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Implementing the flipped classroom model in mathematics class teaching

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The aim with this research was to examine how the implementation of the flipped classroom model (FCM) impacted the quality and durability of 4th grade students' knowledge in the field of measuring and measures when compared to traditional teaching (TT). The research was conducted in Serbia on a sample of 135 students, divided into 3 groups. The 3 groups were a control group (C) which was taught mathematics content using the TT model, an experimental group 1 (E1) which was taught using the FCM, while the students in experimental group 2 (E2) were taught using the FCM – the teacher also provided them with feedback regarding their work and made recommendations for their further development. The results show that the students from the E2 group surpassed those from the C and E1 groups regarding the quality and durability of their knowledge in the field of measuring and measures. There was a statistically significant difference in the knowledge of students in this field at the lower cognitive level among all the groups, while the statistically significant difference at the higher cognitive level was seen between the C and the E2 and between the E1 and the E2 groups.

Keywords: class teaching of mathematics; cognitive levels; flipped classroom model; quasi-experiment with parallel groups

Introduction

Students acquire the basis of mathematical knowledge in lower grades of primary school, therefore the way in which they adopt mathematical notions and the cognitive processes that initiate and develop during the learning process are of extreme importance. The traditional teaching (TT) of mathematics develops skills, procedures, and formulas in students, while the goal of the modern teaching of mathematics is to create mathematical opinion and to implement such knowledge in solving real and everyday problems (Wong & Kaur, 2015). TT is based on the transfer of the teaching content to the student who is listening, remembering and reproducing the teaching content. Typical disadvantages of TT are lecturing, students' passive role in the teaching process and poor interaction between the teacher and the student. Various theories of teaching and learning have developed as a response to the impairments of TT and the changes in society, and have, in turn, led to various modern teaching models. One such model is the flipped classroom model (FCM). This teaching model encourages students to learn independently and to be responsible, while the teaching process is directed towards students' needs and towards the development of their potential (Casem, 2016).

When searching for the model of teaching that would overcome the disadvantages of TT and create more time for the interaction between a teacher and his students, Bergman and Sams (2012) developed the FCM which refers to the independent preparation and learning at home, before the lesson, with the aid of information technologies, followed by class time where the content is reflected upon, discussed and analysed, and tasks are done (Bergman & Sams, 2012). The domain of mathematics teaching leaves open certain issues that refer to the implementation of FCM into class teaching in primary school. With this paper we provide answers to the following questions: "Are students aged 10/11 years old trained to learn independently using digital technologies?; Does the feedback regarding their work impact the improvement of their achievement?; Is the content on measuring and measures suitable for learning through the FCM?"

Literature Review

A large amount of research (Love, Hodge, Corritore & Ernst, 2015; Reidsema, Kavanagh, Hadgraft & Smith, 2017; Strelan, Osborn & Palmer, 2020) exists in which the influence of the flipped classroom model on students' achievements in higher education, in secondary and primary school was examined, but relatively little research has been done on examining the implementation of the FCM in mathematics in primary schools (D'addato & Miller, 2016; Lai & Hwang, 2016; Yang, Lin & Hwang, 2021). Bergman and Sams (2012) conducted research in teaching chemistry in secondary school, which encouraged other teachers and researchers to implement this teaching model and to experiment with it (Bergman & Sams, 2012). The results of their research show that the students liked the FCM as it was closely related to their mode of thinking and communicating; encouraged them to take responsibility for their own learning, and made it possible for them to work at the pace that suited them best (Bergman & Sams, 2012).

Meta-analyses of the FCM research conducted in various fields at higher, secondary, and primary education levels show that the students who were taught through the FCM achieved better knowledge results than the students who were taught through the TT; most of the research that applied the FCM was done at faculties, followed by secondary schools, while the smallest number of such researches studies was done in primary schools (Hew & Lo, 2018; Lo, Hew & Chen, 2017; Strelan et al., 2020; Van Alten, Phielix, Janssen & Kester, 2019; Yang et al., 2021). Strelan et al. (2020) conducted the meta-analysis of 198 types of research in which the FCM was applied at various education levels in various fields. They obtained the following results: the FCM effect rates were closely related when the students were working in groups (g = 0.46) or individually (g = 0.59), while the most effective were the teaching activities aimed at students; the FCM effect was higher among secondary school students (g = 0.64; k = 21) than at higher education (g = 0.48; k = 174). The FCM was most effective in the disciplines that used an active learning approach: humanities (g = 0.98; k = 34), teaching (g = 0.75; k = 7) and engineering (g = 0.72; k = 15), while the lower effect was shown in mathematics (g = 0.35; k = 46) and management (g = 0.38, k = 10). When doing meta-analysis of the works that used the FCM in teaching mathematics, Lo et al. (2017) stated the following three greatest advantages of the FCM: teacher's feedback, peer learning, and more time in the classroom for implementing various students' activities (Lo et al., 2017).

The FCM research in mathematics from 2012 to 2014 was in its initial stage, while from 2015 onward the researchers started to conduct studies and to implement this model more (Yang et al., 2021), mostly in higher education. Relevant literature on FCM shows a small number of papers that focus on the implementation of the FCM in secondary and elementary school in teaching mathematics. The research in secondary schools was mostly about the influence of the FCM on students' achievements (Balaban, Gilleskie & Tran, 2016; Bhagat, Chang & Chang, 2016; Clark, 2015; Katsa, Sergis & Sampson, 2016; Kirvan, Rakes & Zamora, 2015; Lo, Lie & Hew, 2018), on students' motivation (Balaban et al., 2016; Bhagat et al., 2016; Katsa et al., 2016), on students' attitudes and self-confidence (Casem, 2016), and on students' engagement and communication (Clark, 2015; Katsa et al., 2016).

D'addato and Miller (2016) did action research on the FCM among primary school students from poor areas. The FCM brought positive changes regarding students' motivation and responsibility, their activity, and engagement, but also regarding their parents' attitudes towards this model (D'addato & Miller, 2016). Lai and Hwang (2016) applied the FCM in two fourth grade groups in teaching mathematics. This research concluded that self-regulation and self-controlled strategies within the FCM could improve and develop students' self-efficacy, their strategies, planning, and the use of time, which led to more efficient and successful learning and higher achievements than in the conventional approach to the FCM (Lai & Hwang, 2016).

The FCM has been adopted by many countries around the world (Hwang., Yin & Chu, 2019; Yang et al., 2021). No research in this field existed in Serbia except for a few papers in which the FCM was mentioned as an innovative teaching model (Ivanović, 2018; Teofilović & Isailović, 2020). A large number of researchers in the world have shown that the FCM has positive effects on students' achievements (Lo & Hew, 2017). There is a need for further research in implementing the FCM in mathematics in lower grades of primary schools in order to evaluate whether the specific contents are applicable for gaining knowledge by this model at this age (10/11 years) and to establish the way in which feedback and teachers' recommendations will affect students' achievements in future.

Theoretical Framework

The FCM learning means the distribution of learning activities, which enhances the students' active role in the education process (Abeysekera & Dawson, 2015; Katsa et al., 2016), which contributes to the activation of higher cognitive levels (Flumerfelt & Green, 2013), where the teacher's role lies in encouraging students to think, in channelling the discussion among them and in providing them with feedback and advice in order for them to reach quality self-learning (Casem, 2016; Hwang, Lai & Wang, 2015; Van Alten et al., 2019). The FCM facilitates learning, it is also effective, and motivates students using differentiation of instruction during the learning process (Cheng & Weng, 2017). The implementation of the FCM, when compared to the TT, requires more time for preparing the teaching process and for preparing the teaching material (Bergmann & Sams, 2012; D'addato & Miller, 2016; Hwang et al., 2015). In order for a teacher to organise the teaching process in a quality way, it is necessary for the teacher to have a thorough knowledge of the goals and the expected outcomes of the education process, which have been defined by various cognitive stages.

On the grounds of the complexity of the educational achievements, Benjamin Bloom and David Krathwohl (1956) have classified the cognitive domain of knowledge adoption into six categories, from the lowest to the highest: knowledge, understanding, implementation, analysis, synthesis, and evaluation (Bloom &

Krathwohl, 1956). Anderson and Krathwohl (2001) have revised Bloom's taxonomy in the way that they swapped the places of the two last cognitive levels, the synthesis, and the evaluation, and introduced the dimension of cognitive processes that they formulated through verbs. The lower cognitive level comprises the following: knowledge – recognise, remember; understanding – differentiate, calculate, list, compare; implementation – connect, choose, develop. The higher cognitive level comprises analysis – differentiate, segment, organise; evaluation – evaluate, conclude, segment; synthesis – design, construct, develop.

In the FCM students are introduced to the content before they are in the class; in that way they activate the processes from the lower cognitive level, such as knowledge and understanding, while in the classroom the students initiate the process of implementation, which is the highest level of the lower cognitive domain and the processes of analysis, evaluation (Francl, 2014) and the synthesis from the higher cognitive domain (Love et al., 2015; Marshall & DeCapua, 2013). Research in education indicate that active learning initiates students' mental activities, leads to a higher level thinking process, and improves learning outcomes (White, 2011). This teaching model is available to students since it is implemented using information technologies that students are interested in and closely related to. The implementation of the teaching process by the FCM does not require curricula to be remodelled, as is required when project and integrative teaching is implemented. It neither requires additional material funding.

Methodology

The main goal of the research was to examine the impact of the FCM implementation on the quality and durability of the knowledge among fourth grade primary school students regarding measuring and measures when compared to the TT. We also examined the following: the effect of the feedback and teachers' recommendations for further achievement or, in other words, the difference between the FCM with and without the feedback and the recommendations for further achievement on the quality and durability of students' knowledge.

Due to the specific conditions of organising the teaching process in the Republic of Serbia in the 2020/2021 school year (coronavirus disease [COVID-19]), this research has been adapted to the specific conditions and has been carried out respecting all the epidemiological measures. The six classes that participated in this research consisted of more than 15 students, and at the beginning of the school year, each class was subdivided into two subgroups, A and B, where each subgroup was working as a separate class. The classes lasted 30 minutes. The subgroups consisted of 11 to 14 students. Within each subgroup, it was neither allowed to further divide students into smaller groups nor to organise group work. The research sample consisted of 135 fourth grade students from three primary schools in the territory of the Autonomous Province of Vojvodina (the Republic of Serbia). The final grade in mathematics at the end of the third grade and the pre-test in mathematics on students' previous knowledge in measuring and measures (in the first three classes) served as the criterion for forming three equal groups: a control group (C), an experimental group (E1) and an experimental group (E2). Each group consisted of 45 students, and we randomly appointed the C, E1, and E2 groups. The teaching content on measuring and measures was implemented during 2 weeks, or during the 8 working days, in the following teaching units: comparing surface area; surface area of figures; units of measurement for the areas smaller than m^2; and units of measurement for the areas larger than m^2. The experimental part of the programme was implemented using the contents of the unit, measuring and measures, and was done in eight classes with each subgroup – four classes were used for teaching new content and four classes were used to review work. We applied the quasi-experiment with parallel groups. All the subgroups were taught the same content and were given the same mathematical tasks. The classes were presented by the researcher. The control group (two classes – four subgroups) was taught using the traditional method, the experimental group E1 (two classes – four subgroups) was taught using the FCM without any feedback (FCMWAF), while the experimental group E2 (two classes-four subgroups) was taught using the FCM with feedback and recommendations for further achievement (FCMWFR).

In the Republic of Serbia there are no standardised tests that examine the quality and durability of knowledge regarding the contents of measuring and measures. Therefore the pre-test, the post-test, and the re-test were created for the purpose of this research, modelled on Bloom's revised taxonomy (Anderson, Krathwohl & Bloom, 2001). Each test consisted of six tasks of various levels of complexity: three tasks were on the lower cognitive level (knowledge, understanding, applying) and three tasks on the higher cognitive levels (analysing, evaluating, synthesising). The tasks that measured the quality of knowledge at higher cognitive levels carried a higher score than the tasks that measured the quality of knowledge at the lower cognitive levels. The post-test and the re-test checked students' knowledge regarding the contents studied during the experimental part of the programme. The structure and the content of tasks in the post-test and the re-test were similar. The re-test was done following the 5-week period of the post-test.

Some tasks from the post-test of the lower cognitive level domain:

Knowledge level

Add:

$1cm^2$ = _______mm^2; $400mm^2$ = _______cm^2; $1dm^2$ = _______cm^2; 3,000a = _______ha; 3a = _______m^2; 4,800ha = _______km^2; $3km^2$= _______ha; $4{,}000{,}000m^2$ = _______km^2.

Implementation level

The surface area of a room is $25m^2$, while the surface area of a carpet is $900dm^2$. What is the surface area of the room that is not covered with the carpet?

Answer:_______________________________________

During the development classes in the control group, I presented the students with the basic contents and information and introduced them to the basic notions and simple tasks, while students did the tasks regarding the introduced material at home. The review classes consisted of revising the basic notions and tasks. Students in the experimental E1 and E2 groups received lessons through the FCM and were provided with the material (presentations, videos or quizzes) on Google or Edmodo platforms 1 day ahead. The students' homework was to view the posted material and then to do the quiz (Kahoot). At school, the students discussed the material with me and the other students, exchanged opinions and performed tasks in which they implemented the knowledge gained through Google or Edmodo classrooms. At the beginning of the lesson, I gave the students in the E2 group feedback information regarding the quiz that they had completed, in other words, the information about what they did or did not acquire and understand. I also discussed possible dilemmas and gave them individual recommendations regarding further studies of the material and the parts of the presentation or the video that they should revise. After I clarified all the dilemmas and provided additional explanations regarding the material to the E2 group, the students completed the tasks.

The following is a description of the material and of the examples of some of the tasks done within the teaching unit, units of measurement for the areas larger than m^2.

Before the class, the material for online teaching was a PowerPoint presentation and a Kahoot quiz. The PowerPoint presentation consisted of eight slides that listed and explained the measures for an area larger than m^2: acre, hectare and square kilometre. The Kahoot quiz consisted of 10 tasks, seven of them were two-choice tasks and the remaining three were three-choice multiple choice tasks. The time provided for the tasks was 60 seconds. Every task had an equal number of points.

Some of the tasks from the Kahoot quiz:

1) An acre (a) is the area of a square with sides of the following length:
   a) 1m b) 100m c) 10cm d) 10m.

2) The equation 40ha = $4km^2$ is: a) correct b) incorrect.

In direct teaching, the students were doing the tasks and discussing them. The following are some of the examples of the tasks that were done during the class:

1) Order the given areas from the smallest to the largest:
   $800m^2$, 7a, $1km^2$, 4ha, $30{,}000m^2$, 400ha, 500a.
2) Make the following calculations:
   500a + $43{,}000dm^2$ = ________; 7ha + $420{,}000m^2$ = ________; $500km^2$ - 200ha = ________.

Results

On the basis of the single-factor analysis of variance it was concluded that there was no statistically significant difference between the average grade in mathematics at the end of the third grade among students in the C, E1 and E2 groups (p = .796, p > .05), while the Kruskall-Wallis test showed that there was no statistically significant difference in the knowledge of students in the C, E1 and E2 groups at the pre-test (p = .913, p > .05), neither at the lower (p = .863, p > .05) nor at the higher (p = .439, p > .05) cognitive level.

The highest score at the lower cognitive level in the pre-test was achieved by pupils in the C group, while the highest score at the higher cognitive level was achieved by the pupils in the E1 group (cf. Table 1).

Table 1 Total score and the mean score value for all the groups in the pre-test at all cognitive levels

	Groups							
	C		E1		E2		All groups	
Level	Sum of points (S)	*M*	S	*M*	S	*M*	S	*M*
	Lower cognitive level							
Knowledge	259	5.76	220	4.89	209	4.64	688	5.10
Understanding	322	7.16	289	6.42	286	6.36	897	6.64
Implementation	124	2.76	162	3.60	180	4.00	466	3.45
Total	705	15.68	671	14.91	675	15.00	2,051	15.19
	Higher cognitive level							
Analysis	306	6.80	341	7.58	332	7.38	979	7.25
Evaluation	148	3.29	162	3.60	148	3.29	458	3.39
Synthesis	90	2.00	104	2.31	54	1.20	248	1.84
Total	544	12.09	607	13.49	534	11.87	1,685	12.48

The Spearman correlation coefficient was used and it showed the mean correlation between the students' final grades in mathematics at the end of the third grade and the total score that the students had achieved in the pre-test (p = .406, p > .01).

Since all the tests (the pre-test, post-test and re-test) consisted of six tasks, they were in the group of tests with a small number of tasks (less than 10); therefore the reliability of the test was examined using the mean value correlation between the items. All three tests showed reliability (cf. Table 2) since the mean value scale between the items in each of the tests was within the intervals of 0.2 to 0.4 (Briggs & Cheek, 1986).

Table 2 Mean value correlation between the items in the pre-test, post-test and re-test

Test	Mean value correlation	Minimum	Maximum
Pre-test	.308	.117	.534
Post-test	.365	.252	.441
Re-test	.399	.187	.515

Students in all groups achieved the highest score in the post-test, while the students in the C group achieved the lowest score in the re-test. Students in the E1 and E2 groups achieved the lowest score in the pre-test (cf. Table 3).

Table 3 Total score of students of all groups (C, E1 and E2) in the pre-test, post-test and re-test

	Group		
Test	C	E1	E2
Pre-test	1.249	1.278	1.209
Post-test	1.326	1.577	2.006
Re-test	1.202	1.474	1.881

Single factor variance analysis showed that there was a statistically significant difference in the quality of knowledge of students in the post-test between the C and the E2 groups as well as between the E1 and the E2 groups. The analysis also showed that there was no statistically significant difference in the quality of knowledge of students in the C and the E1 groups (cf. Table 4). The difference among the groups' mean value was high; it is expressed by the eta squared indicator and was 0.15. The comparison using Tukey's HSD shows that the mean E2 group value (M = 44.58, SD = 14.55) was significantly different from the mean C group value (M = 29.47, SD = 14.34) and the E1 group value (M = 35.04, SD = 15.42). There was no statistically significant difference in the quality of knowledge of students in the re-test between the C and the E1 groups, but there was a statistically significant difference in the quality of knowledge of students between the C and the E2 groups, but also between the E1 and the E2 groups (cf. Table 3). The mean value difference between the groups was high (Cohen, 1988) and the difference range is expressed by the eta squared indicator, which was 0.17. The comparison using Tukey's HSD shows that the mean value of the E2 group (M = 41.8, SD = 13.69) was significantly different from the mean value of the C group (M = 29.71, SD = 12.62) and the E1 group (M = 32.76, SD = 14.47).

Table 4 Statistically significant difference in the knowledge quality and durability of students in the C, E1 and E2 group in the post-test and the re-test

Cognitive level	(I)Group	(J)Group	Mean difference	*SE*	*p*	95% CI Lower Bound	95% CI Upper Bound
			Post-test				
Lower cognitive level	C	E1	-3.711*	.994	.001	-6.07	-1.35
		E2	-6.911*	.994	.000	-9.27	-4.55
	E1	C	3.711*	.994	.001	1.35	6.07
		E2	-3.200*	.994	.005	-5.56	-.84
	E2	C	6.911*	.994	.000	4.55	9.27
		E1	3.200*	.994	.005	.84	5.56
Higher cognitive level	C	E1	-1.867	2.439	.725	-7.65	3.91
		E2	-8.200*	2.439	.003	-13.98	-2.42
	E1	C	1.867	2.439	.725	-3.91	7.65
		E2	-6.333*	2.439	.028	-12.11	-.55
	E2	C	8.200*	2.439	.003	2.42	13.98
		E1	6.333*	2.439	.028	.55	12.11
All levels	C	E1	-5.578	3.136	.181	-13.01	1.86
		E2	-15.111*	3.136	.000	-22.54	-7.68
	E1	C	5.578	3.136	.181	-1.86	13.01
		E2	-9.533*	3.136	.008	-16.97	-2.10
	E2	C	15.111*	3.136	.000	7.68	22.54
		E1	9.533*	3.136	.008	2.10	16.97
			Re-test				
Lower cognitive level	C	E1	-2.600*	.901	.013	-4.74	-.46
		E2	-5.956*	.901	.000	-8.09	-3.82
	E1	C	2.600*	.901	.013	.46	4.74
		E2	-3.356*	.901	.001	-5.49	-1.22
	E2	C	5.956*	.901	.000	3.82	8.09
		E1	3.356*	.901	.001	1.22	5.49
Higher cognitive level	C	E1	-3.444	2.233	.275	-8.74	1.85
		E2	-9.133*	2.233	.000	-14.43	-3.84
	E1	C	3.444	2.233	.275	-1.85	8.74
		E2	-5.689*	2.233	.032	-10.98	-.40
	E2	C	9.133*	2.233	.000	3.84	14.43
		E1	5.689*	2.233	.032	.40	10.98
All levels	C	E1	-6.044	2.870	.092	-12.85	.76
		E2	-15.089*	2.870	.000	-21.89	-8.29
	E1	C	6.044	2.870	.092	-.76	12.85
		E2	-9.044*	2.870	.006	-15.85	-2.24
	E2	C	15.089*	2.870	.000	8.29	21.89
		E1	9.044*	2.870	.006	2.24	15.85

Note. *p = < 0.5.

Single factor variance analysis has shown that there was a statistically significant difference in the post-test and the re-test in the knowledge at the lower cognitive level among all groups, while at the higher cognitive level there was a statistically significant difference in knowledge between the C and the E2 groups, and between the E1 and the E2 groups (cf. Table 4).

The students in the C group achieved the highest score in the pre-test, a lower score in the post-test and the lowest score in the re-test (cf. Table 2). The results of the Friedman's test show that there was a statistically significant difference in the students' knowledge in the pre-test, the post-test and in the re-test of students in the C group (p = .000 with $p < .001$). Students in the E1 group achieved the highest score in the post-test, lower in the re-test and the lowest score in the pre-test. The E2 students achieved the highest score in the post-test, lower in the re-test and exhibited the lowest score in the pre-test (cf. Table 2). The single factor variance analysis of the repeated measuring was used to compare the results in the pre-test, the post-test and the re-test for the E1 and the E2 groups. Table 5 shows their mean value and the standard deviation. A significant influence of intervention (time) was exhibited (Cohen, 1988) by the E1 group, the Wilks's lambda = .541 F (2. 43) = 17.63, $p < .001$, multivariant partial eta squared = .451, and the students in the E2 group showed the Wilks's lambda =.223 F (2. 43) = 75.07, $p < .001$, multivariant partial eta squared = .777.

Table 5 The descriptive statistical indicator o (the students' knowledge in the post-test and the re-test for the E1 and the E2 groups

	Test		
	Pre-test	Post-test	Re-test
Group		E1	
M	28.40	35.04	32.76
SD	13.224	15.421	14.467
Group		E2	
M	26.87	44.58	41.80
SD	12.731	14.550	13.689

Discussion

The pre-test results show that students in all three groups had more or less equal prior knowledge on measuring and measures and that there was a positive correlation between the final grades in mathematics at the end of the third grade and the score that the students had achieved in the pre-test. The students of all three groups showed mostly balanced knowledge at all cognitive levels in the pre-test, but they had better results in the tasks of the lower cognitive level. Teachers said that the prior knowledge on measuring and measures was gained through traditional teaching methods, which might possibly be the reason for the lower results in the pre-test at higher cognitive levels. The traditional teaching method primarily activates lower cognitive levels such as memorising and understanding (Love et al., 2015; Marshall & DeCapua, 2013).

The E2 students, who were taught using the FCMWFR, achieved a higher score in the post-test and in the re-test compared to the C and the E1 groups, and this difference was of statistical difference; it can, therefore, be concluded that the implementation of this model has impacted the quality and durability of knowledge among the E2 students in the field of measuring and measures. The difference in the quality and durability of the knowledge of students who were taught through the FCMWAF and the FCMWFR implies the significance of the feedback that the students had received from their teachers and of the researcher's recommendations to focus on the parts of the contents that the students re-studied. In a number of papers the FCM was implemented using a formative grading scale at the beginning of the teaching process in the classroom, which impacted the higher achievements in mathematics (Lo & Hew, 2020; Lo et al., 2017).

The students in the E1 group achieved a higher score in both the post-test and the re-test than the C group, but the difference was statistically insignificant. It can, therefore, be concluded that the E1 students, who acquired the knowledge on measuring and measures through the FCMWAF, achieved better quality and durability of the knowledge than the C group students who were taught through TT. This also proves that FCMWAF learning was more efficient than traditional teaching, but the difference in knowledge quality and durability was not statistically relevant. The students in the E1 and E2 groups, who were learning through the FCM, viewed the material and then did the quiz in order to practice and to check their knowledge, which means that it is possible that this factor affected the higher quality and durability of the knowledge of students in these groups. Previous research shows that quizzes have a positive influence on learning outcomes due to the test effect (Dirkx, Kester & Kirschner, 2014; Roediger & Karpicke, 2006), and they also provide teachers with information on the possible difficulties that students encounter while learning independently. It serves as the basis for the teacher to guide the students in their development and to adapt the teaching process to their needs (Abeysekera & Dawson, 2015).

Student's knowledge in the post-test and the re-test at the lower cognitive level among all the groups was statistically significant. The students in the E2 group achieved the highest score, followed by students in the E1 group, whereas the lowest score was achieved by students in the C group. The implementation of the FCM affected the quality and durability of knowledge at lower cognitive levels among students who were taught through the FCM, therefore, it can be concluded that the implementation of this model positively affected the development of knowledge, understanding, and implementation among the students in the two groups (E1 and E2).

The students in the E1 group achieved the highest score in the post-test and the lowest score in the pre-test; this difference was statistically relevant and lead to the conclusion that the implementation of the FCMWAF significantly affected the knowledge quality and durability among the students in this group. The FCM efficiently encouraged students' thinking process in mathematics and raised their self-esteem and engagement (Palmer, 2015).

The students in the E2 group exhibited better knowledge in the post-test and the re-test in the higher cognitive levels compared to the students in C and E1 groups. As the students in E2 group achieved better knowledge results in both the higher and the lower levels in the post-test and the re-test than the students in the E1 group, it can be concluded that the feedback and the recommendations for further development affected the quality and the durability of knowledge in lower and higher cognitive levels among students in the E2 group. Students often need explaining why their answers are wrong in order to avoid making them in the future, but students were rarely provided with such feedback (Woolfolk, Hughes & Walkup, 2013). The students who were taught through the TT model scored the lowest in the re-test; this can be related to the lack of direct guidelines for their homework, which can lead to cognitive overburdening of students in traditional teaching and prevent them from storing the knowledge in their long-term memory (Kirschner, Sweller & Clark, 2006).

Conclusion

The research results show that the use of the FCM positively affected the quality and durability of students' knowledge, particularly the knowledge of those students who were taught measuring and measures through the FCMWFR compared to traditional teaching. The students in the experimental groups (FCM) were guided to learn at home and could read through the uploaded material several times when it was most convenient for them. In the classroom, after having reviewed the contents at home and since there was a small number of them in the groups, the teacher (researcher) had more time for interaction with each student and had more time for individual discussions, which all affected the knowledge and durability of students' knowledge in this field. A few research studies on the impact of the FCM in the teaching of mathematics in lower grades of primary school imply that the implementation of this model has not been analysed sufficiently. Due to the positive effects of other research, including our study, it would be advisable to continue with further analyses and to encourage other researchers to research this topic. Research into the effects of the implementation of the FCM in the teaching of mathematics in lower grades through peer learning, collaborative learning, group learning and team learning would be of great significance. Although the implementation of the FCM led to higher students' achievements, it should be noted that the students were exposed to the novelty effect during a short period (2 weeks). The use of information technologies, new learning approaches, and the new teacher may have affected students' motivation and their higher achievements (Kuykendall, Janvier, Kempton & Brown, 2012). Such possible effects could be neutralised by dedicating more time to such experiments (Bhagat et al., 2016; Clark, 2015) or through action research (Mazur, Brown & Jacobsen, 2015).

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Authors' Contributions

All authors contributed equally to the conceptualisation of the manuscript; SO and MM created the research methodology; SO and AP conducted the data collection; SO and MM contributed to the writing of the original draft of the manuscript; SO and MM wrote, reviewed and edited; SO and AP conducted statistical analyses; SO and MM contributed to visualisation; MM and AP conducted validation and supervision. All authors reviewed the final version of manuscript.

Note

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