On localizing uniform conditioning estimates

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Synopsis

Localized uniform conditioning is a technique developed to spatially locate selective mining unit grades that have been derived using uniform conditioning for the assessment of recoverable resources. The technique has the advantage of producing selective mining unit estimates conforming to the uniform conditioning panel-specific grade-tonnage curve while introducing spatial information at the scale of the selective mining units.

This paper describes an alternative technique to localized uniform conditioning which does not explicitly require the uniform conditioning panel-specific grade-tonnage curve to localize the selective mining unit estimates. The technique can therefore be implemented in mining software where uniform conditioning is not available.

Keywords
uniform conditioning, recoverable resources, change of support, localized UC, Gaussian distribution.

Introduction

During the early stage of a mining project the drill-hole data is frequently located on a relatively large grid, which is generally sufficient for estimating tons and grades using large panels whose dimensions reasonably conform to the drill spacing. However, such drill spacing is insufficient for the estimation of the recoverable resources required to properly assess mining projects at the stage of selective mining.

Uniform conditioning (UC) allows assessing the recovery functions of selective mining units (SMUs) inside a panel by using the estimated panel grade. However, UC does not provide any information about the spatial location of those SMUs, which is important in the mining industry to assess the spatial distribution of recoverable resources.

Localized uniform conditioning (LUC) is a relatively recent technique that has been used to spatially locate the grades of SMUs inside a panel. LUC uses the panel recovery functions given by UC and, through a ranking procedure of direct SMU estimates, derives and assigns SMU grades that reproduce the UC recovery functions (Abzalov, 2006).

This paper presents an alternative approach to LUC that does not require a ranking of direct estimates and does not explicitly use the recovery functions provided by UC to localize the SMU estimates. Comparative results between both approaches are discussed using copper grades from a porphyry deposit.

First, a brief summary of UC and LUC is presented where the concepts and notation used are introduced. Secondly, the alternative technique to LUC is shown in detail, and finally, comparative results for both approaches are presented.

Uniform conditioning

UC is a nonlinear technique for estimating recoverable resources inside a mining panel using the estimated panel grade (Rivoirard, 1994). The key to UC is a conditionally unbiased estimate of the panel grade and a sound change-of-support modelling relating the grades $Z(x)$ at point support, $Z(v)$ at SMU support, and the estimated grade $Z^*(V)$ at panel support.

The discrete Gaussian change-of-support model introduced by Matheron (1976) is used in UC to carry out the change of support. The model expresses $Z(x)$ and $Z(v)$ as functions of two standard Gaussian variables $Y(x)$ and Y_v, as

$$Z(x) = \Phi\big(Y(x)\big) \qquad [1]$$

where Φ is the point anamorphosis function derived from the point support data (Rivoirard, 1994), and

$$Z(v) = \Phi_v(Y_v) \qquad [2]$$

where Φ_v is the SMU anamorphosis function, which is derived via Cartier's relation by further assuming that $Y(x)$ and Y_v have joint Gaussian distribution with correlation coefficient $r > 0$. This implies that Φ_v is given by

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$$\Phi_v(y) = \int_{-\infty}^{+\infty} \Phi\left(ry + u\sqrt{1-r^2}\right) g(u)\, du \qquad [3]$$

where g denotes the standard Gaussian density (Lantuéjoul, 1994; Rivoirard, 1994). The Hermite polynomial expansion of the point support anamorphosis allows the SMU anamorphosis Φ_v to be explicitly computed as

$$\Phi_v(y) = \sum \phi_n r^n H_n(y) \qquad [4]$$

with H_n the Hermite polynomials and ϕ_n the corresponding Hermite coefficients. The coefficient r corresponding to the variance correction factor from point to SMU support is chosen so as to respect the theoretical variance at the SMU support by inverting

$$Var(Z(v)) = \sum \phi_n^2 r^{2n} \qquad [5]$$

An alternative method has been proposed by Emery (2007), which allows the computation of r without inverting Equation [5].

Assuming that a similar approach holds for changing from SMU to panel support (Rivoirard, 1994), the estimated grade $Z^*(V)$ at panel support is expressed as

$$Z^*(V) = \Phi_V(Y_V^*) \qquad [6]$$

where Y_V^* is a standard Gaussian variable and the panel anamorphosis function Φ_V is given by

$$\Phi_V(y) = \sum \phi_n s^n H_n(y) \qquad [7]$$

The correction factor s is chosen so as to respect the variance of $Z^*(V)$ by inverting

$$Var(Z^*(V)) = \sum \phi_n^2 s^{2n} \qquad [8]$$

If it is further assumed that Y_v and Y_V^* have joint Gaussian distribution with correlation R, then $R = s/r$ and the conditional distribution of the Gaussian equivalent SMU grades Y_v given the Gaussian equivalent panel grade Y_V^* is known. For a panel with $Y_V^* = y_V$, this distribution is Gaussian with mean Ry_v and variance $1–R^2$. This key result allows computation of the panel-specific recovery functions given by UC (Rivoirard, 1994). This is the core of the alternative technique to LUC proposed here.

The information effect in UC (Deraisme and Roth, 2001) allows modelling of the fact that the SMUs will ultimately be selected on an estimated SMU grade $Z^*(v)$ instead of the real $Z(v)$ grade.

The modelling is carried out as before by expressing $Z^*(v)$ as a function of a standard Gaussian variable Y_v^*, i.e. $Z^*(v) = \Phi_v^*(Y_v^*)$. The anamorphosis Φ_v^* is given by

$$\Phi_v^*(y) = \sum \phi_n s_1^n H_n(y) \qquad [9]$$

The correction factor s_1 is obtained by inversion as before, so as to respect the theoretical variance of the SMU estimates.

Similarly, assuming that Y_v^* and Y_V^* have joint Gaussian distribution with correlation coefficient $R^* > 0$, the conditional distribution of Y_v^*, given that $Y_V^* = y_V$, is Gaussian with mean R^*y_V and variance equal to $1– (R^*)^2$ (Deraisme, 2001). It is possible to show, by further assuming that Y_v and Y_V^* are conditionally independent given Y_v^*, that R^* is equal to $s/(r * \rho)$ where ρ is obtained by inverting

$$Cov(Z^*(v), Z(v)) = \sum \phi_n^2 r^n s_1^n \rho^n \qquad [10]$$

Therefore, derivation of the UC panel-specific recovery functions accounting for the information effect requires only a minor modification to the correlation value used in the conditional Gaussian distribution.

Localized uniform conditioning

LUC was proposed by Abzalov (2006) as a way to spatially locate the SMU grades inside a panel using the panel-specific tonnage (T) and metal (Q) curves given by UC.

The idea is to first carry out a direct estimation of SMU grades from the drill-hole data. Ranking of these estimates within each panel in increasing order of grade allows the derivation of a set of proportion values, which are used in conjunction with the corresponding panel metal curve to derive the SMU grades that will be used instead of the direct SMU estimates. The process is depicted in Figure 1, assuming a continuous curve for panel-specific tonnage T and metal Q curves. Given an SMU with proportion p, a cut-off z is identified from the tonnage curve. The cut-off z is then used in the metal curve to compute the corresponding metal q associated with the proportion p, and the direct SMU estimate is replaced by the LUC grade given by the ratio between metal q and tonnage p.

In practice UC does not provide a continuous curve for T and Q but a discrete set of tonnage and metal values for a set of predefined cut-offs. In this case, the discrete set of tonnage and metal values is used to define grade classes that are used to assign the SMU grades (Abzalov, 2006). Therefore, it is

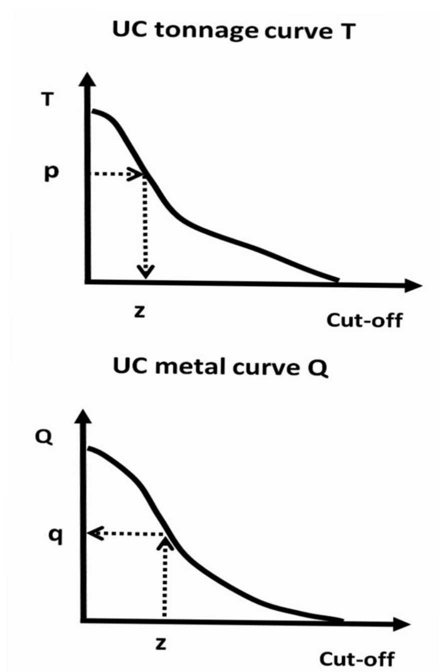

Figure 1—Derivation of SMU grades by LUC assuming a continuous panel-specific tonnage *T* and metal *Q* curves. After ranking the direct SMU estimates inside the panel, a proportion *p* is obtained and used in the tonnage curve to compute the equivalent cut-off *z*. This cut-off value is used in the metal curve to compute the associated amount of metal *q*. The grade to be assigned to the SMU with proportion *p* is *q/p*

extremely important for the use of LUC to properly select a set of cut-offs for which the set of finite panel tonnage and metal values provide a suitable discretization of the panel-specific tonnage and metal curves.

It is worth noting that the mechanism of LUC is not restricted to UC and can be equally used with any recoverable resource technique for which panel-specific tonnage and metal curves can be obtained, for instance multiple indicator kriging (Hardtke *et al.*, 2011). However, incorporation of the information effect is not straightforward when using multiple indicator kriging.

Abzalov (2006) has pointed out some limitations of the LUC process. Particularly important are the number of SMUs per panel and the ability of the direct SMU estimates to provide a reliable ranking criterion within each panel. The former impacts the resolution with which the panel-specific recovery functions can be reproduced. The bigger the number of SMUs per panel, the better the reproduction of the recovery functions will be. The latter is valid within continuous mineralization domains with relative low nugget effect. However, as shown by Harley and Assibey-Bonsu (2007), in the presence of strong short-range variability, LUC does not appear to be capable of producing accurate estimates under the constraint of reproducing the panel-specific tonnage and metal curves given by UC.

Direct LUC

Assuming that UC is a plausible model for assessing recoverable resources, an alternative approach to LUC, referred to as direct LUC, can be derived without explicitly using the panel-specific tonnage and metal curves provide by UC or doing any ranking of the direct SMU grade estimates.

Contrary to LUC, direct LUC explicitly uses the conditional distribution of the Gaussian equivalent SMU grades inside a panel with Gaussian equivalent grade $Y_V^* = y_V$. This distribution is Gaussian with mean λy_V and variance $1-\lambda^2$, where λ is equal to R or R^*, depending on whether the information effect is accounted for or not.

Direct LUC works by using equivalent Gaussian grades at point support to carry out a direct estimate of Gaussian SMU grades, which are then corrected on a panel–by-panel basis to match the conditional distribution given by UC.

The process is summarized as follows:

- Transform the point support data $Z(x)$ into equivalent Gaussian grades $Y(x)$
- Estimate equivalent Gaussian SMU grades y^* using $Y(x)$
- Transform the panel estimates $Z^*(V)$ into equivalent Gaussian panel estimates Y_V^* using the panel anamorphosis function given by Equation [6]
- Correct the distribution of the equivalent Gaussian SMU direct estimates on a panel-by-panel basis to match the conditional distribution of equivalent Gaussian SMU grades knowing the equivalent Gaussian panel grade given by UC.

This correction step is simple and an affine-type correction (Equation [11]) can be used to avoid the ranking of the SMU grades

$$y^*_{corr} = \frac{\sqrt{1-\lambda^2}}{\sigma}(y^* - \mu) + \lambda\, y_V \qquad [11]$$

where μ is the mean of all SMUs inside the panel with Gaussian equivalent grade $Y_V^* = y_V$, σ is the corresponding standard deviation of SMU grades within the panel, and y^*_{corr} is the corrected Gaussian SMU grade.

- Back-transform the corrected Gaussian SMU grades y^*_{corr} using the corresponding SMU anamorphosis function given by Equation [4].

Key to direct LUC is a sound anamorphosis modelling for SMU and panel support as well as the correction of the equivalent Gaussian SMU estimates. In practice, the SMU anamorphosis function given by Equations [4] or [9], if taking into account the information effect, and the panel anamorphosis function given by Equation [6] are input into the process as lookup tables, and direct and back-transformation are carried out using linear interpolation between tabulated values.

It is worth noting that if direct LUC grades are used only to report the proportion of SMUs inside a panel that are above cut-off and their corresponding average grade, then one obtains the UC results.

Case study – a porphyry copper deposit

The deposit is a low-copper (Cu) porphyry deposit with minor molybdenum (Mo) and gold (Au) mineralization. The mineralized domain covers an area of approximately 750 m (east) by 1100 m (north) with depths up to 400 m below surface.

Based on a nominal drill spacing of 80 m (east) by 80 m (north), panel grades were estimated using ordinary kriging with a panel size of 60 m by 60 m by 20 m. The estimation strategy was designed to deliver panel grade estimates with minimal conditional bias. This results in regression slope values ranging from 0.7 to 0.97.

Following the estimation of panel grades, UC, LUC, and direct LUC were undertaken using a SMU size of 10 m (east) by 10 m (north) by 10 mRL.

Comparison between LUC and direct LUC is carried by mean of global and selected panel-specific grade tonnage curves.

Figure 2 shows the global grade-tonnage curve within the mineralized domain obtained using LUC and direct LUC. The latter closely reproduces the results from LUC

A key property of LUC is that regardless of the direct SMU estimated grades, the panel-specific grade-tonnage curve given by UC is always reproduced if there is a large number of SMUs inside the panel. Unlike LUC, direct LUC requires not only a large number of SMUs inside a panel, but also that the corresponding Gaussian SMU direct estimates exhibit a reasonable variability so, after the correction step, its distribution resembles the panel-specific conditional distribution of equivalent Gaussian SMU grades given by UC.

Figure 3 shows the grade-tonnage curves obtained with LUC and Gaussian LUC in a panel containing 72 SMUs. Direct LUC provides a satisfactory match to the grade-tonnage curve given by LUC. Furthermore, as shown in Figure 4, direct LUC provides a more continuous range of grades and does not exhibit the artifact of constant grade values observed in LUC because of the use of grade classes.

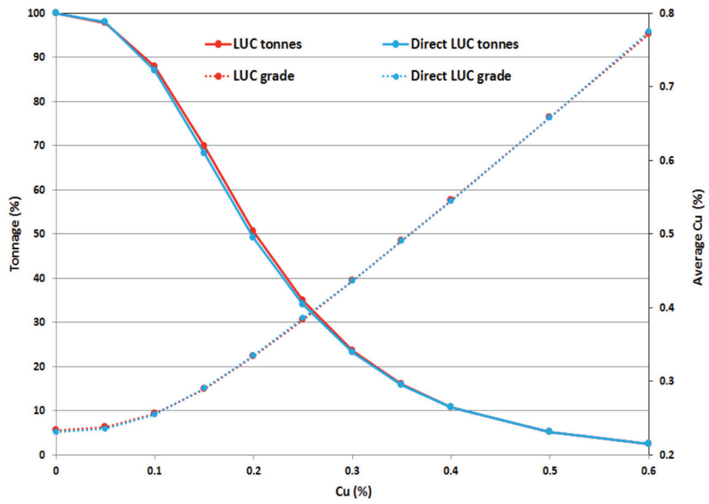

Figure 2—Global grade-tonnage curve for Cu (%) grades obtained with LUC (red) and direct LUC (blue)

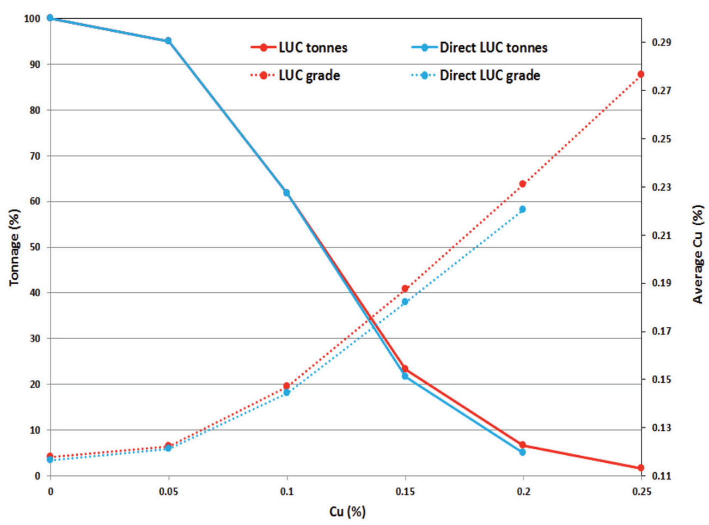

Figure 3—Grade-tonnage curves for 72 SMU Cu grades inside a panel obtained with LUC (red) and direct LUC (blue)

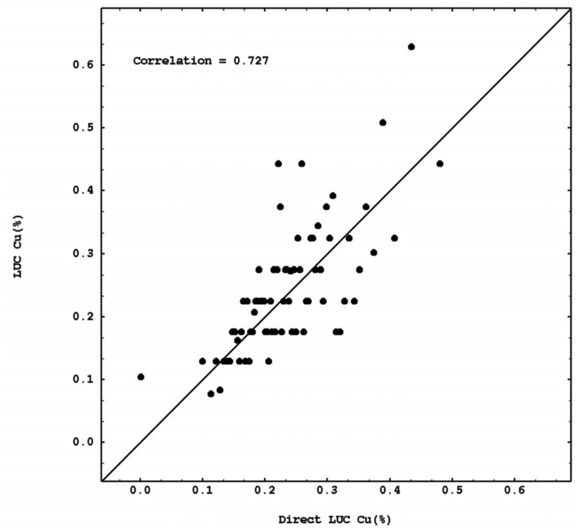

Figure 4—Scatter plot between LUC and direct LUC Cu grades with grade-tonnage curves shown in Figure 3

Discussion

This paper presented the direct LUC approach as an alternative to LUC. The approach does not require a ranking of direct estimates and does not explicitly use the recovery functions provided by UC to localize the SMU estimates. Instead, direct LUC corrects the equivalent Gaussian direct SMU estimates on a panel–by-panel basis to match the panel-specific theoretical distribution given by UC. Final SMU estimates are obtained by back-transforming the corrected SMU grades using the SMU anamorphosis function.

The three main advantages of direct LUC over LUC are:

- Direct LUC does not require carrying out any UC estimate in advance. Therefore, it can be coded as a macro or script in commercial mining software where UC is not available.
- Direct LUC does not rely on a predefined set of cut-off values
- Direct LUC produces on output both UC and LUC results.

However, as pointed out by Abzalov (2006), LUC can easily incorporate external information such as high-resolution geophysical data, which is not the case when using direct LUC.

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