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International comparisons of Foundation Phase number domain mathematics knowledge and practice standards

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Poor mathematics performance in schools is both a national and an international concern. Teachers ought to be equipped with relevant subject matter knowledge and pedagogical content knowledge as one way to address this problem. However, no mathematics knowledge and practice standards have as yet been defined for the preparation of Foundation Phase student teachers in South Africa. To make recommendations for the drafting of such standards for final year Foundation Phase teachers, we compared different policy documents. We performed a document analysis on policy documents from South Africa, The Netherlands, Australia and North Carolina (United States of America), all of which addressed the number domain in mathematics. Our findings indicate that knowledge standards ought to include subject matter knowledge, while practice standards require pedagogical content knowledge, noting that neither of these are fulfilled in the education system in South Africa at present.

Key words: foundation phase; knowledge and practice standards; mathematical knowledge; number domain; pedagogical content knowledge; policy documents; subject matter knowledge

Introduction

Developing competence at all levels of schooling starts early in a learner's life, and is essential for the 21st century. Internationally, different countries took different steps to increase their learners' and teachers' competence and knowledge levels in all subjects. One of these steps involves a movement aimed at developing professional standards for teachers in order to enhance the quality of their preparation, and to promote their life-long learning (Australian Institute for Teaching and School Leadership, 2011). The consequences of this movement are evident in the following: in Ohio, a standards-reform took place in 2004, which led to the defining of standards for teachers and principals for all levels of their career stages, including that of the student teacher (Ohio Department of Education & Educator Standards Board, 2007). In The Netherlands, the drawing up of standards for student teachers is a relatively new undertaking that started in 2008 (Otten, 2009). Furthermore, the African Development Bank Group (2013) points out that a project for the drawing up of standards exists only in Botswana, which makes South Africa part of those countries noted as being without knowledge and practice standards.

In South Africa, the low results obtained in the Annual National Assessments (Department of Basic Education [DBE], 2012) indicate that learners in the Foundation Phase have low competence levels in both mathematics and literacy. According to Jansen (2011), one of the factors that contribute to this state of affairs is teachers' deficits in knowledge, and therefore effective intervention in respect of teacher knowledge is needed.

Mathematics knowledge and practice standards are statements about the knowledge and skills that – in this specific case – a final-year Baccalaureus Educationis (BEd) Foundation Phase student teacher (hereafter referred to as a student teacher) must know, and must be able to apply when entering the teaching profession (Department of Basic Education & Department of Higher Education and Training [DBE & DHET], 2011). These statements are linked to a specific subject or school phase, but are not associated with specific school curriculum statements. In fact, they relate to academic and practical knowledge that is needed to teach a specific subject, and that will allow the student teacher to adapt to potential future curriculum changes (DBE & DHET, 2011).

No specific knowledge or practice standards are defined as guidelines for the development of programmes for the preparation of teachers' overall phases and subjects in South Africa (DBE & DHET, 2011). The lack of standards means that each of the local universities that offer Foundation Phase degree training develops its own curriculum for the preparation of Foundation Phase teachers in mathematics. This state of affairs is problematic, because it implies that not all Foundation Phase teachers are equally well prepared, and that their training may not be of the same quality.

The question that arises is as to what ought to be included in these standards for South African student teachers. In The Netherlands, it was acknowledged that to focus only on subject matter knowledge in the preparation of teachers is not enough (Otten, 2009). This is confirmed by the DBE and DHET (2011) and by Wilson, Floden and Ferrini-Mundy (2001), that all agree that subject matter knowledge and pedagogical content knowledge are important in the preparation of teachers.

In this study, we explored the description of (i.e. what should be included in) knowledge and practice standards from a policy viewpoint. We accepted that subject matter knowledge informs knowledge standards, and that pedagogical content knowledge informs practice standards for South African Foundation Phase teachers, who will be teaching mathematics. Our findings set the scene for the development of a working draft for knowledge and practice standards.

We were guided by the research question, "How can mathematics knowledge and practice standards for the preparation of Foundation Phase teachers be developed from a national and international policy perspective?" The research methodology that we applied to answer this question was a conceptual study. Its aim was to analyse and compare two international countries (Netherlands, Australia), and one US state, namely North Carolina, in terms of knowledge and practice standards, alongside South African policy documents. The findings were integrated in order to set the scene for the development of draft mathematics knowledge and practice standards for the South African Foundation Phase mathematics teacher. These comparisons are only recommendations for further research, and not guidelines in themselves. This article reports on part of a bigger study, where mathematics knowledge and practice standards have been drafted (Human, 2014).

Themes that emerged from the analyses and comparison of school curriculum documents included number sense, explaining answers, reasoning, mental calculations, money, problem solving, place value, fractions, operations and calculations, and general strategies during calculation. According to the standards for teacher education in the countries mentioned above, student teachers should not only hold knowledge about the school curriculum, but should also be familiar with the structure of numbers. They should furthermore know how to reason using numbers during calculations, and should know how to teach this skill effectively to their students.

To the best of our knowledge, this is the first time that these countries' school curriculums and standards for teacher preparation in mathematics have been analysed and compared with one another. Our analysis and comparison provide the basis for our recommendations in respect of the drafting of mathematics knowledge and practice standards in South Africa, and such recommendations are described in detail in the discussion and conclusion at the end of this article. Although we are aware that some authors might view the recommendations as guidelines, this is not the aim of this study. Since the development of standards for all subjects and phases is a grave necessity in South Africa, the methods used to arrive at the recommendations might also be incorporated in the drawing up of standards for the other content areas of mathematics, as well as for other subjects in the different phases (DBE & DHET, 2011).

In the following paragraphs, the background and national policy documents are first explained in more detail, since they provide the backdrop to this study. The different components of the conceptual framework for this study (which is provided next) are social constructivism, mathematics education ideologies and mathematical knowledge for teaching, all of which set the foundation for answering the research question and fulfilling the aim of the study.

Background and National Policy Documents

Since the 1990s, political movements have had an impact on the development of the school curriculum in South Africa (Graven, 2002; Jansen, 1999). Before the political upheavals of the 1990s, the curriculum was seen as a syllabus – a narrow view of curriculum (Graham-Jolly, 2009) – and the teaching approach was behaviouristic in nature (Hackman, 2004). In the 1990s, the curriculum policy debate underwent a critical change, which led to the adoption and development of Outcomes-based Education (Jansen, 1999). This meant that the role of the teacher changed to that of a facilitator of learning, where learners were newly required to be actively engaged with learning in a social context in which they had to construct their own knowledge from experience (Hackman, 2004). This change implicitly influenced the preparation of teachers, especially with regard to the knowledge and practice standards needed for teaching as applicable to this discussion.

According to the Council on Higher Education (CHE) (2011), standards for qualifications have been developed in the higher education sphere, but the different institutions define, interpret and implement these standards in different ways. In the past, 'standards' referred to criteria for admission to a qualification and the maintenance of a staff-student teacher ratio that is appropriate for the effective teaching, assessment and measurement of hierarchical positions of student teachers (CHE, 2011). The CHE argued that the most reliable way of obtaining equality of standards was to introduce a system of national and/or international examinations (CHE, 2011). Such standards should always be valid and reliable, and they should have a general applicability to provide guidelines for the development, implementation and quality assurance of educational programmes (Department of Education [DoE], 2007).

As was the case in the USA (Stykes, 1999), a shortage of teachers compelled tertiary institutions in South Africa to lower the standards that would qualify for admission of teacher-students, so as to increase the number of potential teachers (DBE & DHET, 2011). Hence, the development of knowledge and practice standards is one possible step towards improving the quality of education. The mission of standards is to protect learners from harm (i.e. by not being subjected to ineffective or low-quality education) and to equip teachers to meet the public's expectations (not only to know mathematics, but also to be able to teach it) (Stykes, 1999). Teachers, who have been educated well, perform better in the classroom than those whose training did not prepare them adequately for the task (Ball, Thames & Phelps, 2008; Roth, 1996).

One of the priorities of teacher preparation is to enhance the capacity and competency of student teachers to ensure high-quality education in the school system (DBE & DHET, 2011). The need for intervention in Foundation Phase education is confirmed by:

- the poor performance of Grade (Gr) 3 learners in the 2010 Annual National Assessments when they scored an average mark of 28% for numeracy (DBE, 2011b);
- the poor performance of Grade 3 learners, where in 2011 only 17% of learners achieved at least 50% and in 2012 only 37% learners achieved at least 50% for numeracy (DBE, 2012);
- the fact that teachers often make the same mistakes that learners make (Ryan & Williams, 2007); and
- student teachers showing a gap in their mathematical knowledge when they enrol for further study, since they discontinued specialising in mathematical subjects after the age of 16 years (Goulding, Rowland & Barber, 2002).

The DBE and DHET (2011:4) add that although "…a wide variety of factors interact to impact on the quality of the education system in South Africa, teachers' poor subject matter knowledge and pedagogical content knowledge are important contributors". The *Integrated Strategic Planning Framework for Teacher Education and Development in South Africa 2011-2025* identifies several factors, one of which is teacher preparation, that focuses specifically on subject matter knowledge and pedagogical content knowledge (DBE & DHET, 2011).

In the following section, we discuss the conceptual theoretical framework that provides the foundation for the analysis of the documents. This framework pays attention to the importance and integrated nature of both subject matter knowledge and pedagogical content knowledge in the preparation of student teachers.

Conceptual Theoretical Framework

The three concepts that constitute the conceptual theoretical framework of this study include:

- social constructivism (Ernest, 1998; Kim, 2001; Oldfather, West, White & Wilmarth, 1999);
- the mathematics education ideologies (Ernest, 1991); and
- mathematical knowledge for teaching (Ball et al., 2008).

The relationships between these three concepts are illustrated in Figure 1, and this is followed by discussions in which each of these concepts receives focus. At the end of these discussions, the relationships between the three concepts are explicated.

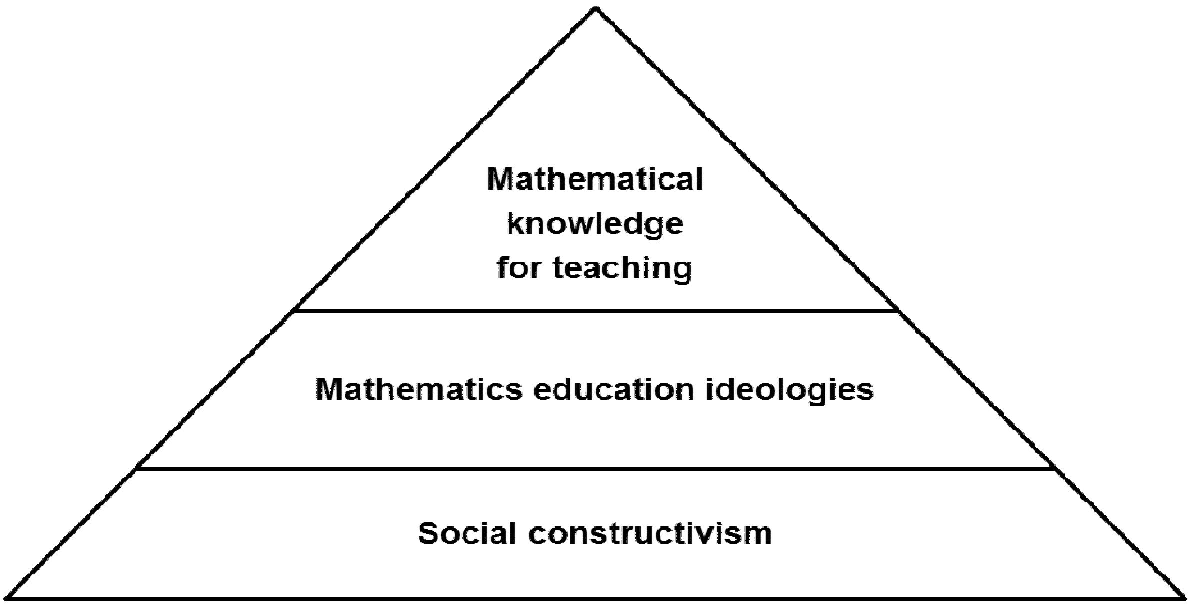

Figure 1 Conceptual theoretical framework (Ball et al., 2008; Ernest, 1991)

Social constructivism

Social constructivism is based on specific assumptions of reality, learning, knowledge (Kim, 2001) and the way in which knowledge is constructed (Cooperstein & Kocevar-Weidinger, 2004). A unique attribute of social constructivism is that learning is seen as the central unavoidable part of the philosophy of mathematics (Ernest, 1998). Knowledge is the outcome of mutual social interactions between people/learners in a social setting, where culture and context are important factors in understanding (Kim, 2001) and where they/learners take responsibility for their own learning (Cooperstein & Kocevar-Weidinger, 2004; Oldfather et al., 1999). Education involves both the mastering of specific knowledge and skills, and the development of the learner's abilities (Dolya, 2010). Furthermore, according to Oldfather et al. (1999), the teacher views learning from the learner's perspective.

Mathematics education ideologies

Different philosophies of mathematics have different influences on the education practice, and this is also the case with regard to mathematical education ideologies (Ernest, 1991). Student teachers would most likely adhere to their own mathematics ideologies, but a public orientation towards mathematics ideologies exists (Ernest, 1991), which influences the practice of mathematics in classrooms. The public orientation would most likely describe the desired education practices in the country.

In this study, mathematical education ideologies form part of the conceptual framework, because they provide the direction in which student teachers ought to be equipped in order to fulfill the DBE (or public) expectation of mathematics education in South Africa. Ernest (1991) describes five mathematics education ideologies, but for this study I will focus only on the two that are relevant in South Africa. Firstly, the progressive educator ideology, where the process of the learner gaining knowledge of mathematical truth, is evaluated (Ernest, 1991). Secondly, I will focus on the public educator, where the philosophy of mathematical knowledge is seen as social constructivism (Ernest, 1991).

A comparison of these ideologies with the South African *Curriculum and Assessment Policy Statements* (CAPS) (DBE, 2011a) appears in Table 1.

Table 1 A comparison of mathematics education ideologies and CAPS

Social group	Progressive educator	Public educator	CAPS (SA curriculum)
View of mathematics	Process view Personalised mathematics Language and human activity	Social constructivism	Unique language Human activity Socially constructing of mathematical ideas and concepts
Theory of the child	Child-centred Progressive view Child viewed as a growing flower and innocent savage	Social conditions view the child as 'clay moulded by environment' and 'sleeping giant'	Learner-centred Promote holistic development Progression from one grade to the next Social conditions
View of ability	Abilities vary but need cherishing		Differentiated activities according to each learner's ability
Mathematical aims	Creativity, self-realisation through mathematics (child-centred)	Critical awareness and democratic citizenship via mathematics	Self-realisation through mathematics Confidence and competence to handle any mathematics situation Creative activity Critical awareness of the role of mathematics in society, environments, cultures and economics
Theory of learning	Activity, play, exploration	Questioning, decision making, negotiation	Play, develop understanding of number and numeracy Interactive Do, speak, demonstrate Develop mathematical thinking
Theory of teaching mathematics	Facilitating personal exploration, preventing failure	Discussion, conflict, questioning of content and pedagogy	Integrated approach Learn through play Facilitator of learning Group work Discussions
Theory of resources	Rich environment to explore		Rich environment with many resources
Theory of assessment in mathematics	Teacher-led internal assessment, avoiding failure	Various modes Use of social issues and content	Various methods Teacher-led internal assessment Grade 3 external assessment

Source: DBE, 2011a; DHET, 2011; Ernest, 1991

Based on Table 1 it seems that the CAPS focus on the mathematics education ideologies of both the progressive and public educator. Although the CAPS have been implemented since 2011 (DBE, 2011a), the intended and the implemented curricula may well differ. These curriculum ideologies and the CAPS nevertheless indicate what kind of teacher is needed in the Foundation Phase classroom in South Africa. Table 1 indicates the mathematics education ideologies on a school level, which create an expectation of how mathematics should be taught in the classroom, but the education ideologies are not descriptive enough in terms of the mathematical knowledge for teaching needed by the student teacher. Therefore, mathematical knowledge for teaching is required in order to describe the expected mathematical knowledge at a student teacher level.

Mathematical knowledge for teaching

Hill, Rowan and Ball (2005) define mathematical knowledge for teaching as the mathematical knowledge that the teacher applies during teaching. Ball et al. (2008) identify two domains of mathematical knowledge for teaching, namely subject matter knowledge, and pedagogical content knowledge (Figure 1). The domain subject matter knowledge consists of three categories, namely: 1) common content knowledge; 2) knowledge at the mathematical horizon; and 3) specialised content knowledge (Ball et al., 2008). Pedagogical content knowledge, on the other hand, entails three categories, namely: 1) knowledge of content and students (learners); 2) knowledge of content and teaching; and 3) knowledge of the curriculum.

Goulding et al. (2002) note that categories within mathematical knowledge for teaching are blurred, because they can be distinguished but not separated. These categories will now be defined, seeing that such definition provides the criteria for making recommendations for knowledge and practice standards.

Common content knowledge refers to mathematical knowledge that people use in their daily lives, the ability to know whether a learner's answer is correct or incorrect and why, and the ability to understand the definition of mathematical concepts (e.g. operations) (Ball et al., 2008; Hill & Ball, 2009). Knowledge at the mathematical horizon refers to the vision to position mathematical concepts on the mathematical horizon and to know how concepts that the teacher imparts at a certain stage relate to broader mathematical ideas, structures and principles (e.g. addition and place value) (Ball & Bass, 2009; Ball et al., 2008). Specialised content knowledge refers to detailed knowledge that people in other professions do not use in their daily lives or occupations. It includes the use of presentations, relationships between symbols and picture representations; how to give a mathematical explanation and how to provide alternative solutions to problems (e.g. representing a number using the symbol, word, picture /diagram or graph) (Ball et al., 2008; Hill & Ball, 2009; Hill et al., 2005).

Knowledge of content and students (learners) refers to the knowledge the teacher should have about the typical mistakes that learners make and how learners at a specific age construct knowledge (Ball & Bass, 2009). Knowledge of content and teaching refers to knowledge of the sequences that the teacher uses to introduce a new concept or method to learners of a specific age group (Ball & Bass, 2009). Knowledge of the curriculum refers to educational aims that the teacher pursues, as well as the policy documents that are set up by government (Ball & Bass, 2009).

Relationship between three concepts in the conceptual theoretical framework

In Figure 1, Social constructivism is illustrated, as the foundation for the conceptual theoretical framework, and serves as the epistemological lens for the conceptual theoretical framework. The mathematics education ideologies (see Figure 1) indicate what the DBE (2011a) expects of learners, and therefore how student teachers ought to facilitate mathematical practice. At school level the expectations (see Table 1) of learners are amongst others to engage in mathematics as a human activity in a social environment, to learn the unique language of mathematics. This expectation is in line with the progressive educator, as well as public educator ideologies. In Figure 1, the final level of the conceptual theoretical framework is the mathematical knowledge for teaching. Mathematical knowledge for teaching builds on the expectations at a school level. The student teacher should not only know mathematics (subject matter knowledge), but should also know how to teach (pedagogical content knowledge) mathematics (Ball et al., 2008).

Research Methods

The research design adopted for this study can be described as a qualitative conceptual study based on an interpretivistic research paradigm. Policy documents were purposefully collected, content analysis was employed and results were compared (Nieuwenhuis, 2007b).

National and International Documents

A comparison was drawn between the South African CAPS (DBE, 2011a) and specific international documents. An international analysis made sense, since changes in demographic conditions and shortages of teachers in specific areas have led to teachers moving around between countries to teach (Townsend & Bates, 2007). Increased globalisation has inspired the need for quality teacher training programmes, prescribed the type of teacher that will be needed in the future (Townsend & Bates, 2007) and caused the comparison of countries' educational achievements (Jansen, 2007). Two such international comparisons are the *Trends in International Mathematics and Science Studies* (TIMSS) (Mullis, Martin, Foy & Arora, 2012) and the *Learning Curve Lessons in Country Performance in Education* (LCLCPE) (The Economist Intelligence Unit [EIU], 2012). South Africa participated in the TIMSS, but not in the LCLCPE. A remarkable finding in a study by the Human Sciences Research Council (HSRC) (2012) was that the performance of the most proficient learners in South Africa in TIMSS 2011 came close to the averages of learners in Singapore, Chinese Taipei, the Republic of Korea, Japan, Finland, Slovenia and the Russian Federation – the top performing countries in the TIMSS. The unfortunate truth was, however, that on average, South Africa's learners came a disappointing second last in TIMSS 2011 (Mullis et al., 2012).

For our study, we have selected two countries and one state located in the USA, which had participated in these studies, namely: The Netherlands (ranked 12th in TIMSS and 7th in the LCLCPE); North Carolina (USA) (ranked 11th in TIMSS and 17th in the LCLCPE); and Australia (ranked 19th in TIMSS and 13th in the LCLCPE). In what follows, an explanation is provided of the reasons why each of these countries was selected.

The Netherlands was selected because this article reports on a study that is part of a bigger project in the South Africa Netherlands Research Programme on Alternative Development (SANPAD). Furthermore, The Netherlands is part of the European Union, which funds the project known as *Developing Scientific Evidence-based Knowledge and Practice Standards for Teacher Preparation Programmes: A Focus on Literacy and Numeracy in English, Setswana and Afrikaans.* The teacher preparation standards in The Netherlands are also more clear than both those of North Carolina (USA) teacher preparation and Australian teacher standards. Documents from The Netherlands that were analysed included: *Kennisbasis rekenen-wiskunde voor de pabo* [Knowledge base in mathematics for the undergraduate teacher] (Otten, 2009) and *Kerndoelen rekenen/wiskunde* [Core goals for mathematics] (Buijs, Klep & Noteboom, 2009).

The USA can be compared to South Africa in various relevant ways. For example, in both countries the educational system is the object of criticism, and it is difficult to attract and keep quality teachers (Bantwini & King-McKenzie, 2011; Jansen, 2007). The USA also played a role in the development of the school curriculum in South Africa (Bantwini & King-McKenzie, 2011) in that North Carolina was one of the states (USA) that took part in TIMSS and incorporated the Common Core State Standards (Accountability and Curriculum Reform Effort [ACRE], n.d.; Mullis et al., 2012). The USA documents that were analysed were the *Teacher Education Specialty Area Standards* (North Carolina State Board of Education [NCSBE], 2009) and the *Common Core State Standards for Mathematics* (Common Core State Standards Initiative [CCSSI], n.d.).

The decision to include Australia in our study stemmed from the fact that the Australian curriculum influenced the development of Outcomes-Based Education in South Africa during the curriculum reform of the 1990s (Jansen, 1999) and thereafter. Two Australian documents were analysed: *Standards for Excellence in Teaching Mathematics in Australian Schools* (The Australian Association of Mathematics Teachers [AAMT], 2006) and *The Australian curriculum: Mathematics* (Australian Curriculum Assessment and Reporting Authority [ACARA], n.d.).

Two more reasons for choosing these documents were language accessibility and the availability of standards, as not all countries have standards for teacher education compiled.

Data Analysis Procedures

We first compared school curriculum documents with regard to the number domain for Grades 1 to 3 learners from South Africa (DBE, 2011a), The Netherlands (Buijs et al., 2009), North Carolina (USA) (CCSSI, n.d.) and Australia (ACARA, n.d.). These documents were presented in table format to determine similarities and differences (Nieuwenhuis, 2007a). Through open coding, we identified categories and themes on similar content that learners should know about, understand and be able to apply (Nieuwenhuis, 2007a).

We then compared teacher standards for Australia (AAMT, 2006), student teacher standards for North Carolina (USA) (NCSBE, 2009), and student teacher standards for The Netherlands (Otten, 2009). The 'mathematical knowledge for teaching' model of Ball et al. (2008) proposed the themes, and by using open coding, we searched for anything relevant that would fit under these themes.

Table 2 A comparison of the number domain requirements of school curriculum documents for Mathematics in Grades 1-3

Theme	South Africa (DBE, 2011a)	USA (CCSSI, n.d.)	Australia (ACARA, n.d.)	Netherlands (Buijs et al., 2009)
Number sense	0-1,000 (Gr 3)	0-1,000 (Gr 2)	0-10,000 (Gr 3)	0-100,000 (Gr 4)
Explain answers and reasoning	Not explicitly indicated	Explicitly indicated	Explicitly indicated	Explicitly indicated
Mental calculations	Recalling facts	Higher order	Higher order – develop strategies	Higher order – develop strategies
Money	Know, value and do problem solving	Only in Gr 2	Do calculations, know other countries' currencies	Describe value of money
Problem solving	Problem-solving techniques and problems in context	Practice standard that should be incorporated in the content standards	Integral part of curriculum	Core standard that develops reasoning skills
Place value	Understand 0-1,000	Understand 0–1,000 (Gr 2)	Understand 0–100,000	Understand the structure of numbers
Fractions	Recognise and name fractions	Recognise, name, show on number line and reason about the size of fractions	Understand and interpret fractions; Understand fractions as a result of division	Understand structure, ratio of fractions, know equivalent fractions and fractions in real-life situations
Operations and calculations	Use the four operations during calculations with numbers 0–1,000	Use the four operations during calculations with numbers 0–1,000; Develop calculation strategies	Use the four operations during calculations with numbers 0–10,000	Use the four operations, emphasis is placed on calculation strategies
General strategies during calculations	Develop techniques and estimate	Understand equal sign and find missing number in equations	Counting on and counting back	Estimation, estimation strategies and use of algorithms

Source: ACARA, n.d.; Buijs et al., 2009; CCSI, 2010; DBE, 2011a

Results

National and International School Policy Documents

National and international school policy documents were compared, and the results are presented in table format (Table 2) above.

Some of the similarities and differences that emerge from Table 2 include the following:

- The number domain and place value ranges are from 0–1,000 in South Africa, while in The Netherlands these ranges are from 0–100,000.
- The CAPS document does not explicitly require explanation and reasoning as well as higher-order thinking skills – yet the other participating countries value this as important. In the CAPS (DBE, 2011a:113), the following statement is made: "the mental mathematics sessions develop learners' number sense; language of Mathematics; reasoning skills; and listening skills." This is the only reference to reasoning in Grade 1-3 Mathematics in South Africa.
- According to the CAPS document, fractions should only be named and recognised, while the other three countries place a high value on reasoning, interpretation and the structure of fractions. This is evident from the following statements: "students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole" (CCSSI, n.d.:21).
- Both South Africa and Australia merely mention the four operations, whereas the USA and The Netherlands include the development of calculation strategies. The following is an example from the Dutch curriculum (Buijs et al., 2009:1): "handig optellen met strategieën zoals [competent use of a range of strategies to add, such as]: rijgen [ordering a pattern] (230 + 90: 230 → 300 → 320); splitsen [expanded notation] (46 + 53 → 90 + 9); compenseren [compensating], (199 + 86: 200 + 86 - 1 of ineens [or immediately] 200 + 85); analogie [analogy] (3000 + 12000 naar analogie van [by analogy of] 3 + 12); verwisselen [order of operation] (2 + 399 → 399 + 2); ... ." An example from the USA's curriculum document (CCSSI, n.d.:15) is: "apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known (commutative property of

addition). To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 (associative property of addition)." Both the Dutch and USA's curriculum include strategies for all four basic operations.

The findings in Table 2 suggest the content that should be considered for providing recommendations of knowledge and practice standards. The education ideologies discussed in the conceptual theoretical framework should also be taken into consideration when the practice standards are formulated, because the ideologies indicate the kind of education practice the DBE (2011a) expects of student teachers. For this reason, these results in Table 1 indicate the 'what' (content knowledge of the number domain) that should be taught in schools, but the education ideologies indicate the 'how' (pedagogical knowledge) of the content should be taught in schools. Next, the results of the analyses of teacher policy documents are presented.

Teacher Policy Documents

The focus of teacher policy documents seems to be based on the 'mathematics knowledge for teaching' model that was proposed by Shulman's model of knowledge for teaching in general (1986), and subsequently researched and refined by Ball et al. (2008) for mathematics knowledge for teaching. These results are described in terms of the third concept of the conceptual theoretical framework in Figure 1, namely mathematics knowledge for teaching. The definitions as given in the section *Mathematical knowledge for teaching* were used as the criteria for analysing the documents. Each of the following results is discussed under the different categories of Mathematical knowledge for teaching.

Common content knowledge

The curriculum policy document from The Netherlands gives more information than the documents of the other countries about the common content knowledge that student teachers need to have. Student teachers should understand place value and be familiar with number notations up to one billion; they should use exponents, negative exponents and scientific calculators, but at the same time be able to do calculations without the use of Information and Communications Technology (ICT); they should be able to do standard algorithms; and must be competent and confident mathematicians (Otten, 2009).

The Netherlands and North Carolina (USA) policy documents concur about some of the types of common content knowledge. Their student teachers should have knowledge about numbers: viz. representations of numbers, relationships between numbers, structure of numbers and number systems (NCSBE, 2009; Otten, 2009). These student teachers should also know and understand operations and calculations. They must understand the relationship between operations; do calculations using properties of addition and multiplication; interpret the results of calculations; do calculations fluently; use negative integers in calculations, and use brackets. They should also be able to do calculations with different kinds of numbers like prime numbers, roots, irrational numbers, real numbers, fractions and decimals (NCSBE, 2009; Otten, 2009). Finally, these student teachers should be au fait with fractions; do calculations with fractions; and understand relationships among fractions; decimal numbers and rounding off (NCSBE, 2009; Otten, 2009).

In Australia, teachers are expected to understand relationships in mathematics as well as the relationship between Mathematics and other subjects (AAMT, 2006).

Knowledge at the mathematical horizon

Only one statement about the mathematical horizon was found in the curriculum of Australia. It referred to teachers' understanding of where the mathematics that they will teach fits into the school Mathematics curriculum (AAMT, 2006).

Specialised content knowledge

The Netherlands policy document is also fairly informative about the specialised content knowledge that is required (Otten, 2009). Their student teachers should be able to reason and verify reasoning: during problem solving; during calculations with fractions and decimals; and during the use of mathematical notations (Otten, 2009). It is expected of these student teachers to be able to use mathematical language for the following: speaking, writing, meaning of numbers, symbols, relationships; integers, formal language, operations, calculations, place value, decimal numbers and whole numbers (Otten, 2009). They should know how to write negative numbers, the 'bigger as' symbol, 'smaller as' symbol, root sign, exponents, fractions, decimal numbers and they should be able to build a repertoire of number networks (Otten, 2009).

In addition, they should demonstrate knowledge and understanding of whole numbers, integers, characteristics of the number system, the decimal number system and other number systems; the relationship between fractions and numbers; how to relate numbers to real-life situations; the relationship between different number systems; and patterns in numbers (Otten, 2009). Representation and modelling of numbers in different ways, using the number line to position numbers and to indicate the number size are also of importance (Otten, 2009), while supporting learners' thinking skills development (by using both context-free and context-bound counting interchangeably) is deemed desirable. Student teachers should know and understand calculations, i.e. properties; reasoning; negative numbers; how and why to use brackets; how to estimate; to know which calculation is the fastest; how to use calculation procedures in complex mathematical situations; choose a solving strategy; how to check for accuracy; how to estimate decimal numbers during use of calculations; and be skilled in all four operations (Otten, 2009). Furthermore, they should be able to do mental calculations fluently, including mental calculations with decimals (Otten, 2009).

The North Carolina (USA) policy document indicates that student teachers should understand and know mathematical content to ensure development in mathematics (NCSBE, 2009).

Knowledge of content and students (learners)

The Australian policy document for teachers refers to knowledge of the content and of the students (learners) – probably because these standards were written for teachers who have been in practice for some time. The aim of their teacher preparation programmes is to lay the foundation for these standards, which should be achieved after a while in practice. The teachers should not only have knowledge about the development of learners and about learning theories that are relevant to mathematics teaching, such as increasing learning opportunities and setting high standards for every learner (AAMT, 2006), they should also know how to take the learners' pre-knowledge into consideration, and be able to develop self-directed learners who enjoy doing mathematics (AAMT, 2006).

The policy documents of Australia and North Carolina (USA) agree that student teachers/teachers should have comprehensive knowledge of the learners: their mental representations of content; preconceived ideas; misconceptions; errors; learning trajectories; social and cultural contexts; and ways in which they learn (AAMT, 2006; NCSBE, 2009).

According to the North Carolina (USA) policy document, student teachers should be able to help learners to develop problem-solving skills; apply different strategies; reflect on the mathematics problem-solving process; communicate mathematical thinking; analyse other learners' mathematical thinking and strategies; use mathematical language to communicate mathematical ideas; construct mathematical relationships; apply mathematics inside and outside the classroom; develop representations of mathematics; and organise mathematical ideas (NCSBE, 2009).

The Netherlands policy document has similar requirements. Student teachers should know how to enable learners to construct mathematical concepts that broaden their knowledge and appreciation of mathematics and that stimulate learners during the process of mathematising (Otten, 2009). It also values abilities such as knowing how to ensure that learners understand the functions, structure and properties of numbers; how to use real-life examples for the exploration of numbers, and how to develop learners' number sense and mental calculations (Otten, 2009).

Knowledge of content and teaching

As far as knowledge of content and teaching is concerned, the Australian policy document for teachers indicates that teachers should be able to involve learners in active learning and to plan coherent learning experiences that give the opportunity for spontaneous self-directed learning (AAMT, 2006). Teachers should be aware of effective mathematical teaching and learning strategies and techniques; they should be able to facilitate learning; and should be able to promote learners' positive attitude towards mathematics (AAMT, 2006).

The Australian and North Carolina (USA) policy documents both demand that student teachers/teachers be able to use ICT during teaching for the discovery of mathematical concepts (AAMT, 2006; NCSBE, 2009). The policy documents of both countries also agree that student teachers should be able to model mathematical thinking, mental calculations and reasoning (NCSBE, 2009; Otten, 2009). The North Carolina (USA) policy document furthermore indicates that student teachers should know and understand the process skills that are required to ensure mathematical development (NCSBE, 2009). It also deems important that student teachers understand that problem solving, reasoning, communication, relationships and representations are integrated over content areas and methods (NCSBE, 2009).

According to the Netherlands policy document, student teachers should have knowledge about teaching numbers, and ought to know how to explain calculations and fractions to their learners (Otten, 2009). They should know how to use the calculator during teaching; how to teach standard procedures; and how to include different learners (Otten, 2009). Lastly, they should be able to use models and schemes for the transition of context-bound to context-free formal calculations and reasoning (Otten, 2009).

Knowledge of the curriculum

With regard to knowledge of the curriculum, both the Australian and North Carolina (USA) policy documents are quite informative. According to the Australian policy document, teachers should have knowledge appropriate to the grade of the learners, and should plan learning experiences that involve substantial mathematics (AAMT, 2006). The Australian and North Carolina (USA) policy documents agree that student teachers/teachers should be able to incorporate teaching strategies, technology and other resources for learning experiences (AAMT, 2006; NCSBE, 2009). The North Carolina (USA) policy document indicates that student teachers should have knowledge about teaching resources, contents and strategies such as sequence of themes, different examples, metaphors, models, tasks, resources and technology (NCSBE, 2009).

Discussion and Conclusion

In an attempt to explore and compare mathematics knowledge and practice standards for the education and training of foundation phase teachers in mathematics, different national and international policy documents were examined to provide insight into what subject matter knowledge and what pedagogical content knowledge is needed for the teaching of numbers by a teacher in his/her first year of practice.

Based on the international comparisons of standards, we recommend that these comparisons not only be done for Foundation Phase mathematics student teachers, but also for all the other subjects in the different phases. Employing these methods, similarities and differences can be detected in the professional standards for student teachers in the different countries. Because we are living in a global society, these similarities and differences should be questioned in terms of the extent to which professional teacher standards should be uniform, and the extent to which they need to be diversified.

Based on our findings regarding the learners' school curriculum, it seems that The Netherlands aims to develop higher-order thinking skills, whereas South Africa is more content-driven (Table 2). High expectations lead to high results, as learners try to keep up with the expectations of their teachers (so long as these expectations are clear, and help and practice materials are offered in a learner-friendly manner).

Our recommendations below are based on the different broad types of mathematical knowledge, mathematical subject matter, and pedagogical content knowledge, which emerged from the studied Mathematics school curriculum policy documents (Table 2). We present the findings as recommendations for further study, regarding the development of knowledge and practice standards. However, it is possible that some authors will view the findings as initial guidelines.

Our findings are supported with reference to Goulding et al. (2002), who have asserted that mathematical knowledge for teaching is complicated and not easily distinguishable. Keeping in mind that knowledge and practice standards are interrelated and can be distinguished but never separated, we firstly recommend that the mathematical subject matter knowledge should inform the mathematics knowledge standards. Mathematical subject matter knowledge includes number sense, place value, operations and calculations, money, fractions and mental calculations. Secondly, we recommend that mathematical pedagogical content knowledge should inform the practice standards. Mathematical pedagogical content knowledge includes the explaining of answers and reasoning, problem solving, and the development of strategies for calculations. It is, however, difficult to clearly draw a line between these themes, because they are integrated. Practice standards should also be informed by education ideologies, namely progressive educator and public educator. Furthermore, the mathematics knowledge and practice standards should not be linked to the school curriculum only, because the latter changes constantly in line with development and research (DBE, 2011a; DBE & DHET, 2011).

Considering the findings regarding the student teachers' standards, The Netherlands is the country whose standards are best distributed in the two domains of mathematical knowledge for teaching. The Netherlands seems to focus on the structure of numbers. Australia seems to emphasise mathematical pedagogical content knowledge, while North Carolina (USA) is the least informative (compared to The Netherlands and Australia) concerning the domains of mathematical subject matter knowledge and pedagogical content knowledge. With regard to knowledge of the curriculum, only Australia and North Carolina (USA) have standards in this regard (AAMT, 2006; NCSBE, 2009).

As far as the development of mathematics knowledge standards is concerned, we recommend that foundation phase student teachers not only harbour a basic knowledge of those elements of mathematics in the school curriculum; but they should also know, understand and be able to apply concepts of numbers, the structure of numbers, properties of addition and multiplication, negative integers, brackets, positive and negative exponents, prime numbers, roots, irrational numbers, real numbers, fractions, decimals, scientific calculators, relationships in mathematics, representations and mathematical language. Student teachers should furthermore be able to reason during mathematical problem solving, during calculations, and during the use of mathematical notations.

Regarding the development of mathematics practice standards, we recommend that student teachers should recognise the social conditions in which the learners grow up, be able to promote holistic development in the mathematics classroom, and have a thorough knowledge of learners and how they learn mathematics in the Foundation Phase. Student teachers ought to have sound knowledge of the different theories of learning that are relevant to mathematics teaching. They should be able to facilitate learning and to adopt a learner-centred approach to teaching numbers. Student teachers should likewise be able to integrate mathematics with other subject areas and real-life examples, which should lead to critical discussions and the development of mathematical thinking and reasoning. Lastly, student teachers should be able to provide a rich environment of resources and assessment methods in mathematics, appropriate to the grade in which the learner is to be found. We believe that these recommendations would usefully guide the drawing up of mathematics knowledge and practice standards as a basis for teacher education principles in the preparation of foundation phase teachers for their future career. Further research is needed to provide guidelines that explicitly state what each of these recommendations implies for the development of knowledge and practice standards in South Africa.

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