Using visual representations to enhance isiXhosa home language learners’ mathematical understanding

Background: Several isiXhosa home language (HL) learners are excluded from meaningful mathematics learning because they are taught in English. Not only do teachers lack epistemological and pedagogical confidence in using multiple languages when teaching mathematics, but there are no mathematical registers for African languages that allow for adequate mathematical teaching and learning. There is a scarcity of research on what constitutes effective mathematics instruction for isiXhosa HL learners in South African language of learning and teaching (LoLT) Grade 1 classrooms.

Aim: The purpose of this study was to explore the experiences of Grade 1 teachers using visual representations to enhance isiXhosa HL learners’ understanding of mathematics in the English-LoLT in Grade 1 classrooms.

Setting: This study was conducted at four primary schools in the Western Cape’s Metro East Education District.

Methods: This study employs a qualitative research approach in conjunction with an adapted interactive qualitative analysis (IQA) systems method to collect in-depth data about current mathematics practices in English LoLT in Grade 1 classrooms. The data were analysed using John Stuart Mill’s analytical comparison technique.

Results: This study found that semiotics such as visual (and concrete) representations assist isiXhosa HL learners to grasp and understand mathematical concepts easily.

Conclusion: This study emphasises the significance of using sufficient visual representation strategies to enhance isiXhosa HL learners’ mathematical understanding in the English LoLT in Grade 1 classrooms.

Contribution: The outcomes of this study can make a positive contribution to current mathematics practice in terms of supporting isiXhosa HL learners in English LoLT in Grade 1 classrooms.

Keywords: English language of learning and teaching (LoLT); Grade 1 classrooms; isiXhosa home language learners; mathematical understanding; visual representations.

Introduction

Language is an important component in the teaching and learning of mathematics, serving both communicative and cognitive functions (Prediger, Erath & Moser Opitz 2019). As a result, in South Africa’s multilingual context, language is critical in teaching and learning, especially during early childhood development (Department of Basic Education South Africa 2010; Sosibo 2015). According to the Department of Basic Education South Africa (2010) and Sosibo (2015), a learner’s native language aids in the organisation of their experiences and thoughts, which are then connected to academic achievement. However, several isiXhosa home language (HLL)-speaking learners in South Africa are excluded from meaningful mathematics learning opportunities because they are taught in English, a language in which they lack fluency (Machaba 2018; Robertson & Graven 2020). This is seen as one of the reasons why South African schools perform poorly in mathematics. Although various research-based initiatives were undertaken, the government’s opportunity to reform African language terminology in 1994 was taken for granted, as the last time they attempted to reform African language terminologies was in 1996. Mtsatse and Combrinck (2018) state that academic and technical accuracy in African languages has since not reached the same level as English or Afrikaans. Therefore, no adequate registers or terminology for mathematics in African languages have been developed to date, which is another reason why
South Africa’s mathematics performance is among the lowest in the world (Essien 2018; Robertson & Graven 2020). Furthermore, regardless of whether isiXhosa HL learners are enrolled in an isiXhosa or English language of learning and teaching (LoLT) classroom, the majority of teachers tend to employ English as the language of instruction for teaching mathematics (Robertson & Graven 2020). However, teaching and learning mathematics in English, which is a second or third language for the majority of the isiXhosa learners, poses major challenges for them. In this sense, isiXhosa learners must not only learn mathematics but also the new language (i.e. English) in which it is taught (Essien 2018; Robertson & Graven 2020).

Although recent research by Robertson and Graven (2020) indicates that teachers lack epistemological and pedagogical confidence in the principled use of multiple languages when teaching mathematics, there are still no proper mathematical registers for African languages that allow for adequate mathematical teaching and learning. As such, it is necessary to acknowledge the unique contexts of current South African classrooms, where most African language learners receive education in English as the language of instruction. Thus, teachers of mathematics should implement appropriate teaching and learning strategies to help English language learners (ELLs) with insufficient fluency in the English-LoLT become proficient users of the English academic register for learning mathematics (Robertson & Graven 2020). Research on what constitutes effective mathematics instruction for isiXhosa HL learners in South African English Grade 1 classrooms is, however, scarce. Hence, this study seeks to explore Grade 1 teachers’ experiences of using visual representations (as a strategy) to enhance isiXhosa HL learners’ understanding of mathematics in English-LoLT Grade 1 classrooms. The introduction is followed by a literature review and a theoretical framework. The research methods and design are outlined, followed by the findings and discussion of the findings. The article finishes with a conclusion and several recommendations regarding visual representations to enhance mathematical understanding.

**Literature review**

**The relationship between language and mathematics**

Mathematics has its own language and means of communication for sharing knowledge (Pimm 1987). Therefore, mathematical language is considered a different ‘register’ within an everyday language, such as English (Le Cordeur & Tshuma 2019). The mathematics register incorporates mathematical terminology used to explain and narrate various mathematical scenarios (Jourdain & Sharma 2016; Le Cordeur & Tshuma 2019). The mathematics register, however, extends beyond just technical terms. In a particular context, the mathematics register also utilises the everyday language (i.e. English-LoLT) to express words, phrases and critical thinking processes (Pimm 1987). The grammatical structure and vocabulary of any specialised language express different thoughts. Therefore, each language has its own mathematical register, which includes different ways of expressing mathematical thinking (Le Cordeur & Tshuma 2019; Ní Riordáin, Coben & Miller-Reilly 2015). Clearly, the intricate ‘register’ of mathematics resembles that of a language and requires the same language-learning abilities as any second or third language. Robertson and Graven (2019) suggest that incorporating second language teaching and learning methods is necessary to promote diversity in mathematics classrooms. This lends mathematics a new dynamic and shows that it cannot be taught without language. Therefore, learning mathematics involves acquiring the specific mathematics register (Setati 2005). Without this facility, learners are incapable of developing or adapting their understandings of mathematics (Meaney 2005). After mastering the mathematics register, learners are able to listen, inquire, debate, read, write and do mathematical operations. However, when many isiXhosa learners in English-LoLT classrooms are unable to tell which register is being used, mathematics and everyday language registers can get in the way of learning and become a significant challenge (Ní Riordáin et al. 2015).

In addition, mathematics is viewed as a type of discourse (or conversation), thus constituting ‘language in use’ (Rymes 2016; Sfard 2012). In this respect, mathematical discourse is more than merely a spoken, written and technical language (Ní Riordáin et al. 2015). Gee (1996) describes discourse as a socially acknowledged relationship between language usage, other symbolic representations and objects of thought, experiences, perceptions and activities. According to Moschkovich (2012:95), ‘Mathematics discourse practices are social, cultural, and discursive because they originate from communities and signify participation in various discourse communities’. Mathematical discourses, like other cognitive discourses, involve symbols, reasoning, resources and interpretations. Depending on the context, words, phrases and texts have many interpretations, purposes and objectives. Mathematics discourses take place within the environment of practices that are linked to communities. Moreover, mathematical discourse practices are built on activities, sense-making, recognition and objectives (Moschkovich 2012). Speaking, writing, gestures, symbols and visual representations are all examples of mathematical discourse (Robertson & Graven 2020). These examples show that mathematical discourse needs a lot of ‘semiotic integration’ to support mathematical understanding and proficiency (Robertson 2017).

**Mathematical proficiency**

To clearly understand what it means to be proficient in mathematics, one must be acquainted with Kilpatrick, Swafford and Findell’s (2001) five interconnected strands for mathematical proficiency. These five strands include procedural fluency, conceptual understanding, adaptive reasoning, strategic competence and a productive disposition. On the basis of these five strands, the Department of Basic Education South Africa (2018) developed a mathematics teaching and learning framework for South Africa as part of its ‘Teaching Mathematics for Understanding’ strategy to
create a well-grounded foundation for a new approach to teaching mathematics and revolutionise how it is taught. This framework, in collaboration with the Curriculum and Assessment Policy Statement, is intended to give South African teachers choices and new ways of thinking about mathematics teaching, learning and assessment in an effort to raise South Africa’s low mathematical ranking (Department of Basic Education South Africa 2018). However, the execution of the five strands of mathematical proficiency within the Department of Basic Education’s most recent teaching and learning framework for mathematics does not accommodate for language diversity when teaching mathematics for proficiency. Therefore, if mathematics teachers do not have suitable teaching and learning strategies in place to scaffold learning for learners with poor fluency in the English-LoLT, language may pose a barrier to ELLs’ understanding and acquiring proficiency in mathematics (Robertson & Graven 2020). Given that many African language learners are not proficient in English-LoLT and mathematics is not taught in their HL, it is crucial to provide meaningful pedagogy for learning mathematics and to devise specialised strategies to help learners with diverse language and mathematical abilities (Essien 2018; Robertson & Graven 2020).

**Support strategies for English language learners’ understanding of mathematics**

Theories addressing the nature of mathematics and the learning of mathematics can be used to outline an approach specifically related to the subject of mathematics instruction. These theories are specific strategies that a teacher employs to facilitate and improve mathematical teaching and learning (Khanal 2015). In addition, Kusumawati and Nayazik (2018) believe that a mathematics learning strategy is a blueprint for conducting mathematics learning activities to attain the intended outcomes. It determines the best appropriate technique, method, theory, resource and learning strategy. Moreover, the mathematics learning strategy is not only built on the learner’s measured ability or skill but also on the educational resources, the learner’s prior knowledge, the available time and resources and the teachers’ attitude and experience. To build an effective blueprint (i.e. learning strategy), a teacher must be knowledgeable about the different types of learning methods and their related advantages and disadvantages. Therefore, it is essential for teachers to recognise the individual diversity in learning and acknowledge the necessity of adopting a balanced approach by adapting their own teaching methods and using a variety of teaching and learning strategies (Khanal 2015).

As mathematics is a language of its own and requires the same language-learning strategies as any other language, experts have differing opinions regarding the definition of language-learning strategies, but the majority of them agree that language-learning strategies are the procedures, actions and methods that learners employ to enhance and improve their language learning. These strategies focus on how learners respond to information and the strategies they use (Hardan 2013). O’Malley and Chamot (1990) categorise language learning strategies into three groups: cognitive, metacognitive and socio-affective strategies. Oxford (1990) also categorises language learning strategies, dividing them into six sub-groups, namely: memory, cognitive and compensating strategies (i.e. direct strategies) and metacognitive, emotional and social strategies (i.e. indirect strategies). In line with the various perspectives on language learning strategies and Essien’s (2018) claim that there are no clear language learning strategies that are relevant to learning mathematics in the early years, Kusumawati and Nayazik (2018) emphasise the critical role and responsibility of teachers in using appropriate teaching and learning strategies (i.e. visual representations) to support the mathematical understanding of ELLs.

**Theoretical framework**

The function that language plays in the acquisition of mathematics in a second language was described using Lev Vygotsky’s (1978) social constructivism theory of learning. According to Piaget’s (1950) stages of cognitive development, the learner in the early years of childhood development is gradually moving from pre-operational to concrete-operational thinking, where learners begin to think logically about concrete events. During this phase, there is rapid development in the cognitive field. Learners can process more information faster, and their memory capacity increases (Piaget & Inhelder 1973; Woolfolk 2007). With the viewpoint of isiXhosa HL learners receiving mathematics education in English as the language of instruction – a language in which they are not all proficient – we sought a theoretical framework that could illustrate how mathematics is learned for proficiency from the standpoint of ELLs (i.e. isiXhosa HL learners) learning mathematics in a second language (i.e. English-LoLT). Thus, the empirical component of this study was analysed using Vygotsky’s learning theory (1978) and Kilpatrick et al.’s (2001) five-stranded model of mathematical proficiency, both of which are concerned with cognitive development and the development of children’s mathematical proficiency. According to Vygotsky’s (1978) theory of learning, interpersonal interactions are the product of cognition, emphasising the importance of language in cognitive growth. This is predicated on the notion that a person’s ability to grasp and make sense of their surroundings is acquired through communication and socio-cultural experiences, both of which influence cognitive development (Das 2020; Robertson 2017). Effective mathematics teaching and learning are based on Kilpatrick et al.’s (2001) five interwoven strands of mathematical proficiency, which are known as ‘conceptual understanding’, ‘procedural fluency’, ‘strategic competence’, ‘adaptive reasoning’ and ‘productive disposition’. Kilpatrick et al.’s (2001) mathematical proficiency model is described by Ramollo (2014) as ‘various strands that contain knowledge about the interaction between the teacher, the learner, and the content and embrace the context to successfully acquire mathematics’. Within this framework, the demands of ELLs
(i.e. isiXhosa HL learners) to become proficient in mathematics should be met within each of the five strands.

In conclusion, Vygotsky’s theory of learning illustrates the importance of language and communication through a multi-semiotic approach to cognitive learning and development. The term ‘scaffolding’ refers to the teacher’s support efforts that are provided to learners based on their individual needs to help them reach their learning goals (Presmeg et al. 2016). In this study, the scaffolding is tailored to the needs of isiXhosa HL learners (i.e. ELLs) with the aim of fostering mathematical proficiency. This means that the teacher must be able to put together and use different semiotic strategies (scaffolds) to help the isiXhosa HL learner become proficient in mathematics (Kilpatrick et al. 2001; Presmeg et al. 2016). Therefore, scaffolding should occur across all five strands in which teachers instruct each learner to become proficient in mathematics (Kilpatrick et al. 2001). Figure 1 illustrates how Vygotsky’s theory of learning and Kilpatrick et al.’s five-stranded model work together to explain how mathematics is learned and how to become proficient at it.

Research methods and design
Study design
This study’s aim – to explore the experiences of Grade 1 teachers’ using visual representations to enhance isiXhosa HL learners’ understanding of mathematics in English-LoLT Grade 1 classrooms – was addressed by employing a qualitative interpretive case study using Northcutt and McCoy’s (2004) interactive qualitative analysis (IQA) systems method approach. Unstructured, open-ended focus group interviews, semi-structured individual interviews and lesson observations were used to explore the selected Grade 1 teachers’ experiences with using visual representations to enhance isiXhosa HL learners’ understanding of mathematics in current English-LoLT Grade 1 mathematics practises.

Setting
This study was conducted in September 2021 at private and public primary schools in the Metro East Education District of the Western Cape. The participating teachers varied from novices to experienced teachers who teach mathematics in Grade 1 English-LoLT classrooms. Although the LoLT of the classrooms is English, more than a quarter of the learners speak isiXhosa as their HL, with 41% representing the greatest proportion of learners whose HL is different from that of the classrooms’ LoLT. The ratio of teachers to learners varied between 1:29 and 1:37.

Population and sampling
A purposive sampling technique was conducted, and a sample was drawn from the population by intentionally selecting four schools located in the Metro East Education District of the Western Cape, with isiXhosa HL learners in English-LoLT Grade 1 classrooms and 11 Grade 1 teachers.


ZPD, zone of proximal development.

FIGURE 1: The interconnection between Vygotsky’s learning theory (1978) and Kilpatrick et al.’s (2001) five-stranded model of mathematical proficiency model.
(nine teachers representing three public primary schools and two teachers representing one independent primary school) as participants for this research study (Cohen, Manion & Morrison 2018). We chose the participants so they could provide a full account of their perceptions, experiences and feelings about using visual representations to enhance isiXhosa HL learners’ understanding of mathematics (Mohajan 2018).

Data collection
Information was acquired using a customised IQA data collection method (Northcutt & McCoy 2004). Unstructured, open-ended focus group interviews and semi-structured individual interviews are generally used to obtain data in original IQA investigations, but data gathering was complemented with observations of mathematics lessons. Consequently, two focus group interviews were conducted to collect data for the development of an interview framework. This framework guided the data collection during the semi-structured individual interviews and mathematics lesson observations that followed (Bargate 2014; Northcutt & McCoy 2004).

Data analysis
Stage 1: Unstructured, open-ended focus group interviews
The data analysis process consisted of three distinct stages. The participants of the unstructured, open-ended, focus group interviews were involved in the initial stage of data processing. During this interactive session, the participants of the focus group brainstormed and recorded on index cards (inductive analysis) their perceptions, experiences and feelings surrounding the research statement, ‘Tell me about your experiences of using visual representations to enhance mathematical understanding’. The brainstorming exercise was followed by a deductive analysis exercise in which the participants sorted and clustered the written cards into groups that represented the themes. Participants provided a descriptive paragraph for each theme. These themes and paragraphs then served as the framework for the individual interviews that followed.

Stage 2: Semi-structured individual interviews
In the second part of the data analysis, the transcribed semi-structured individual interviews were analysed using John Stuart Mill’s analytical comparison technique (Neuman 2014) to look for patterns and try to understand, describe, explain and draw conclusions about the participants’ knowledge, perspectives and experiences with using visual representations to enhance isiXhosa HL learners’ understanding of mathematics.

Stage 3: Observations of mathematics lessons
The third stage involved the analysis of the transcripts of the field observations of mathematics lessons to obtain a chronological perception of what happened in each of the individual interviewed participants’ lessons when they taught mathematics, using the same analytical comparison technique used in the latter stage. By comparing the data analysis of the individual interview transcriptions with the data analysis of the field notes from the mathematics lesson observations, a synthesis was created. After gathering and analysing the data, we carried out a literature review and verified the results.

Interactive qualitative analysis interview techniques, data recording methods and member checking were used to assure the descriptive validity of the data. The interpretive validity, which was intertwined with the descriptive validity, included members contributing their own data and analysis during the focus group interviews. Furthermore, transcripts of individual interviews and classroom observations included both verbal and nonverbal answers elicited from participants, which explained the data provided during the focus group interviews. After the focus group interviews, in which participants defined the themes and subthemes, we undertook a literature review to establish theoretical validity. Literature and direct quotations were used to support the themes and subthemes. Conclusions were reached based on the findings from the data and the literature. Lastly, the evaluative validity was established by drawing conclusions from the data analysis, literature review and theoretical framework.

Ethical considerations
Ethical clearance to conduct this study was obtained from the Cape Peninsula University of Technology (CPUT) Faculty of Education and the Western Cape Education Department (WCED) (No. EFEC 3-12/2019 (CPUT Faculty of Education) No. 20200220-4737 (WCED).

Findings and discussion
The findings of this study are presented as they occurred within each stage of data analysis and are supported by congruent findings between the current research and existing literature.

Stage 1: Results of the two unstructured open-ended focus group interviews
Table 1 depicts the themes, cards (thoughts) and descriptive paragraphs provided by the teacher participants in both focus group interviews. Using John Stuart Mill’s Analytic Comparison technique (Neuman 2014), the analysis of the two focus groups’ themes in the table revealed the first pattern: where that both themes, ‘Visual Aids’ and ‘Holistic and Concrete Learning’, were found to have similar content. Thus, a decision was made to group together these two themes, as they had similar or
complementary content (method of agreement), into one theme, namely, ‘Visual representations’.

Stage 2: Results of the semi-structured individual interviews of the theme ‘Visual representations’

Using the method of agreement and the method of difference, the transcripts of the individual interview participants pertaining to the theme ‘Visual representations’ were analysed.

According to Banse et al. (2016), visual representations, whether concrete objects or visual illustrations, are employed in mathematics education to assist learners establish connections to what they already know as a means of scaffolding and communicating their own thinking. This approach complements Arzarello et al. (2009) concept of the ‘semiotic bundle’, which consists of many semiotic resources (i.e. different semiotic strategies) provided by the teachers to make meaning of mathematics activities, such as printed text, pictures, drawings, gestures and physical objects. Furthermore, Chikiva and Schäfer (2019) claim that visual support strategies will benefit ELLs who do not have language abilities equivalent to their mathematics proficiency. In this regard, participants stated how they employed visual representations, complemented in some cases by the learners’ bodies and/or dramatisation activities:

‘... I have a lot of visual apparatus that helps the learners [isiXhosa learners] because they do not need to read or to understand something in English to be able to do those activities … they can understand and know what to do without necessarily being well-versed in English.’ (P7, Educator, Age 25)

‘... it’s important in order to teach a topic that you do have visual aids for learners to make sense [of mathematics]. I also incorporate either something where I need to act out [or] ... create a scenario.’ (P1, Educator, Age 30)

‘... so you would do a lot of dramatisation and get them to understand the concept.’ (P9, Educator, Age 62)

Participants also discussed their experiences with physically manipulating concrete objects during mathematics instruction, including enhancing the manipulation of concrete apparatus with games. According to Chikiva and Schafer (2019), it is important to give ELLs the chance to play with (or manipulate) real objects because it helps them learn mathematical concepts and terms. Participants stated:

‘... it’s important in order to teach a topic that you do have visual aids for learners to make sense [of mathematics] ... that learners can physically see things that they can physically touch, pack out and understand … especially for learners that struggle to grasp [mathematical] concepts.’ (P1, Educator, Age 30)

‘... when you break down the language into something that is visual or concrete, they are then able to actually understand what it means … and connect a visual image or concrete theme to the language … I enjoy using things like that with the rolling of the dice, and they each get a chance to roll their own dice. It makes it more fun for them and it almost feels like a game, and I think they are then more receptive to learning a new concept.’ (P7, Educator, Age 25)

‘I wish we could teach all concepts using a concrete activity with it because that would be the best way to reach non-English speakers, that is just going to make maths more sense to them.’ (P11, Educator, Age 30)

Participants clearly experienced ‘Visual representations’ as one of the most successful methods (or semiotic resources) for clarifying and simplifying mathematical concepts for isiXhosa HL learners in Grade 1 classrooms where the LoLT is English.

‘I don’t really use anything else besides that [visual aids] ...’ (P1, Educator, Age 30)

Stage 3: Results of the field notes of the mathematics lesson observations of the theme ‘Visual representations’

Not only did the discussions above reveal that all the participants believed in the scaffolding of various visual representation strategies to accommodate mathematical understanding, but they also put their beliefs into practice. The classroom observations showed that teachers used visual representations as a semiotic resource in the form of images, drawings and concrete objects to support the isiXhosa HL learners throughout their mathematical instruction. The method of agreement verified that all participants utilised visual representations to improve their

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**TABLE 1:** Themes, cards (thoughts) and descriptive paragraphs collected during the unstructured, open-ended focus group interviews of Focus Groups 1 and 2.

<table>
<thead>
<tr>
<th>Cards consisting of a brainstorming activity of teachers’ perceptions, experiences and feelings</th>
<th>Themes</th>
<th>Descriptive paragraphs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focus group 1 – Open-ended focus group interview</td>
<td>Having resources makes it easier to teach the concept as learners understand best by visuals [3×]</td>
<td>Visual aids</td>
</tr>
<tr>
<td>Focus group 2 – Open-ended focus group interview</td>
<td>Using concrete activities and objects or apparatus (Learners need to actually ‘feel’ to ‘do’ the problem) [6×]</td>
<td>Holistic and concrete learning</td>
</tr>
</tbody>
</table>

[http://www.sajce.co.za](http://www.sajce.co.za)
learners’ mathematical understanding. Some examples of these kinds of representations are drawing math problems, using pictures (or drawings) to help with number sense and manipulating real objects to help learners understand difficult concepts. Table 2 represents two examples of such activities that were observed during the mathematics lesson observations.

The primary objective of this article was to report on the experiences of Grade 1 teachers’ using visual representations to enhance isiXhosa HL learners’ understanding of mathematics in English-LoLT Grade 1 classrooms. This study found that semiotics such as visual (and concrete) resources (i.e. the ‘math wall’, colourful counters, base ten blocks, playdough, math games, number line, 100-chart, fingers, dice, a whiteboard and other everyday objects in class) assisted the isiXhosa HL learners to grasp and understand mathematical concepts easily.

### Conclusion and recommendations

In this article, Grade 1 teachers’ experiences of using visual representations to enhance isiXhosa HL learners’ understanding of mathematics in English-LoLT Grade 1 classrooms were addressed. The findings discussed in the previous section lead to some recommendations for teachers that emphasise the significance of using sufficient visual representation strategies (i.e. semiotic resources) to assist ELLs’ mathematical understanding within each of the five strands of the mathematical proficiency model. In this regard, it is recommended that teachers utilise visual representations and concrete resources to assist isiXhosa HL learners to grasp and understand mathematical concepts more easily (Chikiwa & Schäfer 2019). This means that these learners should not only look at visual representations but also get the opportunity to manipulate concrete and visual resources. It is further suggested that dramatisation, together with visual representations (i.e. drawing pictures of ‘story sums’ on the board while the teacher also acts out the story), improves isiXhosa learners’ conceptual understanding (Chou 2021). It is further recommended that teachers teach and equip isiXhosa HL learners to solve mathematical problems with more confidence and use visual step-by-step methods to teach mathematics (Banse et al. 2016; Kilpatrick et al. 2001). The results of this study make it possible to have a positive outlook on the study’s outcomes, noting that they may contribute to current mathematics practices in terms of support for isiXhosa HL learners in Grade 1 English-LoLT classrooms, different support systems and their roles and responsibilities for isiXhosa HL learners and teachers in an inclusive education and training system and the current implementation of the mathematics teaching and learning framework in South Africa.

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### Competing interests

The authors declare that they have no financial or personal relationships that may have inappropriately influenced them in writing this article.

### Authors’ contributions

C.L. served as the main supervisor and E.B. as co-supervisor for T.C.’s doctoral study. C.L. and E.B. gave input as co-authors of this article, while T.C. wrote the majority of the article.

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### Data availability

Data sharing is not applicable to this article as no new data were created or analysed in this study.

### Disclaimer

The views and opinions expressed in this article are those of the authors and do not necessarily reflect the official policy or position of any affiliated agency of the authors.

### References


PLACEHOLDER

**TABLE 2: Activities observed during mathematics lessons related to the theme ‘Visual representations’**

<table>
<thead>
<tr>
<th>Participants</th>
<th>Activities observed during participants’ mathematics lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>P9</td>
<td>The teacher posed a problem to the learners. She tells them they have 13 strawberries. She then asked if it was an even or an odd number. The learners replied that it was an odd number. Now the teacher instructed the learners to draw the 13 strawberries like she showed them to do on the board with the previous word problem. The learners drew the dots on their whiteboards. The teacher reminded them to make sure they kept enough space between the groups of strawberries. The teacher instructed the learners to share the strawberries ‘between’ three children. She reminds them to cross one out and put it on the next plate, so they will not get confused.</td>
</tr>
<tr>
<td>P7</td>
<td>Learners had to do sums in their books. The teacher gave one of the isixhosa learners counters to assist him with addition sums. She told the learner to pack out the counters on top of each number accordingly and add them together.</td>
</tr>
</tbody>
</table>