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Multiple cut-off grade optimization by genetic algorithms and comparison with grid search method and dynamic programming

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Synopsis

Optimization of cut-off grades is a fundamental issue for mineral deposits. Determination of optimum cut-off grades, instead of application of a static cut-off grade for the life of a mine, maximizes the net present value. The authors describe the general problem of cut-off grade optimization for multi-mineral deposits and outline the use of genetic algorithms, the grid search method, and dynamic programming for optimal cut-off grade schedules for deposits with up to three constituent minerals. The methods are compared by assessing the results of the implications involved in using them.

Keywords

cut-off grade, sequencing, optimization, multi-mineral deposit, genetic algorithms.

Introduction

Determination of optimum cut-off grades is a fundamental issue in mineral extraction as it assigns the boundaries between ore and waste over time.

The profit from a mining operation is a direct function of the sequences of cut-off grades and associated ore tonnages that define the life-of-mine production schedule. As profit varies with these sequences there will be a sequence, or sequences, that optimize any specified profit criterion. The most widely used cut-off grade optimization criterion is maximum net present value (NPV) of profits. The NPV can be maximized by maximizing profit per unit time. This process necessitates applying, in the early years of operation, the highest cut-off grade that can provide sufficient ore to satisfy the requirements of the processing plant. As time passes, the cut-off grade must be lowered, thereby lowering the opportunity cost. Hence, the highest NPV is achieved.

The objectives of this paper are to develop general methods for determining optimal sequences of cut-off grades for multi-mineral deposits by means of genetic algorithms, to implement this method in computer programs, and to assess the performance of the method. In order to assess the performance of the genetic algorithms method, the grid search method and the dynamic programming method are used and compared with the results of the case of genetic algorithms. The computer programs developed for this purpose are capable of determining optimal sequences of cut-off grades for multi-mineral deposits that contain up to three valuable minerals.

Optimization of cut-off grades for multi-mineral deposits

Mine planning and the financial evaluation of mineral deposits that contain more than one valuable mineral are generally done on the basis of parametric cut-off grades or the equivalents. However, because of problems related to this method, an alternative method of individually optimizing the cut-off grades of the component minerals has been used. The main problem arises from the fact that the revenue and the costs must be calculated on the basis of the average grades of the individual minerals from the calculated equivalent grade. If the constituent minerals are highly correlated, the average grades can be estimated by iteration and by defining some additional parameters for the equivalent grade-tonnage data (Dowd and Xu, 1999). However, if there is very little correlation between the minerals, the validity of the equivalents method is not obvious. Because of the problems the equivalents method brings about, in order to optimize a multi-mineral deposit, the constituent minerals are best dealt with separately.

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Optimization by genetic algorithms

Genetic algorithms constitute a class of stochastic algorithms that use a search method based on the principles of biological genetics and natural evolution. Holland (1975) proposed the basic principles of genetic algorithms. In this approach, individuals of a population are represented as chromosomes and an expanded set of genetic operations takes place. It is presumed that the potential solution of any problem is an individual and can be represented by a set of parameters.

The vocabulary of genetic algorithms is borrowed from genetics science. In nature, each cell of every living organism has a set of chromosomes that make up DNA. Chromosomes are made up of genes, which control different characteristics of an organism. In genetic algorithms, a potential solution to a problem is called an individual or chromosome. Individuals make up a population. Genetic operations, such as crossover, mutation, and reproduction, are also used in genetic algorithms.

Genetic algorithms are particularly suited to the solution of large-scale optimization problems. They belong to the class of probabilistic algorithms but are very different from random algorithms as they combine directed and stochastic searches. Another important property of genetic-based search methods is that they maintain a population of potential solutions. Genetic algorithms can also easily escape from local optima by using genetic operators, such as mutation.

A genetic algorithms flow chart is given in Figure 1.
The basic principles of genetic algorithms are as follows:

1. A set of strings composed of finite elements, generally a binary code, is assigned. Each string refers to a point in the search space or a solution to the problem among the alternatives. Genetic algorithms work on these strings, which are called chromosomes or individuals
2. A first generation, *i.e.* a population, of individuals, is selected. Generally, the selection is done at random
3. The individuals are evaluated on the basis of their return values. Fitness values are assigned to the individuals in order to rank them on the basis of their return values. The values assigned to better solutions result in higher fitness values
4. Some of the individuals are selected on the basis of their fitness values. The individuals with lower fitness values lose in competition
5. Parents are chosen from among the selected individuals. They are crossed over by pairs. The result is two new individuals from each parent
6. Some chromosomes enter a mutation process. That is, one or more digits of a string are changed at random. A new population is ready
7. The process is repeated from step 3 until it converges to a stable value or an assigned number of generations is reached.

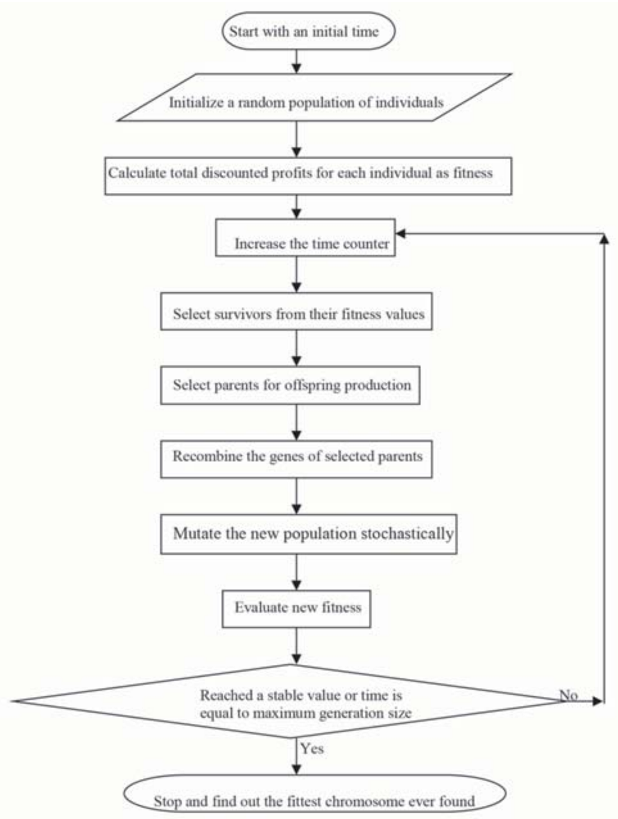

Figure 1—A genetic algorithms flow chart

Major elements of genetic algorithms

Encoding

Genetic algorithms work with representations of solutions. The representation is a symbol string, which carries all the information about the individual. The string has a fixed length and is called a chromosome or individual. The length of the string of an individual depends on the precision requirements. The string can be composed of real decimal numbers or characters, but the most widely used string representation is binary numbers.

The mapping value of a binary string into a real number is straightforward. The binary string is converted into a real number, which is an integer. Then a corresponding real number, which is the mapping value, is found.

In order to initialize an individual composed of binary strings, all bits are to be initialized randomly.

Population

In genetics science, individuals make up a population. The bigger the population, the more extended the search area. However, the number of individuals adversely affects the speed of a computer program based on genetic algorithms.

Evaluation

After initialization of a population made up of binary or real number strings, an evaluation process takes place. Each individual is assigned a fitness value, which is calculated on the basis of objective function for the problem.

Selection

Good individuals with better fitness values are selected in a selection process. Each generation produces new individuals from the current population. Selection is a process of finding how many times each individual from the current population should be copied to generate a new set of solutions or a new population. The process resembles natural selection in that individuals that give better results in the evaluation process have a greater chance of reproducing. The selection process consists of determining the number of times that a particular individual is chosen to have offspring. The selection process can be deterministic or probabilistic.

In deterministic selection, better individuals are determined to have more offspring than poor ones. Individuals with very low fitness values have no chance of survival. Deterministic selection helps to get rid of poor individuals and to generate a quick result.

The roulette wheel method is the most widely used selection method in genetic algorithms. It is a probabilistic method in which individuals with better fitness values are more likely to reproduce, although weak individuals still have a chance of survival. Individuals are represented on the wheel as a proportion of their fitness values.

There are other parameters that can be applied to the deterministic and probabilistic approaches. Scaling is one of these. When the fitness values of individuals of a population are sufficiently distinct, there is no need for any kind of scaling. But if the fitness values are close to each other, which is generally the case as generations pass and most of the individuals have relatively good fitness values, good individuals will lose competitiveness. Scaling is used to improve the situation. The individuals are scaled in order to improve the competition abilities of the good individuals during the selection process. This is generally achieved by subtracting the same number from all the fitness values of the individuals. Consider a problem with only two individuals. Suppose that their fitness values, based on their performances, are 495 and 497. If one of them is to be selected randomly, the chance of the first individual being selected is 49.9% and that of the second is 50.1%. Although one of the individuals is obviously better, the chances of selection are almost the same. However, if the fitness values of the individuals are scaled by subtracting 490 from both, the chances of being selected change to 41.7% and 58.3% respectively.

Another commonly used method of improving the performance of a genetic algorithms process is elitist selection. In genetic algorithms there is always a risk of losing the best individual when generations pass. In elitist selection, the most fit individual, or individuals, after each evaluation phase could be carried to the next generation unchanged (Zalzala and Fleming, 1977).

Genetic operators

As in nature, there are mainly two types of classical genetic operators in genetic algorithms: crossover and mutation.

Crossover

Crossover is the basic operator for the production of new chromosomes. It mimics the sexual reproduction of living organisms. Two parents come together and they produce two infants whose genes resemble those of the parents. The common forms of crossover are 1-point crossover, 2-point crossover, *n*-point crossover, and uniform crossover (Green, 1999).

In 1-point crossover, a crossover point is randomly selected for each couple. Each half of the chromosomes then crosses over to find two new offspring that resemble both parents.

The basic difference in 2-point crossover is that there are two crossover points assigned for each couple. The sections between the two crossover points are swapped for the new individuals.

In *n*-point crossover, there are *n* crossover points. The parts of the strings of the parents between every two crossover points are swapped for the new infants. As a result, the infants would get the parts of the string of a parent between every successive crossover point.

In uniform crossover, a number of points are selected at random. Each selected point is swapped over rather than swapping a part of the string.

Parents are chosen from among the selected individuals randomly according to an explicitly assigned crossover probability.

Mutation

In nature, copying DNA to create offspring can sometimes result in errors. These errors, called mutations, generally do not have a positive effect on the fitness of the individual, although they can sometimes result in beneficial features and they can be passed to succeeding generations via reproduction. Mutation is so important for the evolution of living organisms that without it nature would have been in a vicious circle rather than an evolutionary process.

Genetic algorithms are very different from other stochastic search methods in the method used for searching. Searching starts as a randomly selected population and future solutions depend on mutual relationships of the individuals. Without the mutation process, the search area would be so restricted that finding the global maximum point would be almost impossible in large-scale problems. The search area can be widened gradually by the mutation process and deepened by the crossover process; the features of individuals are improved by the selection process. Individuals evolve gradually until the solutions converge to a maximum point or a predetermined number of generations is reached.

As in the crossover process, mutation points are selected randomly. However, the probability of mutation should be comparatively low, since it is not a common process like crossover. There are different types of mutation. In bit-by-bit mutation, random numbers are generated for each digit of the whole population and, depending on the assigned mutation probability, the digit might be changed. In binary code this is a trivial exercise. If the original digit value were 0, the changed value would be 1, and *vice versa*. In string mutation, however, the mutation probability value is assigned on the string basis. After random numbers have been generated, if a string is mutated, another random number is generated and assigned to the mutation point for the string.

Application of genetic algorithms to cut-off grade optimization

Many cut-off grade optimization problems have huge numbers of local optimum values, which are widely separated from the global optimum point and from each other. Stochastic search methods can easily fail to find the global optimum point for such problems. The real challenge in such problems is finding solutions close to the global optimum point for a restricted time. Genetic algorithms are more robust in this context than many other existing search methods.

Yun *et al.* (1998) applied genetic algorithms to the Jingtieshan iron ore mine in China in order to optimize cut-off grade and minimum average grade, which is a criterion used in China to define ore for mining purposes. They used net present value as a fitness value, binary representation, roulette wheel selection, and 100 iterations (number of generations).

Encoding and evaluation processes used in this paper for the application of genetic algorithms to cut-off grade optimization are described below.

Encoding

Encoding of an individual for the optimization of a single cut-off grade for an ore deposit with only one valuable mineral is straightforward. The string is composed of only one gene, which represents a cut-off grade. The size of the string depends on the number of cut-off grades to be evaluated (searched). If the binary representation is used, the string will be long. A 5-bit string can represent $2^5 = 32$ cut-off grades. To derive real values from the binary code (*i.e.* mapping) the string the formula is:

$$X = X_{\min} + \frac{X_{\max} - X_{\min}}{2^L - 1} * Y$$

where

X: the mapping value

$X_{\min}$: the minimum cut-off grade to be searched for

$X_{\max}$: the maximum cut-off grade to be searched for

L: the length of the binary string

Y: the value of binary representation.

The value of the binary representation for a 5-bit string would be an integer between 0 for string 00000 and 31 for string 11111.

The application of genetic algorithms to the solution of the optimum cut-off grade problem requires a crucial increase in the length of the string. Since in each year in the mine life there might be a different optimum cut-off grade, there should be different genes in the same string. If the mine life is 20 years, the string will be composed of 20 genes, each with a length of five bits, making the total length of the chromosome 100.

Evaluation

In genetic algorithms, every individual is assigned to a fitness value depending on its performance. In cut-off grade optimization, the objective function is maximum NPV. The higher the discounted profit, the better the individual.

Optimization of cut-off grades for multi-mineral deposits by genetic algorithms

The optimization of cut-off grades for multi-mineral deposits is significantly more complex than for single-mineral deposits. A multivariate grade distribution must be used and consequently the dimension of the data increases. This increase in dimension causes an exponential increase in the area to be searched for the optimum.

Besides, genetic algorithms work on representatives of solutions, known as chromosomes. The structures of chromosomes for single-mineral deposits and for multi-mineral deposits differ in that as the number of minerals increases, the length of the related chromosomes increases arithmetically.

The application of genetic algorithms to the optimization of cut-off grades for multi-mineral deposits brings about a further increase of the length of the string. Since for each year of mine life there might be a different optimum cut-off grade, there should be different genes in the same string. In the case of a two-mineral deposit, if the mine life is 20 years, the string would be composed of 40 genes, each five bits in length, making the total length of the chromosome 200.

The genetic algorithms computer program developed in this research work is capable of optimizing cut-off grades for mineral deposits that contain up to three minerals. Binary representation is used and if 32 different cut-off grades are to be searched for each mineral, 5-bit genes must be used. Three minerals require a 15-bit string length. Therefore, if the maximum mine life is 20 years, an ore deposit that contains three minerals requires a string size of 300.

The process of scaling is used in order to improve the selection process. Fitness values are scaled by subtracting the fitness value of the worst individual from the fitness values of all the individuals of the population.

One safeguard has proved necessary to improve the computation results. We know that true maximization of NPV necessitates a sequence of declining cut-off grades. However, only a very small part of randomly selected populations can have cut-off grades in declining order for the life of the mine. Consequently, the algorithm has been changed in such a way that if the depletion rate for a specific year is more than that of the previous year, the cut-off grade for the specified year is set deterministically to that of the previous year. This policy enables the program to search for the optimum among the alternatives that are limited to sequences of declining cut-off grades, and brings about a substantial improvement in the performance of the algorithm.

With respect to the other two methods used in this research, genetic algorithms use four additional parameters that are not directly related to technical or economic constraints. These parameters are population size, generation size, crossover rate, and mutation rate. These parameters have been tested in order to determine an optimum range of control values that will generate the highest discounted profit. As a result of the tests, a population size between 250 and 500, a generation size between 400 and 500, a crossover rate of 20% to 50%, and a mutation rate of 40% to 100% are proved to be reasonable.

For the sake of comparison, the results have been tested by other methods that were used in multi-mineral cut-off grade optimization. These are the grid search method and the dynamic programming method. The grid search method used here is explained by Cetin and Dowd (2013). The use of dynamic programming in multi-mineral cut-off grade optimization used in this work is explained by Cetin and Dowd (2011).

Case study

A case study has been included here to illustrate the application of the software for determining optimal cut-off grades for multi-mineral deposits.

The case study is of a gold, lead, and zinc deposit. The technical and economic data are shown in Table I and Figure 2. The results are given in Table II.

For the sake of comparison, the deposit shown in Figure1 is applied to the grid search method and dynamic programming method. The technical and economic data for the grid search method are shown in Table III and Figure 2. The results are given in Table IV. The technical and economic data for dynamic programming method are shown in Table V and Figure 2. The results are given in Table VI.

Table VII compares the results of the three methods.

The results indicate that all three method give reasonable results but genetic algorithms deliver a better result. Genetic algorithms is a more robust search engine since it can easily escape from a local optimum point by means of crossover and mutation tools, and its natural selection environment.

Conclusions

The paper shows the applicability and robustness of genetic algorithms methods to multi-mineral cut-off grade optimization.

Determination of a complete mine production schedule requires complex modelling of an orebody and the inclusion of access constraints. The work described serves to find broad indications of optimum cut-off grades and a mining sequence that gives optimum discounted profits by using technical and economic constraints only. Detailed mine scheduling that includes physical, or access, constraints is beyond the scope of this research. The orebody is defined by a grade-tonnage distribution, which gives the ore tonnage for different grade intervals. Access constraints are not included, so that any parcel of the orebody is assumed to be immediately accessible. In other words, the grade-tonnage

Table I

Technical and economic data for genetic algorithms for the gold, lead, and zinc deposit

Description	Value
Lower limit of cut-off grades for gold (%)	0
Upper limit of cut-off grades for gold (%)	0.009
Lower limit of cut-off grades for zinc (%)	0
Upper limit of cut-off grades for zinc (%)	3
Lower limit of cut-off grades for lead (%)	0
Upper limit of cut-off grades for lead (%)	1.5
Mining capacity (t/a)	1 200 000
Mineral processing capacity (t/a)	1 000 000
Marketing and/or refining capacity for gold (t/a)	10
Marketing and/or refining capacity for zinc (t/a)	11000
Marketing and/or refining capacity for lead (t/a)	1600
Selling price for gold (dollars per ton)	11 000 000
Selling price for zinc (dollars per ton)	1 400
Selling price for lead (dollars per ton)	600
Marketing and/or refining cost for gold (dollars per ton)	3 000 000
Marketing and/or refining cost for zinc (dollars per ton)	300
Marketing and/or refining cost for lead (dollars per ton)	150
Recovery rate for gold (%)	46
Recovery rate for zinc (%)	80
Recovery rate for lead (%)	85
Variable mining cost of material mined (dollars per ton)	0.4
Variable concentration cost of material processed (dollars per ton)	0.4
Fixed costs (dollars per year)	1 000 000
Discount rate (%)	10
Population size (number of individuals in the population)	500
Number of generations	500
Crossover rate (%)	50
Mutation rate (%)	60

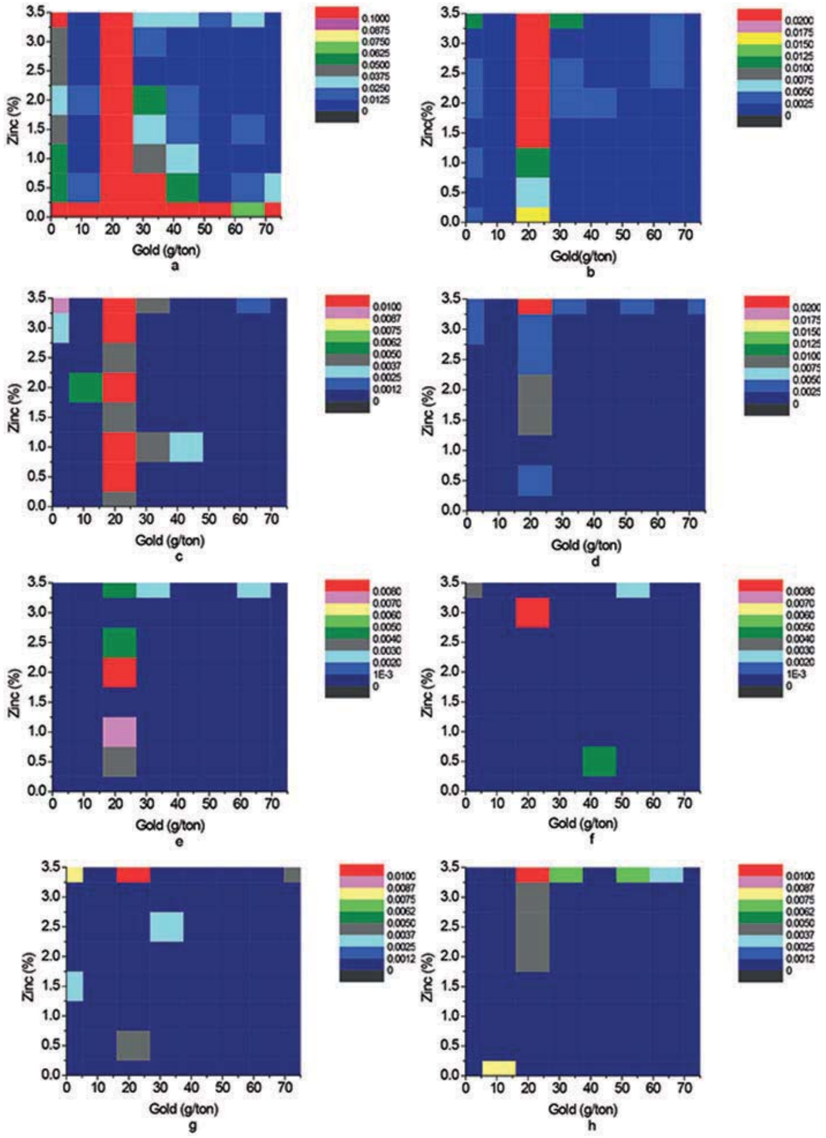

Figure 2—Grade-tonnage distribution for the gold, lead, and zinc deposit. Distributions from the first to the last lead grade intervals can be seen from sections (a) to (h). Figure 2a shows the lead grade interval between 0 and 0.2%, Figure 2b shows the lead grade interval between 0.2% and 0.4%, and Figure 2h shows the lead grade interval 1.4% and higher. The numbers next to the colours indicate the reserve tonnages (in millions of tons)

Table II

Output file for genetic algorithms

The optimum discounted profit is 439.8e + 6 $

Year	Profit ($)	Discounted profit ($)	Depletion (t)	Production (t)	Gold (t)	Zinc (t)	Lead (t)	Gold cut-off grade (%)	Lead cut-off grade (%)	Zinc cut-off grade (%)
1	82 319 950	74 836 319	1 164 693	1 000 000	9	10 564	1526	0.0012	3 000	0.800
2	82 319 950	68 033 017	1 164 693	1 000 000	9	10 564	1526	0.0012	3 000	0.800
3	82 020 659	61 623 335	1 158 748	1 000 000	9	10 582	1526	0.0012	2 600	0.800
4	82 020 659	56 021 214	1 158 748	1 000 000	9	10 582	1526	0.0012	2 600	0.800
5	82 020 659	50 928 376	1 158 748	1 000 000	9	10 582	1526	0.0012	2 600	0.800
6	79 839 492	45 067 312	1 114 896	1 000 000	9	10 342	1507	0.0012	0.800	0.700
7	79 839 492	40 790 283	1 114 896	1 000 000	9	10 342	1507	0.0012	0.800	0.700
8	72 691 453	33 911 099	1 000 000	1 000 000	8	9482	1462	0.0000	2 400	1 100
9	18 589 355	8 463 256	255 730	255 730	2	2425	374	0.0000	2 200	0.800

Table III

Technical and economic data for the grid search method for the gold, lead and zinc deposit

Description	Value
Lower limit of cut-off grades for gold (%)	0
Upper limit of cut-off grades for gold (%)	0,009
Interval between cut-off grade decisions gold (%)	0,0006
Lower limit of cut-off grades for zinc (%)	0
Upper limit of cut-off grades for zinc (%)	3
Interval between cut-off grade decisions zinc (%)	0,2
Lower limit of cut-off grades for lead (%)	0
Upper limit of cut-off grades for lead (%)	1,5
Interval between cut-off grade decisions lead (%)	0,1
Mining capacity (tons per year)	1 200 000
Mineral processing capacity (t/a)	1 000 000
Marketing and/or refining capacity for gold (t/a)	10
Marketing and/or refining capacity for zinc (t/a)	11000
Marketing and/or refining capacity for lead (t/a)	1600
Selling price for gold (dollars per ton)	11 000 000
Selling price for zinc (dollars per ton)	1 400
Selling price for lead (dollars per ton)	600
Marketing and/or refining cost for gold (dollars per ton)	3 000 000
Marketing and/or refining cost for zinc (dollars per ton)	300
Marketing and/or refining cost for lead (dollars per ton)	150
Recovery rate for gold (%)	46
Recovery rate for zinc (%)	80
Recovery rate for lead (%)	85
Variable mining cost of material mined (dollars per ton)	0,4
Variable concentration cost of material processed (dollars per ton)	0,4
Fixed costs (dollars per year)	1 000 000
Discount rate (%)	10

Table V

Technical and economic data for dynamic programming for the gold, lead, and zinc deposit

Description	Value
Lower limit of cut-off grades for gold (%)	0
Upper limit of cut-off grades for gold (%)	0.009
nterval between cut-off grade decisions gold (%)	0.0006
Lower limit of cut-off grades for zinc (%)	0
Upper limit of cut-off grades for zinc (%)	3
nterval between cut-off grade decisions zinc (%)	0.2
Lower limit of cut-off grades for lead (%)	0
Upper limit of cut-off grades for lead (%)	1.5
Interval between cut-off grade decisions lead (%)	0.1
Mining capacity (t/a)	1 200 000
Mineral processing capacity (t/a)	1 000 000
Tonnage interval between decisions to mine or not (t/a)	20 000
Marketing and/or refining capacity for gold (t/a)	10
Marketing and/or refining capacity for zinc (t/a)	11000
Marketing and/or refining capacity for lead (t/a)	1600
Selling price for gold (dollars per ton)	11 000 000
Selling price for zinc (dollars per ton)	1 400
Selling price for lead (dollars per ton)	600
Marketing and/or refining cost for gold (dollars per ton)	3 000 000
Marketing and/or refining cost for zinc (dollars per ton)	300
Marketing and/or refining cost for lead (dollars per ton)	150
Recovery rate for gold (%)	46
Recovery rate for zinc (%)	80
Recovery rate for lead (%)	85
Variable mining cost of material mined (dollars per ton)	0.4
Variable concentration cost of material processed (dollars per ton)	0.4
Fixed costs (dollars per year)	1 000 000
Discount rate (%)	10

Table IV

Output file for the grid search

The optimum discounted profit is 439.1e+6										
Year	Profit ($)	Discounted profit ($)	Depletion (t)	Production (t)	Gold (t)	Zinc (t)	Lead (t)	Gold cut-off grade (%)	Lead cut-off grade (%)	Zinc cut-off grade (%)
1	82 423 897	74 930 816	1 167 041	1 000 000	9	10 572	1527	0.0012	2,600	1,500
2	82 423 897	68 118 923	1 16 7041	1 000 000	9	10 572	1527	0.0012	2,600	1,500
3	82 423 897	61 926 294	1 167 041	1 000 000	9	10 572	1527	0.0012	2,600	1,500
4	82 344 016	56 242 071	1 165 144	1 000 000	9	10 564	1526	0.0012	2,600	1,300
5	82 344 016	51 129 155	1 165 144	1 000 000	9	10 564	1526	0.0012	2,600	1,300
6	80 753 096	45 583 017	1 131 110	1 000 000	9	10 377	1513	0.0006	2,600	1,300
7	80 75 3096	41 439 107	1 131 110	1 000 000	9	10 377	1513	0.0006	2,600	1,300
8	80 30 1619	37 46 1298	1 122 803	1 000 000	9	10 374	1510	0.0006	1,600	1,100
9	5 350 339	2 269 066	74 718	66 991	1	693	101	0.0006	0.800	1,400

Table VI

Output file for dynamic programming

The optimum discounted profit is 434.1e+6										
Year	Profit ($)	Discounted profit ($)	Depletion (t)	Production (t)	Gold (t)	Zinc (t)	Lead (t)	Gold cut-off grade (%)	Lead cut-off grade (%)	Zinc cut-off grade (%)
1	81 144 899	73 76 8090	1 160 000	999 881	9	10584	1525	0.0024	3 000	0.300
2	81 144 899	67 06 1900	1 160 000	999 881	9	10584	1525	0.0024	3 000	0.300
3	81 144 899	60 96 5364	1 160 000	999 881	9	10584	1525	0.0024	3 000	0.300
4	81 14 4899	55 42 3058	1 160 000	999 881	9	10584	1525	0.0024	3 000	0.300
5	79 199 360	49 17 6571	1 120 000	997 909	9	10349	1507	0.0012	2 800	0.200
6	79 19 9360	44 70 5974	1 120 000	997 909	9	10349	1507	0.0012	2 800	0.200
7	79 19 9360	40 64 1795	1 120 000	997 909	9	10349	1507	0.0012	2 800	0.200
8	71 81 4509	33 50 1999	1 000 000	1 000 000	8	9482	1462	0.0000	3 000	0.000
9	20 90 8899	8 867 414	291 151	29 1151	2	2761	426	0.0000	3 000	0.000

Table VII

Cut-off grades over time for the three methods and for the three constituent minerals of the case study

Year	Genetic algorithms gold cut-off grade (%)	Genetic algorithms lead cut-off grade (%)	Genetic algorithms zinc cut-off grade (%)	Grid search gold cut-off grade (%)	Grid search lead cut-off grade (%)	Grid search zinc cut-off grade (%)	Dynamic programm grade (%)	Dynamic programm grade (%)	Dynamic programm grade (%)
1	0.0012	3 000	0.800	0.0012	2 600	1 500	0.0024	3 000	0.300
2	0.0012	3 000	0.800	0.0012	2 600	1 500	0.0024	3 000	0.300
3	0.0012	2 600	0.800	0.0012	2 600	1 500	0.0024	3 000	0.300
4	0.0012	2 600	0.800	0.0012	2 600	1 300	0.0024	3 000	0.300
5	0.0012	2 600	0.800	0.0012	2 600	1 300	0.0012	2 800	0.200
6	0.0012	0.800	0.700	0.0006	2 600	1 300	0.0012	2 800	0.200
7	0.0012	0.800	0.700	0.0006	2 600	1 300	0.0012	2 800	0.200
8	0.0000	2 400	1 100	0.0006	1 600	1 100	0.0000	3 000	0.000
9	0.0000	2 200	0.800	0.0006	0.800	1 400	0.0000	3 000	0.000

distribution is identical for all parts of the orebody and for all parcels of ore.

It is very clear from this work, and that done by others, that maximum NPV can be achieved only by a declining cut-off grades policy. That is, the mining operation should start with a relatively high cut-off grade that declines gradually over the life of the mine. For that reason, and in order to increase the speed of the computations, production schedules that do not have declining cut-off grades are eliminated explicitly in the computer program.

The genetic algorithms method is a very robust search engine. The crossover, mutation and natural selection behaviour of the method ensures that it escape from a local optimum point.

The software developed for this study includes programs for the determination of optimum cut-off grades for multi-mineral deposits by means of the genetic algorithms, grid search method, and dynamic programming are written in C++ code. Although all the programs written are basically for cut-off grade optimization, they are slightly different in terms of data requirements.

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