http://dx.doi.org/10.17159/2411-9717/2017/v117n2a3

A version of Gy's equation for gold-bearing ores

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Synopsis

The two methods for calibrating the parameters *K* and α (alpha) for use in Gy's equation for the Fundamental Sampling Error (FSE)–, Duplicate Sampling Analysis (DSA) and Segregation Free Analysis (SFA)–, are described in detail. A case study using identical broken reef material from a Witwatersrand-type orebody was calibrated using the DSA and SFA methods and the results compared. Classically, the form of Gy's equation for the FSE raises the nominal size of fragments given by d_N to the power of 3. A later modification of Gy's formula raises d_N to the power α (alpha), the latter term being calibrated with the coefficient *K* in the DSA and SFA methods. The preferred value of α for low-grade gold ores used by sampling practitioners in the mining industry is 1.5. A review of calibration experiments for low-grade gold ores using the DSA and SFA methods has produced values of *K* that vary between 70 and 170 and values of α in the range 0.97 to 1.30. The average value for α is shown to be 1, rather than 3 as originally proposed in the classic form of Gy's equation or the industry-preferred 1.5. It is suggested that for low-grade gold-bearing ores the equation for the FSE should raise d_N to a power of 1. Such an equation for the variance of the FSE greatly simplifies the characterization of gold ores, now requiring only the calibration of *K* for a given mass and established fragment size. The implications of the simplified equation for the heterogeneity test are that, provided the fragments have been screened to within a narrow size range, any particular size will return a value for *K* that is acceptable for use in the sampling nomogram.

Keywords
fundamental sampling error, Gy's equation, gold ore characterization.

Introduction

A number of different methods for determining the Fundamental Sampling Error (FSE) have been suggested (Carrasco, 2004; Francois-Bongarçon, 1991, 1992a, 1992b, 1993, 1996, 1998a, 1998b; Francois-Bongarçon and Gy, 2001; Geelhoed, 2005; Gy, 1973, 1979, 1982; Lyman, 1993; Pitard, 1993; Minnitt and Assibey-Bonsu, 2009; Minnitt, Francois-Bongarçon, and Pitard, 2011; Minnitt; 2014). The classic and widely used heterogeneity test for determining the sampling constants was proposed by Gy (Pitard, 1993). It has been described by Carrasco (2005) and Magri (2011), and has been championed by Pitard (2015), but this procedure is not described any further in this paper. Francois-Bongarçon (1988a) investigated the changes in the variance of sample assays due to changes in the fragment size, and the way in which this variance can be applied to the determination of the constants *K* and alpha (α) for use in a modified form of Gy's formula for the FSE shown in Equation [1]. The sampling parameters are substituted into Equation [1] and a graphic, the sampling nomogram, describing the changes in the FSE for different stages of crushing and splitting for a specific ore type at a given grade is compiled.

$$\left(\sigma^{*}_{FSE}\right)^{2} = \frac{Kd_{N}^{\alpha}}{M_{S}} \qquad [1]$$

The method described by Francois-Bongarçon (1991), which is generally known as the Duplicate Sampling Analysis (DSA) method, is widely used in the mining industry and has produced consistently useful results in terms of the sampling nomogram that is applied on mining operations. A second method, referred to as the Segregation Free Method (SFA), has also been suggested but is currently not widely applied (Minnitt *et al.*, 2011; Minnitt, 2014).

The primary argument against the DSA method is that the splitting stage before sample analysis requires that 10–20 kg of particulate material be split into 32 samples. Splitting this material, which includes the complete spectrum of fragment sizes from dust to particles up to 19.0 mm in diameter, is thought to introduce grouping and segregation errors that cannot be eliminated or mitigated. The problem of segregation was avoided through a method proposed by Minnitt *et al.* (2011) and Minnitt (2014), referred to as the Segregation Free Analysis (SFA) method, for calibrating the parameters *K* and α. The SFA method overcomes the related problems of segregation and ambiguity in regard to the exact size of the fragments, but has been

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 ISSN 2225-6253. Paper received Mar. 2016; revised paper received Oct. 2016.

criticized because of its simplicity. Objections raised focus on the single-stage crushing that the material undergoes compared to the multi-stage crushing associated with the DSA method (Minnitt, 2014). The calibration procedures for determining K and α by the DSA and the SFA methods are described and compared. Details of the calculation procedures are almost identical for both methods, but the differences are emphasized.

This paper examines and highlights the differences and similarities in the calibration exercises that have been carried out over the years. As the number of such calibration exercises has increased, there is also growing empirical evidence that the exponent of 1.5 for the nominal fragment size suggested by Francois-Bongarçon (1991) and Francois-Bongarcon and Gy (2001) should in fact be unity. This means that the sampling variance is actually a function of the product of the sampling constant K and the nominal fragment size d_N, divided by the mass. Lyman (1993) proposed a similar equation in which the sampling variance is simply equal to the sampling constant K divided by the sample mass, and has no dependency on the nominal size of the fragments d_N.

Preparation of crushed materials for the DSA and SFA methods

Four bins, each containing 400 to 600 kg of run-of-mine gold-bearing ore from the Target, Tshepong, Joel, and Kusasalethu mines, were provided by Harmony Gold Mining Company Limited. The material from each mining operation was handled separately and was spread out to dry, at which stage all fragments larger than 15 cm diameter were examined and identified. These larger fragments, which invariably consisted of sub-rounded dolerite dyke material or fine-grained, non-mineralized hangingwall quartzite, were removed from the lot. A Boyd crusher was used to crush the dried lot to 95% passing 2.50 cm. A total of 333 kg of the broken ore was divided into two lots, one of 75 kg for the DSA experiment and one of 258 kg for the SFA experiment. The 75 kg of broken ore used for the DSA method was split into six series and each of these series was split into 32 individual samples using a rotary splitter. The 258 kg of broken ore for the SFA experiment was split into 15 series and each of these was then split into 32 samples using a riffle splitter. The author sees no difference between the rotary divider and riffle splitter methods.

All samples were submitted for fire assay using a 50 g aliquot. The choice of aliquot was made to improve the precision of the analyses, but was permissible only because of the very good fluxing and fusion characteristics of the Witwatersrand ores. Reduction of the analytical data to provide points on the calibration curves for the DSA and the SFA methods was similar, apart from slight changes in the fragment sizes used on the curves.

DSA method

The principal difference between the DSA and SFA methods is in the stages and manner of preparation of the crushed particulate ores for fire assay. The DSA method requires a lot varying from 40 kg to 80 kg, depending on how many series are required. For this particular exercise a series of six sub-lots split from the run-of-mine ore from Target mine (Table I), with evenly spread top sizes varying from 2.5 cm to 0.1 mm as shown in Figure 1, was extracted.

The first step was to crush the material to a nominal top size of about 95% passing 2.50 cm. This lot was split into six equal, separate sub-lots, each sub-lot being referred to as a series as shown in Figure 2. The first sub-lot of about 12 to 15 kg may be larger than the following sub-lots simply because a greater mass of material is required for larger fragment sizes. This will ensure a better distribution of fragments per sample during the rotary splitting procedure and will reduce the variance of the large fragment sample masses.

The first of the sub-lots, of 15.0 kg at 2.50 cm, was named Series 1. The remaining five sub-lots (65 kg mass) were recombined and homogenized during crushing to a somewhat smaller sieve size having a nominal top size of 95 % passing 1.90 cm. This lot of about 65 kg was then split

Table I

Series number, mass, and nominal top size of the four sub-lots prepared in the DSA method

Name	Nominal top size	Approximate mass
Series 1	95% passing 2.50 cm	15.0 kg
Series 2	95% passing 1.90 cm	13.0 kg
Series 3	95% passing 1.32cm	13.0 kg
Series 4	95% passing 0.945 cm	13.0 kg
Series 5	95% passing 0.435 cm	13.0 kg
Series 6	95% passing 0.2 cm	13.0 kg

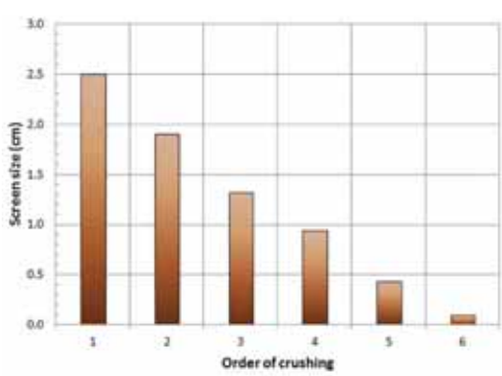

Figure 1—Monotonically decreasing nominal top sizes for the six series used in the DSA method

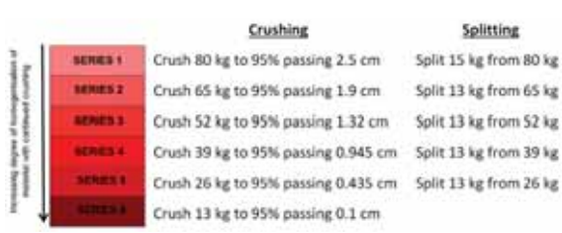

Figure 2—Crushing and splitting protocol for 80 kg of material using the DSA method, emphasizing the increasing degree of homogenization of material with continued crushing

into five sub-lots of about 13 kg each, one of which was chosen at random and named Series 2. The remaining four sub-lots were recombined to give a 52 kg lot at 95% passing 1.90 cm. This material was then crushed to a smaller nominal top size, say 95% passing 1.32 cm. The lot was then split into four sub-lots of 13.0 kg each; one was selected and named Series 3. The remaining three sub-lots of about 13.0 kg each were recombined to give a 39 kg lot, which was crushed to 95% passing 0.945 cm and split three ways. One of the sub-lots, 13 kg by mass, was selected and named Series 4. The remaining two sub-lots with a total mass of 26 kg were recombined and crushed to 95% passing 0.435 cm and split to give two equal sub-lots of 13.0 kg each, one of which was named Series 5. The last 13.0 kg sub-lot was crushed to 95% passing 0.20 cm and termed Series 6. The nominal fragment size and mass of each of the different series, 1 to 6, established in this way are shown in Table 1.

Each of the six series was then split into 32 sub-samples of approximately equal mass (39 g) using a rotary splitter or a riffle splitter. Two of the samples were selected at random from each of the Series 1–4 and tested the granulometry for each of the four size fractions, *i.e.* to check that each size fraction is correctly calibrated. The problems associated with the granulometry test are dealt with by Minnitt *et al.* (2011). The total mass of material used for the granulometry test is small, about 90–100 g, so the test rarely produces results that unquestionably correlate with the given nominal top size of the lot.

Typically, the DSA method of calibration uses broken ore that is progressively reduced from a top size of about 2.5 cm to fragments around 0.1 cm by crushing. Each fraction that is crushed and split out for use in a series contains a complete distribution of fragment sizes, from the range below the nominal top size to dust. For example, a series with a nominal top size of 1.32 cm will contain a complete distribution of fragments that vary in size from 1.32 cm to fine dust less than 75 μm (Minnitt *et al.*, 2011). Previous experiments using the SFA method have demonstrated that the increase in sample variance times mass with increasing fragment size is positive and linear (Minnitt *et al.*, 2011).

The splitting protocol using a standard riffle splitter is shown in Figure 3.

Although it is normal to use three, or at most four, series of split material at different fragment sizes, this particular experiment using the DSA method involved six individual series of material at the fragment sizes listed in Table I.

SFA Method

Material preparation for the SFA method is simpler than for the DSA method. In this particular case a mass of about 258 kg of ore from Target mine was crushed to a nominal top size of 2.50 cm and screened through 15 different screens as shown in Figure 4. In a standard SFA experiment four series at different fragment sizes could be used, but in this particular experiment 15 different size fractions were analysed.

The mass of each screened size fraction is listed in Table II and plotted as a grain size distribution in Figure 5. Each of the series in the SFA experiment at different nominal fragment sizes was then split into 32 samples using a riffle splitter, and each sample was bagged and numbered. The

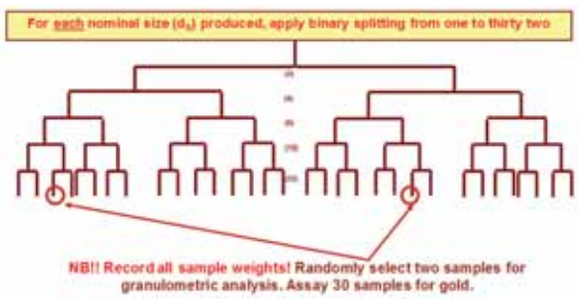

Figure 3—Each series is split into 32 individual samples of approximately equal weight. Nominal top size is tested using two randomly chosen samples

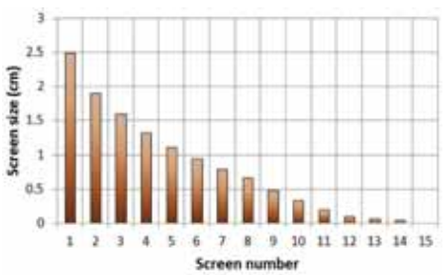

Figure 4—Monotonically decreasing screen sizes from 2.5 cm to <0.05 cm used in screening the particulate ores in the SFA method

Table II

Series number, upper and lower screen sizes, average fragment size (cm), mass (kg), and coefficient of variation (%) for SFA method

Name	Upper screen	Lower screen	Average fragment size	Mass (kg)	CoV
Series 2	2.50	1.90	2.24	18.8	0.35
Series 3	1.90	1.60	1.76	13.5	0.49
Series 4	1.60	1.32	1.47	15.9	0.44
Series 5	1.32	1.12	1.23	10.6	0.26
Series 6	1.12	0.95	1.04	11.7	0.31
Series 7	0.95	0.80	0.88	12.8	0.24
Series 8	0.80	0.67	0.74	19.1	0.20
Series 9	0.67	0.48	0.59	18.4	0.44
Series 10	0.48	0.34	0.42	16.7	0.40
Series 11	0.34	0.20	0.28	22.1	0.29
Series 12	0.20	0.10	0.17	25.6	0.37
Series 13	0.10	0.07	0.09	41.6	0.27
Series 14	0.07	0.05	0.06	26.8	1.20
Series 15	0.05	0.01	0.04	17.2	2.09
Series 16	<0.05	Dust	Dust	4.5	9.10

nominal (average) fragment size in micrometres for a fragment passing between two screens can be calculated using Equation [2].

$$Nominal\ \ fragment\ \ size(\mu m) = \left(\frac{USS(\mu m)^3 + LSS(\mu m)^3}{2} \right)^{0.33} \quad [2]$$

$$USS = Upper\ \ screen\ \ size\ (\mu m)$$

$$LSS = Lower\ \ screen\ \ size\ (\mu m)$$

The actual average fragment size for each series retained between two screens as listed in Table II is systematically

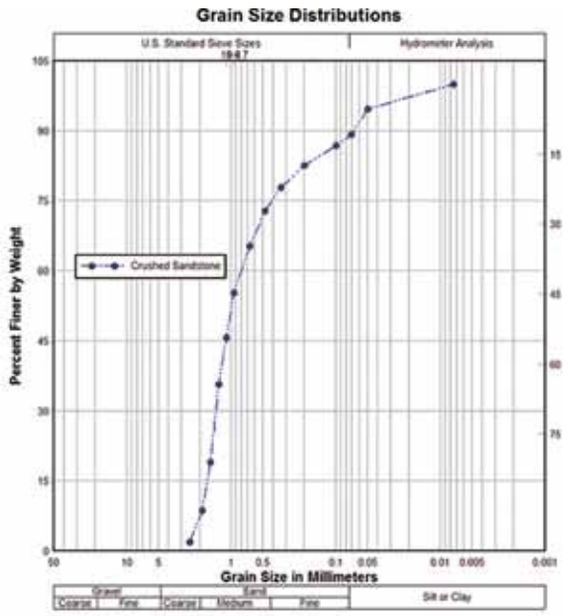

Figure 5—Grain size distribution for the 255 kg of screened ore from Target mine

smaller without any major discontinuities in size. Screening of the broken ore allows the lot to be separated into 15 statistically narrow (clean) size fractions that are then analysed, a method first proposed by Minnitt *et al.* (2011).

The grain size distribution of the screened ore from Target mine used in this investigation is shown in Figure 5, indicating that about 50% of the material (115.7 kg) is less than 1 mm in diameter.

Analysis of the assay data

The lognormal distribution of the assay data required that the means for each series be calculated using a Sichel's *t* estimate. The summary of the statistics for each series in the DSA and the SFA experiments is shown in Tables III and IV, respectively. The fire assay results for the DSA method are given in Appendix 1, and those for the SFA method in Appendix 2.

This data is used in the calibration exercises.

Background theory

The following derivation of the equation for the slope, α, and the intercept, K, is presented in Minnitt *et al.* (2011), but is included here for completeness. The variance in each of the fifteen size fractions analysed must be represented by the same formula if correct estimates of the parameters K and are to be derived from the calibration curve. The ratio of upper and lower screen sizes r, shown in Equation [3], must be reasonably consistent across all screen sizes.

$$r = \frac{d_{MAX}}{d_{MIN}} \qquad [3]$$

This is indeed the case for most of the larger screen sizes used in this particular SFA experiment, as shown in Figure 6, where r is constrained between 1 and 1.5.

The variance of a sample taken, fragment by fragment, from closely sieved material between d_{Min} and d_{Max} is given by Equation [4].

Table III

Fragment size, Sichel's *t* estimate, and number of analyses for the six series of the DSA method

ISA-14-00818 (DSA method)			
Series No.	**Nominal top size (cm)**	**Average Sichel's *t* estimate (g/t)**	**No of analyses**
1	2.50	3.84	32
2	1.90	3.44	32
3	1.32	3.24	32
4	0.95	3.06	32
5	0.42	3.45	32
6	0.20	3.32	32
	DSA average grade	3.09	192 analyses
	Standard deviation	0.83	

Table IV

Fragment size, Sichel's *t* estimate, and number of analyses for the fifteen series of the SFA method

ISA-14-00815 (SFA Method)			
Series No.	**Nominal top-size (cm)**	**Average Sichel's *t* Estimate (g/t)**	**No of Analyses**
1	2.50	3.84	32
2	1.90	3.44	32
3	1.32	3.24	32
4	1.473	6.46	32
5	1.228	6.18	32
6	1.042	7.38	32
7	0.881	5.17	32
8	0.741	5.08	32
9	0.635	5.35	32
10	0.418	5.37	32
11	0.284	4.62	33
12	0.165	4.91	32
13	0.088	4.93	32
14	0.062	4.35	32
15	0.040	4.34	32
16	0.009	5.77	32
	SFA Average grade	5.58	480 Analyses
	Standard Deviation	1.66	

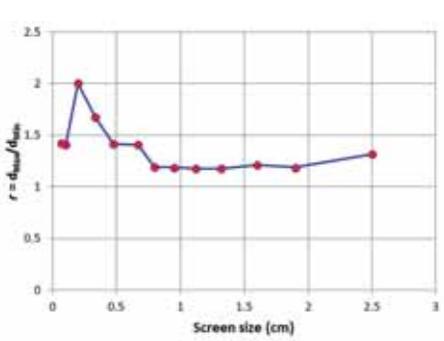

Figure 6—Ratio $r = d_{Max}/d_{Min}$ across the range of fragment sizes used in this SFA experiment (the very lowest screen sizes less than 500 µm are not shown)

$$\sigma^2_{Relative} = \left[\frac{1}{M_S - M_L}\right] cfg'\ell d^3_{MAX} \quad [4]$$

where $g' = g'(r)$ is taken from a curve of the granulometric factor g' *versus* ratio $r = d_{Max}/d_{Min}$ for closely sieved materials, shown in Figure 7.

In this experiment the value of r lies between 1 and 1.5, suggesting a g' value of between 0.6 and 0.7 as acceptable for this experiment. The liberation factor ℓ describes the transition between the liberated, calculable variance and the non-liberated one. The term $f g' d_{Max}^3$ represents the average fragment volume in the fraction $[d_{Min}, d_{Max}]$. The smallest size fraction of ratio r in which the mineral is liberated has a d_{Max} equal to d_ℓ, so in the case of the sampling within size fractions of ratio r, the correct liberation factor is:

$$\ell = \left[\frac{f \cdot g' \times d_\ell^3}{f \cdot g' \times d_{Max}^3}\right]^{b'} = \left[\frac{d_\ell^3}{d_{Max}^3}\right]^{b'} = \left[\frac{d_\ell}{d_{Max}}\right]^{3b'} \quad [5]$$

where $0 < b' < 1$, or, if $\alpha = 3\text{-}3b'$, so:

$$\ell = \left[\frac{d_\ell}{d_{Max}}\right]^{(3-\alpha)} \quad [6]$$

The liberation factor is the ratio of the average fragment volumes at non-liberation and at liberation to a power $3b'$ in Equation [5], and $fg' d_{Max}^3$ is the average fragment volume in the fraction $[d_{Min}, d_{Max}]$. Generally, the mass of the lot is very much larger than the mass of the sample so that $1/M_L$ becomes negligible. Substituting Equation [6] into Equation [4] allows us to rewrite Equation [4] as follows:

$$\sigma^2_{Relative} = \left[\frac{1}{M_S - M_L}\right] cfg'\ell d_\ell^{3-\alpha} d_{Max}^{\alpha} \quad [7]$$

Taking logarithms on both sides of Equation [7], and with the proviso that M_L is much larger than M_S, allows the equation to be rearranged as follows:

$$ln\left[\sigma^2_{Relative} \times M_S\right] = \alpha ln d_{Max} + ln[cfg'\ell d_l^{3-\alpha}] \quad [8]$$

$$ln\left[\sigma^2_{Relative} \times M_S\right] = \alpha ln[d_{Max}] + \{(3-\alpha) ln(d_l) + ln(cfg')\} \quad [9]$$

The two variables that are required to compile the calibration curve are then:

$$ln\left[\sigma^2_{Relative} \times M_S\right] \text{ and } ln[d_{Max}]$$

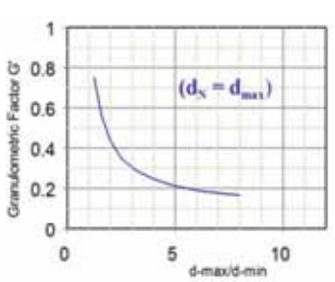

Figure 7—A curve of granulometric factor g' *versus* ratio $r = d_{Max}/d_{Min}$ for the range r = 1 to 8 for closely sieved materials (Francois-Bongarcon, 2010)

So that the slope of the line is given by α, and the intercept is given by:

$$K = ln\left[cfg'\ell d_\ell^{3-\alpha}\right] \quad [10]$$

On this basis it is now possible to extract values for both α and d_ℓ by the fitting of a straight line to the graph (Minnitt *et al.*, 2011).

Reduction of the assay data

The method of reduction of the 32 fire assay results for each of the series in the DSA and SFA experiments is shown in Table V; this particular data is for the DSA Series 3, samples 1 to 32, at a fragment size of 1.32 cm. The logarithmic mean and variance are presented in this way because the Sichel's *t*-estimate for the mean of a logarithmic distribution was used to calculate the mean in grams per ton for each of the individual series. Due to calculating the mean in this way, very few data values are eliminated as outliers. The reduction of analytical data for the SFA method is identical to that for the DSA method.

The data is further reduced to produce the two variables $\ln(\sigma^2 * \text{Mass})$ and $\ln(d_N)$ as shown in Table VI.

Compilation of the calibration curves for the DSA and the SFA methods

The final reduced data for the six series of the DSA method and the fifteen series of the SFA method is compiled in Figure 8.

It is noteworthy that the two curves shown in Figure 7 are almost parallel, with slopes of 1.030 for the DSA curve (red) and 1.123 for the SFA curve. The major difference between these curves is the value for the intercepts that they yield: 5.12 for the DSA curve and 7.83 for the SFA curve, which when transformed back from log space gives values of 167.42 g/t² and 2539.1 g/t² respectively.

Effect on the nomogram

The effect of the differences in K and α for the DSA and SFA methods and their impact on the sampling nomogram is shown in Figure 8. A typical 5 Mt/a gold mining operation is considered as an example. The mine operates 360 days per

Table V

Primary parameters for each individual series (size fraction) in the DSA experiment; these parameters for Series 3, 1.32 cm*

Parameter	Value
Logarithmic mean ($\log_e$ g/t)	1.0412
Sichel's *t* factor for the mean	1.1441
Logarithmic variance ($\log_e$ g/t²)	0.4797
Mean (g/t)	3.2407
Relative std deviation	0.4649
Screen size (cm)	1.32
Average mass (g)	37.20

*This particular data set is from Series 3 of the DSA method for fragment size 95% passing 1.32 cm

Table VI

Derivation of parameters for the calculation of ln(σ²*Mass) and ln(d_N) from each series for points on the calibration curve

Parameter	Value*	Explanation
d_{p95} (cm)	1.320	Calibrated fragment size 95% passing 1.32 cm
Measured multi-stage relative std deviation (RSD)	0.4649	From Table III (Std Dev/Mean)
Measured multi-stage variance	0.2161	RSD squared
Less analytical variance#	0.2080	Residual variance minus analytical variance (0.0081 g/t)
Standardised variance	0.6742	Residual variance x mean
Mass (g)	0.3720	From Table III
Mass assayed (g)	50	Aliquot size from assayers
Single stage variance	0.6640	Standardised variance (0.6742) minus the sub-sampling and analytical variance of the pulp series
$s^2{}^*M_S$	0.2470	
ln($s^2{}^*M_S$)	-1.3984	
ln(d_{Max})	0.2776	

*The values for this particular data set from Series 3 of the DSA method for fragment size 95% passing 1.32 cm.
#Analytical variance for gold between 1 and 50 g/t supplied by Set Point Laboratories, Johannesburg, South Africa

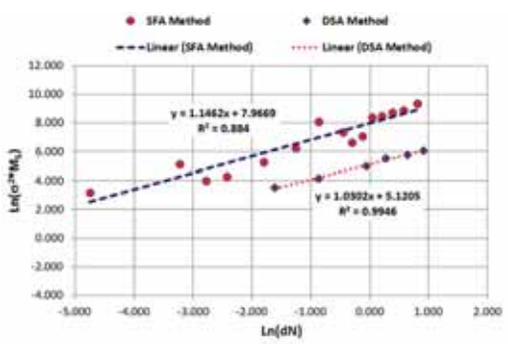

Figure 8—Comparison of calibration curves for the DSA and SFA methods

annum and three shifts per day, equivalent to about 4.6 million grams per shift (Table VII). Assuming that one assay per shift is required it is necessary to reduce the 4.6 million grams to a single 50 g aliquot for fire assay every shift (8 hours).

The details of the crushing and splitting stages required to calculate the nomogram of Figure 8 are presented in Table VIII. The sampling variance is calculated using the K and α values substituted into Equation [1].

The nomograms for the DSA and the SFA methods are compared in Figure 9.

The difference in α is marginal and does not affect the protocol in any significant way, but the difference in K, 167.4 compared to 2539, is significantly large. This difference indicates that the SFA calibration curve produces values of K and α that are more conservative than those with the DSA method. This likewise leads to a more conservative nomogram for the sampling protocol than does the DSA method.

Calculation of the liberation size d_ℓ

The liberation size of the gold can be calculated using data

Table VII

Crushing and splitting sequence in order to obtain a 50 g aliquot every shift (8 hours)

t/a	t/d	Tons/shift	Grams/shift
5 000000	13 888.9	4629.6	4 629 629.6

Table VIII

Mass, fragment size and sampling variance for the DSA and SFA methods

Mass (g)	Fragment size (cm)	DSA method
4 629 629.6	60.0	0.00359
4 629 629.6	20.0	0.00105
462 963.0	20.0	0.0105
462 963.0	1.0	0.00036
23 148.1	1.0	0.00723
23 148.1	0.0300	0.00014
694.4	0.0300	0.00470
694.4	0.0030	0.00035
48.61	0.0030	0.00506
Mass (g)	**Fragment size (cm)**	**SFA method**
4 629 629.6	60.0	0.03721
4 629 629.6	1.0	0.00055
694 444.4	1.0	0.00366
694 444.4	0.1	0.00034
55 555.6	0.1	0.00427
55 555.6	0.0030	0.00012
2055.6	0.0030	0.00311
2055.6	0.0030	0.00311
2055.56	0.0001	0.00005
61.7	0.0001	0.00312
K	167.4	2539
α	1.123	1.03

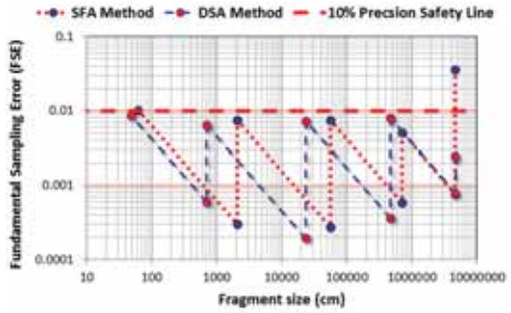

Figure 9—Comparison of the nomograms for the DSA and the SFA methods, indicating a more conservative solution for the SFA method compared to the DSA method

derived from the calibration curves for the DSA and the SFA methods. The specific data, together with an explanation of the relevant calculations, is provided in Table IX. It should be noted that the density for gold used in these calculations is that for a gold-silver amalgam with a density of 16 g/cm³.

Having obtained the estimates for α and K we can now calculate values for d_ℓ, the liberation size, using Equation [11].

$$K = c.f.g.d_\ell^{3-\alpha} \times d_N^{\alpha}$$

$$d_\ell = \left[\frac{K}{c.f.g.d_N^{\alpha}}\right]^{\frac{1}{3-\alpha}} \quad [11]$$

It is important to calculate the liberation size at the nominal fragment size of $d_N = 1$ cm because at this size K is the correct value and $d_N = 1$. Substituting values for K and α for the DSA methods and the SFA method we get:

For the DSA method

$$d_\ell = \left[\frac{167.42}{4163485.52 \times 0.5 \times 0.25 \times 1}\right]^{\frac{1}{3-1.0302}} \times 10000$$

$$d_\ell = 0.00013404^{\frac{1}{1.9698}} \times 10000$$

$$d_\ell = 0.0082365$$

$$d_\ell = 0.000082365 \text{ cm or}$$

$$d_\ell = 82.36 \text{ microns}$$

For the SFA method

$$d_\ell = \left[\frac{2721.66}{2758769.33 \times 0.5 \times 0.25 \times 1}\right]^{\frac{1}{3-1.1419}} \times 10000$$

$$d_\ell = 0.007892^{\frac{1}{1.851}} \times 10000$$

$$d_\ell = 0.0738415$$

$$d_\ell = 0.000738415 \text{ cm or}$$

$$d_\ell = 738.42 \text{ microns}$$

When determining a value for the size of gold grains, *i.e.* the liberation size, it is essential that all the different size fractions (*i.e.* the different series) are derived from the same lot, as they are in this particular case. The liberation size d_ℓ of the gold grains was determined at 82 μm for the DSA method and 738 μm for the SFA method. Generally, a liberation size of 738 μm would appear to be too large for typical Witwatersrand gold-bearing ores, with 82 μm being a far more acceptable grain size. However, no work has been completed to establish the exact size distribution of gold in these ores. The liberation size of the SFA lot should probably been verified using another of the mine samples. There is therefore a concern that the SFA method overestimates K; this may be the subject of further work.

Accumulated evidence from DSA and SFA calibration experiments

The equation for the estimation of Gy's Fundamental Sampling Error has become firmly entrenched in the minerals industry. It is given by (Francois-Bongarçon, 1991):

$$\sigma_{FSE}^2 = \frac{Kd_N^{\alpha}}{M_S} \quad [12]$$

This equation has been used as a method for defining the calibration curves from which values of α and K can be determined. These values are essential in order to compile sampling nomograms for specific ore types from which a sampling protocol can be designed (Minnitt, Rice, and Spangenberg, 2007; Minnitt and Assibey-Bonsu, 2009; Minnitt *et al.*, 2011).

Since 2009 a number of results from experiments using the DSA and SFA calibration methods have been published, and the equations for the trend lines are listed against the source of the data in Table X. Also shown in Table X is a list of the R^2 values for the fit of the trend lines to the data. The behaviour of the data-sets that have been accumulated, and which are listed in Appendix 3, is reviewed.

Table IX

Calculation of the liberation size of gold grains

Parameter	DSA Method	SFA method	Explanation
Grade (g/t)	3.84	5.80	Calculated as the average of 32 assays
g/g (100000)	0.000 003 843	0.0 000 058	Grams per to expressed as gram/gram
ρ/g*	4 163 485.52	275 8769.33	
K (calibrated)	167.42	2 721.66	Calculated from the intercept of the calibration
f	0.5	0.5	Shape Factor
g	0.6	0.25	Granulometry factor
c	4 163 485.52	2 758 769.33	Mineralogical constant
α	1.03	1.14	Calculated from the slope of the calibration curves
Exponent (1/(3-a)	0.51	0.54	
cfg	1 249 045.66	344 846.17	
(*K*/(*f***c***g*)	0.000 134 042	0.00 789	
d_ℓ (cm)	0.010 813 038	0.073 842	
d_ℓ (m)	0.00 010 813	0.000 738	
d_ℓ (mm)	108.13	738.4	Size of liberated gold grains
Density for (gold-silver alloy) 16g/cm³			

A version of Gy's equation for gold-bearing ores

Data listed in Table X is derived from the plots of $\ln(s^2 * M_S)$ versus $\ln(d_{max})$ for DSA and SFA data-sets that are plotted and shown in Figure 10.

The empirical evidence suggests that the average value for α in gold-related calibration curves shown in Figure 10 is 1.026, which is sufficiently close to unity to suggest that the value should in fact be 1.00. Such a value means that Equation [1] suggested by Francois-Bongarçon (1996) should in fact be written as shown in Equation [13].

$$\sigma^2_{FSE} = \frac{Kd_N}{M_S} \qquad [13]$$

Such a change in the formula for the FSE results in a considerable simplification in the calculation of the error and in the methods for calibrating a value for K in Equation [13].

Implications for the heterogeneity test

The heterogeneity test (HT) is a standard industry practice that allows the sampling constants, in particular a value for K, to be determined for the purpose of designing and optimizing sample preparation protocols for different types of mineralization. Characterization of mineral size distribution, mineral associations, modes of occurrence, and sampling characteristics of the ores should precede the HT. The standard HT is performed by controlling d_N to a size as close to 1 cm as possible so that the value of d_N^3 is close to unity; as it turns out the size of fragments between screen sizes of 0.63 and 1.25 cm is 1.05 cm. The mass of each sample is controlled to an exact value so that M_S is also known exactly. The variance is then calculated from the 100 or so fire assays of samples collected from this particular size range, leaving K as the only unknown which is solved for in Equation [1].

A distinction needs to be drawn between the use of symbols K and C in the equations defining the inherent heterogeneity (of the lot) IHL and the FSE, depending on how the exponent of the nominal fragment size is specified. If the nominal size d has an exponent of 2.5 ($d^{2.5}$), or α, where $\alpha = 3\text{-}x$, (d^{3-x}), the appropriate symbol is K. If the exponent of d is 3(d^3), then the appropriate symbol is C. According to Pitard (2009), Gy's earlier literature defined the constant factor of constitution heterogeneity (IH_L) as shown in Equation 14:

$$IH_L = f \times g \times c \times \ell \times d_N^3 = C \times d_N^3 \qquad [14]$$

Because the liberation factor is a function of d_N the constant C, the product of four factors including ℓ, changes as d_N changes. For practical purposes it is customary to express IH_L as shown in Equation [14], with little doubt that the exponent of d is $\alpha = 3$, unless the liberation factor is modelled as a function of d itself. FSE is therefore a function of the sampling constant C, the cube of the nominal size of the fragments (d^3_N), and the inverse of the mass of the sample (M_S), giving the familiar formula derived by Gy (1979) and shown in Equation 15:

$$s^2_{FSE} = \frac{Cd_N^3}{M_S} \qquad [15]$$

In regard to the HT we define C_ϕ as the sampling constant for a specific size fraction ϕ, *i.e.* for a single stage of comminution in the sampling process, identified by subscript ϕ. The single stage error variance is defined as:

$$\sigma^2_\phi = \left(\frac{1}{M_\phi} - \frac{1}{M_L}\right) C_\phi d_\phi^3 \qquad [16]$$

where variables in the equation represent the mass of sample (M_ϕ), mass of lot (M_L), nominal fragment size (d_ϕ) and sampling constant (C_ϕ) for a specific or single stage variance in a sampling protocol. Equation [16] can be rearranged (Minnitt and Assibey-Bonsu, 2009) to indicate that the sampling constant K does not change from one stage of comminution to another, so that:

$$K = C_\phi \sqrt{d_\phi} \qquad [17]$$

However since Equation [13] is now considered applicable for the derivation of FSE, and the exponent of d is now unity, the value for K should be the same for any particular size fraction that one may choose to use. No longer is it necessary to use a fragment size close to 1 cm; any fragment size should give the same value for K.

The proposed change in the formula for the FSE also means that the simple HT championed by Gy and Pitard is also simplified and indicates that the only factor needed to establish an acceptable nomogram is a calibrated value for K.

Table X

Trendline equations and R^2 values for DSA and SFA data sets

Data Set	Straight line equations	R^2
DSA_Lily_GM_2009	y = 0.8131x + 2.51	0.836
DSA_Mponeng_GM_2009	y = 1.5161x + 4.14	0.993
DSA_Kloof_GM_2009	y = 0.9323x + 6.01	0.742
DSA_Nyankanga_GM(BIF)_2014	y = 0.8666x + 5.24	0.928
DSA_Geita_GM_2014	y = 0.7501x + 4.78	0.992
DSA_Star_and_Comet_GM_2014	y = 0.9877x + 4.29	0.958
DSA_Nyankanga_GM(DIO)_2014	y = 0.9522x + 2.59	0.997
DSA_Target GM_2014	y = 1.0302x + 5.12	0.995
SFA_Lily_GM_2011	y = 1.1395x + 3.67	0.952
SFA_Target_GM_2014	y = 1.1462x + 7.97	0.884
SFA_Tshepong_GM_2015	y = 1.1517x + 0.374	0.941
Average of α values	1.026	

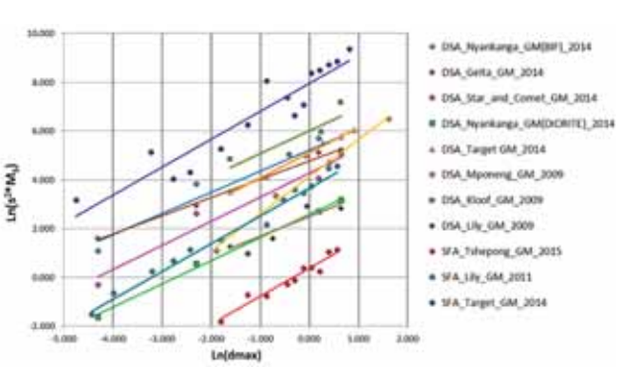

Figure 10—Plot of $\ln(s^{2*}MS)$ *versus* $\ln(d_{Max})$ for DSA and SFA data-sets

Simulated changes in *K* and α for the DSA model

A model for low- (1 to 5 g/t) and high- (6 to 22 g/t) grade ores was simulated to examine the behaviour of values for the coefficient *K* and for the exponent α. For any given fragment size, low-grade ores will generally have lower variance than higher grade ores. The changes in *K* are large for changes in grade, whereas there are only minor changes in the exponent α for high- and low-grade ores, as shown in Figure 11.

The model shown in Figure 11 indicates that *K* decreases from about 7 to 2 in low-grade ores, and from about 32 to 2 in high-grade ores, as the grade increases from 2 to 22 g/t. Thus the FSE must increase as the grade of the ore increases, meaning that for higher grade ores a sample of larger mass is required. Values for the exponent α change from 0.96 to 1.03, a marginal change around a value of 1.0, in support of the empirical evidence that a value of unity should be applied in heterogeneity studies and in the construction of nomograms.

Further indications that the value of the exponent in the Francois-Bongarçon (1992) version of Gy's formula is unity is provided in Figure 12. Actual data for the exponent from a number of different gold mines where calibration exercises had been carried out are plotted against the corresponding mean grade gold grades in Figure 12.

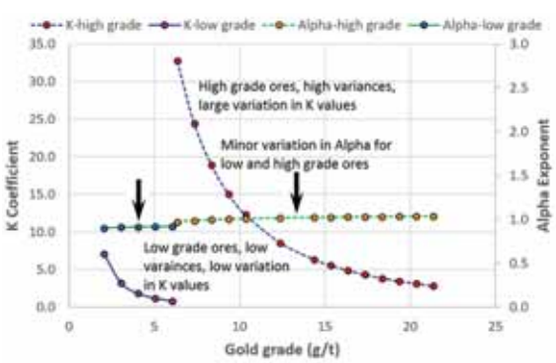

Figure 11—Regions of high- and low-grade ore showing significant changes in the values for coefficient *K*; minor variations in the exponent α around a value of unity occur in high- and low-grade ores

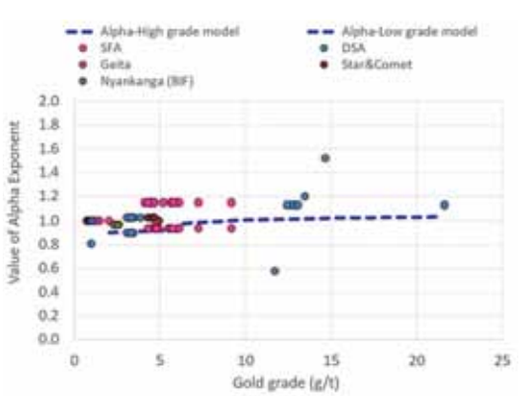

Figure 12—Plot of α against the mean ore grade for various mines using the DSA and SFA methods of calibration

The α values are generally between 0.8 and 1.2 and lie around the α-high grade and α-low grade model lines (dashed) shown in Figure 12. Outliers are present at 0.58 and 1.54, but no clear explanation for these values can be offered. The average value for 83 exponents was 1.009.

Conclusion

This paper compares two different approaches, the DSA and the SFA, to the calibration of the sampling parameters *K* and α for use in the formula proposed by Gy (1979) for the estimation of the Fundamental Sampling Error. The importance of these comparisons is to demonstrate that while they may produce different values for *K*, the values for the exponent α for both methods are almost identical and close to unity, as indicated by an analysis of eleven DSA and SFA tests on twelve different gold-bearing ores.

This conclusion has significant implications for future heterogeneity tests in that it indicates that provided the fragment size of the material used for the test is closely screened, any size ratio should produce the same value for the sampling constant *K*. Furthermore, the nomograms produced for sampling protocols from constants derived from the calibration of *K* and α will be identical.

Acknowledgements

The author acknowledges the permission of the Australasian Institute of Mining and Metallurgy (AusIMM) to publish some of the material presented in a paper at the Sampling 2014 Conference held in Perth. Reviewer's comments resulted in a minor extension to this paper, for which they are thanked.

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Appendix 1: Series number, sample number, mass, and gold grade for the six DSA series of analyses (Analysis certificate ISA-14_00818 supplied by SGS South Africa (Pty) Limited)

Size:	2.5 cm	Assay
Sample No	Mass (g)	(ppm)
Series 1 Sample 1	412	4.77
Series 1 Sample 2	412	2.17
Series 1 Sample 3	412	2.56
Series 1 Sample 4	412	2.78
Series 1 Sample 5	412	8.09
Series 1 Sample 6	412	3.33
Series 1 Sample 7	412	2.78
Series 1 Sample 8	412	2.92
Series 1 Sample 9	412	7.50
Series 1 Sample 10	412	1.97
Series 1 Sample 11	412	7.88
Series 1 Sample 12	412	7.28
Series 1 Sample 13	412	4.89
Series 1 Sample 14	412	2.44
Series 1 Sample 15	412	2.03
Series 1 Sample 16	412	1.90
Series 1 Sample 17	412	2.38
Series 1 Sample 18	412	2.51
Series 1 Sample 19	412	3.61
Series 1 Sample 20	412	2.78
Series 1 Sample 21	412	3.10
Series 1 Sample 22	412	1.77
Series 1 Sample 23	412	3.58
Series 1 Sample 24	412	5.88
Series 1 Sample 25	412	7.35
Series 1 Sample 26	412	5.94
Series 1 Sample 27	412	2.40
Series 1 Sample 28	412	2.20
Series 1 Sample 29	412	2.58
Series 1 Sample 30	412	5.20
Series 1 Sample 31	412	2.99
Series 1 Sample 32	412	0.43

Size:	1.90 cm	Assay
Sample No	Mass (g)	(ppm)
Series 2 Sample 1	381	4.50
Series 2 Sample 2	381	2.01
Series 2 Sample 3	381	3.55
Series 2 Sample 4	381	1.86
Series 2 Sample 5	381	2.90
Series 2 Sample 6	381	1.70
Series 2 Sample 7	381	2.19
Series 2 Sample 8	381	3.87
Series 2 Sample 9	381	3.55
Series 2 Sample 10	381	3.10
Series 2 Sample 11	381	6.57
Series 2 Sample 12	381	3.92
Series 2 Sample 13	381	1.74
Series 2 Sample 14	381	5.26
Series 2 Sample 15	381	3.10
Series 2 Sample 16	381	5.97
Series 2 Sample 17	381	4.40
Series 2 Sample 18	381	1.22
Series 2 Sample 19	381	7.23
Series 2 Sample 20	381	3.75
Series 2 Sample 21	381	1.18
Series 2 Sample 22	381	8.26
Series 2 Sample 23	381	1.61
Series 2 Sample 24	381	1.97
Series 2 Sample 25	381	2.14
Series 2 Sample 26	381	2.17
Series 2 Sample 27	381	4.18
Series 2 Sample 28	381	1.62
Series 2 Sample 29	381	3.50
Series 2 Sample 30	381	2.67
Series 2 Sample 31	381	4.45
Series 2 Sample 32	381	4.32

Size:	1.32 cm	Assay
Sample No	Mass (g)	(ppm)
Series 3 Sample 1	372	6.17
Series 3 Sample 2	372	2.87
Series 3 Sample 3	372	1.54
Series 3 Sample 4	372	1.72
Series 3 Sample 5	372	1.68
Series 3 Sample 6	372	5.52
Series 3 Sample 7	372	2.00
Series 3 Sample 8	372	3.04
Series 3 Sample 9	372	1.96
Series 3 Sample 10	372	2.78
Series 3 Sample 11	372	3.88
Series 3 Sample 12	372	2.69
Series 3 Sample 13	372	3.28
Series 3 Sample 14	372	3.80
Series 3 Sample 15	372	4.52
Series 3 Sample 16	372	5.38
Series 3 Sample 17	372	3.84
Series 3 Sample 18	372	4.78
Series 3 Sample 19	372	6.99
Series 3 Sample 20	372	1.82
Series 3 Sample 21	372	4.51
Series 3 Sample 22	372	4.67
Series 3 Sample 23	372	1.49
Series 3 Sample 24	372	3.88
Series 3 Sample 25	372	2.75
Series 3 Sample 26	372	3.42
Series 3 Sample 27	372	1.09
Series 3 Sample 28	372	1.83
Series 3 Sample 29	372	1.56
Series 3 Sample 30	372	1.74
Series 3 Sample 31	372	1.78
Series 3 Sample 32	372	2.51

Appendix 1 *(continued)*

Size:	0.95cm	Assay
Sample No	Mass (g)	(ppm)
Series 4 Sample 1	368	3.54
Series 4 Sample 2	368	2.78
Series 4 Sample 3	368	2.31
Series 4 Sample 4	368	2.23
Series 4 Sample 5	368	8.23
Series 4 Sample 6	368	3.98
Series 4 Sample 7	368	2.67
Series 4 Sample 8	368	2.14
Series 4 Sample 9	368	2.84
Series 4 Sample 10	368	2.16
Series 4 Sample 11	368	3.08
Series 4 Sample 12	368	3.10
Series 4 Sample 13	368	4.29
Series 4 Sample 14	368	2.26
Series 4 Sample 15	368	2.66
Series 4 Sample 16	368	2.46
Series 4 Sample 17	368	2.95
Series 4 Sample 18	368	3.40
Series 4 Sample 19	368	2.03
Series 4 Sample 20	368	3.15
Series 4 Sample 21	368	2.67
Series 4 Sample 22	368	2.95
Series 4 Sample 23	368	2.43
Series 4 Sample 24	368	3.40
Series 4 Sample 25	368	3.68
Series 4 Sample 26	368	1.39
Series 4 Sample 27	368	3.44
Series 4 Sample 28	368	3.53
Series 4 Sample 29	368	2.17
Series 4 Sample 30	368	3.51
Series 4 Sample 31	368	1.78
Series 4 Sample 32	368	4.23

Size:	0.42 cm	Assay
Sample No	Mass (g)	(ppm)
Series 5 Sample 1	368	3.86
Series 5 Sample 2	368	3.43
Series 5 Sample 3	368	3.12
Series 5 Sample 4	368	5.74
Series 5 Sample 5	368	2.37
Series 5 Sample 6	368	3.41
Series 5 Sample 7	368	4.85
Series 5 Sample 8	368	3.14
Series 5 Sample 9	368	3.32
Series 5 Sample 10	368	4.19
Series 5 Sample 11	368	2.83
Series 5 Sample 12	368	4.41
Series 5 Sample 13	368	3.10
Series 5 Sample 14	368	3.50
Series 5 Sample 15	368	2.45
Series 5 Sample 16	368	3.40
Series 5 Sample 17	368	3.65
Series 5 Sample 18	368	3.88
Series 5 Sample 19	368	3.98
Series 5 Sample 20	368	4.08
Series 5 Sample 21	368	3.43
Series 5 Sample 22	368	4.25
Series 5 Sample 23	368	3.26
Series 5 Sample 24	368	3.48
Series 5 Sample 25	368	1.62
Series 5 Sample 26	368	2.55
Series 5 Sample 27	368	3.04
Series 5 Sample 28	368	4.49
Series 5 Sample 29	368	2.77
Series 5 Sample 30	368	4.11
Series 5 Sample 31	368	1.82
Series 5 Sample 32	375	2.59

Size:	0.2 cm	Assay
Sample No	Mass (g)	(ppm)
Series 6 Sample 1	375	2.11
Series 6 Sample 2	375	2.94
Series 6 Sample 3	375	4.83
Series 6 Sample 4	375	3.05
Series 6 Sample 5	375	3.02
Series 6 Sample 6	375	2.54
Series 6 Sample 7	375	4.08
Series 6 Sample 8	375	3.79
Series 6 Sample 9	375	3.15
Series 6 Sample 10	375	3.96
Series 6 Sample 11	375	3.67
Series 6 Sample 12	375	4.21
Series 6 Sample 13	375	2.80
Series 6 Sample 14	375	2.83
Series 6 Sample 15	375	2.98
Series 6 Sample 16	375	2.94
Series 6 Sample 17	375	3.16
Series 6 Sample 18	375	2.83
Series 6 Sample 19	375	3.97
Series 6 Sample 20	375	4.47
Series 6 Sample 21	375	3.93
Series 6 Sample 22	375	3.93
Series 6 Sample 23	375	3.58
Series 6 Sample 24	375	2.88
Series 6 Sample 25	375	4.21
Series 6 Sample 26	375	3.33
Series 6 Sample 27	375	3.32
Series 6 Sample 28	375	3.94
Series 6 Sample 29	375	2.50
Series 6 Sample 30	375	2.68
Series 6 Sample 31	375	2.91
Series 6 Sample 32	375	2.57

Appendix 2: Series number, sample number, mass, and gold grade for the fifteen SFA series of analyses (Analysis certificate ISA-14_00815 supplied by SGS South Africa (Pty) Limited)

Size:	2.5-1.9	Assay
Sample No	Mass (g)	(ppm)
Series 2 Sample 1	0.121	0.27
Series 2 Sample 2	0.164	0.48
Series 2 Sample 3	0.089	0.46
Series 2 Sample 4	0.106	1.30
Series 2 Sample 5	0.136	0.15
Series 2 Sample 6	0.125	1.32
Series 2 Sample 7	0.094	3.76
Series 2 Sample 8	0.117	0.07
Series 2 Sample 9	0.053	0.75
Series 2 Sample 10	0.161	0.23
Series 2 Sample 11	0.132	0.21
Series 2 Sample 12	0.135	0.96
Series 2 Sample 13	0.235	0.08
Series 2 Sample 14	0.095	0.28
Series 2 Sample 15	0.133	0.33
Series 2 Sample 16	0.162	0.81
Series 2 Sample 17	0.083	1.67
Series 2 Sample 18	0.083	1.25
Series 2 Sample 19	0.113	140
Series 2 Sample 20	0.159	5.46
Series 2 Sample 21	0.115	3.31
Series 2 Sample 22	0.103	0.64
Series 2 Sample 23	0.209	9.58
Series 2 Sample 24	0.111	0.40
Series 2 Sample 25	0.157	2.02
Series 2 Sample 26	0.213	0.89
Series 2 Sample 27	0.114	161
Series 2 Sample 28	0.114	3.72
Series 2 Sample 29	0.127	1.80
Series 2 Sample 30	0.067	2.38
Series 2 Sample 31	0.119	0.54
Series 2 Sample 32	0.183	0.47

Size:	1.9-1.6	Assay
Sample No	Mass (g)	(ppm)
Series 3 Sample 1	0.604	7.71
Series 3 Sample 2	0.599	5.57
Series 3 Sample 3	0.666	2.83
Series 3 Sample 4	0.623	1.59
Series 3 Sample 5	0.594	8.45
Series 3 Sample 6	0.601	4.51
Series 3 Sample 7	0.495	58
Series 3 Sample 8	0.640	0.73
Series 3 Sample 9	0.517	6.77
Series 3 Sample 10	0.557	1.67
Series 3 Sample 11	0.545	0.30
Series 3 Sample 12	0.520	4.84
Series 3 Sample 13	0.539	0.41
Series 3 Sample 14	0.486	7.26
Series 3 Sample 15	0.512	0.74
Series 3 Sample 16	0.500	18.9
Series 3 Sample 17	0.506	4.11
Series 3 Sample 18	0.444	1.87
Series 3 Sample 19	0.597	0.32
Series 3 Sample 20	0.688	27
Series 3 Sample 21	0.586	1.68
Series 3 Sample 22	0.477	29
Series 3 Sample 23	0.411	19.2
Series 3 Sample 24	0.437	4.28
Series 3 Sample 25	0.505	25
Series 3 Sample 26	0.431	8.35
Series 3 Sample 27	0.466	1.55
Series 3 Sample 28	0.363	4.69
Series 3 Sample 29	0.444	7.99
Series 3 Sample 30	0.489	13.5
Series 3 Sample 31	0.471	9.00
Series 3 Sample 32	0.519	5.96

Size:	1.6-1.32	Assay
Sample No	Mass (g)	(ppm)
Series 4 Sample 1	0.879	1.09
Series 4 Sample 2	1.093	3.87
Series 4 Sample 3	0.812	8.10
Series 4 Sample 4	0.830	0.73
Series 4 Sample 5	0.881	5.95
Series 4 Sample 6	0.975	4.18
Series 4 Sample 7	0.736	7.05
Series 4 Sample 8	0.960	2.28
Series 4 Sample 9	0.779	24
Series 4 Sample 10	0.840	2.45
Series 4 Sample 11	0.849	0.76
Series 4 Sample 12	0.917	2.55
Series 4 Sample 13	0.852	1.16
Series 4 Sample 14	0.859	10.6
Series 4 Sample 15	0.785	1.64
Series 4 Sample 16	0.829	3.29
Series 4 Sample 17	0.793	13.7
Series 4 Sample 18	0.769	0.99
Series 4 Sample 19	0.850	15.3
Series 4 Sample 20	0.928	51
Series 4 Sample 21	0.707	15.1
Series 4 Sample 22	0.813	0.43
Series 4 Sample 23	0.693	4.91
Series 4 Sample 24	0.778	1.65
Series 4 Sample 25	0.800	1.44
Series 4 Sample 26	0.736	6.95
Series 4 Sample 27	0.835	6.59
Series 4 Sample 28	0.829	3.18
Series 4 Sample 29	0.640	1.05
Series 4 Sample 30	0.871	4.69
Series 4 Sample 31	0.796	5.76
Series 4 Sample 32	0.791	1.45

Appendix 2 *(continued)*

Size:	1.32-1.12	Assay
Sample No	Mass (g)	(ppm)
Series 5 Sample 1	1.292	1.97
Series 5 Sample 2	1.473	6.05
Series 5 Sample 3	1.288	2.28
Series 5 Sample 4	1.403	6.70
Series 5 Sample 5	1.279	0.97
Series 5 Sample 6	1.317	2.02
Series 5 Sample 7	1.248	7.13
Series 5 Sample 8	1.309	3.81
Series 5 Sample 9	1.242	4.81
Series 5 Sample 10	1.409	3.16
Series 5 Sample 11	1.291	4.55
Series 5 Sample 12	1.279	3.72
Series 5 Sample 13	1.341	5.85
Series 5 Sample 14	1.324	17.4
Series 5 Sample 15	1.225	3.63
Series 5 Sample 16	1.326	8.40
Series 5 Sample 17	1.261	5.59
Series 5 Sample 18	1.309	1.90
Series 5 Sample 19	1.264	6.27
Series 5 Sample 20	1.285	12.2
Series 5 Sample 21	1.232	5.58
Series 5 Sample 22	1.262	2.50
Series 5 Sample 23	1.344	4.68
Series 5 Sample 24	1.277	3.19
Series 5 Sample 25	1.241	2.78
Series 5 Sample 26	1.230	6.45
Series 5 Sample 27	1.247	15.5
Series 5 Sample 28	1.111	1.85
Series 5 Sample 29	1.173	1.76
Series 5 Sample 30	1.341	15.2
Series 5 Sample 31	1.303	6.61
Series 5 Sample 32	1.344	21

Size:	1.12-0.95	Assay
Sample No	Mass (g)	(ppm)
Series 6 Sample 1	0.878	1.74
Series 6 Sample 2	0.893	2.80
Series 6 Sample 3	0.855	3.03
Series 6 Sample 4	0.820	4.67
Series 6 Sample 5	0.784	9.30
Series 6 Sample 6	0.823	8.30
Series 6 Sample 7	0.785	1.75
Series 6 Sample 8	0.777	2.62
Series 6 Sample 9	0.726	3.59
Series 6 Sample 10	0.767	11.6
Series 6 Sample 11	0.738	4.88
Series 6 Sample 12	0.756	10.2
Series 6 Sample 13	0.790	1.42
Series 6 Sample 14	0.807	14.8
Series 6 Sample 15	0.736	9.59
Series 6 Sample 16	0.802	3.23
Series 6 Sample 17	0.761	10.5
Series 6 Sample 18	0.797	7.30
Series 6 Sample 19	0.846	3.88
Series 6 Sample 20	0.851	2.51
Series 6 Sample 21	0.786	2.25
Series 6 Sample 22	0.781	19.3
Series 6 Sample 23	0.728	7.63
Series 6 Sample 24	0.748	3.13
Series 6 Sample 25	0.756	27
Series 6 Sample 26	0.878	5.33
Series 6 Sample 27	0.732	25
Series 6 Sample 28	0.798	2.39
Series 6 Sample 29	0.745	4.23
Series 6 Sample 30	0.790	6.39
Series 6 Sample 31	0.700	2.19
Series 6 Sample 32	0.796	9.32

Size:	0.95-0.80	Assay
Sample No	Mass (g)	(ppm)
Series 7 Sample 19	0.649	2.25
Series 7 Sample 18	0.682	5.85
Series 7 Sample 22	0.635	5.64
Series 7 Sample 32	0.684	4.64
Series 7 Sample 7	0.671	3.25
Series 7 Sample 6	0.715	1.91
Series 7 Sample 1	0.673	1.54
Series 7 Sample 31	0.683	2.45
Series 7 Sample 16	0.704	5.49
Series 7 Sample 8	0.734	5.33
Series 7 Sample 29	0.635	5.59
Series 7 Sample 17	0.679	3.99
Series 7 Sample 5	0.645	5.35
Series 7 Sample 26	0.683	6.02
Series 7 Sample 30	0.700	8.34
Series 7 Sample 12	0.649	2.39
Series 7 Sample 20	0.677	2.72
Series 7 Sample 4	0.757	1.68
Series 7 Sample 10	0.702	1.27
Series 7 Sample 13	0.636	4.63
Series 7 Sample 9	0.622	7.58
Series 7 Sample 28	0.720	1.00
Series 7 Sample 11	0.656	11.80
Series 7 Sample 3	0.706	6.48
Series 7 Sample 2	0.771	8.04
Series 7 Sample 14	0.667	3.46
Series 7 Sample 24	0.709	13.30
Series 7 Sample 21	0.613	5.55
Series 7 Sample 25	0.642	2.68
Series 7 Sample 15	0.660	3.48
Series 7 Sample 23	0.671	2.35
Series 7 Sample 27	0.682	1.07

Size:	0.8-0.67	Assay
Sample No	Mass (g)	(ppm)
Series 8 Sample 7	0.480	4.40
Series 8 Sample 23	0.467	2.62
Series 8 Sample 26	0.526	3.34
Series 8 Sample 22	0.525	2.48
Series 8 Sample 10	0.545	5.76
Series 8 Sample 8	0.526	3.45
Series 8 Sample 15	0.488	0.73
Series 8 Sample 20	0.524	1.84
Series 8 Sample 4	0.558	9.71
Series 8 Sample 25	0.489	1.80
Series 8 Sample 2	0.581	10.40
Series 8 Sample 32	0.457	4.08
Series 8 Sample 3	0.540	6.00
Series 8 Sample 6	0.559	4.59
Series 8 Sample 28	0.532	2.30
Series 8 Sample 12	0.560	4.22
Series 8 Sample 17	0.514	4.15
Series 8 Sample 16	0.485	8.15
Series 8 Sample 1	0.536	5.30
Series 8 Sample 14	0.523	2.45
Series 8 Sample 30	0.514	6.92
Series 8 Sample 31	0.473	1.66
Series 8 Sample 19	0.485	0.96
Series 8 Sample 5	0.513	10.50
Series 8 Sample 13	0.479	2.53
Series 8 Sample 21	0.522	1.53
Series 8 Sample 29	0.469	8.12
Series 8 Sample 27	0.506	3.97
Series 8 Sample 18	0.551	6.92
Series 8 Sample 9	0.507	4.82
Series 8 Sample 11	0.494	5.25
Series 8 Sample 24	0.496	2.70

Size:	0.67-0.475	Assay
Sample No	Mass (g)	(ppm)
Series 9 Sample 9	0.261	7.59
Series 9 Sample 25	0.251	9.79
Series 9 Sample 22	0.268	4.89
Series 9 Sample 17	0.265	19.40
Series 9 Sample 28	0.263	3.93
Series 9 Sample 32	0.245	3.63
Series 9 Sample 31	0.240	1.96
Series 9 Sample 11	0.239	2.98
Series 9 Sample 29	0.233	0.57
Series 9 Sample 10	0.274	1.89
Series 9 Sample 7	0.254	1.76
Series 9 Sample 15	0.229	3.37
Series 9 Sample 14	0.255	2.65
Series 9 Sample 23	0.233	2.32
Series 9 Sample 13	0.236	2.03
Series 9 Sample 21	0.248	6.04
Series 9 Sample 8	0.257	0.91
Series 9 Sample 12	0.251	29.00
Series 9 Sample 26	0.259	5.22
Series 9 Sample 6	0.273	17.40
Series 9 Sample 5	0.242	2.77
Series 9 Sample 3	0.244	0.85
Series 9 Sample 19	0.259	2.47
Series 9 Sample 27	0.232	17.50
Series 9 Sample 16	0.258	0.79
Series 9 Sample 1	0.270	3.57
Series 9 Sample 30	0.270	5.77
Series 9 Sample 2	0.299	1.38
Series 9 Sample 20	0.245	1.77
Series 9 Sample 24	0.244	8.62
Series 9 Sample 4	0.271	1.52
Series 9 Sample 18	0.288	1.41

Size:	0.475-0.34	Assay	Ln
Sample No	Mass (g)	(ppm)	grade
Series 10 Sample 30	0.586	7.08	
Series 10 Sample 24	0.579	6.28	1.84
Series 10 Sample 5	0.603	3.16	1.15
Series 10 Sample 32	0.550	12.80	2.55
Series 10 Sample 22	0.586	1.82	0.60
Series 10 Sample 26	0.594	5.09	1.63
Series 10 Sample 31	0.536	10.50	2.35
Series 10 Sample 27	0.536	3.98	1.38
Series 10 Sample 20	0.590	3.63	1.29
Series 10 Sample 29	0.568	23.00	3.14
Series 10 Sample 23	0.550	21.00	3.04
Series 10 Sample 17	0.590	3.95	1.37
Series 10 Sample 28	0.592	10.20	2.32
Series 10 Sample 3	0.587	9.30	2.23
Series 10 Sample 15	0.565	3.36	1.21
Series 10 Sample 19	0.548	4.04	1.40
Series 10 Sample 25	0.571	2.99	1.10
Series 10 Sample 9	0.586	5.47	1.70
Series 10 Sample 12	0.598	3.38	1.22
Series 10 Sample 8	0.602	2.62	0.96
Series 10 Sample 16	0.580	10.70	2.37
Series 10 Sample 6	0.614	1.99	0.69
Series 10 Sample 18	0.634	2.84	1.04
Series 10 Sample 2	0.652	1.66	0.51
Series 10 Sample 1	0.611	3.61	1.28
Series 10 Sample 14	0.613	2.19	0.78
Series 10 Sample 13	0.566	2.58	0.95
Series 10 Sample 7	0.579	3.02	1.11
Series 10 Sample 21	0.575	2.62	0.96
Series 10 Sample 4	0.619	0.83	-0.19
Series 10 Sample 11	0.566	2.27	0.82
Series 10 Sample 10	0.617	1.94	0.66

Appendix 2 *(continued)*

Size:	0.335-0.2	Assay
Sample No	Mass (g)	(ppm)
Series 11 Sample 17	0.376	2.47
Series 11 Sample 25	0.383	7.59
Series 11 Sample 6	0.413	3.80
Series 11 Sample 15	0.369	3.90
Series 11 Sample 22	0.386	4.49
Series 11 Sample 32	0.375	1.70
Series 11 Sample 10	0.421	4.31
Series 11 Sample 1	0.399	2.91
Series 11 Sample 27	0.356	6.85
Series 11 Sample 13	0.380	2.21
Series 11 Sample 11	0.384	2.84
Series 11 Sample 8	0.398	5.26
Series 11 Sample 18	0.417	2.82
Series 11 Sample 3	0.391	7.22
Series 11 Sample 4	0.417	1.78
Series 11 Sample 30	0.383	7.83
Series 11 Sample 29	0.367	1.18
Series 11 Sample 7	0.380	2.97
Series 11 Sample 21	0.379	6.01
Series 11 Sample 5	0.398	13.50
Series 11 Sample 31	0.351	4.48
Series 11 Sample 24	0.368	2.04
Series 11 Sample 26	0.404	6.34
Series 11 Sample 12	0.391	4.87
Series 11 Sample 19	0.372	1.64
Series 11 Sample 23	0.361	5.11
Series 11 Sample 9	0.402	2.47
Series 11 Sample 14	0.394	7.27
Series 11 Sample 28	0.377	4.17
Series 11 Sample 2	0.432	4.06
Series 11 Sample 16	0.376	4.79
Series 11 Sample 20	0.389	2.15

Size:	0.2-0.1	Assay
Sample No	Mass (g)	(ppm)
Series 12 Sample 2	0.402	3.76
Series 12 Sample 8	0.372	1.88
Series 12 Sample 16	0.333	3.40
Series 12 Sample 11	0.333	3.37
Series 12 Sample 9	0.357	3.89
Series 12 Sample 6	0.378	2.72
Series 12 Sample 27	0.327	4.60
Series 12 Sample 4	0.383	2.37
Series 12 Sample 3	0.353	2.66
Series 12 Sample 28	0.351	6.24
Series 12 Sample 1	0.376	2.62
Series 12 Sample 5	0.359	4.60
Series 12 Sample 18	0.387	4.25
Series 12 Sample 13	0.336	7.07
Series 12 Sample 31	0.316	6.20
Series 12 Sample 26	0.367	2.51
Series 12 Sample 7	0.340	5.57
Series 12 Sample 12	0.356	4.00
Series 12 Sample 21	0.345	5.94
Series 12 Sample 22	0.369	6.66
Series 12 Sample 32	0.339	4.88
Series 12 Sample 25	0.349	4.96
Series 12 Sample 17	0.364	7.70
Series 12 Sample 30	0.358	6.28
Series 12 Sample 19	0.352	5.37
Series 12 Sample 15	0.325	4.40
Series 12 Sample 10	0.368	3.27
Series 12 Sample 24	0.348	3.73
Series 12 Sample 20	0.369	8.87
Series 12 Sample 14	0.362	5.62
Series 12 Sample 23	0.323	4.29
Series 12 Sample 29	0.327	5.02

Size:	0.1-0.071	Assay
Sample No	Mass (g)	(ppm)
Series 13 Sample 27	0.291	4.69
Series 13 Sample 21	0.310	3.60
Series 13 Sample 12	0.325	5.74
Series 13 Sample 23	0.301	6.65
Series 13 Sample 2	0.350	5.20
Series 13 Sample 19	0.311	3.99
Series 13 Sample 7	0.311	3.86
Series 13 Sample 26	0.320	4.67
Series 13 Sample 25	0.301	4.72
Series 13 Sample 32	0.305	6.20
Series 13 Sample 30	0.321	5.67
Series 13 Sample 6	0.343	3.14
Series 13 Sample 20	0.322	5.00
Series 13 Sample 29	0.295	5.00
Series 13 Sample 24	0.312	5.95
Series 13 Sample 28	0.306	5.80
Series 13 Sample 8	0.329	7.75
Series 13 Sample 1	0.331	5.56
Series 13 Sample 9	0.323	3.80
Series 13 Sample 22	0.327	4.10
Series 13 Sample 13	0.303	2.89
Series 13 Sample 31	0.289	4.75
Series 13 Sample 5	0.321	3.40
Series 13 Sample 18	0.337	4.36
Series 13 Sample 11	0.308	3.93
Series 13 Sample 3	0.327	3.86
Series 13 Sample 16	0.322	2.27
Series 13 Sample 15	0.304	4.65
Series 13 Sample 10	0.349	4.15
Series 13 Sample 4	0.348	3.98
Series 13 Sample 17	0.317	5.09
Series 13 Sample 14	0.335	3.95

Size:	0.071-0.05	Assay
Sample No	Mass (g)	(ppm)
Series 14 Sample 24	0.168	3.95
Series 14 Sample 30	0.174	7.00
Series 14 Sample 26	0.183	6.41
Series 14 Sample 28	0.174	4.98
Series 14 Sample 7	0.165	6.43
Series 14 Sample 23	0.155	3.55
Series 14 Sample 15	0.157	2.93
Series 14 Sample 31	0.143	4.46
Series 14 Sample 6	0.189	4.89
Series 14 Sample 10	0.186	3.63
Series 14 Sample 18	0.186	4.29
Series 14 Sample 27	0.156	5.49
Series 14 Sample 14	0.184	4.56
Series 14 Sample 1	0.187	3.92
Series 14 Sample 25	0.171	3.03
Series 14 Sample 19	0.163	4.60
Series 14 Sample 32	0.161	5.37
Series 14 Sample 20	0.178	3.72
Series 14 Sample 11	0.162	4.13
Series 14 Sample 22	0.179	4.25
Series 14 Sample 21	0.167	4.38
Series 14 Sample 8	0.179	4.37
Series 14 Sample 13	0.164	2.93
Series 14 Sample 29	0.154	1.94
Series 14 Sample 16	0.173	3.95
Series 14 Sample 9	0.172	1.96
Series 14 Sample 4	0.185	3.74
Series 14 Sample 17	0.175	2.90
Series 14 Sample 12	0.175	4.57
Series 14 Sample 3	0.176	1.94
Series 14 Sample 5	0.172	3.30
Series 14 Sample 2	0.200	4.16

Size:	0.05-0.0095	Assay
Sample No	Mass (g)	(ppm)
Series 15 Sample 27	0.375	3.74
Series 15 Sample 21	0.393	4.73
Series 15 Sample 8	0.422	3.98
Series 15 Sample 18	0.437	5.03
Series 15 Sample 28	0.395	6.02
Series 15 Sample 26	0.413	6.81
Series 15 Sample 25	0.389	4.79
Series 15 Sample 22	0.411	1.74
Series 15 Sample 9	0.438	3.52
Series 15 Sample 1	0.432	5.26
Series 15 Sample 29	0.387	5.10
Series 15 Sample 13	0.407	6.64
Series 15 Sample 3	0.429	3.85
Series 15 Sample 16	0.381	4.24
Series 15 Sample 15	0.361	4.16
Series 15 Sample 14	0.436	4.03
Series 15 Sample 19	0.388	5.19
Series 15 Sample 2	0.475	2.22
Series 15 Sample 7	0.392	4.32
Series 15 Sample 23	0.359	6.04
Series 15 Sample 4	0.445	1.66
Series 15 Sample 11	0.409	3.10
Series 15 Sample 17	0.425	4.90
Series 15 Sample 10	0.460	6.77
Series 15 Sample 32	0.372	2.87
Series 15 Sample 31	0.360	2.37
Series 15 Sample 5	0.423	1.53
Series 15 Sample 20	0.413	2.23
Series 15 Sample 30	0.408	3.81
Series 15 Sample 12	0.419	6.12
Series 15 Sample 24	0.399	5.72
Series 15 Sample 6	0.456	5.27

Size:	<0.05	Assay
Sample No	Mass (g)	(ppm)
Series 16 Sample 2	0.646	4.88
Series 16 Sample 20	0.532	2.35
Series 16 Sample 29	0.563	5.13
Series 16 Sample 1	0.620	5.54
Series 16 Sample 18	0.583	7.12
Series 16 Sample 12	0.592	5.57
Series 16 Sample 3	0.587	9.18
Series 16 Sample 23	0.564	6.12
Series 16 Sample 22	0.638	5.99
Series 16 Sample 17	0.543	6.43
Series 16 Sample 31	0.509	5.67
Series 16 Sample 16	0.551	4.99
Series 16 Sample 30	0.586	6.61
Series 16 Sample 28	0.581	6.62
Series 16 Sample 4	0.640	6.35
Series 16 Sample 6	0.605	5.30
Series 16 Sample 11	0.550	5.16
Series 16 Sample 21	0.604	4.95
Series 16 Sample 9	0.577	6.63
Series 16 Sample 8	0.591	4.67
Series 16 Sample 26	0.578	5.87
Series 16 Sample 25	0.570	5.14
Series 16 Sample 15	0.541	5.13
Series 16 Sample 10	0.598	7.50
Series 16 Sample 27	0.534	6.33
Series 16 Sample 13	0.567	6.14
Series 16 Sample 14	0.596	6.46
Series 16 Sample 19	0.510	5.36
Series 16 Sample 32	0.549	4.79
Series 16 Sample 5	0.586	5.31
Series 16 Sample 24	0.615	5.27
Series 16 Sample 7	0.552	6.98

Appendix 3: Compilation of data used to construct the calibration curves in Figure 10

DSA_Lily_GM_2009	
ln(dmax)	ln(s2*Ms)
0.642	2.826
-0.051	2.932
-0.744	1.587
-1.609	1.248

DSA_Mponeng_GM_2009	
ln(dmax)	ln(s2*Ms)
1.609	6.508
0.405	4.717
-0.693	3.364
-1.897	1.095

DSA_Kloof_GM_2009	
ln(dmax)	ln(s2*Ms)
0.642	7.175
0.239	5.95
-0.4	5.034
-1.609	4.847

DSA_Nyankanga_GM(BIF)_2014	
ln(dmax)	ln(s2*Ms)
0.642	5.238
0.182	5.716
-2.303	3.839
-4.313	1.099

SFA_Lily_GM_2011	
ln(dmax)	ln(s2*Ms)
0.57	4.56
0.39	4.49
0.04	3.74
-0.13	3.48
-0.30	3.60
-0.53	3.18
-0.88	2.17
-1.26	0.99
-1.80	1.56
-2.43	1.14
-2.78	0.69
-3.20	0.25
-3.98	-0.64
-4.43	-1.51

SFA_Tshepong_GM_2015	
ln(dmax)	ln(s2*Ms)
0.567	1.144
0.387	1.056
0.205	0.245
0.041	0.401
-0.126	0.382
-0.300	-0.112
-0.454	-0.270
-0.871	-0.746
-1.260	-0.703
-1.801	-1.817
-1.260	-0.703
-1.801	-1.817

DSA_Geita_GM_2014	
ln(dmax)	ln(s2*Ms)
0.642	5.126
0.182	5.121
-2.303	2.935
-4.313	1.597

DSA_Star_and_Comet_GM_2014	
ln(dmax)	ln(s2*Ms)
0.642	5.038
0.182	4.058
-2.303	2.632
-4.313	-0.305

DSA_Nyankanga_GM(DIO)_2014	
ln(dmax)	ln(s2*Ms)
0.642	3.178
0.182	2.706
-2.303	0.569
-4.313	-1.617

DSA_Target GM_2014	
ln(dmax)	ln(s2*Ms)
0.92	6.06
0.64	5.77
0.28	5.51
-0.05	5.00
-0.87	4.14
-1.61	3.52

DSA_Tshepong_2015	
ln(dmax)	ln(s2*Ms)
0.92	6.10
0.64	6.02
0.28	5.26
-0.05	4.91
-0.87	4.07
-1.61	3.65

SFA_Target_GM_2014	
ln(dmax)	ln(s2*Ms)
0.806	9.376
0.567	8.859
0.387	8.734
0.205	8.500
0.041	8.373
-0.126	7.076
-0.300	6.642
-0.454	7.372
-0.871	8.074
-1.260	6.252
-1.801	5.273
-2.431	4.311
-2.776	4.032
-3.224	5.134
-4.754	3.167
-3.224	5.134
-4.754	3.167