GOAL PROGRAMMING MODEL FOR PRODUCTION PLANNING 
IN A TOOTHPASTE FACTORY 

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ABSTRACT 

The GP model was developed for production planning in a toothpaste factory. Two objectives were distinguished: minimization of processing cost, and maximization of the capacity utilization of production facilities. Two priority structures were used to explore the trade-off options. When processing cost minimization was assigned the first priority, the utilizations of Processing Plant 1 and Filling Machine 2 were 20.32\% and 0.18\% respectively. When capacity utilization was assigned first priority, the processing cost increased by 7.55\% but capacity utilization improved. The least utilized facility was Filling Machine 1 with a utilization of 43.85\%.

OPSOMMING 

Die GP-model is ontwikkel vir die beplanning van produksie van ‘n tandepastafabriek. Twee doelwitte is gestel vir die beplanning naamlik minimisering van produksiekoste, en maksimering van prosesbesetting. Wanneer minimisering van koste gebruik word as eerste prioriteit is die besettingswaardes van prosesseringaanleg 1 20.32\% en van vulmasjien 2 0.18\%. Wanneer prosesbesetting gebruik word as eerste prioriteit neem die proceskoste toe met 7.55\% en styg die prosesbesetting. Vulmasjien 1 se gevolglike besetting was 43.85\%.
1. INTRODUCTION

The problem that is the focus of this paper is the development of a model for material mix in a multi-stage multi-facility production system. This type of problem is often encountered in food, drug, and chemical industries. It has to do with the determination of the quantities of raw materials that will enter each facility at each stage of production, such that the cost of production, capacity under-utilization, etc. will be minimized. The solution of the model will give the material mix for each facility at each stage of production, such that the decision maker (DM) is presented with solutions that give the best possible compromise between the objectives.

The decision-making process utilized in this study considers two key objectives of the firm: (i) minimizing the total cost of production, and (ii) maximizing the capacity utilization of the production facilities. Since these objectives may be mutually incompatible, it may be impossible to optimize with respect to all the objectives (Sundraham, [1], Zanakis et al [2], and Zaloom, et al, [3]). So the decision process will concern itself with trying to find the best possible solution, given the existing conditions. The technique of goal programming (GP) was developed for just such a situation (Charnes and Cooper [4], Giannikos [5], Lawrence et al [6], Animesh Biswas et al [7], Esfandiari Bijan [8], Hadi Gokcen et al [9], and Baykasoghu A.I et al, [10]). GP enables an organization’s problem to be analyzed in terms of the separate and often conflicting objectives inherent in many real world decision problems.

2. THE GOAL PROGRAMMING (GP) APPROACH

The basic approach of GP is to establish a specific numeric value (aspiration level) for each objective, formulate an objective function for each objective, and then seek a solution that minimizes the weighted sum of the deviations of these objective functions from their respective target level. There are two cases of GP. One, called non-pre-emptive GP, is where all of the goals are of roughly comparable importance. The other, called pre-emptive GP, is where there is a hierarchy of priority levels for the goals, so that the goals of primary importance receive first priority attention, those of second priority receive second priority attention, and so forth (Hillier and Lieberman, [11]).

To use the GP procedure, it is necessary to consider the structural or technological constraints and objectives. For each constraint, possible deviations are stated, and for each objective a target level is set. The objective is to minimize the stated constraint deviations and variations from the target levels. The GP procedure provides a method for minimizing these deviations and for dealing with them in the rank order specified, while not violating the technological constraints.

3. PROCESS DESCRIPTION

The production process of toothpaste can be generally considered as composed of two major stages: (i) premix, and (ii) processing.

At the premix stage three major raw materials are mixed together in sealed mixing vessels (premix vessels) to avoid aeration of the paste. The raw materials for this
stage are water, glycerin, and carboxymethylcellulose (CMC). At the processing stage four other raw materials are added; *flavours*, abrasives, preservatives, and moisturizing agents (MA). The processing plant uses a highly effective vacuum mixer with a mixing and dispersing system, which can be used for each individual toothpaste formulation. Since mixing is done in sealed vessels, losses are negligible. The paste is pumped into a feed hopper of the filling machine. The major production facilities, with their corresponding maximum capacities and production cost coefficients, are shown in Table 1. The process flow diagram is presented in Figure 1.

4. **MATHEMATICAL MODEL OF THE PROBLEM**

4.1 **Assumptions of the model**

The process flow diagram of the factory under study in this paper is schematically depicted in figure 1 and the following assumptions are set to construct the mathematical model of the problem.

(i) A single product is produced but many raw materials are required. The raw material number is denoted by $i$, ($= 1, 2, 3... I$).

(ii) The production stages consist of work centres in which several machines that perform similar functions are located. The machine number is denoted by $j$, ($= 1, 2... J$). The work centres are sequenced in the production technological order. The stage number is denoted by $k$, ($= 1, 2, 3... K$).

(iii) Owing to the difference in the model and age of the machines, the unit production cost ($c_{jk}$) varies from machine to machine within a stage.

(iv) Stage $k$ immediately follows stage $k-1$. In-process inventory is not allowed, and losses during production are negligible.

(v) There is no limitation on the availability of raw materials.

4.2 **Notations**

- $x_{ijk}$: The quantity of the $i^{th}$ raw material fed into the $j^{th}$ facility of the $k^{th}$ stage of production
- $y_{jk}$: The quantity of intermediate product fed into the $j^{th}$ facility of the $k^{th}$ stage of production
- $c_{jk}$: The unit production cost of the $j^{th}$ facility of the $k^{th}$ stage of production

4.3 **Objectives of the model**

The two key objectives considered are:

(i) Minimization of the total sum of production costs

(ii) Maximization of the capacity utilization of the production facilities

4.3.1 *The cost minimization objective $Z_1$:*

Table 1 provides the production cost coefficients of the major production facilities. This is done by setting the unit variable cost of premix vessel 3 (PM3) equal to 1.0. Based on this, the other unit variable costs are then normalized with respect to this
coefficient. The indexing system is adopted for convenience in the mathematical and computer manipulations. The total production cost is the sum of the products of the unit variable costs and the quantity of material processed by each facility. The criterion is:

\[
\text{Minimize } Z_1 = 2(x_{111} + x_{211} + x_{311}) + 1.2(x_{121} + x_{221} + x_{321}) + (x_{131} + x_{231} + x_{331}) + \\
2(y_{12} + x_{412} + x_{512} + x_{612} + x_{712}) + 1.8(y_{22} + x_{422} + x_{522} + x_{622} + x_{722}) +
\]

simplified as:

\[
\text{Minimize } Z_1 = 2x_{111} + 2x_{211} + 2x_{311} + 1.2x_{121} + 1.2x_{221} + 1.2x_{321} + x_{131} + x_{231} + x_{331} + \\
2y_{12} + 2x_{412} + 2x_{512} + 2x_{612} + 2x_{712} + 1.8y_{22} + 1.8x_{422} + 1.8x_{522} + 1.8x_{622} + 1.8x_{722} + \\
1.4y_{32} + 1.4x_{432} + 1.4x_{532} + 1.4x_{632} + 1.4x_{732} + 1.6y_{42} + 1.6x_{442} + 1.6x_{542} + 1.6x_{642} + \\
1.6x_{742} + 0.3y_{13} + 0.45y_{23} + 0.20y_{33}
\]

or:

\[
\text{Maximize } Z_1 = -(2x_{111} + 2x_{211} + 2x_{311} + 1.2x_{121} + 1.2x_{221} + 1.2x_{321} + x_{131} + x_{231} + x_{331} + \\
2y_{12} + 2x_{412} + 2x_{512} + 2x_{612} + 2x_{712} + 1.8y_{22} + 1.8x_{422} + 1.8x_{522} + 1.8x_{622} + 1.8x_{722} + \\
1.4y_{32} + 1.4x_{432} + 1.4x_{532} + 1.4x_{632} + 1.4x_{732} + 1.6y_{42} + 1.6x_{442} + 1.6x_{542} + 1.6x_{642} + \\
1.6x_{742} + 0.3y_{13} + 0.45y_{23} + 0.20y_{33})
\]

\[4.3.2 \text{Maximization of capacity utilization } Z_2\]

The capacity utilization function is the summation of individual utilization factor (i.e. load divided by maximum capacity). See Table 1 and Figure 1.

\[
\text{Maximize } Z_2 = (x_{111} + x_{211} + x_{311})/9600 + (x_{121} + x_{221} + x_{321})/14400 + (x_{131} + x_{231} + x_{331})/24000 + \\
(y_{12} + x_{412} + x_{512} + x_{612} + x_{712})/25000 + (y_{22} + x_{422} + x_{522} + x_{622} + x_{722})/25000 + \\
(y_{32} + x_{432} + x_{532} + x_{632} + x_{732})/40000 + (y_{42} + x_{442} + x_{542} + x_{642} + x_{742})/30000 + \\
y_{13}/80000 + y_{23}/45000 + y_{33}/20000
\]

For convenience we multiply the capacity utilization function above by the capacity of Processing Plant 3 (pp3). The choice of PP3’s capacity for the normalization is arbitrary. The purpose is to make the mathematical and computer manipulations easy. The simplified objective function after normalizing is:

\[
\text{Maximize } Z_2 = 4.17x_{111} + 4.17x_{211} + 4.17x_{311} + 2.78x_{121} + 2.78x_{221} + 2.78x_{321} + \\
1.67x_{131} + 1.67x_{231} + 1.67x_{331} + 1.6y_{12} + 1.6y_{22} + 1.6x_{412} + 1.6x_{422} + 1.6x_{512} + 1.6x_{522} + \\
1.6x_{612} + 1.6x_{622} + 1.6x_{712} + 1.6x_{722} + y_{32} + x_{432} + x_{532} + x_{632} + x_{732} + \\
1.33y_{42} + 1.33x_{442} + 1.33x_{542} + 1.33x_{642} + 1.33x_{742} + 0.5y_{13} + 0.89y_{23} + 2y_{33}
\]
<table>
<thead>
<tr>
<th>Stage of production</th>
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<th>Normalized cost coefficient/kg of material processed</th>
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<td>Premix</td>
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<td>2.00</td>
</tr>
<tr>
<td></td>
<td>Premix Vessel 2 (PM2)</td>
<td>14400</td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td>Premix Vessel 3 (PM3)</td>
<td>24000</td>
<td>1.00</td>
</tr>
<tr>
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<td>Processing Plant 2 (PP2)</td>
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<td>1.80</td>
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<td>Processing Plant 3 (PP3)</td>
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<td>Processing Plant 4 (PP4)</td>
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<td>Filling Machine 2 (FM 2)</td>
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<td>0.45</td>
</tr>
<tr>
<td></td>
<td>Filling Machine 3 (FM 3)</td>
<td>20000</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Table 1: Major production facilities with corresponding capacities and cost coefficients

4.4 Constraints of the problem

In addition to the objectives associated with this problem, the model structure for this decision process will consist of the following constraint types:

(i) Available production capacity of each facility at each stage of production
(ii) Material proportion constraints
(iii) Material balance

Raw materials are in sufficient supply, hence there is no limitation on raw material availability.

4.4.1 Capacity constraint

The total amount of materials fed into a facility should not exceed the capacity of the facility.

\[
x_{111} + x_{211} + x_{311} \leq 9600 \quad (PM1) \tag{6}
\]
\[
x_{121} + x_{221} + x_{321} \leq 14400 \quad (PM2) \tag{7}
\]
\[ x_{131} + x_{231} + x_{331} \leq 24000 \quad \text{(PM3)} \quad (8) \]
\[ y_{12} + x_{412} + x_{512} + x_{612} + x_{712} \leq 25000 \quad \text{(PP1)} \quad (9) \]
\[ y_{22} + x_{422} + x_{522} + x_{622} + x_{722} \leq 25000 \quad \text{(PP2)} \quad (10) \]
\[ y_{32} + x_{432} + x_{532} + x_{632} + x_{732} \leq 40000 \quad \text{(PP3)} \quad (11) \]
\[ y_{32} + x_{432} + x_{532} + x_{632} + x_{732} \leq 30000 \quad \text{(PP4)} \quad (12) \]
\[ y_{13} \leq 80000 \quad \text{(FM1)} \quad (13) \]
\[ y_{23} \leq 45000 \quad \text{(FM2)} \quad (14) \]
\[ y_{33} \leq 20000 \quad \text{(FM3)} \quad (15) \]

It is the decision of management to utilize the full capacity of the factory. In the existing design of the factory, the premix stage is the production bottleneck. Thus the full capacity of the premix stage implies the full capacity of the factory.

\[ x_{111} + x_{211} + x_{311} + x_{121} + x_{221} + x_{321} + x_{131} + x_{231} + x_{331} = 48000 \quad (16) \]

### 4.4.2 Material proportion constraints

The raw materials required for Stage 1 (Premix stage) are carboxymethylcellulose, distilled water, and glycerin.

(i) Carboxymethylcellulose (CMC) constraint: It is required that the quantity of CMC fed into each of the premix vessels be 10% by weight of glycerin fed into that facility. For Premix Vessel 1:

\[ x_{111} / x_{311} = 0.1 \quad (17) \]

This constraint is not linear but can be linearized as follows:

\[ x_{111} - 0.1x_{311} = 0 \quad \text{(PM1)} \quad (18) \]

Similarly, CMC constraints for premix vessels 2 and 3 are:

\[ x_{121} - 0.1x_{321} = 0 \quad \text{(PM2)} \quad (19) \]
\[ x_{131} - 0.1x_{331} = 0 \quad \text{(PM3)} \quad (20) \]

(ii) Distilled water constraint: It is required that distilled water fed into each premix vessel be 130% by weight of glycerin fed into it. Following the same procedure as above, the distilled water constraint is given by:

\[ x_{211} - 1.3x_{311} = 0 \quad \text{(PM1)} \quad (21) \]
\[ x_{221} - 1.3x_{321} = 0 \quad \text{(PM2)} \quad (22) \]
\[ x_{231} - 0.1x_{331} = 0 \quad \text{(PM3)} \quad (23) \]

The raw materials needed for Stage 2 (the processing stage) are moisturizing agents, preservatives, abrasives, and flavours.
Moisturizing agent (MA) constraint: The quantities of MA fed into each of the processing plants of Stage 2 must be 6.25% by weight of intermediate product from the premix stage (Stage 1) fed into it.

Figure 1: Process flow diagram for toothpaste production

For Processing Plant 1 (PP1):
\[ x_{412} / y_{12} = 0.0625 \]
After linearization we get:
\[ x_{412} = 0.0625y_{12} = 0 \] (PP1) \[ (24) \]
Similarly, for Processing Plants 2, 3, and 4 we get:
\[ x_{422} = 0.0625y_{22} = 0 \] (PP2) \[ (26) \]
\[ x_{432} = 0.0625y_{32} = 0 \] (PP3) \[ (27) \]
\[ x_{442} = 0.0625y_{42} = 0 \] (PP4) \[ (28) \]
(iv) Preservatives constraints: It is required that the quantities of preservatives fed into each processing plant be 1.042% by weight of intermediate product from Stage 1 fed into each.

\[ x_{512} - 0.01042y_{12} = 0 \] \hspace{1cm} (PP1)  
\[ x_{522} - 0.01042y_{22} = 0 \] \hspace{1cm} (PP2)  
\[ x_{532} - 0.01042y_{32} = 0 \] \hspace{1cm} (PP3)  
\[ x_{542} - 0.01042y_{42} = 0 \] \hspace{1cm} (PP4)

(v) Abrasives constraints: The quantities of abrasives fed into each processing plant must be 96% by weight of the intermediate product from the premix stage fed into it.

\[ x_{612} - 0.96y_{12} = 0 \] \hspace{1cm} (PP1)  
\[ x_{622} - 0.96y_{22} = 0 \] \hspace{1cm} (PP2)  
\[ x_{632} - 0.96y_{32} = 0 \] \hspace{1cm} (PP3)  
\[ x_{642} - 0.96y_{42} = 0 \] \hspace{1cm} (PP4)

(vi) Flavour constraints: The quantity of flavour fed into a processing plant must be 5.21% by weight of the intermediate product from the premix stage.

\[ x_{712} - 0.0521y_{12} = 0 \] \hspace{1cm} (PP1)  
\[ x_{722} - 0.0521y_{22} = 0 \] \hspace{1cm} (PP2)  
\[ x_{732} - 0.0521y_{32} = 0 \] \hspace{1cm} (PP3)  
\[ x_{742} - 0.0521y_{42} = 0 \] \hspace{1cm} (PP4)

4.4.3 Material Balance Constraints

Since loses are negligible, the quantity of material fed into the premix stage is equal to the output of that stage. Also, in-process inventory is not allowed, so all the output of the premix stage is fed into the facilities of the processing stage. Figure 1, the process flow diagram, shows the materials inputted to each facility. The junctions are introduced for convenience and to help clarify the process flow diagram. With the aid of Figure 1, the material balance constraints can now be formulated.

Material balance at Junction a:

\[ x_{111} + x_{211} + x_{311} + x_{121} + x_{221} + x_{321} + x_{131} + x_{231} + x_{331} = y_{12} + y_{22} + y_{32} + y_{42} \] \hspace{1cm} (41)

simplified as:

\[ x_{111} + x_{211} + x_{311} + x_{121} + x_{221} + x_{321} + x_{131} + x_{231} + x_{331} = y_{12} + y_{22} + y_{32} + y_{42} = 0 \] \hspace{1cm} (42)

Material balance at Junction b:

\[ y_{12} + x_{412} + x_{512} + x_{612} + x_{712} + y_{22} + x_{422} + x_{522} + x_{622} + x_{722} + y_{32} + x_{432} + x_{532} + x_{632} + x_{732} - y_{13} - y_{23} - y_{33} = 0 \] \hspace{1cm} (43)
5. MODEL APPLICATION

5.1 Aspiration level

The ideal solutions of the cost minimization and maximum capacity utilization objectives were determined individually before the statement of goals. This is because a priori determination of goals could be too difficult or too arbitrary without a prior exploration of potentials provided by the two objectives. If the goals are too low, a suboptimal and even dominated solution might be computed. The optimum value of the cost objective, $Z_1^*$ was taken as the target level for the cost goal, and that of the capacity utilization objective $Z_2^*$ was set as the target level for the maximum capacity utilization goal.

5.2 Pre-emptive GP

We consider two cases of pre-emptive GP with two different priority structures:

Case 1: Cost minimization was the overriding objective, so the following priorities were established.

$P_1$ (Priority 1): Minimize the deviation from the minimum cost goal
$P_2$ (Priority 2): Minimize the deviation from the maximum capacity utilization goal

Case 2: Capacity maximization was the overriding objective

$P_1$ (Priority 1): Minimize the deviation from the maximum capacity utilization goal
$P_2$ (Priority 2): Minimize the deviation from the minimum cost goal

The GP model is given as:

Minimize $a = g_1(d_1^+), g_2(d_2^-)$

subject to:

$2x_{111} + 2x_{211} + 2x_{311} + 1.2x_{121} + 1.2x_{221} + 1.2x_{321} + x_{131} + x_{231} + x_{331} + 2y_{12} + 2x_{412}$
$+ 2x_{512} + 2x_{612} + 2x_{712} + 1.8y_{22} + 1.8x_{422} + 1.8x_{522} + 1.8x_{622} + 1.8x_{722} + 1.4y_{32} + 1.4x_{432} + 1.4x_{532} + 1.4x_{632} + 1.4x_{732} + 1.6y_{42} + 1.6x_{442} + 1.6x_{542} + 1.6x_{642} + 1.6x_{742} + 0.3y_{13} + 0.45y_{23} + 0.20y_{33} - d_1^+ = Z_1^*$ (cost goal constraint) (44)

$4.17x_{111} + 4.17x_{211} + 4.17x_{311} + 2.78x_{121} + 2.78x_{221} + 2.78x_{321} + 1.67x_{x_{131}} + 1.67x_{x_{231}} + 1.6x_{x_{311}} + 1.6x_{x_{512}} + 1.6x_{x_{612}} + 1.6x_{x_{712}} + 1.6y_{x_{22}} + 1.6x_{x_{422}} + 1.6x_{x_{522}} + 1.6x_{x_{622}} + 1.6x_{x_{722}} + 1.6y_{x_{32}} + 1.33x_{x_{432}} + 1.33x_{x_{532}} + 1.33x_{x_{632}} + 1.33x_{x_{732}} + 0.5y_{x_{13}} + 0.89y_{x_{23}} + 2y_{x_{33}} + d_2^- = Z_2^*$ (capacity utilization goal constraint)
plus the structural constraints:

(Production facilities capacity constraints)
(Material proportion constraints)
(Material balance constraints)

where \( g_1, g_2 \) are goals one and two respectively

\[ d_1^+, d_2^- \] are deviations from goals one and two respectively

6. DISCUSSION OF RESULTS

The results of the model are presented in Tables 2 and 3. A corollary summary is given in Table 4. The allocation of materials to production facilities in the premix stage is the same for the two cases. The premix stage proves to be the bottleneck in the whole process. This is shown by the 100% utilization of all premix vessels (Table 4). When cost minimization was given first priority, the total processing cost (normalized) was \( \$247,678.40 \). The value of the deviational variable \( d_1^+ = 0 \) shows that the cost goal was achieved, while \( d_2^- = 29420 \) shows that the capacity utilization of production facilities was underachieved. The capacity utilization goal was underachieved by 8.2%. The utilization of Processing Plant 1 (PP1) and Filling Machine 2 (FM 2) are 20.32% and 0.18% respectively. FM2 is almost idle. It is justifiable to comment that the design of the production system is far below optimal, since the full utilization of the capacity of the premix stage corresponds to serious under-utilization of PP1 and FM2. This poor system design is not only technically unhealthy, it reduces the economic efficiency of the facility and the production system as a whole.

When capacity utilization of production facilities was assigned first priority, the capacity utilization goal was achieved \( (d_2^- = 0) \). The value \( d_1^+ = 18689 \) indicates that the cost goal was overachieved by \( \$18,689 \). This translates into an increase of 7.55% in the processing cost. However, the utilization of the least-utilized facility was 43.85% as against 0.18% when cost minimization was assigned first priority.

It should be emphasized that any model is a means to an end, not an end in itself. The decision maker (DM) will be helped by the model to decide which particular production plan to select. As the result of the GP model is fed back to the DM, additional inputs - such as goals, priorities, etc. - are created. In doing so the DM alters the model results by considering factors that were not initially included in the model.
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<thead>
<tr>
<th>Stage</th>
<th>Raw material</th>
<th>Monthly allocation to facility (kg)</th>
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<tbody>
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<td>Premix</td>
<td></td>
<td>Premix Vessel 1</td>
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<tr>
<td></td>
<td>CMC</td>
<td>400</td>
</tr>
<tr>
<td></td>
<td>Water</td>
<td>5200</td>
</tr>
<tr>
<td></td>
<td>Glycerin</td>
<td>4000</td>
</tr>
<tr>
<td>Processing</td>
<td>Processing Plant 1</td>
<td>Processing Plant 2</td>
</tr>
<tr>
<td>Intermediate product from stage 1</td>
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<td>Moisturizing Agent</td>
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<tr>
<td>Preservatives</td>
<td>25.4</td>
<td>124.0</td>
</tr>
<tr>
<td>Abrasives</td>
<td>2339.5</td>
<td>11510.4</td>
</tr>
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<td>Flavour</td>
<td>127.0</td>
<td>624.7</td>
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<td>Paste</td>
<td>80000</td>
<td>81</td>
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Table 2: Monthly material allocation to production facilities when the cost minimizing goal was assigned first priority

<table>
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<tr>
<th>Stage</th>
<th>Raw material</th>
<th>Monthly allocation to facility (kg)</th>
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<td>Flavour</td>
<td>624.7</td>
<td>624.7</td>
</tr>
<tr>
<td>Paste</td>
<td>35081</td>
<td>45000</td>
</tr>
</tbody>
</table>

Table 3: Monthly material allocation to production facilities when the maximization of capacity utilization goal was assigned first priority

207
<table>
<thead>
<tr>
<th>Facility name</th>
<th>Cost goal as first priority</th>
<th>Capacity utilization goal as first priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premix Vessel 1</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Premix Vessel 2</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Premix Vessel 3</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Processing Plant 1</td>
<td>20.32</td>
<td>100</td>
</tr>
<tr>
<td>Processing Plant 2</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Processing Plant 3</td>
<td>100</td>
<td>50.20</td>
</tr>
<tr>
<td>Processing Plant 4</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Filling Machine 1</td>
<td>100</td>
<td>43.85</td>
</tr>
<tr>
<td>Filling Machine 2</td>
<td>0.18</td>
<td>100</td>
</tr>
<tr>
<td>Filling Machine 3</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>(d_1^* = 0)</td>
<td></td>
<td>(d_1^* = 18689)</td>
</tr>
<tr>
<td>(d_2^* = 29420)</td>
<td>(d_2^* = 0)</td>
<td></td>
</tr>
</tbody>
</table>

\[
\text{Cost} = 247678.4 \quad \text{Cost} = 266367.68 \text{ (7.55\% increase)}
\]

Table 4: Summary of the percentage utilization of production facilities with associated costs

7. CONCLUSION

The model has served its purpose of determining the material mix for each facility at each stage of production. The exploration of the various trade-off options was made by using different priorities to generate the two possible solutions. The model offers the decision maker flexibility, since the conflicting objectives can be simultaneously considered.

REFERENCES


