



# Power laws, demography and entrepreneurship in selected South African regions

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Statistically significant Pareto-like log-log rank-size distributions were recorded for population and enterprise agglomeration in the towns of three different regions of South Africa, and are indicative of skewed distributions of population and enterprise numbers in regional towns. There were no distinct differences between groups of towns of regions from different parts of the country. However, the regional agglomerations differed from those of groups of towns randomly selected from a database. Regions, therefore, appear to have some uniqueness regarding such agglomerations. The identification of Zipf-like links between population and enterprise growth in regional towns still does not fully explain why some towns grow large and others stay small and there is a need to further explore these issues. The extreme skewness in population and enterprise numbers of different towns' distributions should, however, be considered in local economic development planning and execution.

## Significance:

- This contribution illustrates that the populations and enterprises of South African regional towns are distributed in orderly ways (called Pareto distributions) that result in some being large/many and others small/few.

## Introduction

Accounting for the way populations are distributed over different geographical locations and their evolution over time is important. The distribution of populations across geographical areas is not random<sup>1</sup>: there is a strong tendency toward agglomeration, i.e. populations are concentrated within common restricted areas such as cities, which results in a few large cities and many smaller cities. This striking pattern of geographical agglomeration is called Zipf's law for cities.<sup>2</sup> For instance, the size distribution of cities in the USA is startlingly well described by a simple power law,<sup>3</sup> which essentially states that the probability that the size of a city is greater than some  $S$  is proportional to  $1/S$ . Zipf's law is a special case of a Pareto distribution.<sup>4</sup>

Although normal (Gaussian) distributions and related quantitative methods are still relevant for a significant portion of organisational research, the increasing discovery of power laws signifies that Pareto rank-frequency distributions are pervasive and indicative of non-linear organisational dynamics.<sup>4</sup> Researchers ignoring Pareto distributions risk drawing false conclusions and promulgating useless advice to practitioners. Most managers face extremes, not averages.<sup>4</sup> Morudu and du Plessis<sup>5</sup> reported Pareto rank-frequency distributions for population, employment and gross economic value addition data of municipalities in South Africa. Their results hint of a limited impact of national policies such as GEAR (the Growth, Employment and Redistribution), ASGISA (Accelerated and Shared Growth Initiative for South Africa) and NGP (the New Growth Path) plans since 2001, and the marginal spatial impact of local economic development plans on economic variables at municipal level.

Patterns of a few large towns and many smaller ones have also been observed in regional studies of South African towns.<sup>6-12</sup> It is unknown if these patterns adhere to a rank-size distribution consistent with Zipf's law. Considering the above warning<sup>4</sup> and the implications reported for municipalities<sup>5</sup>, it is necessary to investigate the possible presence of Pareto (Zipf-like) population and/or enterprise distributions in the towns of South African regions. The presence of linear regularities (proportionalities) in the form of statistically significant correlations between the demographic and entrepreneurial characteristics of South African towns<sup>7-19</sup> could signal the possible presence of Pareto rank-size distributions in regions.

Knudsen<sup>20</sup> examined the size distribution of cities in Denmark by way of three questions: (1) Does Zipf's law apply to the population distribution of Danish cities? (2) What are the implications of Zipf's law for models of local growth? (3) Is there a Zipf's law for firms? Knudsen found that Zipf's law applies to Danish cities and that the size pattern of more than 14 000 Danish production companies follows a rank-size distribution consistent with Zipf's law. He did not examine the distribution patterns of the number of enterprises in Danish cities.

Knudsen's approach<sup>20</sup> provided guidance to this investigation about possible Pareto population and/or enterprise distributions in the towns of South African regions, i.e. the use of questions to examine the distribution patterns. The following questions were examined: (1) Does Zipf's law or Pareto rank-size distributions apply to the population distribution of towns in different regions of South Africa? (2) If Zipf's law (or a Pareto rank-size distribution) applies to town populations, does it also apply to the number of enterprises in these towns? (3) If Zipf's law (or a Pareto rank-size distribution) applies to populations and/or enterprises, do different regions differ from one another (in other words, do they have uniqueness)? (4) Should such distributions, if they exist, be considered in local economic development planning and support? The purpose of this contribution is to provide answers to these questions.

## Approach used in this contribution

Previously studied towns of three different regions of South Africa were selected for this analysis (Table 1). The first group includes 12 towns of the Eastern Cape Karoo (EC Karoo) that have previously been extensively

studied.<sup>6,9</sup> These studies included an analysis of data covering almost a century (1911 to 2006).<sup>11</sup> The second group includes 29 Karoo towns included in a recent Shale Gas Strategic Environmental Assessment of the Karoo.<sup>21</sup> Further study of these towns could provide information essential for decision-making about shale gas production. The third group includes the towns of the Gouritz Cluster Biosphere Reserve (GCBR) in the southern Cape.<sup>22</sup>

**Table 1:** The towns of the Eastern Cape Karoo (EC Karoo), the strategic environmental assessment (SEA) study area and the Gouritz Cluster Biosphere Reserve (GCBR). Towns that fall into two of the three regions are indicated in bold.

No.	EC Karoo	SEA study area	GCBR
1	<b>Aberdeen</b>	<b>Aberdeen</b>	Albertinia
2	<b>Cradock</b>	Beaufort-West	Barrydale
3	<b>Graaff-Reinet</b>	Burgersdorp	Calitzdorp
4	<b>Hofmeyer</b>	Carnarvon	De Rust
5	<b>Jansenville</b>	Colesberg	Great Brak River
6	<b>Middelburg</b>	<b>Cradock</b>	Heidelberg
7	<b>Pearston</b>	Fort Beaufort	Ladismith
8	<b>Somerset East</b>	Fraserburg	Montagu
9	<b>Steynsburg</b>	<b>Graaff-Reinet</b>	Mossel Bay
10	Steytlerville	<b>Hofmeyr</b>	Oudtshoorn
11	Venterstad	<b>Jansenville</b>	<b>Prince Albert</b>
12	Willowmore	Klipplaat	Riversdal
13		Lady Frere	Stilbaai
14		Laingsburg	Swellendam
15		Loxton	Uniondale
16		Merweville	
17		<b>Middelburg</b>	
18		Murraysburg	
19		Nieu-Bethesda	
20		Noupoort	
21		<b>Pearston</b>	
22		<b>Prince Albert</b>	
23		Queenstown	
24		Richmond	
25		<b>Somerset East</b>	
26		<b>Steynsburg</b>	
27		Sutherland	
28		Victoria West	
29		Williston	

## Population and enterprise distribution in cities

Why do cities exist, and why do they vary in size? These fundamental questions have received a considerable amount of attention from regional

and urban economists in recent years.<sup>23</sup> Cities are thought to arise to give consumers easy access to a large variety of goods or because of the 'external' effects of consumer location or because of the advantages of proximity of consumers to their workplaces. Although these reasons are probably important, Brakman et al.<sup>23</sup> believe they do not explain why cities are spread out unevenly across space or explain why systems of cities exist.

Already in 1682, Alexandre Le Maitre remarked on a systematic pattern in the size distribution of French cities.<sup>1</sup> Power laws were discovered more than a hundred years ago by Vilfredo Pareto.<sup>24</sup> Felix Auerbach, in 1913, and George Kingsley Zipf, in 1949, formally established an empirical regularity: the sizes of the large cities are inversely proportional to their ranks.<sup>1</sup> The proportionality of rank and size implies a power distribution with exponent equal to one – a phenomenon that became known as Zipf's law for cities, a special case of a Pareto distribution. It is a striking pattern of agglomeration that may well be the most accurate regularity in economics and it holds for many countries and dates.<sup>2</sup> If a sample of cities is ranked according to population size and presented as a graph of log population size (independent variable) and log rank (dependent variable), a straight line with slope -1 indicates a Zipf distribution.<sup>23</sup> If the slope is higher than one, cities are more dispersed than predicted by Zipf's law, and if the slope is less than one, cities are more even-sized than the prediction.<sup>23</sup>

The reason for the existence of Zipf's law for population distribution is still rather obscure and Krugman<sup>3</sup> remarked: 'At this point we are in the frustrating position of having a striking empirical regularity with no good theory to account for it.' However, Gabaix<sup>2</sup> stated that the reason why cities become large is essentially because of inertia in the creation of jobs: the number of new jobs is roughly proportional to the number of existing jobs. Eeckhout<sup>1</sup> remarked that once population mobility is understood, the underlying economic mechanisms can be examined. Agglomeration and residential mobility of the population between different geographical locations are tightly connected to economic activity; the evolution of the population across geographical locations is an extremely complex amalgam of incentives and actions taken by many individuals, enterprises and organisations.

A pattern of some large towns combined with a number of smaller towns is also present in the three regions under consideration here. However, it is not known if there are Pareto rank-size regularities as far as populations are concerned, and, if present, whether they adhere to Zipf's law or differ from one another.

What about enterprise development? Axtell<sup>25</sup> mentioned that Gibrat reported a lognormal distribution of the sizes of French industrial firms in 1931. Such a distribution was later also recorded in the UK.<sup>26</sup> Axtell<sup>27</sup> reported that the distribution of US firm sizes closely followed the Pareto distribution with an exponent near unity, i.e. the Zipf distribution. Knudsen<sup>20</sup> reported that the size distribution of Danish production companies had a strong fit with a Pareto rank-size distribution with an exponent of 0.741.

No record was found that the distribution patterns of the number of enterprises in countries or regions have received research attention. Given the statistically significant relationships between population sizes and enterprise numbers frequently recorded for South African towns,<sup>7-19</sup> it is clear that town size (measured by the population number), the creation of jobs, and thus enterprise numbers and entrepreneurial decisions, have similar variance patterns. It is, therefore, also necessary to examine if the enterprise numbers of regions have Pareto rank-size distributions. If they do, such regularities could be used in development decisions and support as suggested by Andriani and McKelvey<sup>4</sup> and used by Morudu and du Plessis<sup>9</sup>.

## EC Karoo towns

The EC Karoo and its 12 towns (Table 1) provided the focus of a number of studies.<sup>6,9,11</sup> This area is a sub-region of the Karoo, wholly included in the Eastern Cape Province. Its selection for study by Nel and Hill<sup>6</sup> was based on the availability of comparable and continuous census data records over an extended period. They identified the following trends

based on nearly 100 years of continuous records: shifts in agricultural production, small town and rural population change, and evolving small town economies. The 12 towns were also used in a study of enterprise proportionality phenomena in small towns of the EC Karoo.<sup>9</sup> The towns' enterprises were identified, enumerated and classified into different business sectors. Statistical analyses were used to examine the enterprise dynamics. Regional proportionalities, i.e. fairly constant ratios between business sector enterprise numbers and total enterprise numbers in the towns, were used to construct a 'regional enterprise structure'.

The 12 towns have also been used to address the question of whether the proportionality between population numbers and enterprise numbers in South African towns was present at earlier times. Access to the century-long database of Nel and Hill<sup>6</sup> enabled such a study.<sup>11</sup> Ten data sets were extracted in which the years of the population estimates and enterprise counts in a specific data set differed by at most 2 years. Proportionalities were present over the century and a detailed picture of the relationship between population dynamics and enterprise dynamics was developed.<sup>11</sup> The Nel and Hill<sup>6</sup> data sets also lend themselves to examination of the time-dependence of rank-size distributions of the population and enterprise numbers of the EC Karoo. This region was chosen as the first region in the present study.

## Karoo towns

The Karoo, which occupies some 40% of the surface area of South Africa, has a continuous census record and a network of small towns of differing sizes, which made it a suitable area to research aspects of small town development.<sup>28</sup> In addition, shale gas development in the Karoo is being considered by the South African government. An area of 171 811 km<sup>2</sup> of the Central Karoo, delimited by the applications for exploration rights for shale gas lodged by different companies, plus a 20-km buffer, constituted the study area of a strategic environmental assessment that considered shale gas development in the Karoo.<sup>21</sup> The study area includes 29 towns (Table 1).

The Karoo was slowly urbanised after 1785 when its first town, Graaff-Reinet, was founded.<sup>29</sup> By 1850 only 10 of the selected 29 towns had been founded, but, with a couple of exceptions, all had been founded by 1900. These towns exhibit a spread of population and enterprise sizes that raises a question about the possible presence of rank-size distributions.

There is an overlap of nine towns between the groups of the EC Karoo and the strategic environmental assessment study area (Table 1). This overlap was considered acceptable because comparison of the results for the two groups would help to determine if specific rank-size distributions, if present, are typical of a specific region. The shale gas development strategic environmental assessment study area, with its 29 towns, was selected as the second region in this study.

## The Gouritz Cluster Biosphere Reserve

The GCBR is located in the southern Cape area of South Africa. It is globally unique as it is the only area in the world where three recognised biodiversity hotspots converge: the Fynbos, Succulent Karoo and Maputaland-Tongoland-Albany hotspots.<sup>22</sup> Two mountain ranges (the Swartberg Mountains in the north and the Langeberg/Outeniqua Mountains in the south) separate the GCBR into two separate geographical sub-regions. To the north and nestled between the Swartberg and the Langeberg/Outeniqua/Tsitsikamma mountain ranges lies a semi-arid to arid valley, the Little Karoo.<sup>30</sup> In the south lies a more verdant coastal plain bordered by the Langeberg mountains in the north and the Indian Ocean in the south.

Urbanisation of the GCBR was also slow. Only one town, Swellendam, had been founded by 1800 for administrative purposes.<sup>29</sup> By 1850 there were only five towns but thereafter the pace of urbanisation increased to the extent that all 15 of the GCBR towns (Table 1) had been founded by 1900. Most of these towns were founded to cater for a rural population's needs of religious services and not for commercial reasons.<sup>29</sup> The GCBR was selected as the third region of this study.

## Methods

### Population size rank-size analyses

Population numbers for 1946 were obtained from a government report which provides information from 1904 to 1970.<sup>31</sup> Population numbers for 2001 and 2011 of the towns were sourced from a German website.<sup>32</sup> Population estimates for 2014 or 2016 were based on 2011 data extended by the growth rate between 2001 and 2011.

The towns of each of the regions for a specific year were ranked from highest to lowest according to their population numbers. The following regression was then calculated for each time period and region:

$$\text{Log rank}_{1,2,\dots,n} = a - b(\text{log population size}_{1,2,\dots,n}) \quad \text{Equation 1}$$

where  $a$  is the intercept,  $b$  is the regression coefficient and  $n$  is the number of towns in a region. Microsoft Excel software was used for the calculations.

### Enterprise numbers rank-size analyses

Enterprise numbers for 1946/1947 and 2013/2014 or 2015/2016 of towns of all three regions were determined according to the methods of Toerien and Seaman<sup>7</sup>, using telephone directories for the specific year.

The towns of the different regions for a specific year were ranked from highest to lowest according to their enterprise numbers. The following regression was then calculated for each time period and region:

$$\text{Log rank}_{1,2,\dots,n} = a - b(\text{log enterprise numbers}_{1,2,\dots,n}) \quad \text{Equation 2}$$

where  $a$  is the intercept,  $b$  is the regression coefficient and  $n$  is the number of towns in a region. Microsoft Excel software was used for the calculations.

### Time dependence of rank-size distributions

Once statistically significant Pareto rank-size distributions were recorded, it became necessary to test the time dependence of such distributions. The database of Nel and Hill<sup>6</sup> was used to extract population and enterprise data for the 12 EC Karoo towns and their associated rural areas for selected years during 1911 to 2004. Twelve of the years were selected for the population analyses and ten years were selected for the enterprise analyses.

### Are regions unique?

Once statistically significant Pareto rank-size distributions were recorded, it also became necessary to test the uniqueness of a region's population and enterprise distribution. It is possible that the towns in a region are analogous to a random selection of towns from a database. To test this possibility, six random selections were made of towns from a database of 206 South African towns that contains, among others, 2011 population and recent enterprise data for each town. The selected towns and their population and enterprise numbers are presented in Table 2. The rank-size distributions of population and enterprise numbers of each selection were analysed as described earlier.

It was hypothesised that if there are links between the towns of a region resulting in unique rank-size distributions, the same would not be observed for random selections of towns. The null hypothesis was that a random selection of towns from the database would not result in statistically significant rank-size distributions and/or dissimilar regression coefficients.

## Results

### Population rank-size distributions

Pareto power laws describe the rank-size relationships of populations of the towns of the three regions for the respective years (Table 3). Figure 1 illustrates this relationship for the towns of the GCBR in 2013/2014. In all cases, except the GCBR towns in 1946, more than 90% of the variation was explained (see  $R^2$  in Table 3). There is clearly significant orderliness in the population agglomeration patterns of the South African regions investigated.

**Table 2:** Six groups of 15 towns each randomly selected to examine their rank-size distributions

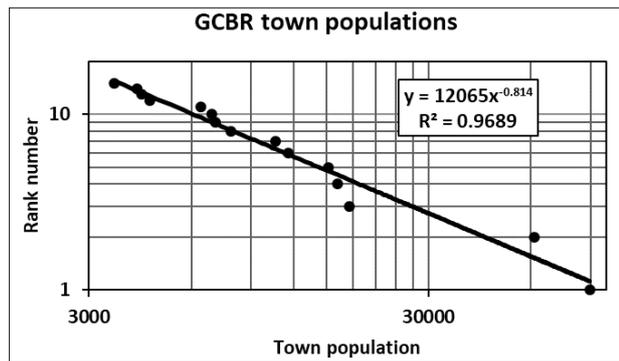
No.	Town	Population	Enterprises	Town	Population	Enterprises	Town	Population	Enterprises
<b>Group 1</b>			<b>Group 3</b>			<b>Group 5</b>			
1	Alexander Bay	1736	55	Alexander Bay	1736	55	Boshof	8509	36
2	Augrabies	3627	41	Britstown	5145	27	Bredasdorp	15 524	274
3	Botshabelo	181 712	203	Carnarvon	6612	78	De Aar	29 990	223
4	Brandvlei	2859	22	Gariepdam	1568	22	Edenburg	6460	26
5	Christiana	20 882	137	Hennenman	24 355	120	Fraserburg	3029	35
6	Groblershoop	4938	51	Hertzogville	9423	26	Garies	2105	26
7	Hartswater	10 465	295	Keimouth	291	30	Hertzogville	9423	26
8	Hendrina	15 871	85	Middelburg (EC)	18 861	174	Jagersfontein	5729	20
9	Hofmeyr	3680	21	Phuthaditjhaba	54 661	409	Kroonstad	97 780	701
10	Jansenville	5612	75	Richards Bay	252 968	2126	Memel	7142	32
11	Ladismith	7127	108	Springfontein	3699	20	Middelburg (EC)	18 861	174
12	Ladybrand	25 816	258	Sutherland	2836	52	Norvalspont	1198	8
13	Norvalspont	1198	8	Vanderkloof	1228	18	Paul Roux	6152	17
14	Paul Roux	6152	17	Victoria West	8254	88	Springfontein	3699	20
15	Wepener	9553	37	Winterton	6030	117	Steynsburg	7212	42
<b>Group 2</b>			<b>Group 4</b>			<b>Group 6</b>			
1	Beaufort-West	71 011	489	De Rust	3566	54	Bethlehem	76 667	993
2	Carnarvon	6612	78	Fauresmith	3628	20	Daniëlskuil	13 597	85
3	Edenburg	6460	26	Kleinmond	6634	210	Douglas	20 083	127
4	Fort Beaufort	25 668	108	Koffiefontein	10 402	39	Gansbaai	11 598	254
5	Greyton	2780	59	Lady Frere	4024	35	Gariepdam	1568	22
6	Keimouth	291	30	Lime Acres	4408	42	Hotazel	1756	16
7	Koffiefontein	10 402	39	Loeriesfontein	2744	29	Koppies	13 803	68
8	Lime Acres	4408	42	Memel	7142	32	Memel	7142	32
9	Phalaborwa	109 468	543	Mossel Bay	89 430	1949	Nieu-Bethesda	1540	58
10	Prieska	14 246	108	Odendaalsrus	63 743	189	Nieuwoudtville	2093	30
11	Reitz	20 183	133	Parys	45 746	506	Sannieshof	11 016	84
12	Sutherland	2836	52	Reddersburg	4886	26	Thabazimbi	28 847	323
13	Swellendam	17 537	398	Schweizer-Reneke	41 226	224	Viljoenskroon	31 468	143
14	Vredendal	18 170	351	Somerset East	18 825	200	Vosburg	1259	16
15	Wepener	9553	37	Viljoensdrif	751	18	Williston	3368	32

The exponent of Zipf's law for the population distribution of cities is normally -1 or close to it.<sup>23</sup> The regions investigated here do not rigidly exhibit Zipf's law because their coefficients are lower than -1 (Table 3). Their Pareto rank-size distributions are nevertheless reasonably close to Zipf's law and predict that a lower-ranked town in the three regions investigated here would have from 55% to 62% of the population of a town just above it in the rank (Table 3).

To examine the time dependence of the population rank-size distribution, use was made of the database for the 12 EC Karoo towns of Nel and Hill.<sup>6</sup> Urban and rural population data recorded by Nel and Hill<sup>6</sup> of 12 different years between 1911 and 2004 for the 12 towns were subjected to the

same rank-size analyses described earlier. The results are presented in Table 4.

With the exception of 1911, more than 90% of the variation for every year was explained (see R<sup>2</sup> in Table 4). The results substantiate the finding that the population rank-size distribution follows a Pareto power law (Table 4) and indicate that the relationship holds true over time. Changes in the regression coefficient show a definite pattern. It changed from -1.2 in the early 1900s to about -0.9 by the early 2000s. As a consequence, the population ratio of a lower-ranked town to a town ranked just above it has changed from about 40% in the early 1900s to just over 50% by the early 2000s (Table 4).



**Figure 1:** Example of a power law describing the rank-size relationship of population numbers (in 2011) of towns in the Gouritz Cluster Biosphere Reserve (GCBR), South Africa. The GCBR is used as an example.

**Table 3:** Population rank-size distributions of three South African regions for 1946 and 2011. Towns were ranked according to population sizes, and then regressed as  $\log_{10}$  values against  $\log_{10}$  values of rank numbers.

Region	Time period	Correlation	R <sup>2</sup>	Regression coefficient	n	Ratio (%) <sup>†</sup>
Eastern Cape Karoo	1946	-0.97*	0.943	-0.87	12	54.7
	2011	-0.96*	0.914	-0.86	12	55.1
Strategic environmental assessment study area	1946	-0.97*	0.943	-0.86	28	55.1
	2011	-0.96*	0.919	-0.75	29	59.1
Gouritz Cluster Biosphere Reserve	1946	-0.90*	0.808	-0.70	15	61.6
	2011	-0.97*	0.944	-0.75	15	59.1

\*Statistically significant at  $p < 0.01$ .

<sup>†</sup>The percentage ratio of the population of a lower ranked town to the population of the town ranked just above it.

**Table 4:** Population rank-size distribution analyses spanning the period 1911 to 2004 of the 12 Eastern Cape Karoo towns. Towns were ranked according to population sizes of their urban and rural areas, which were regressed as  $\log_{10}$  values against  $\log_{10}$  values of rank numbers.

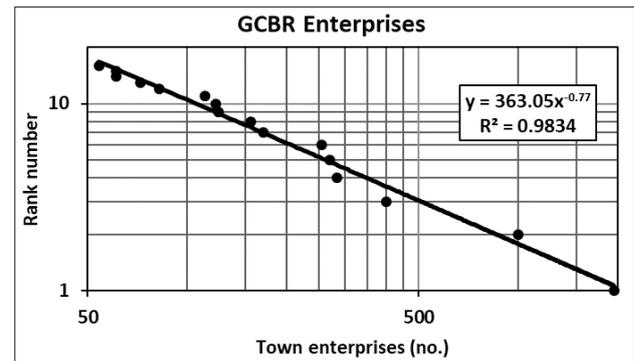
Year	Correlation	R <sup>2</sup>	Regression coefficient	n	Ratio (%) <sup>†</sup>
1911	-0.91*	0.836	-1.19	12	44
1921	-0.95*	0.902	-1.27	12	42
1936	-0.96*	0.915	-1.19	12	44
1946	-0.96*	0.918	-1.11	12	46
1951	-0.96*	0.917	-1.07	12	48
1960	-0.96*	0.926	-1.07	12	48
1970	-0.96*	0.924	-1.09	12	47
1980	-0.97*	0.942	-1.03	12	49
1985	-0.96*	0.922	-0.99	12	50
1991	-0.95*	0.906	-0.93	12	50
2001	-0.96*	0.915	-0.9	12	54
2004	-0.96*	0.915	-0.89	12	54

\*Statistically significant at  $p < 0.01$ .

<sup>†</sup>The percentage ratio of the population of a lower ranked town to the population of the town ranked just above it.

### Enterprise number rank-size distributions

The rank-size relationships of enterprise numbers in 1946/1947 and 2013/2014 of the towns of the three regions and the ranks of their enterprise numbers are also described by power laws (Table 5). Figure 2 shows the relationship for the GCBR towns in 2013/2014. Except for the GCBR towns in 1946/1947, more than 90% of the variation was explained for both time periods (see R<sup>2</sup> in Table 5). This result is not unexpected given the often-observed linear relationship between population sizes and enterprise numbers of South African towns.<sup>7-19</sup> If the rank-size population distribution of towns in a region is described by a power law, the enterprise number rank-size distribution of the region should exhibit the same pattern.



**Figure 2:** Example of a power law describing the rank-size relationship of enterprise numbers (in 2013/2014) of towns in the Gouritz Cluster Biosphere Reserve (GCBR), South Africa. The GCBR is used as an example.

**Table 5:** Enterprise number rank-size distributions of three South African regions for 1946 and 2011. Towns were ranked according to their enterprise numbers, and regressed as  $\log_{10}$  values against  $\log_{10}$  values of rank numbers.

Region	Time period	Correlation	R <sup>2</sup>	Regression coefficient	n	Ratio (%) <sup>†</sup>
Eastern Cape Karoo	1946/1947	-0.96*	0.924	-0.98	12	51
	2015/2016	-0.96*	0.918	-0.68	12	63
Strategic environmental assessment study area	1946/1947	-0.95*	0.907	-1.12	28	46
	2015/2016	-0.98*	0.951	-0.76	29	59
Gouritz Cluster Biosphere Reserve	1946/1947	-0.92*	0.847	-0.88	15	54
	2013/2014	-0.99*	0.980	-0.77	15	59

\*Statistically significant at  $p < 0.01$ .

<sup>†</sup>The percentage ratio of the population of a lower ranked town to the population of the town ranked just above it.

The ratios observed for the enterprise distribution of the three regions indicate that lower-ranked towns in the three regions have from 50% to over 60% of the enterprises of the towns ranked just above them (Table 5). The ratios of all towns in 2015/2016 were higher than those of 1946/1947, suggesting a shift over time. Based on these ratios there is no clear distinction between the different regions (Table 3).

To examine the time dependence of the enterprise number Pareto rank-size distributions of the EC Karoo towns, use was also made of the database for the 12 EC Karoo towns of Nel and Hill<sup>6</sup>. Enterprise numbers recorded by Nel and Hill<sup>6</sup> of 10 years between 1904 and 2000 for the

12 towns were subjected to the same rank-size analyses described earlier. The results are presented in Table 6.

Pareto-like rank-size distributions of enterprises in the EC Karoo towns were observed between 1904 and 2000. Pareto rank-size distributions appear to be enduring and time-independent characteristics. The ratios observed for the enterprise distributions indicate that lower-ranked towns in the region varied from 42% to over 64% of the enterprises of the towns ranked just above them (Table 6). A definite shift in the ratio was observed after 1951: lower-ranked towns progressively had more enterprises relative to higher-ranked towns. This shift could be because lower-ranked towns had over time increased populations relative to higher-ranked towns (Table 3).

**Table 6:** Enterprise number rank-size distributions of Eastern Cape Karoo towns for the period 1904 to 2000. Towns were ranked according to enterprise numbers, which were regressed as  $\log_{10}$  values against  $\log_{10}$  values of rank numbers.

Year	Correlation	R <sup>2</sup>	Regression coefficient	n	Ratio (%) <sup>†</sup>
1904	-0.96*	0.927	-1.06	12	48
1911	-0.96*	0.926	-1.26	12	42
1921	-0.95*	0.907	-1.22	12	43
1935	-0.95*	0.900	-1.05	12	48
1951	-0.97*	0.935	-1.16	12	45
1961	-0.97*	0.948	-0.91	12	53
1970	-0.96*	0.923	-0.86	12	55
1980	-0.97*	0.941	-0.85	12	55
1990	-0.95*	0.912	-0.74	12	60
2000	-0.95*	0.905	-0.65	12	64

\*Statistically significant at  $p < 0.01$ .

<sup>†</sup>The percentage ratio of the population of a lower ranked town to the population of the town ranked just above it.

### Distributions of randomly selected groups of towns

The rank-size population distributions of six randomly selected groups of South African towns exhibited Pareto-like properties and the correlations

are statistically significant (Table 7). The regression coefficients varied from -0.48 to -0.70 with an average of  $-0.58 \pm 0.073$ , which is clearly lower than the regression coefficients of the three selected regions (Table 3). The average is also much lower than the century-long coefficients of the towns of the EC Karoo (Table 4). The ratios of more than 60% of lower-ranked towns to towns ranked just above them are consequently higher than those of the three regions (compare Table 7 with Tables 3 and 4). In randomly selected groups of towns, population numbers tend to be more evenly distributed.

The rank-size enterprise distributions of the selected groups also exhibited Pareto-like properties and the correlations are statistically significant (Table 7). The regression coefficients of the randomly selected groups varied from -0.56 to -0.71 with an average of  $-0.64 \pm 0.0059$ , which is somewhat lower than the regression coefficients of the three selected regions (Table 5). These coefficients are also lower than the century-long coefficients of the EC Karoo towns except for the year 2000. As a consequence, the ratios of lower-ranked towns to towns ranked just above them are higher (compare Tables 5 and 7). The enterprise numbers of randomly selected groups of towns were more evenly distributed than those of the three regions.

Because the database of more than 200 towns contains large as well as small towns, the random selection of 15 towns from it should yield a group that has a spread of town sizes (measured in terms of populations or enterprise numbers). This spread could lead to the recording of statistically significant log-log rank-size distributions, as was observed (Table 7). The coefficients of the randomly selected groups were, however, lower and their ratios higher than those of the three regions, which suggests that population agglomeration and enterprise development patterns of regions do have uniqueness. Expressed differently: what happens in one town of a specific region influences what happens in other towns of that region. Regions, however, do not appear to differ from one another.

Returning to the questions initially raised in this contribution, the following can be concluded:

- Statistically significant log-log rank-size distributions apply to population agglomerations of towns in different regions of South Africa.
- Such rank-size distributions also apply to the number of enterprises in these towns.
- Based on the relationships observed, there is no clear distinction between different regions. However, the regional agglomerations differed from those of groups of towns randomly selected from a database (compare Tables 5 and 7). Regions, therefore, appear to have some uniqueness regarding such agglomerations.

**Table 7:** The rank-size distributions of populations and enterprise numbers of six groups of 15 towns each randomly selected from a large database

Group	Populations				Enterprises			
	Correlation	R <sup>2</sup>	Regression coefficient	Ratio (%) <sup>†</sup>	Correlation	R <sup>2</sup>	Regression coefficient	Ratio (%) <sup>†</sup>
Group 1	-0.98*	0.951	-0.62	65	-0.93*	0.861	-0.69	62
Group 2	-0.88*	0.776	-0.48	72	-0.96*	0.918	-0.71	61
Group 3	-0.96*	0.979	-0.59	66	-0.99*	0.979	-0.59	66
Group 4	-0.95*	0.896	-0.55	68	-0.98*	0.959	-0.56	68
Group 5	-0.97*	0.931	-0.70	62	-0.97*	0.939	-0.61	66
Group 6	-0.94*	0.877	-0.56	68	-0.98*	0.970	-0.65	64

\*Statistically significant at  $p < 0.01$ .

<sup>†</sup>The percentage ratio of the population of a lower ranked town to the population of the town ranked just above it.

## Discussion

Population distribution across geographical areas is not random: there is a strong tendency toward agglomeration.<sup>1</sup> Why then are there large and small towns? This question led Christaller to theorise about the centrality of towns, based on the services that towns deliver to their hinterlands.<sup>33</sup> Christaller argued about a system of central places that exhibits a hierarchical principle: any goods supplied in a central place of order  $i$  is also supplied in all central places of order  $j > i$ . Centrality became an important issue in studies of South African towns.<sup>34</sup>

However, Eaton and Lipsey<sup>35</sup> argued that Christaller's theory of central places is simultaneously a theory of the location and agglomeration of economic activity in which there is no force creating agglomeration, in which agglomeration serves no purpose, and in which no firm ever chooses a location. Despite this criticism, economic geographers and regional economists remain interested in the reasons for uneven regional development. They still ask why economic growth does not lead to similar levels of prosperity, employment and welfare across space (for example see Gardiner et al.<sup>36</sup>); a question of relevance also about South African regions.

A number of questions were posed in this study that can now be answered. Whereas population agglomeration patterns in many countries are subject to Zipf's law, a power law with Pareto-like characteristics<sup>1,2</sup>, this study has demonstrated that Pareto-like population and enterprise distributions close to Zipf's law are present in the towns of different South African regions (Tables 3 and 5) – a fact hitherto unknown. Morudu and du Plessis<sup>5</sup> reported Zipf-like distribution of the population, employment and economic value addition characteristics of South African municipalities. The latter are organisations that are man-made constructs, often containing more than one town, which do not necessarily reflect the 'natural' way in which agglomeration phenomena evolve in towns of a region.

The Pareto distribution types encountered in this study have endured over a long time even when the regression coefficients progressively changed (Tables 3 and 6). The patterns recorded in regions differ from that of randomly selected groups of towns, suggesting that there is some uniqueness in the orderly way in which regions give rise to population and enterprise agglomeration patterns.

This study has revealed an additional dimension of the regularities observed between population and/or enterprise characteristics in South African towns.<sup>7-19</sup> Previously, linear relationships between population and enterprise numbers and enduring power law relationships between total enterprise numbers and the enterprise richness (total number of enterprise types) of South African towns<sup>15,19</sup> were recorded. This study recorded log-log agglomeration patterns for the population and enterprise numbers of regional towns. Population growth and distribution, and enterprise development and distribution, are clearly highly orderly processes. This fact should be factored into local economic development planning and support, as was also suggested by Morudu and du Plessis<sup>5</sup>.

The similarities in the variance patterns for population and enterprise distributions observed in this study, raise a 'chicken or egg' scenario, that is, does population growth precede enterprise development, or does enterprise development precede population growth? Fujita and Thisse<sup>37</sup> argue that consumer behaviour predicts agglomeration because consumers face search costs and have incomplete information about the retail landscape, so they find it efficient to patronise larger centres. Firms cluster because of consumer behaviour and benefit from demand externalities by locating in the larger centres. Their thinking, therefore, implies that population growth precedes enterprise development. Eaton and Lipsey<sup>35</sup> also commented that because the clustering of heterogeneous firms facilitates multipurpose shopping, it allows consumers to economise on shopping costs.

However, the contrary – that enterprise development could precede population growth – should also be considered. Gabaix<sup>2</sup> stated that the creation of jobs is important and people are attracted to where there are jobs. This implies that enterprise growth has to take place

before population growth results from immigration of people seeking employment. Eeckhout<sup>1</sup> remarked that agglomeration and residential mobility of the population between different geographical locations are tightly connected to economic activity and that the evolution of populations across geographical locations is an extremely complex amalgam of incentives and actions taken by many individuals, enterprises and organisations.

Fransen<sup>29</sup> remarked that most towns in the former Cape Colony in South Africa were not founded for commercial reasons. So-called 'church towns' developed around churches that were built to satisfy the needs of farming communities for religious services. In many cases following the building of a church, members of a congregation built 'town houses' for use when the rural families attended church services. Regular gathering of people in these settings created informal markets where goods were exchanged or bartered. These markets attracted entrepreneurs and the establishment of enterprises followed. In this case, population growth preceded enterprise growth. However, over time, the presence of enterprises and the possibility of finding employment, attracted more people to the fledgling towns, thereby growing the local market.

Population growth and enterprise development, therefore, seem to proceed hand in hand. On the one side, an entrepreneur might start a new business and if it is successful, it contributes to employment, which enhances the image of the town as a place to find employment. On the other side, a new immigrant attracted to the town enhances its total disposable income, thereby increasing the ability of the town to 'carry' more enterprises. This interdependence probably causes the similarities of the population and enterprise distribution patterns. This fits in with Eeckhout's suggestion that agglomeration and residential mobility of the population between different geographical locations are tightly connected to economic activity.<sup>1</sup>

However, a better understanding of the links between population and enterprise growth in regional towns still does not explain fully why some towns grow large and others stay small. Krugman's<sup>3</sup> lament that there is not a good theory to account for the striking empirical regularities in population agglomeration patterns observed, which is now also the case for enterprise agglomeration patterns, still applies. There is a need to further explore these issues. The extreme skewness in population and enterprise numbers of different towns' distributions should, however, be considered in local economic development planning and execution.

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