics while also developing competencies related to CT. Given the high digital divide in South Africa, the findings from this study contribute towards a low-cost method for developing CT in mathematics education that aligns to the CAPS curriculum. By knowing how CT practices manifest in pen-and-paper problem-solving, teachers may design lessons that can target the development of CT using everyday teaching practices of solving mathematical problems using pen and paper. To my knowledge, such an investigation has not been previously conducted. This explicit integration of CT practices in mathematics classrooms can improve mathematical thinking and problem-solving in learners (Rich et al., 2019). In the next phase of this project, I plan to further this study by exploring if incorporating the ways in which CT practices manifest in pen-and-paper problem-solving in a real classroom can enhance student problem-solving competence and how.

Limitations

Due to the small number of participants (n = 5), and the qualitative nature of the study, the findings are not generalisable. Another discernible weakness is the focus on only four CT practices: abstraction, decomposition, algorithmic thinking, evaluation, and not observing other important CT practices such as pattern recognition in PSTs' pen-and-paper problem-solving. This may be the result of the types of problems that were posed to the PSTs.

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Author's contribution

S.C.M. is the sole author of this research article.

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Data availability

The data that support the findings of this study are available from the corresponding author, S.C.M., upon reasonable request. Due to the nature of the data, restrictions apply to the availability of these data to protect participant confidentiality and comply with ethical approval conditions.

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