Introduction

Indirect techniques to estimate recoverable resources during medium- to long-term planning derive the unknown selective mining unit (SMU) distribution estimates from the observed distribution of relative large kriged blocks (panels). The drawback of the indirect methods is that only the probability distribution of the SMUs within local panels can be derived, but not their individual spatial locations within the panel. A localized multivariate uniform conditioning (LMUC) post-processing technique has been proposed to enhance the indirect uniform conditioning by localizing the results at the SMU scale.

UC consists of estimating the grade distribution on SMU support within a panel, conditioned to the estimated panel grade, usually based on ordinary kriging (OK) or simple kriging (SK) with local mean to accommodate a possible lack of stationarity (i.e. when the average grade varies within the deposit). The general framework that forms the basis of UC is the discrete Gaussian model of change of support, based in particular on the correlation between Gaussian-transformed variables. The mining industry’s acceptance of the UC method has been apparent for several years, and a good reconciliation is generally found between UC medium- to long-term estimates and production data.

The UC method has been extended to the multivariate case, where the correlations between main and secondary variables can be calculated on any support after transformation into Gaussian space (Deraisme et al., 2008). As the correlations between the different grades and the main element are taken into account in the multivariate uniform conditioning (MUC), the local scale estimates for the other multiple-commodity metals contained in the tonnage, which in this case are assumed to depend only on the main commodity grade, are immediately obtained from the metals attached to those tonnages. The resultant local SMU estimates are referred to as localized multivariate uniform condition estimates. The correlations between the different elements on the SMU support are reproduced by means of the correlations introduced in the multivariate change-of-support model. In addition, a rigorous formulation of the information effect on panel grade distribution has been developed that allows the heterogeneity of the expected production data configurations to be taken into account when estimating future SMU recoverable grades.

As highlighted above, the disadvantage of traditional indirect UC is that the outputs consist of panel-local grade-tonnage curves representing a number of non-localized SMUs within these panels. Therefore, it is difficult in practice to use these models for underground...
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and open pit mine planning that requires a grade model on the SMU support. Abzalov (2006) proposed a solution using the grade-tonnage functions from the indirect univariate UC and then decomposing the panel-specific grade-tonnage data into a suite of individual SMU-sized units within the respective panels, according to a ranking of the main element grade estimate of the SMUs.

Deraisme and Assibey-Bonsu (2011) extended Abzalov’s proposal to the MUC case for medium- to long-term recoverable estimates for mine planning on a porphyry copper-gold deposit in Peru. A multivariate conditional simulation (MCS) comparative study with MUC by Deraisme and Assibey-Bonsu (2012) also showed that the fundamental assumption of the MUC technique, whereby the correlations of the secondary elements are dependent solely on the main element, is acceptable in practice. Although MCS takes into account the complete set of correlations, a significant amount of time is required for MCS; also, it does not readily provide a base model for practical mine planning as several equiprobable simulations are generated in the process.

This paper presents a brief review of the UC, MUC, and LMUC techniques and provides a production reconciliation case study for the same porphyry copper-gold deposit that was evaluated using the new LMUC technique (Deraisme and Assibey-Bonsu, 2011). In this case study the long-term LMUC estimates are compared to the corresponding production blast-hole grade control model, as well as the final LMUC estimates are compared to the corresponding ground truth.

Discrete Gaussian model applied to recoverable resource estimation

Review of the discrete Gaussian model

Let $v$ be the generic SMU and $Z(v)$ its grade, which will be used for the selection at the future production stage.

The recoverable resources above cut-off grade $z$ for such blocks are:

- the ore $T_{i,v,z} = 1_{Z_{i,v} \geq z}$
- the metal $g_{i,v} = Z_{i,v}^{*} 1_{Z_{i,v} \geq z}$

where $1_{Z_{i,v} \geq z}$ stands for the grade indicator at cut-off $z$, i.e.:

- $1_{Z_{i,v} \geq z} = 1$ if $Z(v) \geq z$;
- $1_{Z_{i,v} < z} = 0$ if $Z(v) < z$

We use here the discrete Gaussian model for change of support (Rivoirard, 1994). A standard Gaussian variable $Y$ is associated with each raw variable $Z$. Let $Z_{i,v} = \Phi_{v}(Y_{i,v})$ be the sample point anamorphosis. The block model is defined by its block anamorphosis $Z(v) = \Phi_{v}(Y_{i,v})$, given by the integral relation:

$$
\Phi_{v}(Y) = \int \Phi(\gamma + \sqrt{-1r^{2}}u) g(u) du
$$

where $r$ is the change of support coefficient $r$ is obtained from the variance of blocks.

Then, the global resources at cut-off $z$ are:

$$
E[T(z)] = E[1_{Z_{i,v} \geq z}] = E[1_{Y_{i,v} \geq z}] = 1 - G_{Y}(z)
$$

where $g$ and $G$ are the standard Gaussian probability density function (p.d.f.) and cumulative distribution function (c.d.f.), and $Y$ is the Gaussian cut-off related to $z$ through $z = \Phi_{v}(Y)$.

Review of the uniform conditioning in the univariate case

UC by panel grade (Rivoirard, 1994) aims at estimating the recoverable resources on a generic selection block $v$ randomly located within a large block or panel $V$, conditioned on the sole panel grade, or for more generality, the panel grade estimate $Z(V)^{*}$. Tonnage and metal at cut-off $z$ are then:

$$
E[T_{i,v}(z)] = E[1_{Z_{i,v} \geq z} | Z(V)^{*}] = E[1_{Y_{i,v} \geq z} | Z(V)^{*}] = 1 - G_{Y}(z)
$$

The estimation of the metal at zero cut-off must then satisfy the relation: $E[Z(v) | Z(V)^{*}] = Z(V)^{*}$. This implies that the panel grade estimate $Z(V)^{*}$ has to be conditionally unbiased i.e.

$$
E[Z(v) | Z(V)^{*}] = Z(V)^{*}
$$

The model also assumes that the Gaussian anamorphosis of $Z(V)^{*}$ is linked to that of $Z(v)$:

$$
Z(V)^{*} = E[\Phi_{v}(Y_{i,v}) | Y_{v}] = \Phi_{v}(Y_{v})
$$

Hence the relationship:

$$
S = r \rho_{v,y} = r \text{cor}(Y_{v}, Y_{y})
$$

This is used to compute the correlation (‘cor’) between the block and the panel estimate:

$$
\text{cor}(Y_{v}, Y_{y}) = \rho_{v,y} = S / r
$$

The ore tonnage and metal at cut-off $z = \Phi_{v}(Y)$ are then

$$
E[1_{Y_{i,v} \geq z}] = 1 - G_{Y}(z)
$$

$$
E[1_{Y_{i,v} = z} | Y_{v}] = 1 - G_{Y}(z)
$$

$$
\Phi_{v}(Z) = \Phi_{v}(Y_{i,v}) + \sqrt{1 - (\rho_{v,y})^2} u g(u) du
$$

Uniform conditioning in the multivariate case

MUC consists of estimating the recoverable resources of blocks $v$ in panel $V$ from the panel estimates $Z_{i,V}^{*}$, $Z_{j,V}^{*}$, ...

The problem is simplified by making the following assumptions ($i$ denotes the index of a secondary variable 2, 3, ...

- $Z_{i}(v)$ is conditionally independent of $Z_{i,V}^{*}$ given $Z_{i,V}^{*}$, and so the UC estimates for the main variable correspond to the univariate case
- Similarly, $Z_{i}(v)$ is conditionally independent of $Z_{j,V}^{*}$ given $Z_{j,V}^{*}$.
The development of the equations makes practical computations achievable (Deraisme et al., 2008).

The important point is that the multivariate model requires correlations between all variables and one main variable. The choice of that variable is then of prime importance, particularly because the correlations between the secondary commodities are not directly modelled but are partly inferred through their respective relations with the main variable. It should be noted that the panel estimates must be calculated using co-kriging. As highlighted above, simple co-kriging has been used for the case study in this paper. The aim was to avoid conditional biases that were observed for the ordinary co-kriging estimates as a result of the limited resource drilling data on a relatively large grid.

Case study

Geology

The case study is based on a porphyry copper-gold deposit in Peru. The mineralization is found in intrusive rocks within sedimentary rocks. Oxidation, weathering, leaching, and subsequent secondary enrichment have led to the formation of four mineral domains with distinct different metallurgical behaviours.

The uppermost domain is the oxide domain. It is characterized by the complete removal of copper mineralization through oxidation and leaching. Gold mineralization within the oxide domain is characterized by some improvement in grade and is free-milling due to the complete breakdown of primary sulphide minerals.

All of the ore beneath the oxide domain makes up parts of the sulphide zone, which is separated into three domains on the basis of degree of oxidation and consequent change in sulphide mineralogical composition. The sulphide zone domains are, from top to bottom, the mixed domain, the supergene domain, and the hypogene domain. The supergene domain is an enriched copper blanket comprising chalcocite-covellite-chalcopyrite (Figure 1).

The production reconciliations presented in this paper covered mainly the supergene and hypogene domains, which have significant economic importance on the mine. The variables studied were gold (AUTOT), total copper (CUTOT), and net smelter return (NSR).

Database and analysis

The resource drilling data on average was on 25×25 m to 50×100 m drill spacing. The samples were composited on a 2 m basis and used to derive the LMUC estimates. The initial MUCs were based on simple co-kriging of 40 m × 40 m × 10 m panels assuming 10 m × 10 m × 10 m SMUs. The SMUs were based on equipment capacities and mining selectivity as applied at the mine. Figure 2 provides the drill-hole layout for the Annulus hypogene domain.
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In addition to the resource drilling data, a comprehensive 6 m × 5 m blast-hole data grid was available from mining. The blast-hole data was not used for the MUC/LMUC resource estimates. The three variables AUTOT, CUTOT, and NSR generally have a positively skewed distribution as shown in the hypogene annulus domain (Figure 3), with coefficients of variation from 0.65 to 0.85. Significant correlations of about 0.7 are observed between gold and copper. Declustering weights have been applied to calculate histograms and variograms. Figure 4 shows one of the typical variograms in the hypogene domain.

As proposed by Abzalov (2006) and based on the extended multivariate UC work done by Deraisme and Assibey-Bonsu (2011), the grade-tonnage functions from the indirect multivariate UC were decomposed into a suite of individual 10×10×10 m SMU-sized units within the respective panels. These decomposed 10×10×10 m SMU estimates are referred to as localized multivariate uniform conditioning (LMUC) estimates as highlighted above. The main advantage of the LMUC approach is to derive non-smoothed SMU grades with variability closer to the future production SMU block grades.

Change-of-support models for MUC and LMUC

The distribution of 2 m composites has been modelled using a Gaussian anamorphosis function decomposed into Hermite polynomials. The change of support on SMUs is then achieved; the coefficients are calculated according to the selected main variable. The interpretation of these coefficients (Table 1) as coefficients of correlation between different variables.
variables in the Gaussian space shows that the correlations between block values are slightly higher than the correlations on composites.

In providing the co-kriging panel conditioning estimates required for the MUC/LMUC, significant conditional biases were observed with ordinary co-kriging (OK) as demonstrated by the large negative kriging efficiencies (KEs) and poor slopes of regression associated with a substantial number of the OK-based estimates in Figure 5. The conditional biases observed for the OK estimates are a result of the limited available resource data. These significant conditional biases observed with the OK estimates have adverse consequences on ore and waste selection for mine planning as well as financial planning. As a result, simple co-kriging with local means was used for the panel conditioning in all cases.

Basis for the production reconciliations

The LMUC recoverable estimates were compared with the corresponding ‘actual’ grade control (GC) block values based on the available comprehensive 6 m x 5 m blast-hole data grid. The LMUC estimates were also compared with plant production data. Reconciliations have been analysed on a monthly, quarterly, and annual basis. The efficiency of the LMUC reconciliations is measured on the basis of the spreads of percentage errors defined as:

\[
\text{Percentage Error} = \frac{(\text{Actual} - \text{Estimate} - 1) \times 100}{\text{Actual}}
\]

Actual represents either in situ GC block estimates based on 6 m x 5 m blast-hole data or plant production data (PD); and Estimate is the corresponding LMUC resource estimates before production.

<table>
<thead>
<tr>
<th>NSR</th>
<th>CUTOT</th>
<th>AUTOT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Punctual variance (anamorphosis)</td>
<td>270.45</td>
<td>0.076</td>
</tr>
<tr>
<td>Variogram sill</td>
<td>128.191</td>
<td>0.045</td>
</tr>
<tr>
<td>Real block variance</td>
<td>147.905</td>
<td>0.035</td>
</tr>
<tr>
<td>Real block support correction (( h ))</td>
<td>0.7754</td>
<td>0.69</td>
</tr>
<tr>
<td>Kriged block support correction (( s ))</td>
<td>0.7754</td>
<td>---</td>
</tr>
<tr>
<td>Kriged-real block support correction</td>
<td>1</td>
<td>---</td>
</tr>
<tr>
<td>Main-secondary block support correction</td>
<td>---</td>
<td>0.8733</td>
</tr>
</tbody>
</table>

Figure 4—Experimental and modelled variograms for CUTOT and AUTOT variables

Figure 5—Scatter diagram of kriging efficiency versus slope of regression, ordinary co-kriged (OK) grade of Au, showing substantial number of OK panel estimates with significant negative kriging efficiencies and poor slopes of regression
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Results

As highlighted above, the main advantage of the LMUC approach is to derive non-smoothed SMU grades with variability closer to the future production SMU block grades. Table II shows the LMUC-estimated SMU dispersion variances against the corresponding ‘actuals’ based on final production blast-hole data. The table shows that the LMUC dispersion variance estimates compare well with the ‘actuals’. Figure 6 further illustrates that the smoothing effect due to information effect has appropriately been improved by the LMUC technique. However, as shown later, on an individual block basis the assigned LMUC grades sacrifice local accuracy, as noted also by Journel et al. (2000): ‘It appears that global accuracy (semivariogram reproduction) cannot be obtained without sacrificing local accuracy. [Proper] kriging, notwithstanding its smoothing effect, remains the best local estimator’. (See also Assibey-Bonsu et al., 2008).

Table III shows the production reconciliation of the monthly LMUC resource estimates with the corresponding plant results. The reconciliation results in Table III are provided on the basis of the spreads of the percentage errors. The analyses of the spreads of the monthly percentage errors show upper and lower 10% confidence limits of –12%/+10%, –6%+/+14%, and –8%/+8% for tons, gold grade, and copper grade respectively (the lower and upper 10% confidence intervals have been read directly off the histogram of the percentage of errors as observed over the production period). Figure 7 further shows the analyses of the spreads of errors in a graphical form. The figure shows that during the monthly production periods the percentage errors were well within the above confident limits. (The top two benches of the LMUC resource model incorporate what is referred to on the mine as a ‘short-term model’, which is discussed later in this paper).

The results further show percentage errors of +6%, +2%, and -7% on a quarterly (ie 3 monthly) basis for tons, gold grade, and copper grade respectively (Table IV). The mine reports production results on a quarterly basis to shareholders. Over an annual production period, the observed percentage errors were –1%/+3%, demonstrating the narrowing of the observed percentage errors over the annual period.

Table II

SMU dispersion variance of ‘actual’ versus LMUC estimates

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimated LMUC dispersion variance</th>
<th>‘Actual’ dispersion variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>0.33</td>
<td>0.38</td>
</tr>
<tr>
<td>Copper</td>
<td>0.08</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table III

Distribution of monthly percentage errors between resource model and plant production over 12-month period

<table>
<thead>
<tr>
<th>Tons Grade</th>
<th>Limits</th>
<th>Limits</th>
<th>Limits</th>
<th>Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lower</td>
<td>Upper</td>
<td>Lower</td>
<td>Upper</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>-12%</td>
<td>10%</td>
<td>-6%</td>
<td>14%</td>
</tr>
<tr>
<td>Gold</td>
<td>-8%</td>
<td>8%</td>
<td>-8%</td>
<td>8%</td>
</tr>
<tr>
<td>Copper</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6—Typical example of CUTOT estimated grades on one bench. Anticlockwise from the top: kriged panels, kriged SMUs, SMUs indirectly estimated by LMUC.
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Table V also shows the production reconciliation of the LMUC resource estimates with the corresponding ‘actual’ GC block values for the entire reconciliation period. As per the plant reconciliation, these results are on the basis of percentage distribution of errors. Both the 2011 and the updated 2012 LMUC resource models were used for the reconciliation over the same production period. The table shows that the resource models compare well with the grade control model (e.g., when compared to the internationally accepted 15% errors for an annual production period – see Stoker, 2011).

Reconciliation of LMUC and grade control models in the short-term model area

The mine replaces the first two benches of the LMUC resource model with what is termed a short-term (ST) model. The ST model is developed by extrapolating the blast-hole data from the mined-out areas and is used as interim short-term estimates for the first two benches (i.e. before blast-hole data become available in the short-term model area). The observed reconciliations of the LMUC model with the GC and ST models show similar good results (Tables VI and VII, see also Table IV above on errors for respective periods).

Furthermore, Figures 8 and 9 show the regression of the LMUC model SMU estimates on the corresponding ‘actual’ GC block values. As highlighted previously, the GC values are based on the available comprehensive 6 m × 5 m blast-hole data grid in the GC area. The figures demonstrate a reasonable general agreement of the LMUC individual block estimates with the corresponding follow-up production data, with correlations of 0.65 and 0.62 for Au and Cu respectively.

Table V

<table>
<thead>
<tr>
<th>Period</th>
<th>Tons</th>
<th>Grade</th>
<th>Gold</th>
<th>Copper</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 months</td>
<td>-0.6</td>
<td>9.6</td>
<td>5.2</td>
<td></td>
</tr>
<tr>
<td>6 months</td>
<td>-0.6</td>
<td>6.5</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td>Annual</td>
<td>-0.6</td>
<td>0.6</td>
<td>-1.8</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 months</td>
<td>-0.1</td>
<td>2.5</td>
<td>-5.6</td>
<td></td>
</tr>
<tr>
<td>6 months</td>
<td>-0.4</td>
<td>6.5</td>
<td>0.9</td>
<td></td>
</tr>
</tbody>
</table>

The average global errors between the follow-up and the LMUC model are within acceptable limits (<6%, Tables VI and VII).

However, the individual LMUC selective mining block estimates, based on simple co-kriging conditioning (SK) using local means, show some conditional biases as reflected by the slope of regressions of 0.7 and 0.52 for Au and Cu respectively (Figures 8 and 9). The conditional biases are a result of the limited available resource data used for the LMUC resource estimates as well as certain geological model changes on waste and ore contacts, which were updated.

Figure 7–Distributions of percentage errors (tons, Au and Cu grades) for monthly reconciliation (resource model vs plant)

Figure 8—LMUC resource model vs GC in short-term model area (Au)

Figure 9—LMUC resource model vs GC in short-term model area (Cu)
Conclusions

- Gaussian models (in this case multivariate uniform conditioning, MUC) used for calculating recoverable resources provide consistent results in modelling the change of support and the information effect in the multivariate case.

- The production reconciliation results show the overall advantage gained by using localized multivariate uniform conditioning (LMUC) estimates based on SK co-kriging as demonstrated by the narrow spreads of the monthly percentage errors. The central 80% confidence limits of the monthly production errors were -12%/+10%, -6%/+14%, and -8%/+8% for tons, gold, and copper grades respectively. The case study also showed percentage errors of +6%/+2%/-7% on a quarterly basis for tons, and copper and gold grades respectively. The narrowing of the observed confidence limits is also observed as shown by the reduced observed average percentage errors of -1%/+3% for the plant production reconciliations on a macro- or long-term production basis.

- The study further showed that on a local production scale (and especially for short- to medium-term planning), regression effects and conditional biases were still evident with the assigned LMUC individual SMU estimates, thus sacrificing local accuracy. Significant conditional biases were particularly evident with the ordinary co-kriging estimates, which were mainly due to the limited data that was available for the LMUC resource estimates (limited obtainable data is typical of all long-term and project resource estimates). In this regard, the simple co-kriging estimates based on local means showed more efficient panel conditioning estimates for the purpose of the MUC/LMUC resource assessment and the reconciliations.

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References


