A MODEL FOR QUALITY MANAGEMENT IN A SUPPLY CHAIN WITH A RETAILER AND A MANUFACTURER

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ABSTRACT

This paper presents a model to study quality management in a supply chain system with one manufacturer and one retailer. The manufacturer invests in production quality, and the retailer compensates to improve service quality. The model is analysed using game theory.

OPSOMMING

’n Toevoerketting waar gehaltebestuur aangewend word deur ’n enkele vervaardiger en ’n enkele verbruiker word gemodelleer via die speltheorie. Die vervaardiger en die verbruiker neem deel aan besluitneming oor optimum befondsing van die bestuursaksies.
1. INTRODUCTION

Quality has been defined as fitness for use, or the extent to which a product successfully serves the purpose of consumers [21]. Improving quality is an important factor in achieving competitive advantage for companies, and it is attended to extensively in today’s fast-paced and increasingly competitive market [39]. Various aspects of quality have been investigated, two of which are product quality and service quality. A product, and the service offered to customers who buy the product, must meet or exceed customers’ expectations. The emphasis on improving the quality of products and services has been increased by firms in reaction to enhanced competitive environments. In other words, product and service quality have been recognised as playing a crucial role in success and survival in today’s competitive market.

Papers dealing with production and service quality are briefly considered below.

Efficient mechanisms are investigated in research into service quality [18,19,31] in order to offer the best customer service. Retailers in a supply chain may be focusing on areas in their operations that might give them an advantage over their competitors. Therefore, investment in service quality is a way to enhance the efficiency of the entire supply chain. Several papers considering product quality deal with pay back-warranty contracts for the purpose of sharing costs caused by poor quality of various sections of the supply chain that can influence product quality positively or negatively [6,32,33,25,20,5,44,34,2,3,40,24]. Certain papers consider designing quality control processes. These papers present models consisting of determining batch size, order quantity, sampling size, randomly drawn from a lot, and critical value for accepting or rejecting the lot to minimise total cost [38,29,8,36]. Other papers study the relationship between quality and inventory control, and determine the lead time, order quantity, and probability of the production processes being out of control for the purpose of minimising the total aggregated cost of setup, ordering, adjustment of the production processes, and holding [14,43]. Some papers are related to the trade-off between price and quality for selection of a supplier by a customer [37,23,16]. More papers investigate the effect of price and quality on demand, and others how demand changes by variation of price, quality, brand diversity, and location of factory come about [7,28,10,17,22,26,12,15,27,11,9,42,35]. Finally there are papers that incorporate quality in designing supply chain networks [13,1,41,30].

As reviewed above, there is little work in the literature that considers the relationship between product and service quality in a supply chain. This motivates us to present a model to consider product quality and service quality in a supply chain with a manufacturer and a retailer. The manufacturer invests in production quality and the retailer compensates to improve the service quality. The model is analysed by game theory.

The paper is organised as follows. The model is presented in Section 2. Sections 3 and 4 give the Nash and Stackelberg equilibriums of the model. Section 5 of the paper presents some interesting managerial results.

2. MODELING

The gross profits of the retailer and the manufacturer \( \pi_r(a,q), \pi_m(a,q) \) are determined as:

\[
\begin{align*}
\pi_r(a,q) &= \rho_r S(a,q) - a \\
\pi_m(a,q) &= \rho_m S(a,q) - q
\end{align*}
\]

where the decision variables, parameters, and function \( S(a,q) \) are defined below.

(i) Decision variables:

\( a \) is the retailer’s investment in improving service quality.
\( q \) is the manufacturer’s investment in improving production quality.

(ii) Parameters:

- \( \rho_m \) is the manufacturer’s marginal profit for each unit to be sold, and is a positive constant.
- \( \rho_r \) is the retailer’s marginal profit for each unit to be sold, and is a positive constant.

(iii) Sale function:

\( S(a,q) \) is a single period sale function.

In the above model, the following sale function is considered:

\[
S(a,q) = a - \beta a^{-\gamma}q^{-\delta} , \quad a \geq a_0 \text{ and } q \geq q_0
\]

where the parameters are interpreted as follows:

- \( a \) is a positive constant and is the sale saturation asymptote. On the other hand, when either or both the production and service quality investments tend to infinity, \( S(a,q) \) tends to the constant \( a \).
- \( \gamma \) is a positive constant, which is the elasticity of the service quality.
- \( \delta \) is a positive constant, which is the elasticity of the production quality.
- \( \beta \) is a positive constant, and determines the impact of the production and service quality investments on market demand.
- \( a_0 \) is a lower bound for service quality, and is a non-negative number.
- \( q_0 \) is a lower bound for production quality, and is a non-negative number.

It should be noted that the parameters \( a_0 \) and \( q_0 \) somehow depend on each other - i.e. for every \( q \geq q_0 , a \) must be greater than \( a_0 \) such that \( S(a,q) \geq 0 \) for \( a \geq a_0 \) and \( q \geq q_0 \).

Considering

\[
S(a,q) \geq 0 \iff a \geq \left( \frac{\beta}{\alpha q} \right) \frac{1}{\gamma},
\]

we should have \( a \geq \overline{a}(q) \) with \( \overline{a}(q) = \left( \frac{\beta}{\alpha q} \right) \frac{1}{\gamma} \). The inequalities \( q \geq q_0 \) and \( a \geq \overline{a}(q) \) imply \( a \geq \sup_{q \geq q_0} \overline{a}(q) \), and by observing that \( \overline{a}(q) \) is non-decreasing with respect to \( q \) we obtain \( a \geq \overline{a}(q_0) \), which means that \( a_0 \) must satisfy \( a_0 \geq \overline{a}(q_0) \) to guarantee that \( S(a,q) \) is non-negative for \( a \geq a_0 \) and \( q \geq q_0 \). To preserve the full possible range of values for \( a \), we suppose \( a_0 = \overline{a}(q_0) \).

In the next two sections the model is analysed for two scenarios. The first scenario is that the manufacturer and the retailer act sequentially, where the manufacturer is the leader and the retailer is the follower. This scenario is studied in Section 3 by using game theory, and the related equilibrium point, called the Stackelberg equilibrium, is obtained. The second scenario is that the manufacturer and the retailer act simultaneously. This scenario is considered in Section 4, and the associated equilibrium point, called the Nash equilibrium, is obtained. In the rest of the paper, for simplicity, we frequently use the auxiliary parameters \( E,G,H,F \) defined below:
3. SEQUENTIAL MOVE

In this section we study the model for the first scenario that the manufacturer and the retailer sequentially move, where the manufacturer is leader and the retailer is follower. The leader first chooses the strategy \( q \), and the follower then observes this decision and makes his own strategy choice \( a \). In this scenario the model is analysed by obtaining the associated Stackelberg equilibrium point. To this end, we first find the optimal value of \( a \) which optimises the retailer gross profit, and then by substituting it in the manufacturer’s gross profit, we obtain the optimal value of \( q \).

By defining \( a_i(q) = \arg \max_{a>0} (\pi_r(a,q)) \), the Stackelberg equilibrium point can be found by solving the following problem:

\[
\max_q \pi_m = \pi_m(a_i(q), q) \\
\text{s.t.} \quad q \geq q_0
\]

The function \( \pi_r(a,q) = \rho_r S(a,q) - (\alpha - \beta a^{-\gamma} q^{-\delta}) a \) is concave for \( a \geq a(q_0) \), so we have:

\[
a_i(q) = \max \left\{ a(q_0), \frac{\gamma \rho_r \beta}{q^\gamma} \right\}.
\]

By considering \( a_i(q) \), the Stackelberg equilibrium point can be obtained by solving the following two problems:

\[
\max_q \pi_m = \rho_m \alpha - \rho_m \beta q^{-\delta} \left( \frac{\gamma \rho_r \beta}{q^\gamma} \right)^{\frac{1}{\gamma}} - q \\
\text{s.t.} \quad q \geq q_0 \quad \text{(Problem 1)}
\]

\[
\max_q \pi_m = \rho_m \alpha - \rho_m \beta q^{-\delta} \left( \frac{\gamma \rho_r \beta}{q^\gamma} \right)^{\frac{1}{\gamma}} - q \\
\text{s.t.} \quad q \geq q_0 \quad \text{(Problem 2)}
\]

By solving Problems 1 and 2, and comparing their optimal objective values, we find the Stackelberg equilibrium \( (q_i, a_i) \), which is presented in Table 1 under nine sets of conditions. In this table the following parameters are considered:
\[
L = \left(1 + \frac{\delta}{\rho_m \alpha q_0^2}\right)^{\frac{1}{\beta \delta}} \frac{\rho_m}{\rho_m \alpha q_0^2} \left(\frac{\beta}{\rho_m \alpha q_0^2}\right)^{\frac{1}{\beta \delta}} - q_o \left(\frac{\rho_r \beta}{H}\right)^{\frac{1}{\beta \delta}}
\]
\[
M = \frac{\rho_m}{\rho_m \alpha q_0^2} \left(\frac{\beta}{\rho_m \alpha q_0^2}\right)^{\frac{1}{\beta \delta}} - q_o \left(1 - \left(\frac{\rho_r \beta}{H}\right)^{\frac{1}{\beta \delta}}\right)
\]
\[
N = \frac{\rho_m}{\rho_m \alpha q_0^2} \left(\frac{\beta}{\rho_m \alpha q_0^2}\right)^{\frac{1}{\beta \delta}} + q_o \left(\frac{\rho_r \beta}{H}\right)^{\frac{1}{\beta \delta}} - \left(1 + \frac{\rho_m \delta}{1 + \gamma} \left(\rho_m \alpha q_0^2\right)^{\frac{1}{\beta \delta}}\right) \left(\rho_m \alpha q_0^2\right)^{\frac{1}{\beta \delta}}
\]

<table>
<thead>
<tr>
<th>#</th>
<th>Conditions</th>
<th>Stackelberg equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((1 + \gamma)^{1-\gamma} \rho_m^{1+\gamma} \rho_r^{-\gamma} &gt; E)</td>
<td>(a_s = (\rho_m \alpha q_0^2)^{\frac{1}{\beta \delta}})</td>
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<td></td>
<td>((1 + \gamma)^{1-\gamma} \rho_m^{1+\gamma} \rho_r^{-\gamma} = F)</td>
<td>(a_s = \frac{1}{\beta \delta} (\rho_m \alpha q_0^2)^{\frac{1}{\beta \delta}})</td>
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<tr>
<td></td>
<td>(\rho_r &gt; H)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(L \leq 0)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(\rho_r &gt; H)</td>
<td>(a_s = (\rho_m \alpha q_0^2)^{\frac{1}{\beta \delta}})</td>
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<td></td>
<td>((1 + \gamma)^{1-\gamma} \rho_m^{1+\gamma} \rho_r^{-\gamma} \rho_m^{-\delta} \leq F)</td>
<td>(a_s = \frac{1}{\beta \delta} (\rho_m \alpha q_0^2)^{\frac{1}{\beta \delta}})</td>
</tr>
<tr>
<td></td>
<td>(L \leq 0)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(\rho_r &gt; H)</td>
<td>(a_s = (\rho_m \alpha q_0^2)^{\frac{1}{\beta \delta}})</td>
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<tr>
<td></td>
<td>((1 + \gamma)^{1-\gamma} \rho_m^{1+\gamma} \rho_r^{-\gamma} \rho_m^{-\delta} \leq F)</td>
<td>(a_s = \frac{1}{\beta \delta} (\rho_m \alpha q_0^2)^{\frac{1}{\beta \delta}})</td>
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<tr>
<td></td>
<td>(L \geq 0)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(\rho_m \leq G)</td>
<td>(a_s = q_o)</td>
</tr>
<tr>
<td></td>
<td>(\rho_r &lt; H)</td>
<td>(a_s = \frac{1}{\beta \delta} (\rho_m \alpha q_0^2)^{\frac{1}{\beta \delta}})</td>
</tr>
<tr>
<td>5</td>
<td>((1 + \gamma)^{1-\gamma} \rho_m^{1+\gamma} \rho_r^{-\gamma} &gt; E)</td>
<td>(a_s = \frac{1}{\beta \delta} (\rho_m \alpha q_0^2)^{\frac{1}{\beta \delta}})</td>
</tr>
<tr>
<td></td>
<td>(\rho_r &gt; H)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(N \leq 0)</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>((1 + \gamma)^{1-\gamma} \rho_m^{1+\gamma} \rho_r^{-\gamma} &gt; E)</td>
<td>(a_s = \frac{1}{\beta \delta} (\rho_m \alpha q_0^2)^{\frac{1}{\beta \delta}})</td>
</tr>
<tr>
<td></td>
<td>(\rho_r &gt; H)</td>
<td></td>
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<tr>
<td></td>
<td>(N \leq 0)</td>
<td></td>
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<tr>
<td>7</td>
<td>((1 + \gamma)^{1-\gamma} \rho_m^{1+\gamma} \rho_r^{-\gamma} \rho_m^{-\delta} \leq E)</td>
<td>(a_s = \frac{1}{\beta \delta} (\rho_m \alpha q_0^2)^{\frac{1}{\beta \delta}})</td>
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<tr>
<td></td>
<td>(M \geq 0)</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>((1 + \gamma)^{1-\gamma} \rho_m^{1+\gamma} \rho_r^{-\gamma} \rho_m^{-\delta} \leq E)</td>
<td>(a_s = \frac{1}{\beta \delta} (\rho_m \alpha q_0^2)^{\frac{1}{\beta \delta}})</td>
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<td></td>
<td>(M \geq 0)</td>
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<tr>
<td>9</td>
<td>(\rho_m &gt; G)</td>
<td>(a_s = \frac{1}{\beta \delta} (\rho_m \alpha q_0^2)^{\frac{1}{\beta \delta}})</td>
</tr>
<tr>
<td></td>
<td>(\rho_r &lt; H)</td>
<td>(a_s = \frac{1}{\beta \delta} (\rho_m \alpha q_0^2)^{\frac{1}{\beta \delta}})</td>
</tr>
<tr>
<td></td>
<td>(\rho_r^{1+\gamma} \rho_m^{-\delta} \leq F)</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Stackelberg equilibrium point
4. SIMULTANEOUS MOVE

In this section the model is investigated for the second scenario that the manufacturer and the retailer act simultaneously. In this scenario we can analyse the model by obtaining the Nash equilibrium point for the model. By defining

$$a^*_r(q) = \arg\max_{a_r \in A_r} (\pi_r(a_r, q)),$$

$$a^*_m(q) = \arg\max_{a_m \in A_m} (\pi_m(q, a_m))$$

the Nash equilibrium point \((q^*_m, a^*_m)\) is the solution of the following system:

$$\begin{align*}
q^*_m &= a^*_r(q^*_m) \\
a^*_m(q) &= a^*_m(q^*_m) \\
a, q &\geq a_0, q \geq q_0
\end{align*}$$

The Nash equilibrium point, i.e. the solution of the above system, exists, and is given in Table 2 under four sets of conditions.

<table>
<thead>
<tr>
<th>#</th>
<th>Conditions</th>
<th>Nash equilibrium point</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\rho_r \leq H) (\rho_m \leq G)</td>
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<tr>
<td></td>
<td>(q^<em>_m = q_0) (a^</em>_r = \beta \frac{1}{(\gamma q_0^*)^{\frac{1}{\gamma}}})</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(\rho_m &gt; G) (\rho^+_m \rho^-_m \leq F)</td>
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<tr>
<td></td>
<td>(q^<em>_m = \left(\frac{(\gamma q_0^</em>)^{\frac{1}{\gamma}}}{\beta}\right)) (a^<em>_r = \beta \frac{1}{(\gamma q_0^</em>)^{\frac{1}{\gamma}}})</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(\rho_r &gt; H) (\rho^+_m \rho^-_m \leq E)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(q^<em>_m = q_0) (a^</em>_r = \left(\frac{(\gamma q_0^*)^{\frac{1}{\gamma}}}{\beta}\right))</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(\rho_r^+ \rho^-_m \leq F) (\rho^+_m \rho^-_m &gt; E)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(q^<em>_m = \left(\frac{(\gamma q_0^</em>)^{\frac{1}{\gamma}}}{\beta}\right)) (a^<em>_r = \left(\frac{(\gamma q_0^</em>)^{\frac{1}{\gamma}}}{\beta}\right))</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Nash equilibrium point

5. CONCLUSIONS AND COMMENTS

In Sections 3 and 4, the model of Section 2 is analysed by game theory for the two scenarios of sequential move and simultaneous move. Tables 1 and 2 respectively present the Stackelberg equilibrium point for the scenario of sequential move, and the Nash equilibrium point for the scenario of simultaneous move. These results help both manufacturer and retailer to choose suitable strategies for their investment values on product and service quality. Moreover, from these tables it may be seen how changing the parameters of the model affects the investment values in each scenario. For example, for the parameters \(\rho_m\) and \(\rho_r\), we have:
\[
\frac{\partial q_s^*}{\partial \rho_m} = 0, \quad \frac{\partial q_s^*}{\partial \rho_r} \geq 0
\]

\[
\frac{\partial a_s^*}{\partial \rho_r} \geq 0, \quad \frac{\partial q_s^*}{\partial \rho_r} \geq 0 \text{ (except for Case 6 in Table 1)}
\]

\[
\frac{\partial a_N^*}{\partial \rho_m} = 0, \quad \frac{\partial q_N^*}{\partial \rho_m} \geq 0
\]

\[
\frac{\partial a_N^*}{\partial \rho_r} \geq 0, \quad \frac{\partial q_N^*}{\partial \rho_r} \leq 0
\]

These show that the manufacturer’s investments in quality, i.e. \( q_m^* \) and \( q_i^* \), positively depend on its marginal profit \( \rho_m \), and that the retailer’s investments in service quality, i.e. \( a_m^* \) and \( a_i^* \), positively depend on its marginal profit \( \rho_r \). In other words, if they wish to increase their marginal profits, they cannot decrease their investments in quality. In addition, \( \rho_r \) affects the manufacturer’s investments, while \( \rho_m \) does not affect the retailer’s investments. This means that if the manufacturer decides to increase its marginal profit, there is no need for an increment in the retailer’s investment in service quality in both scenarios. However, if the retailer increases its marginal profit, in the scenario of simultaneous move the manufacturer should not increase its investment in production quality, while in the scenario of sequential move the retailer cannot decrease its investment except for Case 6 in Table 1.

6. REFERENCE


[38] Tapiero, C.S. 2007. Consumers risk and quality control in a collaborative supply


