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Eleven Grade 1 teachers' understandings of mathematical language in a South African context

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Background: Fluency in mathematical language is essential for learning mathematics. Teachers must understand and use their diverse mathematical knowledge, including language and communication difficulties inherent to mathematics instruction. According to recent South African research, Grade 1 teachers are not equipped to utilise learners' linguistic skills for efficient learning of mathematics.

Objectives: This research investigates South African Grade 1 teachers' mathematical language perceptions, experiences, and feelings. These Grade 1 teachers' transcripts were analysed to discover their understanding of the language of mathematics.

Method: Exploratory, descriptive, and contextual research designs were used in conjunction with an adapted interactive qualitative analysis technique. Focus group interviews, individual interviews, and lesson observations, together with a purposive sampling technique, were used to gather the data from both public and private primary schools.

Results: The results showed that Grade 1 teachers view mathematics as a separate language with its own vocabulary and register. The findings highlighted the need to simplify the language of mathematics to enhance understanding.

Conclusion: This research concluded that language is essential to mathematics learning and that mathematics has its own register, which is acquired like any other additional language. To help isiXhosa learners understand mathematics in English, scaffolding strategies must be aligned with their linguistic demands.

Contribution: This article provides important recommendations for teachers who need to recognise the reality that English is the lingua franca and ensure isiXhosa home language-speaking learners receive the necessary support to acquire actual proficiency in the academic register of English for mathematical language learning.

Keywords: English Grade 1 classrooms; Grade 1 teachers; isiXhosa home language-speaking learners; mathematical language learning; scaffolding strategies; South African context; understanding of mathematical language.

Introduction

One of the primary reasons for South African learners' poor performance in mathematics is that the significant role of language in education is overlooked in curriculum and teacher-training courses, resulting in a lack of language awareness. As a result, inadequate teaching strategies lead to language difficulties in curriculum areas such as mathematics (Robertson & Graven 2020:86). One of the crucial imperatives that was taken on by the South African government in 1994 was language planning. In this regard, as a way for the government to carry out this responsibility, the Department of Basic Education agreed to draft linguistic policies and identify strategies to improve the state of African languages (Department of Basic Education 2010; Yu & Dumisa 2015). In order to promote and develop African languages, the Pan-South African Language Board (PanSALB) was subsequently established. The PanSALB was founded as a statutorily mandated authority to oversee and enforce the implementation of all 11 official languages of South Africa. The PanSALB has organised lexicography components for each official language to ensure terminology uniformity and promote multilingualism (Mtsatse & Combrinck 2018:22). However, despite the importance of language planning on the reunification agenda, it was evident that much of the African language terminology had not been updated since the apartheid era and is still in use today (Mtsatse & Combrinck 2018:22). Moreover, according to Webb (2013), the condition of African languages has not changed significantly since 1996. Mtsatse and Combrinck (2018:22) claim that the government has produced much too little evidence that South Africa's language policy has effectively promoted the 'graphisation' and 'codification' of African languages. Mtsatse and Combrinck assert that African languages have not been used meaningfully outside of formal schooling frameworks, such as in parliament, courts, academic institutions, schools, and the media in general. Due to the absence of official social systems in which African languages are utilised, it appears that Afrikaans and English maintain a higher linguistic status than African languages (Mtsatse & Combrinck 2018). Van Laren and Goba (2013) and Mtsatse and Combrinck claim that the standardisation of African languages is an ongoing difficulty because it is difficult to locate precise, widely accepted vocabulary for mathematics translated from English into an African language. It seems unlikely that academic registers in all African languages will be established in the near future, especially due to the influence and status of English as the lingua franca (Essien 2018; Robertson & Graven 2020). Consequently, concerns regarding equal educational opportunities to become a successful academic participant in mathematics and the necessary language skills required to maximise learners' potential to really make sense of mathematics still persist (Engelbrecht et al. 2016; UNESCO 2016; UN 2018).

Significance of the study

Robertson and Graven (2020) claim that sustained and systematic investment is crucial in devising strategies to ensure that African language-speaking learners are provided the necessary support to achieve actual proficiency in the academic register of English for mathematical learning. Recent research, however, shows that there is a gap between what 'good practice' might look like in South African classrooms, building on what already happens in them, and the reality of these classrooms and how teachers perceive and deal with African language-speaking learners within their classrooms (Essien 2018; Mtsatse & Combrinck 2018; Robertson & Graven 2020).

This article is derived from a full doctoral thesis. However, as the aim of this research was to explore, describe, and understand what Grade 1 teachers' understandings of mathematical language are in a South African context, the researchers only report on one of the sub-research questions, namely: 'What are Grade 1 teachers' understandings of mathematical language in the context of South Africa?' In answering this research question, we add to a body of knowledge from which recommendations for Grade 1 teachers and their understanding of mathematical language were developed, allowing them to provide the necessary mathematical language support to isiXhosa home language-speaking learners receiving mathematics education in an English Grade 1 classroom.

The article begins with an introduction, followed by an explanation of the significance of the study, a literature review, and a theoretical framework. The research design and method are described, followed by the results and deliberation of the findings. The article concludes with a conclusion and recommendations.

Literature review

Understanding the relationship between language and mathematics

Mulaudzi (2016) highlights the significance of language for effective mathematical teaching and learning. He emphasises that the language of learning and teaching (LoLT), through which mathematics is learned, creates the basis for mathematical teaching and learning to evolve in that language. Extensive research confirms that many language qualities influence cognitive development (Barwell 2016; Das 2020; Kotzé 2016; Robertson & Graven 2020; Sfard 2012). Despite the seeming complexity and inconsistency of the literature on the relationship between language and mathematics, there is a consensus that thinking occurs in some language or another (Robertson 2017). Vygotsky (1962) stressed that language and thought are inextricably linked and interdependent. Therefore, it can be argued that external talk brings a thought to life, whereas inner speech focuses energy on words to facilitate thought processing. In light of the latter premise, language influences the thought process. However, the act of a thought has its own structure, while the act of translating the thought into language is challenging. This is not an unconscious process because thought only develops and is expressed through language. Signs outside mediate thought, while word meanings internally mediate thought (Vygotsky 1962). Bruner (1975) highlights the significance of language as a medium for thought as well as its influence on cognitive development. Consequently, thought is tightly bound to language and must adapt to it (Robertson 2017).

The mathematics register is a distinct manner of using language and communicating information (Lee 2006; Pimm 1987). Thus, mathematical language is considered as a distinct 'register' inside an everyday language (e.g. English – 'any language that has evolved spontaneously in humans via usage and repetition without conscious design or premeditation') (Le Cordeur & Tshuma 2019:107). In addition to the specialised vocabulary used in mathematics, the mathematics register also includes the language used to describe or explain a specific sort of mathematical problem (Jourdain & Sharma 2016; Le Cordeur & Tshuma 2019). On the other hand, the mathematics register encompasses a great deal more than just lexicon and technical terms. It also includes phrases, words, and reasoning approaches in a specific context, all of which are expressed using everyday language (e.g. English) (Pimm 1987). The grammatical structure and lexicon of a specialised language (i.e. mathematics) allow for the communication of a wide range of concepts. Consequently, each language has its own mathematical register, which incorporates the various methods in which mathematical meaning is communicated in that language (Ní Ríordáin, Coben & Miller-Reilly 2015; Le Cordeur & Tshuma 2019).

As can be seen, the complicated 'register' of mathematics is comparable to that of a language, and, as such, it needs similar language-acquisition skills. In line with this perspective, Robertson and Graven (2019) argue that mathematics teachers must discover ways to incorporate second-language teaching and learning strategies into their pedagogical repertoires to increase inclusivity in mathematics classrooms. This adds a new layer to mathematics education and reinforces the view that mathematics content cannot be taught without language. Learning mathematics comprises the acquisition of the mathematics register automatically (Setati 2005). According to Meaney (2005), this enables learners to convey their mathematical reasoning with ease; 'without this fluency, learners are limited in their ability to develop or rearrange their mathematical understandings'. Once they have mastered the mathematical register, learners will be able to listen, ask questions, and converse, as well as read, record, and engage in mathematics. Similarly, registers can be utilised in a variety of domains, as well as in any LoLT. Often, subtly, mathematics and ordinary language registers may be inhibited in an educational environment. Learners must therefore be able to recognise each register in order to decide which is being used at any given time, which is a difficult task for many English language learners (ELLs) in multilingual classrooms such as South Africa. Thus, it is essential to understand how different languages and registers function, as well as the fact that the usage of numerous languages and registers can aid a learner's improvement in mathematical understanding (Ní Ríordáin et al. 2015).

Mathematics is a discourse and a type of communication, which is more than just language (Sfard 2012). Rymes (2016:5) describes discourse as 'language in use'. Gee (1996) defines discourse as:

> 'A socially accepted association among ways of using language, other symbolic expressions, and artefacts of thinking, feeling, believing, valuing, and acting that can be used to identify oneself as a member of a socially meaningful group or social network, or signal [*that one is playing*] a socially meaningful role.' (p. 131)

According to this explanation, discourses involve more than the use of specialised terminology and spoken or written language. Communities, viewpoints, values, and beliefs all contribute to discourse in this regard (Ní Ríordáin et al. 2015). Moschkovich (2012a:95) defines mathematics discourse practices as 'social, cultural, and discursive because they come from communities and mark membership in different discourse communities' to emphasise the notion that discourses are anchored in sociocultural practices. Furthermore, mathematical discourses are cognitive in character since they make use of thought, signs, tools, and meanings. Words, phrases, and texts have different meanings, uses, and intentions depending on the context. Mathematical discourses occur within the framework of practices that are connected to communities. Moschkovich (2012a) continues by stating that mathematical discourse practices are generated through actions, sense-making, focus of attention, and aims, and are anchored in social practices. As a result, multi-semiotic systems are included in mathematical discourse (e.g. speech, text, gestures, symbols, and visual images) (Arzarello et al. 2009; Robertson & Graven 2020).

Mathematical registers

The difficulty of functioning within English language registers is one of the most significant obstacles that ELLs experience while attempting to learn mathematics in an English-LoLT classroom (Robertson & Graven 2020). Halliday (1978) defines the mathematical register as a set of meanings suitable for a particular linguistic feature, along with the terminology and structures that represent these meanings. Thus, the register of mathematics is a sense of mathematical language-specific meanings. In mathematics classrooms, multiple registers are utilised. To succeed in mathematics, learners must not only be comfortable with and educated about their everyday English register, but also be fluent in several mathematical registers (Essien 2018; Moschkovich 2005; Schleppegrell 2011). For the understanding of mathematical registers and the ability to switch between them, strong language and metalinguistic skills are necessary. These skills are necessary for learners to communicate with their peers and cope with more complex mathematics (Mandy & Garbati 2014; Planas & Setati-Phakeng 2014; Robertson & Graven 2020).

As a new type of language that must be taught and mastered, mathematical registers provide a major challenge for ELLs (Setati 2008). This is a specific challenge in South Africa, where mathematics achievement is inadequate due to ELLs' inadequate language skills, as they must master a specialised mathematical language to comprehend the mathematics curriculum's content (Department of Basic Education 2011; Mulaudzi 2016; Sibanda & Graven 2018; Robertson & Graven 2020). The ELL must not only seek to learn in English while also learning to speak English but must also operate within the English mathematical register before mastering everyday English (Essien 2018; Mandy & Garbati 2014; Robertson & Graven 2020). Therefore, in order to fully understand mathematics in a classroom where English is the LoLT, ELLs may require more processing time compared to English mother-tongue speakers (Mulaudzi 2016; Setati 2008). Consequently, these learners may fail to keep up in mathematics because they spend too much time attempting to understand and switch between registers (Jourdain & Sharma 2016; Machaba 2018).

Furthermore, understanding mathematics-specific vocabulary is required for learners to be proficient in mathematics (Moschkovich 2005; Mulaudzi 2016; Robertson & Graven 2020). This mathematical vocabulary is not used in everyday English registers; hence, speakers of other languages have difficulty understanding it (Jourdain & Sharma 2016). Not only are the technical words and vocabulary of mathematics crucial for learners' capacity to comprehend and interpret mathematics, but they also have a significant impact on their future mathematical development (Robertson & Graven 2020). Learning to operate in different registers poses significant difficulties for ELLs in this regard. This is particularly true when learning vocabulary because the meaning of words varies across registers (Mulaudzi 2016; Robertson 2017).

Everyday English versus mathematical English

Regarding mathematical teaching and learning, learners already have a vocabulary. Although some LoLT terms may be known to ELLs, they may have different connotations in the context of mathematics (Morgan et al. 2014). Machaba (2018) contends that non-proficient learners of the LoLT will struggle to distinguish between the correct mathematical meaning of a common word and everyday English phrases. ‘Volume, multiplication, parallelogram, operation, even, and odd’ are examples of these terms (Ní Ríordáin et al. 2015:14). Therefore, it is crucial that both the learner and the teacher explore the numerous meanings and interpretations of mathematical terminology for them to comprehend the intended meanings and language patterns (Mulwa 2014). In addition to the terminology used in everyday English, mathematics includes several of its own unique terminologies (Le Cordeur & Tshuma 2019). Therefore, if learners wish to be successful in mathematics, they must learn these terms, their basic mathematical meanings, and how to use them in different situations.

Teaching and learning mathematics in South African classrooms

The phenomenon of multilingualism can be observed in classrooms all around the world. However, only major regional or national languages are used in the majority of classrooms, often for pragmatic or political reasons (Gandara & Randall 2019). In the multilingual setting of South Africa, the presence of historically multilingual communities is seen in these schools (Barwell et al. 2016; Gandara & Randall 2019). In spite of this, the majority of South African learners study mathematics in English, as this is the dominant language that the learners’ parents ‘buy into’ as the medium of teaching (Robertson & Graven 2020). Consequently, there are many learners in South African schools who are not proficient in the LoLT, which is often English (Dale 2015; Robertson 2017; Robertson & Graven 2018).

Mulaudzi (2016) and Essien (2018) assert that English classrooms where mathematics is not taught in the learners’ home language (HL) are contexts in which there is a need for meaningful pedagogy on mathematics learning as well as the development of specialised skills that provide learners with a variety of opportunities to learn mathematics in such classrooms. In this context, Essien (2018) explicates Cummins’s (1979) claim that constructive cognitive development is achieved when a learner reaches a particular level of linguistic competence in a second language. For this reason, it can be assumed that language-deficient learners will be deprived of cognitive development. Furthermore, Essien found that learners in South African classrooms whose HL differs from the LoLT are unfamiliar with the linguistic structures encountered during mathematics instruction. Learners need to familiarise themselves with the structure of the mathematical language. This, however, necessitates that these learners understand both mathematical concepts and the language in which they are embedded. As a result, teachers are challenged to find a balance between English as the LoLT and the language of mathematics.

Theoretical framework

The interconnection between Vygotsky’s learning theory and the five strands of mathematical proficiency

Both Vygotsky’s (1978) learning theory and Kilpatrick Swafford and Findell’s (2001) five-stranded model of mathematical proficiency, which place emphasis on the relationship between cognitive development and the general development of children’s mathematical proficiency, were used to interpret the empirical part of this research.

The mathematical proficiency model developed by Kilpatrick et al. (2001:5) consists of five ‘intertwining strands’ of mathematical proficiency, namely: ‘conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition’. Ramollo (2014:14) defines these strands as ‘various strands that incorporate knowledge about the interaction between the teacher, the learner, and the content and embrace the context for learning mathematics successfully’. Thus, each strand should address the needs of ELLs (i.e. isiXhosa HL-speaking learners) to become proficient in mathematics. Teaching for mathematical proficiency, on the other hand, necessitates the same interconnected strands from both teachers and learners. Thus, in order to teach for mathematical proficiency, a teacher must have the following skills:

- conceptual understanding of the core knowledge of mathematics, learners, and pedagogical strategies required for teaching;
- procedural fluency in carrying out basic instructional procedures;
- strategic competence in planning effective instruction and solving problems that arise while teaching;
- adaptive reasoning in rationalising a decision; and
- a positive attitude (productive disposition) toward mathematics, teaching, learning, and improving mathematics practice.

The five strands of mathematical proficiency of Kilpatrick et al. (2001) and the learning theory of Vygotsky (1978) were combined to fit the context of South Africa since the majority of South African learners enter school speaking a language other than the LoLT, which presents a challenge for mathematics teaching and learning to most learners (Essien 2018; Robertson & Graven 2019).

A fundamental connection between Vygotsky’s (1978) theory of learning and Kilpatrick et al.’s (2001) five strands of mathematical proficiency is the development of cognitive competencies. The learning theory incorporated within Kilpatrick et al.’s five strands of mathematical proficiency is particularly applicable to this research since learners with inadequate LoLT fluency may regard language as a barrier to comprehending and becoming proficient in mathematics

(Robertson & Graven 2020). According to Prediger et al. (2019), intellectual resources, such as language or other physical resources, serve an important role in teaching and learning for both communicative and conceptual reasons. Within each strand, Vygotsky's theory of learning demonstrates 'the relevance of language in learning and cognitive development'. In this regard, the teacher applies intellectual and physical resources (depending on the needs of the learner) as scaffolds (support strategies) to enhance learners' mathematical understanding (Presmeg et al. 2016).

Research design and method

The researchers conducted an interpretive qualitative case study using Northcutt and McCoy's (2004) interactive qualitative analysis (IQA) systems method to address the sub-research question: 'What are Grade 1 teachers' understandings of mathematical language in the context of South Africa?' With the help of unstructured, open-ended focus group interviews, semi-structured individual interviews, and lesson observations, the researchers were able to identify a population, select a sample from that population, and then explore, describe, and understand teachers' understandings of mathematical language in current mathematics practices in the context of South African English Grade 1 classrooms.

Setting

The researchers conducted this study in 2021 at public and independent primary schools in the Western Cape, Metro East Education District. The participating teachers ranged from newly qualified teachers to more experienced teachers teaching mathematics in English-LoLT Grade 1 classrooms. Although the LoLT of the classrooms is English, more than a quarter of the learners speak isiXhosa as their HL, of which 41% is the highest percentage of learners whose HL is not that of the classrooms' LoLT. The teacher-learner ratio ranged between 1:29 and 1:37.

Participants

All Grade 1 teachers in the Western Cape constituted the study's population. Using the technique of purposive sampling, the researchers selected four schools and 11 Grade 1 teachers (nine teachers representing three public primary schools and two teachers representing one independent primary school) as participants for this study (Creswell 2018; Cohen, Manion & Morrison 2018; Okeke & Van Wyk 2015). The participants were selected so as to offer the researchers a rich and comprehensive narrative of their perspectives, experiences, and feelings regarding their understanding of the language of mathematics in a South African context (Cohen et al. 2018; Mohajan 2018).

Data collection

The researchers acquired their data using an adapted IQA data collection method. In this sense, original IQA research often collects data using two techniques, such as unstructured, open-ended focus group interviews and semi-structured individual interviews (Northcutt & McCoy 2004). Nonetheless, the researchers introduced a third data collection approach, namely field observations of mathematics lessons. Due to the COVID-19 pandemic's restrictions on social distancing, we conducted two separate face-to-face, unstructured, open-ended focus group interviews instead of just one, with a total of 11 Grade 1 teacher participants (i.e. six Grade 1 teachers participated in Focus Group 1, and five Grade 1 teachers participated in Focus Group 2). These unstructured, open-ended focus group interviews informed the interview framework. The interview framework then guided the researchers' data collection through the face-to-face mathematics lesson observations of 6 of the 11 individual participants (i.e. three Grade 1 teachers from Focus Group 1 and three Grade 1 teachers from Focus Group 2). After that, the interview framework helped guide the six semi-structured individual interviews, which took place online with the same six teachers whose mathematics lessons were observed.

Data analysis

The data analysis process consists of three distinct steps. Figure 1 depicts the data analysis process to show the relationship between the three processes.

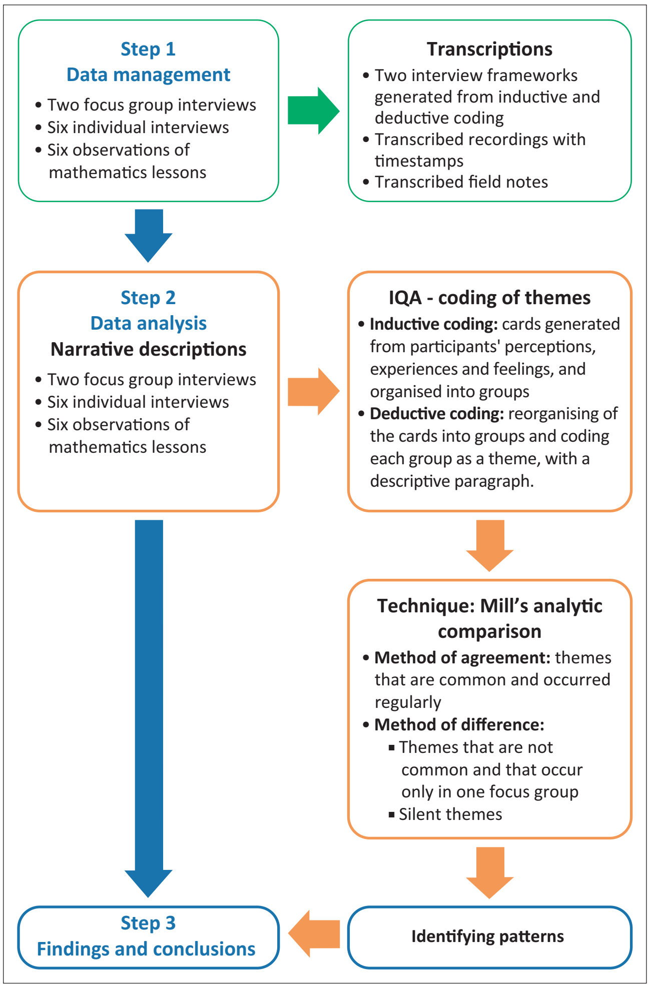

FIGURE 1: Qualitative data analysis process.

The 11 participants of the unstructured, open-ended focus group interviews were involved in the first step of data analysis. During this interactive session, the participants of the focus group generated and recorded on index cards (inductive analysis) their perceptions, experiences, and feelings regarding the research statement (based on the sub-research question), namely: 'Tell me what you think or feel or call to mind when I use the term "mathematical language"'. The brainstorming exercise was followed by a deductive analysis exercise in which the participants sorted and clustered the written cards into groups that represented the topics. The written ideas were accompanied by descriptive paragraphs that served as the framework for the interview.

The second step of the data analysis consisted of analysing the transcribed semi-structured individual interviews using John Stuart Mill's analytic comparison technique as an analytical tool to detect patterns among themes (Neuman 2014). The third step entailed analysing the transcripts of the field observations of mathematics lessons again, using Mill's analytic comparison technique (Neuman 2014) as an analytical tool to get a chronological understanding of what transpired in each participant's mathematics lesson. The data analysis of the individual interview transcriptions was compared to the data analysis of the field notes from the mathematics lessons to create a synthesis. After gathering and analysing the data, the researchers performed a literature check, using recent and pertinent research as a backdrop for articulating and formulating the findings. The data was verified using Auberbach and Silverstein's (2003) criterion of transferability and Maxwell's five categories of validity in qualitative research (Maxwell 2016). The descriptive validity of the data utilised in this study was ensured using IQA interviewing techniques, data-gathering techniques, and member checking. Along with the descriptive validity, the interpretative validity includes the focus group interview members contributing their own data and analysis. Additionally, the verbal and non-verbal replies acquired from the participants to validate the information they provided during the focus group interviews were incorporated into the transcripts of the individual interviews and classroom observations. The researchers conducted a literature review to establish the theoretical validity after the focus group interview participants had identified the themes and sub-themes. Literature and direct quotations were used to support the themes and sub-themes. Consequently, conclusions were reached based on the findings of the data and the literature. Inferences were drawn from the data analysis, literature review, and theoretical framework to confirm the evaluative validity.

Ethical considerations

The researchers sought ethical approval from the Cape Peninsula University of Technology (CPUT) and the education authorities (Western Cape Education Department [WCED]) before beginning the investigation. The participants were given informed consent forms to sign and the researchers made it clear that their involvement was voluntary and that they could withdraw from the processes at any time. Participants remain anonymous and are simply identifiable by a number, for example Participant 1. Audio recordings and transcripts will be preserved in a secure safe for at least 5 years after the completion of this research.

Results

The results of the two unstructured, open-ended focus group interviews, semi-structured individual interviews, and classroom observations are described in the three sections that follow.

Results of the two unstructured open-ended focus group interviews

By employing Mill's analytic comparison technique (Neuman 2014), the analysis of the two focus groups' themes revealed the first pattern: the researchers found that both themes had similar content or flash cards, but they had different names, namely 'Mathematical language' and 'The language'. Thus, the researchers decided to group together the two themes as they had similar or complementary content (method of agreement) into one category, namely: 'Mathematical language'. The theme, cards (thoughts), and descriptive paragraph indicated by the 11 teacher participants in both focus group interviews are shown in Table 1 and Table 2.

Results of the semi-structured individual interviews of the category 'Mathematical language'

The semi-structured individual interviews were analysed to look for patterns. The researchers analysed the individual interview transcripts from the six participants individually to determine the following:

- if the participant agreed with the interview framework's theme (method of agreement),
- if the participant disagreed with the interview framework's theme (method of difference), and
- if the participant elaborated on or modified the interview framework's theme (method of difference).

As previously mentioned, the category 'Mathematical language' was developed by grouping the themes 'Mathematical language' and 'The language' derived from the

TABLE 1: The theme 'Mathematical language' generated by Group 1's open-ended focus group interview.

Group 1 – Open-ended focus group interview		
Cards consisting of a brainstorming activity of teachers' perceptions, experiences, and feelings	**Theme**	**Descriptive paragraph**
• Simplifying mathematical terms (2×) • Numbers and symbols (4×) • Story sums? Concentration • Numbers, patterns and relationships, patterns, Space and shape, measurement, data (2×) • It is a language of its own (like music) (2×) • Relatable terms first thereafter formal terms (2×) • Relating concepts to learners' own experiences • Speak slowly	Mathematical language	'Maths is a language of its own. It includes symbols, signs, numbers, patterns, space and shape, measurement, data handling etcetera. When introducing a new mathematical concept, speak slowly, clearly and use repetition as well as visual aids. Simplify words and relate it to their everyday lives so they can make meaning of it'.

TABLE 2: The theme 'The language' generated by Group 2's open-ended focus group interview.

Group 2 – Open-ended focus group interview		
Cards consisting of a brainstorming activity of teachers' perceptions, experiences, and feelings	**Theme**	**Descriptive paragraph**
• 'Buddy' system • We sing Xhosa counting songs to help them feel included + relevant • LoLT is English: Learners • understand English although they can speak isiXhosa • Comprehension of what is being asked and understanding concepts (3×) • Simple understandable language • Words that mainly relate to understanding of how to do maths (2×) • Using body language or signs alongside a new term • Overwhelmed with what is expected (2×) • Difficult to comprehend for learners (who are more concrete or visual) • Mathematical terminology (3×) • Numbers • Learning how to write or say and/or read number names • Introduce vocabulary alongside a concrete example • Using a problem-solving approach	The language	'The language is all about understanding which may incorporate words, signs or symbols and rhythm. Start with (and revise) prior knowledge. Make use of body or sign language while introducing a concept with an everyday problem or experience. Introduce a symbol alongside a term. There must be a relationship between a visual concept and the verbal instruction. Pair up a non-English speaker with an English speaker.'

two focus group interviews (see Table 1 and Table 2). When the researchers deliberated on the category 'Mathematical language', the six participants, based on their personal perceptions, experiences, and feelings, expressed a variety of perspectives on their understanding of mathematical language. Participants' responses were as follows:

> 'It is a language of its own' (Participant 1, Educator, 30 years).

> 'It's a language of its own; I mean, we're teaching the children maths and then they see it as numbers' (Participant 5, Educator, 28 years).

> 'But as far as mathematical language is concerned, we basically have to start right at the beginning. So, no matter what language they're speaking or just say predominant language, even the English children are learning it [*mathematics*] for the first time.' (Participant 9, Educator, 62 years)

> 'So, language is … when we teach maths in the Grade 1 classroom and not only for those learners that don't speak English as their first language, but children don't speak maths. Maths language is something you teach them whether they are English speakers or not first language English speakers, and that is how you build any maths concept for a child, and from there you build onto those concepts.' (Participant 11, Educator, 30 years)

Le Cordeur and Tshuma (2019) contend that mathematical language is perceived as a distinct register within an everyday language (e.g. English), and that learning mathematics entails the acquisition of the mathematics register itself (Setati 2005; Sibanda & Graven 2018). Therefore, the isiXhosa HL-speaking learner must learn both the LoLT, which is English, and the language of mathematics at the same time (Sibanda & Graven 2018).

Two of the participants agreed that mathematics has its own terminology, which is in line with the participants' statements above. However, they stressed that mathematical terminology is difficult:

> 'When I think of the language with regard to mathematical, or mathematics, the mathematical language, … then the first that … come to mind… is the mathematical terminology, … and the specific terminology we use when teaching a new concept … division of maths … part of mathematics [*content area*], that falls under mathematics. For example, addition, subtraction or objects, 3D objects, those kinds of terminologies. …these terms are a big word that might sound challenging for learners, … that can sometimes hinder them because they just think of this big word and what it means, overwhelms them.' (Participant 7, Educator, 25 years)

> 'I also think that it's [mathematical language] about simplifying the terms because sometimes they get so overwhelmed. … You got to plus this and minus this, or if you even bring up the word divide, they're like looking at you like, "what does that mean?"' (Participant 3, Educator, 24 years)

According to Jourdain and Sharma (2016) and Le Cordeur and Tshuma (2019), the specialised vocabulary employed in mathematics is one characteristic of the mathematics register. This unique terminology is used to describe or explain a specific type of mathematical problem. Ní Ríordáin et al. (2015) assert that learners from various cultural or linguistic backgrounds frequently struggle to comprehend mathematical problems due to a lack of familiarity with these terms.

In addition to the quotes provided by the participants thus far, they also acknowledged the significance of language to the understanding of mathematics. Mulaudzi (2016) argues that the LoLT in which mathematics is learned creates the foundation for the development of mathematical instruction and learning in that language. To be successful in mathematics, learners must not only be comfortable with and well informed of the everyday English register (i.e. LoLT), but also be fluent in numerous mathematical registers (Essien 2018; Moschkovich 2005; Schleppegrell 2011). Three participants emphasised the significance of comprehending the LoLT when introducing mathematical ideas:

> '... [*A*]nd with mathematics you need to understand the language that it's being taught in [i.e., English-LoLT]. So once you understand the language mathematics is taught in – concepts and all of those things that is part of mathematics become easier to understand.' (Participant 1, Educator, 30 years)

> 'The learners do not necessarily understand the language itself [*Language of Mathematics*] and/or if it is learners that perhaps struggle with English as it might not be their home-language, … they then struggle to understand the [*mathematical*] concepts.' (Participant 7, Educator, 25 years)

> 'The language [LoLT] and the concept goes hand in hand. If they don't get the language, they're not going to get the [*mathematical*] concept; and if they don't hear them often enough, then they may not be able to use it and remember it.' (Participant 11, Educator, 30 years)

This demonstrates that language (both the LoLT and the language of mathematics) plays a role in fostering mathematical comprehension (Barwell et al. 2016; Das 2020; Presmeg et al. 2016; Robertson & Graven 2020; Vygotsky 1962).

In addition, a participant's response validated Le Cordeur and Tshuma's (2019) assertion that the language of mathematics necessitates similar pedagogic procedures to those employed when learning any other language. Participant 9 stated the following:

> 'If I teach say Afrikaans, I would always put a visual with a word and we do a lot of repetition, so for mathematical language we would do the same. We would introduce a concept, we would give the concept a name, and then we would work around that concept and all the children are learning the same mathematical language.' (Participant 9, Educator, 62 years)

Participant 9's perspective above is consistent with Robertson and Graven's (2019) argument that, in order to promote inclusivity in mathematics classrooms, mathematics teachers must find ways to incorporate second-language teaching and learning strategies.

In conclusion, the method of agreement reveals that all individual interviewees agreed that mathematics has its own language with its own vocabulary (i.e. mathematical terminology). In addition, they have agreed that language plays an essential role and must be simplified for mathematical understanding. The method of difference, on the other hand, showed that only one person thought that mathematical language could be learned in the same way as another language.

Results of the field notes of the mathematics lesson observations of the category 'Mathematical language'

As one of the aims of this research study was to explore, describe, and understand what Grade 1 teachers' understandings of mathematical language are in a South African context, the researchers could not obtain a full picture of the category 'Mathematical language' by just observing Grade 1 teachers teaching mathematics. Nevertheless, to a certain extent, the method of agreement revealed that the participants used visual representations for mathematical concepts, which form part of the language of mathematics. In this regard, all the teacher participants displayed mathematical information on a wall with all the mathematical terms together with pictures so that learners who speak isiXhosa could understand the mathematics vocabulary.

Moreover, the researchers found that Participants 1, 7, and 9 made the mathematics terms easier to understand by exposing learners to different words that meant the same thing. Participants 5 and 11 made the language easier when they introduced word sums, for example, by using simple, everyday English that was relevant to learners' everyday lives. Participant 3 made the language easier to understand by giving learners the chance to revise terminology as a group before they did an individual activity about the same thing. The observations of these activities are displayed in Table 3.

TABLE 3: Activities observed during mathematics lessons related to the category 'Mathematical language'.

Participant	Activities observed during participants' mathematics lessons
Participant 1	When the teacher introduced the activity of 'bonds' to the learners, she talked about tens and units, but also mentioned to the learners that units are also referred to as ones.
Participant 7	Before the teacher embarked on an addition activity, she first asked the learners what addition is. The learners answered that 'you plus'. The teacher acknowledged their answer and told the learners that addition also means you add or make something more.
Participant 9	The teacher read the word sum to the learners: 'If I have 8 sweets and eat 2 of them, and I eat another 3, how much do I have left?' Before the learners were left to solve the problem, she first asked them what kind of sum it is. Then she asked them why they said it is a minus sum. She asked the learners to think about the sum and picture it in their heads. 'If I eat 2 sweets, what happens? It gets less. If I eat three more sweets, it gets less.'
Participant 5	When the teacher taught story sums, she used learners' names in the classroom and related the story to their everyday experiences – as the teacher said, for example, when she taught a sharing story sum, she would use real objects (e.g. real lollipops) to share among the learners, as everyone likes sweets.
Participant 11	Instead of just asking learners which number is greater than 5, the teacher simplified the language and related it to their everyday life by asking: 'If she has five sweets and they have three sweets, who has more?'
Participant 3	The teacher asked learners a few before and after questions (e.g. 'What comes before 12?'; 'What comes after 12?') As soon as the teacher saw that everyone was on par with the terms before and after, she gave them an individual activity to complete in this regard.

Participant 9 asserted that mathematics can be taught and learned in the same way as any other language can be. In this instance, the method of difference revealed that all the participants were using various language strategies (e.g. semiotic systems), which are also used in second or third language acquisition. For example, all the teachers employed visual representations to help learners learn the language and concepts of mathematics.

Through analysing the transcripts of the classroom observations for the category 'Mathematical language' and comparing them to the transcripts of the individual interviews, the method of agreement showed that all six participants agreed that mathematics is made up of different terminology. They also agreed that language is a key part of understanding mathematics, which is why the participants tried to make mathematical terms easier to understand. During the analysis of the individual interview transcripts, the method of difference showed that only Participant 9 mentioned that strategies for teaching mathematics in a second language could be used. In line with what Participant 9 said above, it was noticed that all the other interview participants used second-language didactic strategies in their mathematics lessons without even realising it.

Deliberation on the findings

According to the data in Table 4, the category 'Mathematical language' comprises the theme 'The language (of mathematics)'. All participants had a firm grasp of the concept that mathematics has its own register (mathematical terminology) and should be learned like any other language. In this regard, the participants asserted that the language of mathematics is a new language that both English-speaking and isiXhosa-speaking Grade 1 learners must learn. However, participants believed that isiXhosa learners lag behind English-HL learners because they must first comprehend the language in which mathematics is taught (i.e. English LoLT) before they can make sense of mathematics itself (Mulaudzi 2016; Sibanda & Graven 2018). In addition to the fact that mathematics has its own register, two participants stressed that mathematical terminology can be challenging for some learners because these terms are unknown to those from other cultural or linguistic backgrounds (Ní Ríordáin et al. 2015). As a result, the participants acknowledged the importance of language in mathematical understanding and emphasised the importance of understanding the English language when mathematical concepts are introduced (Barwell 2016; Bruner 1975; Das 2020; Moschkovich 2012a; Presmeg et al. 2016; Robertson & Graven 2020; Sfard 2008; Vygotsky 1962). Consequently, participants stressed the importance of simplifying (scaffolding) mathematical language for better understanding. They also emphasised the need for learners to hear both the English LoLT and the language of mathematics regularly in order to grasp and apply them when mathematical concepts are introduced (Essien 2018). Another participant was of the opinion that the language of mathematics necessitates the same pedagogical techniques utilised when learning any other language (Le Cordeur & Tshuma 2019; Robertson & Graven 2019).

According to Vygotsky (1931), language is associated with cognitive development and occurs in a sociocultural environment. In addition, based on Vygotsky's theory of learning, Moschkovich (2012b) claims that the language of mathematics refers to communicative skills that are necessary and adequate for participation in mathematical discourse. In this context, Moschkovich (2012a) argues that the idea of 'language' must be expanded to include the interaction of multi-semiotic systems engaged in mathematical communication (i.e. natural language, mathematics symbol systems, and visual representations). Therefore, the processes of language development and conceptual mathematical understanding are interdependent and interconnected (Council of the Great City Schools [CGCS] 2016). To support isiXhosa HL-speaking learners in becoming proficient in English for mathematical learning, teachers must provide specific scaffolds (i.e. multi-semiotic systems) during mathematical discourse to match isiXhosa HL-speaking learners' linguistic demands, thus fostering their acquisition of English mathematical proficiency (CGCS 2016; Walshaw 2017).

TABLE 4: Sub-research question, research objective and deliberation on the findings.

Sub-research question	Research objective	Category: Mathematical language	
		Theme	Findings
'What are Grade 1 teachers' understandings of mathematical language in the context of South Africa?'	The researchers used a qualitative data collection method to establish what (explore) Grade 1 teachers' understandings of mathematical language are in a South African context.	The language (of mathematics).	**Finding 1:** It was understood by all the participants that mathematics constitutes its own language, with a specific vocabulary and register. **Finding 2:** The language of mathematics needs to be simplified for mathematical understanding.

Conclusion and recommendations

As was previously stated, this research was conducted using the theoretical framework of Vygotsky's (1978) learning theory, which was incorporated into the five strands of the mathematical proficiency model by Kilpatrick et al. (2001). This article reports on teachers' understandings of the mathematical language in the context of South Africa and findings are concluded and connected to the applicable strand 'conceptual understanding' only, in which teachers need to have a conceptual understanding of what the mathematical language entails to provide the necessary mathematical language support to isiXhosa HL-speaking learners receiving mathematics education in an English Grade 1 classroom. Table 5 depicts the conclusion and recommendations.

The primary objective of this article was to report on Grade 1 teachers' understandings of mathematical language in the South African context. This study found that teachers must recognise that mathematics has its own register and is learnt like any other second or third language (Le Cordeur & Tshuma 2019). In this regard, mathematics, like any other language, has its own mathematical discourse (i.e. terminology, grammar, syntax, word order, synonyms, conventions, idioms, abbreviations, sentence and paragraph structures), which is a

TABLE 5: Conclusion of the findings based on Vygotsky's theory of learning and Kilpatrick et al.'s strands of 'conceptual understanding' of the mathematical proficiency model.

Development strand for mathematical proficiency	Recommendations for teachers
Conceptual understanding **Teacher:** Conceptual understanding of core mathematics knowledge, learners, and instructional practices required for teaching. **Learner:** Understanding or proficiency in mathematical concepts, operations, and relationships.	**Category: Mathematical language** • Teachers must know that mathematics has its own register (mathematical terminology) and is learnt like any other second or third language (Le Cordeur & Tshuma 2019). • IsiXhosa-HL speaking learners first need to understand the English LoLT before they can make sense of mathematics itself (Mulaudzi 2016; Sibanda & Graven 2018). Thus, more time is recommended for isiXhosa-HL learners to acquire mathematical proficiency (Mulaudzi 2016; Setati 2008). • Teachers need to acknowledge that mathematical terminology can be challenging for learners from other cultural or linguistic backgrounds (Ní Ríordáin et al. 2015). • Teachers need to understand the significant role that language plays in learning mathematics, and, therefore, it is recommended that the language of mathematics needs to be simplified (scaffolded) for isiXhosa learners to support their mathematical understanding (Vygotsky 1931). • Scaffolding strategies during mathematics discourse must be tailored to the isiXhosa learners' individual language needs to promote mathematical proficiency in the English LoLT (CGCS 2016; Vygotsky 1931, Walshaw 2017). • Strong language and metalinguistic skills are necessary to communicate mathematically (Robertson & Graven 2020). In this regard, it is recommended that learners need to hear both the English LoLT and the language of mathematics regularly in order to grasp and apply them when mathematical concepts are introduced (Essien 2018).

LoLT, Language of learning and teaching.

specialised academic language (third language) that isiXhosa-HL learners must acquire in addition to the everyday English language register (second language) in which mathematics is taught (Jourdain & Sharma 2016; Ledibane, Kaiser & Van der Walt 2018; Essien 2018). According to Ledibane et al. (2018), learners who do not speak English as their first language and are learning an academic language (i.e. mathematics) experience many of the same difficulties as those learning English as a second language, and thus learning skills similar to those used in learning any other language are required. Thus, a scaffolded approach of using multi-semiotics (i.e. natural language, mathematical symbol system, and visual representations) to supporting isiXhosa learners' individual language needs should be implemented to support their mathematical understanding and to promote mathematical proficiency in the English LoLT (CGCS 2016; Vygotsky 1931; Walshaw 2017). The reality is that English is the lingua franca in most schools, and we should ensure that isiXhosa HL-speaking learners receive the necessary support in acquiring proficiency in the academic register of English for mathematical language learning.

Due to the global COVID-19 pandemic, many educational institutions were unwilling to take part in the research as the teachers already had 'too much on their plates'. This study was limited to a small group of Grade 1 teachers in the Western Cape of South Africa and focused solely on Grade 1 teachers teaching mathematics to isiXhosa HL-speaking learners in English-LoLT classrooms. Therefore, this research study does not provide generalisable findings to the wider Grade 1 teacher population. Furthermore, one of the original data collection methods of the IQA systems method, which is face-to-face individual interviews, was replaced with online interviews as the researchers had to limit any unnecessary face-to-face interaction due to the COVID-19 pandemic's restrictions on social distancing at the time.

The researchers suggest that education programmes for pre-service and in-service teachers be developed and implemented in accordance with the findings of this study and monitor the outcomes thereof so as to inform current mathematics practice. We also suggest that additional research be conducted to determine how the recommendations made in this study can support other teachers (the wider Grade 1 population). Furthermore, we suggest additional investigations into issues such as the terminology used in mathematics for African languages as well as the availability and accessibility of bilingual teaching materials that could assist learners with limited mathematical proficiency in the LoLT.

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C.L. served as the main supervisor and E.B. as co-supervisor for T.C.'s doctoral study. C.L. and E.B. gave input as co-authors of this article, while T.C. wrote the majority of the article.

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Data availability

The authors affirm that the supporting data for this study's conclusions are included in this article.

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