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Teacher learning about probabilistic reasoning in relation to teaching it in an Advanced Certificate in Education (ACE) programme

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I report on what teachers in an Advanced Certificate in Education (ACE) in-service programme learned about probabilistic reasoning in relation to teaching it. I worked 'on the inside' using my practice as a site for studying teaching and learning. The teachers were from three different towns in the Northern Cape province and had limited teaching contact time, as is the nature of ACE programmes. Findings revealed a complicated picture, where some teachers were prepared to consider influences of their intuitive probabilistic reasoning on formal probabilistic reasoning when it came to teaching. It was, however, the 'genuineness' of teacher learning which was the issue that the findings have to address. Therefore a speculative, hopeful strategy for affecting teacher learning in mathematics teacher education practice is to sustain disequilibrium between dichotomies such as formal and intuitive probabilistic reasoning, which has analogies in content and pedagogy, and subject matter and method.

Introduction

The research question — what do teachers learn about probabilistic reasoning in relation to teaching it in an Advanced Certificate in Education (ACE) programme? — is the focus in this article. It has several interrelated and overlapping levels. It is concerned with the importance of studying teacher learning in a teacher education programme, connecting probabilistic reasoning and probabilistic reasoning teaching, and finally, studying teacher learning in relation to children's probabilistic reasoning. To address these different levels there is initially a review of literature on teacher education and mathematics teacher education.

Teacher learning is a key research area in teacher education practice. In the current climate of curriculum policy in South Africa all teachers find themselves in situations where they learn the policy rhetoric associated with 'outcomes-based education'. For example, they talk about assessment standards, learning outcomes and continuous assessment, to name but a few. South African curriculum policy is quite ambitious about the 'kind of teacher that is envisaged' (Department of Education (DoE), 2003). For instance, teachers are to mediate, interpret and design learning programmes and materials. These are examples of policy images of teachers (Jansen, 2001). How do teachers learn to teach in ways that are aligned with such ambitious policy rhetoric? More importantly, what is the role of teacher educators in fostering teacher learning in such a policy environment? In policy debates in the United States, Ball and Cohen (1999) call for 'interweaving' (Ball & Bass, 2000) of content and pedagogy in teaching and learning to teach. They contend that teacher educators should not only be interested in what teachers must know, but also how they must be able to use knowledge (Ball & Bass, 2000) as they learn to teach. In their writing 'learning' and 'teaching' are deliberately put together because of the notion of 'interweaving.' They argue that knowing how to teach entails more than simply applying prior understandings. Teacher educators must therefore take seriously the notion of teacher learning when it comes to aligning curriculum policy and classroom practice. The fact of the matter is that curriculum reform in South Africa has no relation with pedagogical reform (Jansen, 2001). For teacher educators therefore the critical issue remains: how are teachers going to learn to teach in ways that reflect curriculum policy?

As a way forward Ball and Cohen (1999) propose 'closing the gap' in teacher education with a focus on developing and using knowledge 'in practice'. This proposal means closing the gaps between subject matter and method, and between content and pedagogy. This view builds on Dewey (1904/1964), who articulates the tension between subject matter and method, and points out a sophisticated and subtle relationship between the two. Separating the two in teacher education programmes reduces teaching practice to the use of clearly stated recipes. In another sense it means there is a need for teachers to 'learn in and from practice' (Ball & Cohen, 1999:10). What they mean by 'in practice' is not to be understood in a narrow, physical sense, e.g. in a school or in a university setting. Wilson and Berne (1999) recommend that teacher learning be activated, rather than bound and delivered in the form of recipes and models. They regard creating and sustaining disequilibrium as a requirement for teacher learning. On a similar point Lord (1994) theorises that a 'critical collegiality' will help teachers learn by increasing their comfort with high levels of ambiguity and uncertainty, which will be regular features of teaching for understanding.

In mathematics teacher education practice we need to understand better what it means to teach both mathematics and teaching in the same programme (Adler, Ball, Krainer, Lin & Novotna, 2005). How do teachers learn both mathematics, probability as in this study, and teaching in teacher education? This 'gap' is an instantiation of Dewey's (1904/1964) content and pedagogy and subject matter and method dichotomies. In the Teacher Development Experiment, Simon (2000) worked on teachers' mathematical development and their pedagogical development. An enduring problem in mathematics teacher education is to build both mathematics and teaching identities in teachers (Adler *et al.*, 2005). This is a specific response to Jansen's (2001) policy images of teachers, which is more general. A teacher could have a very clear understanding of probability, but that would not necessarily mean that he or she would be able to apply that during teaching. In a Deweyan sense the teacher would have to understand the subject matter of the probability in relation to method. This point is taken up next in the case of probability in the mathematics curriculum.

Probabilistic reasoning research

Research reveals that the subject matter of formal, mathematical probability has its 'psychical roots' (Dewey, 1904/1964:162) in intuitive or subjective probability. For instance, Konold (1989) points out that probabilistic reasoning is fraught with misconceptions or strong prior conceptions that are at odds with formal conceptions of probability. This has implications for the 'method' of teaching and learning of probabilistic reasoning. For the purposes of this article, 'probabilistic reasoning' refers to those instances in the teaching and learning of probability concepts or notions where explanation and reasoning are required. In the teaching of probability notions there should be a consideration of the nature and influence of 'subjective probabilities in the development of formal probability concepts', in particular cases where inferences from the former come into conflict with those based on the latter (Hawkins & Kapadia, 1984:350). Teachers must learn to become aware of, and extend, the two probabilities when it comes to teaching children. In fact Hawkins and Kapadia (1984) emphasise that the counter-intuitive nature of even simple probabilities needs to be borne in mind when teaching probability to children. In this study many of the teachers wanted to know how they might teach probability to their learners. Furthermore, Hawkins and Kapadia (1984) emphasise the importance of developing a better understanding of growth and communication in probabilistic notions. They see subjective probability as an expression of personal belief or perception and also as a precursor to formal probability. An example of ignoring the psychical roots of probabilistic reasoning is where the scientist's formal mathematical probability is transposed as the subject matter into the teaching situation, bound and delivered (Wilson & Berne, 1999). This means ignoring intuitive or subjective probability, i.e. separating subject matter from method. The result is producing skill in action independent of any engagement of thought (Dewey, 1916/1966:178). It implies having the skill of computing formal probabilities without understanding why and how particular probability formulas come about. Teachers who wish to learn to develop their awareness and that of children or learners in this regard in and through their teaching constantly struggle against situations where formal knowledge comes into conflict with students' intuitive knowledge (Lampert, 1985; 1990; 2001).

Hawkins and Kapadia (1984) recognize no 'harsh dividing line' between the two, a move consonant with Dewey's (1904/1964) call for studying subject matter in ways that took it back to its 'psychical roots'. Similarly Fischbein and Gazit (1984) argue for a teaching programme that aims at developing and improving probabilistic intuitions for probability concepts along with formal mathematical probability concepts. They suggest providing learners with frequent opportunities to experience stochastic situations actively, even emotionally. Their argument is consonant with Lord's (1994) call for enabling teachers to deal with high levels of ambiguity and uncertainty, in this case, probabilistic reasoning. Hawkins and Kapadia (1984:358-359) also refer to 'misconceptions' and give the famous historical example of the possibility of obtaining a head and a tail when tossing two coins. They observe that a number of mathematicians have assigned a probability of 1/3 as they have erroneously assumed an equally likely sample space of three possibilities (two heads, two tails, or a head and a tail). Bennie (1998) refers to this example under 'distinguishing outcomes' and found that teaching that involved systematic listing and classroom discussion was useful in trying to counter this misconception.

In South Africa little work has been done in terms of gathering information from teachers at the in-service level when it comes to stochastics, i.e. probability and statistics. Laridon (1995) studied intuitive probability concepts in South African adolescents, while Kazima (2000) and Kazima and Adler (2006) studied students' perceptions of fairness in probability games. Quite some time ago Shaughnessy (1992) pointed out the need to unravel teachers' probability concepts as an area in which little or no research exists and for which data could be of assistance to those making decisions regarding the professional development needs of teachers.

On the subjects in the study and the teaching context

The subjects in the study were intermediate and senior phase teachers who were registered for an ACE which was administered by a higher education institution in the Western Cape. There were 50 teachers in total enrolled in a mathematics education module in the ACE programme, scattered over three teaching venues. They came from rural and urban areas in the Northern Cape province of South Africa. They all had a Grade 12 or matriculation certificate and three years of teacher training college education. Their experience in the classroom ranged from being novices to mid-career teachers. They had never done any tertiary-level study in mathematics in general, nor had they done any tertiary-level courses on data handling, specifically courses in statistics and probability. They may be described as 'generalists' with a professional training mainly in pedagogy. They formed part of the majority of teachers in the South African education system, amounting to about 77%, who have a three-year post-school level or a Relative Education Qualification Value (REQV) of 13 (a Diploma in Education), which obviously impacts on school mathematics reform. The policymaking community is well aware of this phenomenon and has called for adequate planning to ensure that recruitment drives and programme design take into account the actual needs of the school sector, in terms of scarce subject areas (Mathematics, Physical Science and Technology) and the capacity of teachers to implement outcomes-based approaches to teaching, learning and assessment. For example, the current qualification framework has raised the minimum qualification requirement for all new teachers from a three-year post-school level (REQV 13) to a four-year professional degree level (REQV 14) (DoE, 2005). Through ACE programmes the Department of Education of the South African government makes funds to 'upgrade' and 'reskill' available to a selected number of teachers who do not have a university qualification to learn to teach the different 'learning outcomes' (LOs) in the mathematics component of the Revised National Curriculum Statement (RNCS) (DoE, 2003). For this reason it is important to study how these teachers with an REQV of 13 learn to reflect on their learning about probabilistic reasoning, albeit on a small scale as is the case in this study.

Teachers enrolled in the ACE programme took modules in all of the five LOs in the mathematics component of the RNCS as well as other elective modules. According to the RNCS, the LOs are

- numbers, operations and relationships;
- patterns, functions and algebra;
- space and shape;
- measurement; and
- data handling.

I taught a module called Mathematics for Teaching on 'Learning Outcome 5' (LO5) on data handling (DoE, 2003) to the teachers. This was the first and only module that I taught them. It was also the last module in the mathematics sequence of the ACE programme according to bureaucratic arrangements. On completion of the module the teachers had no further contact with me. It was one of several modules that they had to take to earn an ACE. The module had a total of four contact sessions of three hours each, followed by a final examination.

The module was offered in three towns, P, Q, and R in the Northern Cape province in South Africa. I was part of a team of lecturers who taught other modules in the ACE programme who came to these towns, with rented cars. During a period of one week starting on a Monday and ending on a Friday I taught the module in the three different towns P, Q and R. This meant that I travelled between these towns during that particular week. During the same week teachers in the different towns had lectures ranging from the Mathematics for teaching module on a particular day(s) to other modules offered by the other lecturers in the team. The contact sessions for the module occurred once a month over a three-month period, giving a total contact time of 18 hours for each town. Table 1 shows in which towns the module was offered and on which days of the week and the total contact time for the complete module.

I had not taught previous *Mathematics for teaching* modules on the other LOs to the same cohort of teachers.

On method and data

In this study 'method' has two meanings. One is related to my teaching of the module on data handling, and the other on the way I went about researching my teaching and presenting my findings. In terms of the latter I 'worked on the inside' using my own teaching of the module as a site to study teaching and learning (Ball, 2000). Such a research genre requires 'distance' (Adler *et al.*, 2005), i.e. critical perspective in terms of reporting and analysing findings. Later on in the article this perspective will be used in a discussion in relation to the findings. In teaching the module an overall method I used was to incor-

Table 1

Day of the week	Town	Contact time per month	Contact time at the end of 3-month period
Monday	P	3 hours	$3 \times 6 = 18$ hours
Tuesday	P	3 hours	
Wednesday	Q	3 hours	$3 \times 6 = 18$ hours
Thursday	Q	3 hours	
Friday	R	6 hours	$3 \times 6 = 18$ hours

porate statistical reasoning and probabilistic reasoning with an explicit focus on 'for teaching' in line with the title of the module and literature on the importance of 'interweaving content and pedagogy' in teacher education programmes (Ball & Cohen, 1999; Ball & Bass, 2000; Adler *et al.*, 2005). Furthermore, I attempted to follow, with major modifications, the guidelines expounded by Simon (2000), such as addressing questions about 'genuineness' and 'legitimacy' when it comes to presenting teachers' self-reports as data or findings.

To elaborate further on the method I used in my teaching, a brief exposition of the policy rhetoric on data handling, probability in particular, follows. According to the RNCS, the 'learning outcome' for data handling states that:

> [the] learner will be able to collect, summarise, display and critically analyse data in order to draw conclusions and make predictions, and to interpret and determine chance variation (DoE, 2003).

This policy statement can be connected to literature on the teaching and learning of probabilistic reasoning at the school level. Units from the Connected Mathematics Project (Lappan, Fey, Fitzgerald, Friel & Phillips, 1997) such Data around us, What are my chances and How likely is it? turned out to be useful in terms of giving meaning to this policy statement. These units are from a middle grades curriculum project in the United States and had to be changed to match local conditions. One of my explicit goals for this module was to direct the teachers' attention to ambiguity and complexity in stochastics in line with the literature reviewed earlier on. I had hoped to find out what they learned about probabilistic reasoning during the module and how they reflected on their own learning about it with respect to their future teaching. The reason for the latter was my research interest in teacher learning with respect to curriculum policy in general and probabilistic reasoning in particular.

The data in this study include the following:

- My reflective notes on the mathematics I taught and my teaching;
- The subjects' educational and biographical backgrounds from the higher

education institution that administered the ACE programme;

- Data from the Department of Education on teachers with a post-school level REQV 13, reported earlier on (DoE, 2005);
- Teachers' responses to the questionnaire that I administered during the last contact session (see Appendix).

Teachers' discussions and debates with me and with their peers in the ACE programme should be seen as happening 'in practice,' meaning that they were capable of 'learning from practice'. This line of reasoning is consonant with the literature on teacher learning reviewed earlier on. For example, during the middle month of the three-month teaching contact session, I designed a set of tasks that highlighted the differences between subjective or intuitive probability, and formal or mathematical probability. The theoretical intent of this design was to put theories of teacher learning 'in harm's way', i.e. to test theories of teacher learning with respect to the counter-intuitiveness of probabilistic reasoning. These tasks represent instances where I sought alignment with policy rhetoric about learners having to 'critically analyse, make predictions, and to interpret and determine chance variation' (DoE, 2003). In the context of my teaching the 'learners' should be viewed as the teachers taking the module.

In one particular set of tasks the teachers grappled with ways intuitive probability interacts and collides with formal probability which leads to Pascal's Triangle as a central object. Pascal's Triangle has significant mathematics encoded in it that unifies many of the different 'learning outcomes' in the RNCS. Examples of questions that the tasks included are:

> What is the probability of getting a one head / one tail, when tossing?
> - One coin?
> - Two coins?
> - Three coins?
> - Four coins and so on.
>
> Explain your reasoning in each case.

The prompts have purposes, namely, mathematical development and pedagogical development with respect to probabilistic reasoning. Another example is:

> What is the probability that there will be one boy in a family of …
> - One child?
> - Two children?
> - Three children?
> - Four children?

A summary of formal probability in the investigations in the case of the coins can be presented in the form of a summary as in Figure 1, which leads to Pascal's Triangle (Figure 2).

One coin		H 1 way	T 1 way
Two coins	HH 1 way	HT TH 2 ways	TT 1 way

Figure 1

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

. . . . . .

Figure 2

In the investigation on the coins a significant incident occurred which gave rise to the idea of researching teacher learning of probabilistic reasoning in relation to teaching it. In this incident all the teachers in the three different teaching venues assumed that the probability of getting a head and a tail when tossing two fair coins is 1/3. This incident captured a misconception in probabilistic reasoning (Hawkins & Kapadia, 1984). A fair coin is one where the formal probability of getting a heads or tails when tossing the coin is the same. It was then that I developed the idea of a questionnaire as a means to generate data on teacher learning. The prompts in the questionnaire attempted to focus the teachers' attention on the misconception which occurred during my teaching of the coins investigation elaborated in the above. It specifically required teachers to spell out their reasoning and how that reasoning might be taken into account should they teach a similar investigation where intuitive and formal probability interact. It should be noted that not all the teachers in the different towns completed the questionnaire.

The written responses of the teachers TA, TB, TC and TD are presented as findings because they reflect an interesting variation on what teachers who were enrolled in the ACE module on data handling learned. One of the teachers wrote the following:

> P(head and tail) = 1/3. Why do you think this is so?
>
> *Nobody really knew how to reason about this.*

This excerpt is indicative of the extent of the engagement and disequilibrium the teachers experienced when the researcher taught probability.

Findings

Variation in teacher learning with respect to the first prompt ranged from figuring out how to distinguish between different outcomes (heads and tails) and finding a way out of intuitive probability to formal or mathematical probability by interpreting the numbers in Pascal's Triangle. The first prompt focused on promoting teachers' mathematical development with respect to probabilistic reasoning. It required the teachers to give reasons why the misconception occurred:

> In the case of two coins there was a **misconception** about the probability of getting a head and a tail. Everyone in the class agreed that this P(head and tail) = 1/3. Why do you think this was so?

Below is what the teachers wrote as a response:

TA *Because we did not take the position of the coin into consideration. HT/TH is the same but position plays a role. Hence the confusion.*
TB *I was not aware that the order of the coins must also be considered.*
TC *Intuitive thinking without actual activities.*
TD *It now seems stupid with the understanding of Pascal's Triangle.*

TA and TB had learned that the psychical roots of their probabilistic reasoning based on intuitive notions were in conflict with mathematical probability. They gave reasons for the misconception in terms of formal or mathematical probability, namely, the position or order of the outcomes, heads/tails and tails/heads. Their responses showed their mathematical development in probabilistic reasoning. They had learned to distinguish between the outcomes heads/tails and tails/heads in the case of tossing two coins. TA regarded the tension between intuitive and mathematical probability when tossing two coins as '*confusion*.' TB became '*aware that the order of the coins must also be considered*' if mathematical probability were to be considered. TA and TB realised the mathematical significance of distinguishing between heads/tails and tails/heads. Evidence for the psychical roots of probabilistic reasoning is captured in the words '*confusion*' and '*aware that the order of the coins must also be considered*'. During their investigation of finding the probabilities when tossing several coins, they experienced the sophisticated and subtle relationship between intuitive and formal probability. Here the subject matter of probabilistic reasoning became interwoven with their method. In their method they had an opportunity to notice the difference between heads/tails and tails/heads. This they were not aware of during the investigation itself.

Teacher learning was activated to a point that reveals disequilibrium as can be seen in TC's and TD's responses. During the coin-tossing investigation I resisted transposing the scientist's formal or mathematical probability as the subject matter into the teaching situation, bound and delivered. Not all the teachers readily accepted the distinction between heads/tails and tails/heads. They only agreed after seeing how the structure in the positions of the heads and tails leads to Pascal's Triangle. Teachers' '*intuitive thinking*' as indicated by TC prevailed at the beginning when they settled for P (head and tail) = 1/3. I encouraged all the teachers in each of the teaching venues in the different town to discuss and to confer whether positions of the heads and tails mattered through '*actual activities*' as TC wrote. TC's response highlighted the psychical roots of the subject matter of probability, i.e. the influence of intuitive probabilities — '*intuitive thinking*' — in the development of mathematical probabilities. During the investigation the teachers tossed different numbers of coins and recorded the outcomes. These can be considered as '*actual activities*' although not sufficient for them to be convinced that the P(head and tail) = 1/2 according to mathematical probability. There was thus the disequilibrium between 'intuitive thinking' and 'actual activities'. In the case of tossing two coins the teachers would have had to do a simulation via information technology using the law of large numbers to be convinced P(head and tail) = 1/2 when tossing two coins. No information technology was used during the teaching of the ACE module. What is evident from the responses of TA, TB and TC was that the teachers had opportunities to experience a stochastic situation actively and even emotionally. Disequilibrium in the form of '*confusion*' as reported by TA was captured in TD's response, where she or he wrote '*It now seems stupid with the understanding of Pascal's Triangle*'. This is an emotive response, which Fischbein and Gazit (1986) suggest as a means to improve intuitive probability along with mathematical probability. TD's response appears to indicate a resolution in the disequilibrium — '*It* ***now*** *seems stupid with the understanding of Pascal's Triangle*' (emphasis added). Pascal's Triangle is a mathematical object that can be interpreted as a mathematical summary of the formal probability outcomes when tossing one coin, two coins, three coins, and so on. A particular understanding of the numbers in Pascal's Triangle also connects it to algebra, probability and combinations. TD's response shows that he or she has an understanding of the row 1 2 1 in Pascal's Triangle:

HH	HT	TT
	TH	
1	2	1
way	ways	way

It can be inferred that TD had acquired such an understanding.

After the coin-tossing investigation all the teachers in the different towns were able to distinguish between different outcomes, but not without some uneasiness and disequilibrium prevailing. From my notes I recall posing the question: what is the probability that there will be one boy in a family of four children? Many teachers answered 1/4, while others said 1/16. The latter answer is the mathematical probability and is counter-intuitive for those, whether they are adults or children, who do not know the subtlety and sophistication in probabilistic reasoning. For this question the teachers had brief exchanges during which some teachers pointed out how intuitive and mathematical probability '*come together*' and how they can be '*confusing*'.

The second prompt,

> How do you think this misconception will affect your teaching of the 2 coins probability?

aimed at finding out whether the teachers would consider the psychical roots of intuitive probabilistic reasoning in the case of the 2 coins when it came to teaching. Below is a list of teacher responses to this prompt:

TA *It only broadened the way I think things were, but now I have to take into account the role of the positioning of the coins.*

TB *The learners will first have to figure out on their own, before I will lead them to the correct way of probability.*

TC *This might result in giving wrong information to learners, because you did not test the validity of your information.*

TD *The misconception was straightened out and a mutual understanding was reached and thus no misconception after thorough examples and explanations from the lecturer.*

For the first prompt TA noted that he or she would take into consideration the position of the coins. For the second prompt TA wrote that the misconception had '*broadened*' the way he or she thought '*things were*' and learned to discern the 'role of the positioning of the coins' and may take it into account when it comes to teaching. This could mean that he or she would enable children to experience the subtle and sophisticated relationship between intuitive and mathematical probability. It is hard to say, because there were no follow-up interviews with any of the teachers in the different towns. TB's response, however, was more explicit about a teaching strategy, i.e. '*learners will first have to figure out on their own*'. It seemed as if he or she thought of mathematical probability as '*the correct way of probability*'. Alternatively, 'the correct way of probability' could mean that this teacher would have his or her learners 'figure out on their own' and thus experience the psychical roots of intuitive probabilistic reasoning and its influence on mathematical probability. TB appeared to be receptive to an organised and deliberate investigation (Lord, 1994) of probabilistic reasoning when it comes to teaching it in the case of tossing 2 coins. This could also show a willingness to avoid a rush towards mathematical probability. On the other hand, what TB wrote may simply be due to the effect of discussion and debate that occurred during my teaching of probabilistic reasoning. In relation to teaching, TC saw the misconception as '*giving wrong information to learners*'. 'Wrong information' could point to the belief that mathematical probability is the 'right information' despite the fact that all the teachers experienced the influence of subjective or intuitive probabilistic reasoning in the development of mathematical probability, i.e. in their experience there was no harsh dividing line between the two. '*Validity of information*' could mean that the teachers tell their learners that there are two possible answers — P(head/tail) = 1/2 and P(head/tail) =1/3 — and that they must decide which is correct and that they must support their answers with reasons. Testing 'the validity of your information' would therefore be an interesting way to explore probability.

During my teaching, mathematical probability only became evident after experimentation and discussion. Teachers then checked the validity of each other's information and some agreed while others disagreed. My teaching was not simply a case of giving information about 'correct' or mathematical probability. I designed instruction so that teachers would engage and encounter a tension between intuitive and mathematical probability. TC would be correct in terms of mathematical probability, meaning that the misconception should be avoided if mathematical probability were to be the sole objective of teaching. '*Test*[ing] *the validity of your information*' was what happened during my teaching when the teachers debated whether the order or position of the coins mattered. For the first prompt TC wrote that the misconception came about because of '*intuitive thinking without actual activities*'. It could be that he continued to see mathematical probability as result of the '*actual activities*'. TD saw the misconception '*straightened out*' through '*thorough examples and explanations from the lecturer*'. His/her use of '*mutual understanding*' could be an indication that an understanding came about where the influence of intuitive probability on mathematical probability could not be ignored when one teaches an investigation of the 2 coins probability for the first time with no awareness of the '*correct*' probability.

It was hard to infer what TC and TD would actually do when their learners said the probability of getting a heads and a tails when tossing two fair coins is 1/3, according to intuitive probability. Furthermore, there are also no data on their actual teaching of the 2 coins probability. TC's learning seemed to be pulled in the direction of mathematical probability. Phrases such as '*wrong information*' and '*validity of information*' are evidence for this claim. Also, it seemed especially unlikely that TD would see a role for intuitive probability in the development of mathematical probability concepts, especially in instances where inferences from the former are in conflict with those of the latter. Phrases such as '*The misconception was straightened out*' and '*thorough examples and explanations from the lecturer*' support this claim, but cannot be viewed as conclusive evidence. If TC and TD were to teach in ways more or less similar to their own experiences in probabilistic reasoning, then it would be very likely that they would encounter this misconception, and would then have to address it in their classes. The third prompt aimed at getting to such a classroom situation.

The third prompt was more explicit in terms of asking teachers what they would do in their teaching because of the use of the word 'address':

> What would you do to address this misconception when you one day teach this 2 coins probability problem?

TA *A practical demonstration would be ideal for the learners to see the position of the coins.*

TB *I would let the learners 'play' around to find out the probabilities and let them write the findings. Further, I would let them 'mark' the coins as coin 1 and coin 2.*

TC *I will give then practical exercises to do; they will have to engage in tossing coins practically and record the findings, make observations and come to conclusions.*

TD *Let the learners come up with a response; if they're incorrect, confess that I did the same mistake and show them how Pascal's Triangle can help them with the coin problem.*

If a '*practical demonstration*' amounted to showing children in a straightforward way that the position of heads/tails is different from tails/heads, then it means that formal probability will be reached very quickly in terms of teaching it. TA would thus not be prepared to educate children or learners about the tension between intuitive and formal probability. On the other hand, if it meant providing learners with opportunities to '*play*,' '*make observations*' and '*come to conclusions*' or '*come up with a response*' and thus experience the ambiguity and uncertainty in the misconception, then it would imply explicitly educating learners about the tension between intuitive and formal probability. '[M]*ark the coins as coin 1 and coin 2*' was not what I did during his teaching.

This was a suggestion that some teachers came up with as a way to resolve the tension between intuitive and formal probability in the case of tossing two or more coins. In fact, TA, TB, TC and TD's collective responses to the third prompt could be viewed as evidence for ways they might counsel children. In one way or another they want to counsel children on the tension between intuitive or informal and formal probabilistic reasoning. Guidelines that I, together with the teachers, came up with were 'play,' 'make observations,' 'to come to conclusions' and 'let the learners come up with a response'. These are consistent with developing informal conceptions of probability. They are also pedagogically and psychologically responsive to ways of fostering children's conceptions of probability and take into account difficulties that some children might encounter in probabilistic reasoning.

TD's '*confess that I did the same mistake*' regarding the misconception showed evidence of a certain "comfort level [with] ambiguity and uncertainty" (Lord, 1994) and an admission of the psychical roots of intuitive probability in relation to formal probability. This claim is especially evident in the use of the word '*mistake*'. This is the same teacher who wrote: '*it now seems stupid with the understanding of Pascal's Triangle*'. What was still not clear from TD's writing in the third response was whether he/she would organize probabilistic reasoning teaching in the case of the coins so that Pascal's Triangle comes at the end of several investigations or whether it comes out of thin air as a way to cope with the tension between intuitive and formal probabilistic reasoning. Understanding the meanings of the numbers in Pascal's Triangle would certainly help in coping with the ambiguity and uncertainty in the misconception, leading to the correct answer of 1/2 according to formal probability in the case of tossing two coins.

The fourth prompt was further aimed at structuring teacher learning on the interplay between intuitive and formal probability regarding the outcomes of tossing two fair coins:

> What was difficult or unclear about the 2 coins probability question?

TA *Position of the coins.*
TB BLANK
TC *There was nothing unclear; it's just that we did not really think deeply on what was really asked.*
TD *Logical thinking was needed and after a hard day's work, logical thinking goes down the drain.*

If the teachers in all the classes were told explicitly to consider the position of the coins, they would probably have been able to distinguish heads/tails as different from tails/heads. This was not what I did at the beginning when he introduced probabilistic reasoning when tossing a different number of coins. The importance of the positions of the coins came as a result of discussion and debate, in line with the idea of designing, experimenting and studying teacher learning with respect to tension between intuitive or subjective and formal probabilistic reasoning. TA clearly indicated that he/she was now aware of the position of the coins and saw this as a difficulty that caused '*confusion*' (see TA's response to the first prompt). He/she was more aware of a method of resolving this difficulty by focusing on the position of the coins through '*a practical demonstration*' (see TA's response to the third prompt). TB did not have a written response, which I only realised afterwards. TC's articulation — '*we did not really think deeply*' — seemed to show an awareness that he/she had become sensitized to what was '*really asked*'. Did TC show signs of a habit of thought where he or she might '*think deeply*' on what was really asked? It is difficult to say. We can become 'unclear' even in the case of simple probabilities because of their counter-intuitive and ambiguous natures. TD's reference to '*logical thinking*' could imply an awareness that was analytical and amenable to distinguishing between outcomes such as heads/tails and tails/heads. This could mean discerning intuitive aspects of probabilistic reasoning from formal ones. On the other hand, using 'logical thinking' could mean getting straight to formal probability. The question was whether TC's and TD's reports were merely expressions of their desire to cope with or to ignore the misconception they as part of the class encountered during my teaching? What was evident, however, from all the teachers' responses to the prompts was that some teachers learned that there is a subtle and sophisticated relationship between intuitive and formal probability because of their experiences during my teaching. This is consonant with Dewey's notion of the 'psychical roots' of subject matter and method, content and pedagogy.

What we have in these limited excerpts is evidence of teacher learning in probabilistic reasoning in relation to teaching it. They are instances of 'closing the gap' between intuitive and formal probability as reflected in the teachers' and children's probabilistic reasoning as reviewed in the literature.

Discussion

Having made these arguments about teacher learning, however, a couple of caveats are in order. First, should the findings reported be taken seriously? After all they are based on a small-scale qualitative study. If we consider Shaughnessy's (1992) call for the need to unravel teachers' probability concepts, then a small-scale study like this one is a good place to start. The findings provide us with knowledge of effects of putting teacher learning theories with respect to probabilistic reasoning and its teaching 'in harm's way'. The findings drew on the empirical and on literature related to how teachers learn in general and how they learn some of the counter-intuitiveness associated with probabilistic reasoning in relation to teaching it in particular. In teaching the module ACE Mathematics for teaching I designed ways to promote the development of the teachers as a means to study their development. A credible explanation for the teacher learning reported in this study can therefore be attributed to the focused investigation on the tension between intuitive and formal probability. Most of the teachers in the different towns P, Q, R, and S admitted that they had never taught probabilistic reasoning in ways that they experienced during the module. For example, when asked to comment, one of them wrote the following:

(f) Comment on any part of what you learned in DH.

To be honest, I've never taught Data Handling/Probability for more than 2 days. I, firstly, got bored, but now I'm looking forward to it.

Also, a careful read of the written responses shows how the four teachers became aware of the tension between intuitive and formal probabilistic reasoning and what they hypothetically might want to do to address this tension or misconception when it comes to teaching it to children.

A second reason for taking the findings seriously is because they provide us with opportunities to better understand the particular cases of ACE programmes in which mathematics education modules are offered. It seems natural that the interest in particularisation — small-scale studies such as this one — precedes generalisation, i.e. large-scale studies of teacher learning of probabilistic reasoning in relation to teaching it. The findings are a good starting point for working with perhaps the same teachers, in particular because they can compare their situation with what they wrote then. The findings can give principals, education bureaucrats and policy makers an authentic view of the limitations of ACE programmes and what is possible within them. Moreover, the findings can be used to show policy makers how complex teachers' learning about probabilistic reasoning really is.

How does one defend the fact that the findings are about four teachers out of a total of fifty teachers who registered for the Mathematics for Teaching module in the ACE programme? This concern is also connected to qualitative small-scale studies. Not all the teachers completed and handed in the questionnaire. It must be borne in mind that my teaching happened in 'real time' as far as the ACE programme was concerned. Sustained contact time with the teachers during and beyond the ACE programme was not possible. Some of the teachers taught and live in outlying rural towns. There was no large-scale, external funding to support the research reported.

A third reason why the findings should be taken seriously is because they are about a teacher population, albeit very small, that is a subset of the vast majority of teachers at the REQV 13 level in South Africa. To date we know very little about how this teacher population understands and learns probabilistic reasoning as expounded in the current curriculum policy of the South African Department of Education. ACE programmes are specifically targeted at such teachers as a means to familiarise them with the policy statements in RNCS. In terms of mathematical and pedagogical development the findings of the four teachers give an idea of what REQV 13 level teachers might wish to do beyond the 'upgrading' and 'reskilling' of the ACE programme. Moreover, at a practical level these findings speak directly to the types of problems that teachers might have to address in the course of their work when they wish to introduce probabilistic reasoning to children. If the teachers in the study were to go a route similar to their experience during the teaching experiment, they could arrive at what is called Pascal's Triangle. An insightful understanding of Pascal's Triangle will show that it can serve to unify several of the separately stated so-called learning outcomes in the mathematics curriculum. They are likely to revisit their own experiences in probabilistic reasoning encountered in the ACE programme in which the literature connects to those of children's probabilistic reasoning. The latter was a definite concern many of the teachers raised during discussions.

More importantly, it is necessary to adopt a sceptical stance, i.e. 'distance' towards the findings because they are self-reported data. In other words, there is a need to regard the findings as 'speculative.' How should one understand 'speculative' in the case of these findings? In the teacher learning self-report data, notions of legitimacy and genuineness come into play. The latter are taken from Simon's (2000) research on the development of mathematics teachers. It is legitimate for the teachers to respond to the prompts in the questionnaire, i.e. the teachers are the appropriate persons to report on their learning. However, it should be noted that in the context of my pedagogy in the module Mathematics for Teaching the teachers were likely to develop conceptions of the idealised participant. I pointed out to the teachers how the subject matter of formal probability has its psychical roots in intuitive probability. Most of the teachers therefore became aware of their intuitive probabilistic reasoning in the case of tossing different numbers of coins and how that differed from formal or mathematical probability. I made them aware of the possibility that children might experience the same tension between intuitive and formal probability in the case of the tossed coins and other probability concepts in general. The teachers' various written statements should therefore not be seen as evidence of understanding. As particular findings these statements have to be treated with a critical perspective because I was 'working on the inside' studying my own teaching.

Specifically the genuineness of the self-reported data in the findings should be questioned. For example, in a final comment written at the end of the questionnaire, one of the teachers wrote the following:

Thank you — I understand this for the first time in my life !!

This looks like the teacher recorded his or her insight with respect to the complexity of probabilistic reasoning in relation to teaching it. It has to be treated with scepticism, however, and cannot be viewed as a deep personal commitment to understanding the complexity of probabilistic reasoning. In research where one 'works on the inside' there is always the seduction of simple-minded enthusiasm. Is this teacher saying that he or she is now convinced of the ambiguity or counter-intuitive nature — the psychical roots — of even simple probabilities when it comes to teaching? It is hard to say definitively. Data that reflect teachers' involvement in teaching and/or learning situations involving probability concepts from which inferences can be made are needed. The design of ACE programmes does not enable the occurrence of such situations.

Conclusion

Noting the limitations of ACE programmes in general and their noble goals of 'upgrading' and 'reskilling' teachers with an REQV 13, these findings do illuminate our understanding of possibilities about how to align teacher learning with respect to probabilistic reasoning in relation to teaching it. If we were to move beyond speculation, then sustaining disequilibrium between dichotomies, such as subject matter and method and intuitive and formal probability, appears to be a viable option in terms of studying teachers' 'learning to teach' probabilistic reasoning.

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Appendix

I am interested in finding out your understanding of the kind of teaching I am doing in Data Handling (DH) in Learning Outcome 5. I am therefore asking your permission to respond to my questions regarding my teaching of DH.

Do you agree to participate in this research project?

Please circle: Yes No

You do not have to fill in your name anywhere.

Please take a few minutes of your time to respond to the following questions.

I am interested to find out

- ✓ What has helped in your learning of probability?
- ✓ What has hindered in your learning of probability?

Background

How many years have you been teaching?

What grades do you currently teach?

How many kilometres do you drive to attend this class?

1. In the case of 2 coins there was a **misconception** about the probability of getting a head and a tail. Everyone in the class said that this P(head and tail) = 1/3. Why do you think this is so?

2. How do you think this misconception will affect your teaching of the 2 coin probability?

3. What would you do to address this misconception when you one day teach this 2 coin probability problem?

4. What was difficult or unclear about the 2 coin probability question?

5. About the 3 coin probability question?

6. What ideas come in the way of understanding probability questions?

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