Localized uniform conditioning (LUC): method and application case studies

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Synopsis

A new method, localized uniform conditioning (LUC), was proposed in 2006 for modelling grades of small blocks of ore when data spacing is too broad for their accurate modelling by the linear regression based techniques, such as kriging (Abzalov, 2006). It represents a modified uniform conditioning (UC) technique that calculates the grade distribution functions for the large panels. LUC uses partitioning of the panels onto the small blocks and then ranks them in increasing order of grade. Based on the block ranks, a single grade value can be deduced for each block from the UC model of the grade-tonnage relationships of the corresponding panel.

After being first presented in 2006, the LUC method has been implemented in ISATIS© (commercial software) and became one of the common approaches for grade estimation when data spacing is broad in comparison with the estimated block size. Several years of study on the LUC method and its application to different geological environments, have allowed identification of the strengths and weaknesses of the method, which are as follows:

- ➤ The method produces accurate grade-tonnage functions, which are in a good accordance with a volume-variance relationship principles
- ➤ An initial ranking of the selective mining unit (SMU) blocks can be made by direct kriging from the sparse data grid. Therefore, the LUC method can be particularly useful at the early stages of exploration and mining project evaluations when sparsely distributed data is often the only available information
- ➤ Accuracy of the local estimation depends on the SMU ranking techniques. When ranking performed by direct kriging of the SMU blocks their spatial distribution is approximate. When the variogram of the studied variable is characterized by a large nugget effect, the block ranks produced by kriging can significantly differ from their 'true' distribution
- ➤ Block ranking can be improved using auxiliary data, either geophysical or geochemical. This allows use of the LUC method for integrating different data sets. In particular, LUC can be used for grade control in open pits by integrating resource definition data (e.g. drill-hole assays) and blast-hole assays. The latter are used for the block ranking.

Keywords

geostatistics, localized uniform conditioning, LUC, resource modelling.

Introduction

It is well known in the geostatistical community that techniques based on linear regression are unsuitable for modelling grades of small blocks when the data spacing is too broad in comparison with the estimated block sizes (Armstrong and Champigny, 1989; Ravenscroft and Armstrong, 1990; Pan, 1998). To overcome this problem, a new method, localized uniform conditioning (LUC), was proposed (Abzalov, 2006). The LUC method represents a modified uniform conditioning (UC) technique. It calculates the grade distribution functions for the large panels by a conventional UC method and then uses partitioning of the panels onto the small blocks and ranks them in increasing order of grade. Based on the block ranks and using the calculated grade-tonnage relationships of the panels as a guide, a single grade value is deduced for each block. In other words, the proposed method localizes the UC model results; it is therefore called localized uniform conditioning (LUC).

After first presentation in 2006 the LUC method was implemented in ISATIS© (Bleines *et al.*, 2001) and became one of the common approaches for grade estimation when data spacing is too broad in comparison with the estimated blocks size. Several years of continuing studies of the LUC method, applying it in different geological environments, have allowed an assessment of the strengths and weaknesses of the method, which are presented in this paper.

Method

Uniform conditioning (UC)

Uniform conditioning (UC) is a nonlinear geostatistical technique for calculating tonnage (T_v) and mean grade (M_v) of recoverable resources distributed in a large panel (V) as the small blocks of size (v) representing a partitioning of this panel (Figure1).

In geostatistical terms the UC technique (Rivoirard, 1994; Chiles and Delfiner, 1999; Wackernagel, 2002) consists of calculating a conditional expectation of a nonlinear function

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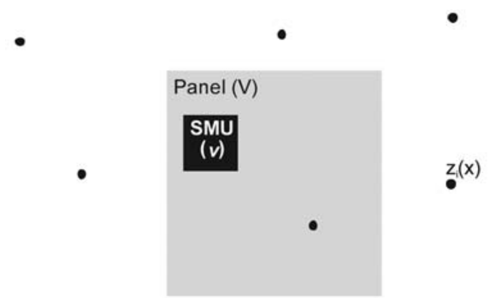

Figure 1—Distribution of the selective mining units of support (v) in the panel (V). Data nodes z(x) are denoted by black dots

$\Psi(Z(v))$ of the blocks (v) with respect to the corresponding panel grade Z. In other words the UC method assumes that the grade of the panel ($Z(V)$) is known. In practise, as the true panel grades are not available they are substituted in the UC models by the $Z(V)^*$ panel grades estimated by ordinary kriging (OK).

Estimation of a nonlinear function $\Psi(Z(v))$ from the available point (i.e. sample) variable $Z(x)$ uses a discrete Gaussian point-block model (Rivoirard, 1994; Chiles and Delfiner, 1999; Wackernagel, 2002). The UC method requires calculation of two such models. The first model is a point-to-selective mining unit (SMU [v]) anamorphosis (Equation [1]).

$$Z(v) = \phi_v Y(v) = \sum_{k=1}^{\infty} \frac{\varphi_k}{k!} r^k H_k(Y(v)) \qquad [1]$$

The second model is a point-to-panel (V) anamorphosis (Equation [2]).

$$Z(V) = \phi_v(Y(V)) = \sum_{k=1}^{\infty} \frac{\varphi_k}{k!} s^k H_k(Y^*(V)) \qquad [2]$$

Both anamorphoses are calculated using the discrete Gaussian point-block correction approach (Appendix). Based on these models the recoverable tonnage (T) and contained metal (Q) are calculated (Equations [3] and [4].

$$T_v(z_C) = E\left[I_{Z(v)\geq z_C} \middle| Z^*(V)\right] = E\left[I_{Y_v \geq y_C} \middle| Y_V^*\right] = 1 - G\left\{\frac{y_C - \frac{s}{r} Y_V^*}{\sqrt{1-\left(\frac{s}{r}\right)^2}}\right\} \qquad [3]$$

$$Q_v(z_C) = E\left[Z(v) I_{Z(v)\geq z_C} \middle| Z^*(V)\right] = \sum_{k=1}^{N} \left(\frac{s}{r}\right)^k H_k(Y_V^*) \sum_{j=1}^{N} \varphi_j r^j \int_{y_C}^{+\infty} H_k(y) H_j(y) g(y) dy \qquad [4]$$

where $Y_V^* = \phi_V^{-1}(Z^*(V))$ and $y_C = \phi_v^{-1}(z_C)$

Finally, the mean grade (M) of the recovered mineralization whose SMU grades are above a given cut-off (z_C) is estimated (Equation [5]).

$$M_{v(z_C)} = \frac{Q_v(z_C)}{T_v(z_C)} \qquad [5]$$

Applying several cut-off values (z_C), a complete grade-tonnage distribution can be constructed for each studied panel. However, distribution of the selective mining unit (SMU) blocks in the panels is not modelled by a conventional UC approach, which is a major disadvantage of the method restricting its practical application for estimation of the mineable resources and evaluating mining projects.

Localized uniform conditioning (LUC)

The localized uniform conditioning (LUC) method was developed with an intention to overcome the limitations of conventional UC (Abzalov, 2006). It calculates the grade distribution functions for the large panels by a conventional UC method and then localizes the UC model results.

Partitioning the panel on the small blocks

The first step is to split (partition) the panel on sub-cells whose size are equal to that chosen for estimation of small blocks (Figure 2a). Usually these are the blocks whose size matches the proposed selectivity of the mining method and therefore they are referred as selectively mineable units (SMU).

Ranking SMU blocks in increasing grade order

The SMU blocks distributed in each panel should be ranked in order of increasing grade. This is the underlying concept of the LUC method. It is obvious that an accurate ranking would require high-density information, such as high-resolution geophysics. However, reasonably accurate rankings of the SMU blocks in the panels can be deducted from the spatial distribution patterns of the grade values, such as zoning or grade trends. The latter approach is particularly relevant for continuous mineralization, characterized by a low nugget effect, such as occurs in stratiform base metal sulphide or iron oxide deposits. Spatial grade distribution patterns are easily recognized by geoscientists in many stratiform deposits at the early stages of exploration, when drill spacing is still too broad for direct accurate modelling of grades of small block, but sufficient for identification of the major distribution trends.

The global distribution features of the grade variables exhibiting a strong continuity can be reconstructed by interpolating available data nodes using any conventional linear interpolator, such as ordinary kriging (OK). In other words, it is suggested that direct kriging of the small blocks can be used to rank them approximately in increasing order

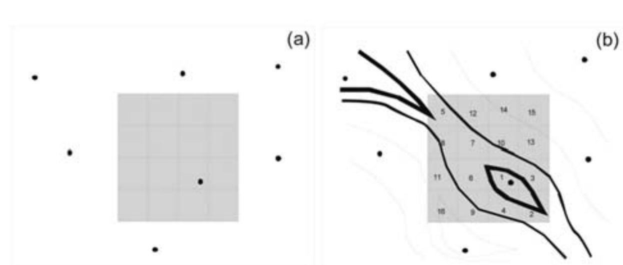

Figure 2—Partitioning of the panels on SMU blocks and their ranking. (a) Splitting (partitioning) of a panel on 16 SMU blocks; (b) definition of the SMU ranks

of grade in the panels, even when the drill spacing is too wide for non-biased SMU grade estimation. The proposed approach to ranking of the SMU blocks in the panels using a linear estimator is schematically shown in Figure 2b.

The validity of the obtained grade ranks depends on the complexity of the grade distribution patterns. It is obvious that further studies are required to quantify the limitations of application of linear estimators (e.g. OK) for ranking the SMU size blocks. At this stage it is assumed that the above assumption is applicable to grade variables whose spatial distribution satisfies a border effect condition and which are also characterized by a low nugget effect and exhibit a good continuity at their variogram origins.

The OK-based ranking of SMU blocks can be further enhanced using suitable high-resolution geophysical techniques. The precision of geophysical methods is usually insufficient for a quantitative interpretation of the geophysical responses; however, it can be adequate for a relative ranking of the SMU blocks in the panels. These reconstructed distribution patterns are finally used for the definition of the grade relationships between SMU blocks.

Defining the grade classes and estimating their mean grades

The next step is discretization of the UC model on the grade classes and estimation of the mean grade of every grade class. The grade classes are defined for each panel using the relationships between the cut-off grade and the tonnage of recoverable mineralization at the given cut-off estimated by the UC technique (Figure 3a). The grade class is the portion of the panel whose grade is higher than a given cut-off (z_i) but lower than the next largest cut off (z_{i+1}). In other words, each grade class is defined by its lower (z_i) and upper (z_{i+1}) cut-off values, and at the same time by corresponding T_i (z_i) and T_{i+1} (z_{i+1}) values representing a recoverable tonnage at lower and upper cut-offs defining the given grade class (Equation [6]).

$$GC_i \subset \{T_i(Z_i),\ T_{i+1}(Z_{i+1})\}\ and\ GC_i \subset \{Z_i, Z_{i+1}\};\quad [6]$$

where T_i (z_i) is recoverable tonnage at cut-off (z_i) and T_{i+1} (z_{i+1}) is recoverable tonnage at cut-off (z_{i+1}).

The mean grade of each grade class (MGC_i) is deduced from the UC model estimating grade of recoverable mineralization at the different cutoff values (Figure 3b).

Assigning grade to SMU blocks according to their rank

The mean grades of the grade classes can be assigned to the SMU blocks by matching their ranks with the grade classes. To do so it is necessary to convert the SMU ranks to the grade classes (Figure 3c). This is deduced from the relationships between the SMU rank and the proportions of the panel (Equation [7]).

$$SMU_{RANK} \subset (T_{RANK}, T_{RANK+1}) \quad [7]$$

where T_{RANK} is the proportion of the panel distributed in SMU blocks whose rank is equal or lower than ($RANK$), and T_{RANK+1} is proportion of the panel distributed in SMU blocks having higher rank ($RANK$+1).

The grade class are determined for each SMU_{RANK} by comparing its (T_{RANK}, T_{RANK+1}) intervals with the intervals of the grade classes {T_i (z_i), T_{i+1} (z_{i+1})} (Figure 3c). SMU_{RANK} will be assigned grade class (TGC_i) if the (T_{RANK} - T_{RANK+1}) intervals falls within the limits (T_i - T_{i+1}) of the grade class (GC) (Figure 3c).

Finally, a mean grade (MGC_i) of each class is transferred to the SMU_{RANK} blocks by matching their grade class indexes (MGC_i and TGC_i) (Figure 3d).

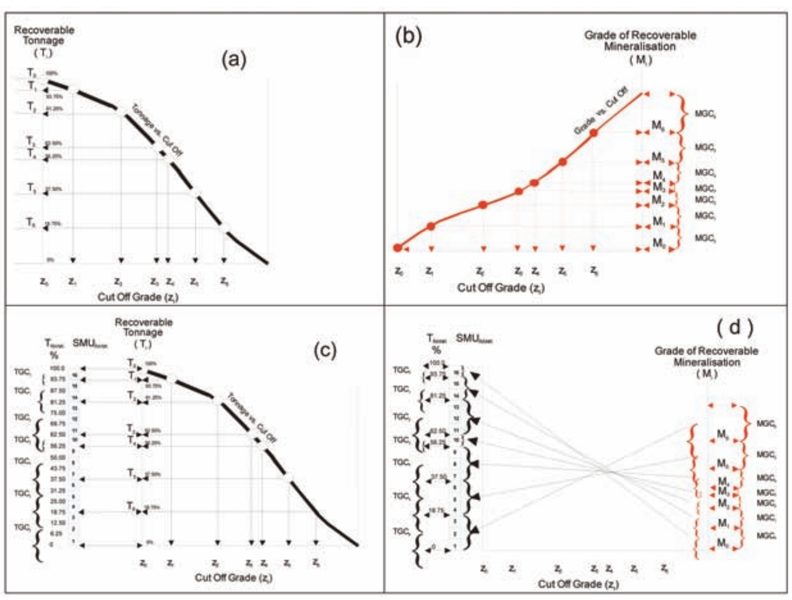

Figure 3—Definition of the grade classes and assigning the grades to the SMU blocks. The example uses 16 SMU blocks in a panel and 6 cut-off values used in the UC model. (a) Definition of the grade classes from UC results. Grade class (GCi) represents a portion of mineralization distributed in the panel as the SMU size blocks which grade lies within the range of > Z_i and < Z_{i+1}. Z_i - cut off values, T_i - tonnage of mineralization above the cut-off Z_i expressed as proportion (%) of the panel; (b) definition of the mean grades (MGC_i) of the grade class (GC_i); (c) assigning the grade class indexes (TGC_i) to the *SMU* blocks falling within the range from T_i to T_{i+1}; (d) assigning the mean grades (MGC_i) of the grade class (GC_i) to the *SMU* blocks whose index (TGC_i) matches the grade class (GC_i)

Implementation of the LUC methodology

The procedure of localizing the UC model results and assigning a single value to the SMU blocks (Figure 3) assumes an exact match between grade class intervals $\{T_i(z_i), T_{i+1}(zi_{+1})\}$ and intervals of SMU bocks (T_{RANK}, T_{RANK+1}), which is readily achieved in practice.

Researchers designing the computerized scripts for implementation of the LUC approach need to consider the cases when the range of SMU (T_{RANK} - T_{RANK+1}) does not precisely matches that of the grade classes (T_i - T_{i+1}). The problem can be partially overcome by using a large number of grade classes. Personal experience shows that a good match between grade-tonnage relationships estimated by the conventional UC method and by the LUC approach is achieved when 50 grade classes are used. Further improvement can be achieved if the mean SMU grade is estimated by weighting grades of the classes to their proportions of the SMU. This approach was used by the author in the case studies described in the following sections.

Case studies

Iron ore deposit

The LUC method was tested on pisolitic iron ore mineralization in the eastern Pilbara, Western Australia (Hall and Kneeshaw, 1990; Abzalov *et al.*, 2010). The resources of the deposit were defined by drilling using the grids as follows:

Measured: 100 x 50 m
Indicated: 200 x 100 m
Inferred: 300 x 200 m.

However, it has been recognized that use of large blocks, such as 100 x 50 x 10 m, for definition of Measured Resources and Proved Reserves can lead to a substantial underestimation of the actual variability of the orebody, which is mined with a selectivity of approximately 25 x 25 x 10 m (Abzalov *et al.*, 2010). As a consequence, using the large blocks for the reserve model can cause incorrect estimation of the recoverable mineralization. For example, if <2.6% Al_2O_3 is a metallurgically acceptable impurity threshold, then modelling grade distribution as 100 x 50 x 10 m blocks would overestimate recoverable tonnage by 3.7% in comparison with the model estimated using 25 x 25 x 10 m blocks, which matches the mining selectivity (Figure 4).

Direct estimation of the small blocks by kriging is not feasible because of the large distances between the drill-holes. Therefore, in order to obtain more accurate estimation of the recoverable resources it was decided to test the LUC method.

The exercise was based on a detailed study area that was drilled at 50 x 50 m centres and contained 8121 samples. The drill data was sampled in order to create a more sparsely distributed subset, with the drill-holes distributed at 100 x 50 m centres, which matches the grid used for definition of Measured Resources. The subset, containing 4801 samples, was used to generate block models through application of the LUC technique to estimate the Al_2O_3 grade distributed as blocks of 25 x 25 x 10 m in size (Figure 5a). For comparison, Al_2O_3 grades of the same blocks were estimated by OK applied to the same subset of the data, distributed as 100 x 50 m centres (Figure 5b).

The LUC model exhibits significantly higher resolution than OK model constructed using the same data (Figure 5). The resolution of the LUC method matches the mining selectivity and therefore is suited for detailed production planning at this project.

The model was validated by averaging sample and the block grades into the large panels and plotting both grades against the centres of the panels (Figure 6). In the current study the grades of the LUC model blocks have been averaged into 100 m wide panels drawn across the entire deposit. The average block grades are compared with the average grades of the all drill-hole samples contained in the same 100 m wide panel. Distribution of the average grades on the spidergram shows that LUC model reconciles well when compared with the drill-hole samples when data are averaged by the large panels (Figure 6).

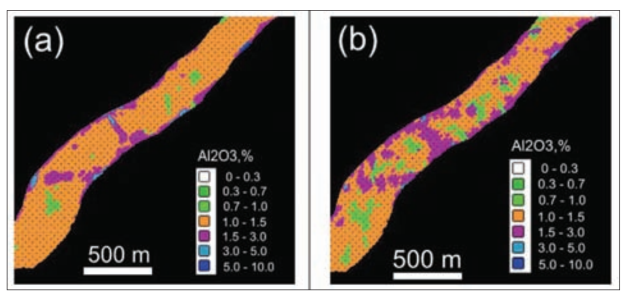

Figure 5—Examples of the block models constructed using subsets of the drill-holes distributed at 100 x 50 m centres. (a) OK model; (b) LUC model

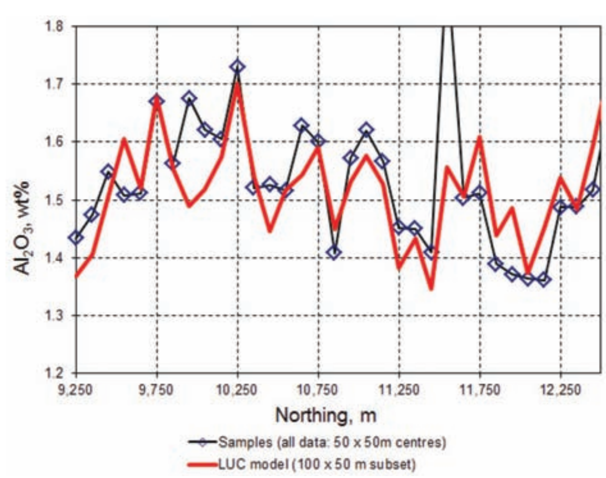

Figure 6—Spidergram showing distribution of the Al_2O_3 grades along the strike of mineralization. Grade values are averaged by 100 m wide slices drawn across the entire orebody

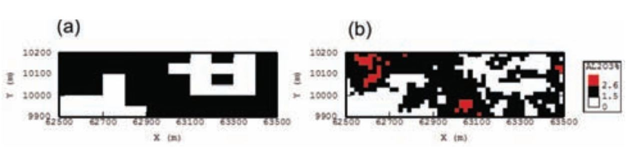

Figure 4—Maps showing spatial location of the mineralization at the different cut-offs. (a) Modelled as 100 x 50 x 10 m blocks; (b) modelled as 25 x 25 x 10 m blocks

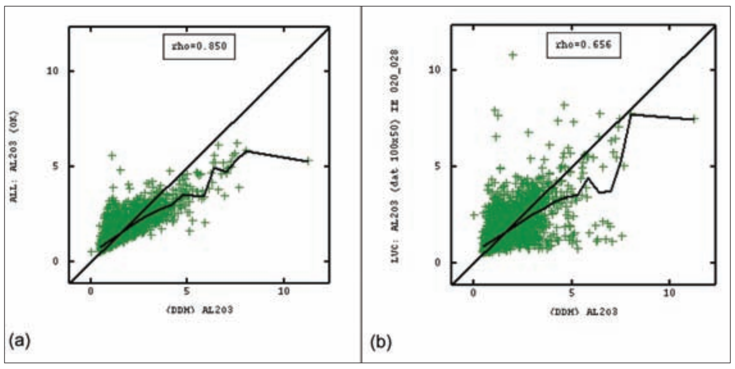

Figure 7—Scatter diagram of Al_2O_3 grades in the modelled blocks vs. drill sample grades contained in the same block. 1:1 line (straight) and conditional expectation line are shown on the diagrams for reference. (a) Ordinary kriging (OK) model estimated using all data; (b) LUC model constructed using a 100 x 50 m subset of the drill-holes

Comparison of the local estimates shows that correlation of the block grades estimated by LUC method with sample grades is 0.66, which is lower than the 0.85 correlation obtained for OK blocks (Figure 7). However, the LUC model is constructed using only 4801 samples out of the 8121 samples that were used for the OK model (Figure 7), therefore a lower precision in the local grade estimation is not unexpected. The error level is possibly acceptable considering that SMU grades are estimated from the drill-holes centred at 100 x 50 m (Figure 7).

Bauxite deposit

The study was undertaken as part of the long-term mine planning at the Weipa bauxite operation in Queensland, Australia. Evaluation of the project expansion required creation of a 3D model of the project area (Abzalov and Bower, 2009). The bauxite seam had to be represented as 0.5 m high slices, which represents the mining selectivity at the operation. However, direct kriging of the 3D blocks was impossible because most of the drill-holes (1419 holes) had been drilled in the 1970s, and at that time the holes were sampled in a 2D format where a single sample is taken for entire thickness of the seam. 3D data was available only from 117 holes which had been sampled as continuous strings of 0.2 m samples (Figure 8).

The 2D model was converted into a 3D model using the LUC method. The bauxite model was discretized to panels of 500 x 500 x thickness (m) and then a grade-tonnage relationship was estimated for each panel using the UC method. The 500 x 500 x 0.5 m blocks were ranked using the 3D samples of the 117 holes (Figure 8) and their grades deduced from the UC model using the LUC technique (Figure 9).

Application of the LUC methodology allowed us to reconstruct a vertical profile of the bauxite seam maintaining the total sum of the contained metal (Figure 9) and, at the same time, adhering to the principals of the volume-variance relationships.

Discussion and conclusions

Since it was first presented in 2006 the LUC method has been implemented in ISATIS© (commercial software) and has become one of the common approaches for grade estimation

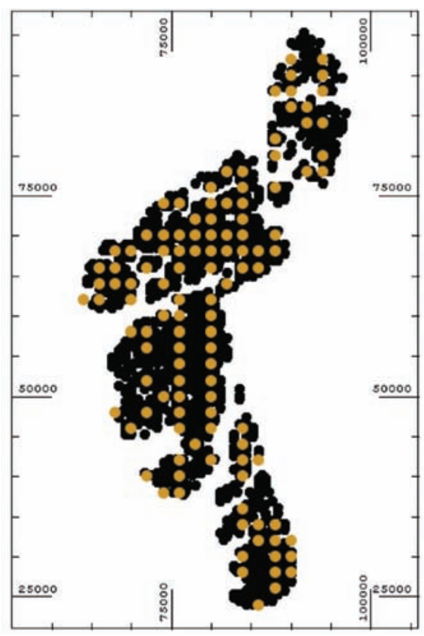

Figure 8—Map of the Weipa bauxite project showing distribution of the drill-holes. Black - holes where a single sample was taken for full thickness of the bauxite seam (2D data); light brown - drill-holes that were sampled as continuous strings of 0.2 m samples (3D data)

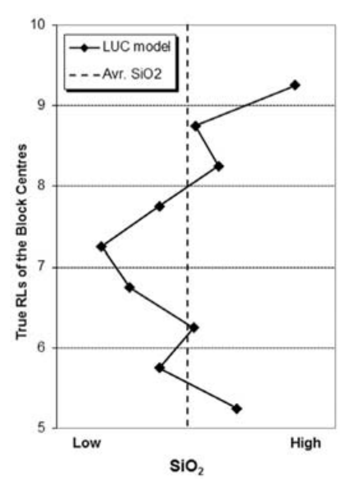

Figure 9—Example (500 x 500 x 5.5 m bauxite panel) showing the vertical SiO_2 trend generated from 2D model by the LUC technique

when data spacing is too broad in comparison with the estimated block size. It is one of the geostatistical methods used for recoverable resource estimation. In comparison with other methods, such as disjunctive kriging, service variables, and residual or multiple indicator kriging, LUC is relatively simple and benefits from its simplicity.

The key feature of the LUC approach is the ability to partition the panel into the small blocks (SMU) and estimate their grades maintaining the volume-variance relationship. The procedure (Abzalov, 2006) is not directly attached to uniform conditioning and can be applied like a post-processing algorithm to any recoverable resource estimate. The same approach was recently applied for localization of recoverable resources estimated by indicator kriging (i.e. localized indicator kriging). In order to choose between Gaussian-based algorithms, such as LUC, and indicator-based algorithms (e.g. LIK), the border effect needs to be checked and tested by estimating the ratios of indicators (Abzalov and Humphreys, 2002). If the ratio of indicators cross-variogram changes regularly with distance, the Gaussian-based models are applicable.

The accuracy of the local estimation depends on the SMU ranking techniques. When ranking is performed by direct kriging of the SMU blocks their spatial distribution is approximate. Accuracy of localization of the SMU grades decreases when the variogram of the studied variable is characterized by a large nugget effect.

Block ranking can be improved using auxiliary data, either geophysical or geochemical. This allows the use of the LUC method for integrating different data-sets, which will enhance the practicality of the LUC technique. However, this requires further investigation in order to obtain a better understanding of the strengths and limitations of the technique when it is applied in the multivariate environment.

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Notations

$Z(v)$	variable of interest (metal) which is estimated as the blocks of size (v)
$Y(v)$	Gaussian variable with mean 0 and variance 1
ϕ_v	Gaussian anamorphosis
H_k	Hermite polynomial
k	coefficient of a Hermite polynomial expansion
r	point-to-block correlation coefficient
$T_v(z_C)$	tonnage of mineralization recovered at the cut off (z_C) at the support (v)
$Q_v(z_C)$	quantity of contained metal (z) recovered at the cut off (z_C) and at the support (v)
$M_v(z_C)$	mean grade of mineralization recovered at the cut off (z_C) and at the support (v)

Appendix

Discrete Gaussian point-block model

The distribution of the SMU (v) grades can be expressed using a Hermite polynomial expansion

$$Z(v) = \phi_v Y(v) = \sum_{k=1}^{\infty} \frac{\varphi_k}{k!} r^k H_k(Y(v))$$

where k are coefficients established in the normal score transformation (Gaussian anamorphosis), $Y(v)$ is the Gaussian variable with mean 0 and variance 1, and r is the point-to-block correction coefficient.

The underlying assumption of the above equality is that pairs of Gaussian transformed values $Y(x)$ (point anamorphosis) and $Y(v)$ (block anamorphosis) are bi-Gaussian linearly correlated values with a correlation coefficient r. This coefficient is unknown and needs to be calculated. The procedure for calculating the point-to-block correction coefficient r is as follows.

The first step is to calculate a point anamorphosis (i.e. normal score transformations) $Z(x) = \phi\,(Y(x))$.

The next step is to calculate an empirical point variogram $g(h)$ using the available data $Z(x)$ (i.e. samples) and fit the variogram model.

From the point variogram $g(h)$ of $Z(x)$, the point-to-block correction coefficient r of the block (v) anamorphosis $Z_v(x) = \phi_v\,(Y(v))$ can be calculated using the following geostatistical relationship between the variance of $Z(v)$ and block anamorphosis function:

$$Var\big(Z(v)\big) = Var\big(\phi_v Y(v)\big) = \sum_{k=1}^{\infty} \frac{\varphi_k^2}{k!} r^{2k}$$

At the same time the variance of $Z(v)$ is equal to a block covariance $\bar{C}(v,v)$, which can be easily calculated from the variogram model:

$$Var\big(v\big) = \bar{C}\big(v,v\big) = \gamma\big(\infty\big) - \bar{\gamma}\big(v,v\big)$$

Therefore, using the above relationships the final equation for calculating the point-to-block correction coefficient r can be expressed as follows:

$$(\gamma(\infty) - \bar{\gamma}(v,v)) = \sum_{k=1}^{\infty} \frac{\varphi_k^2}{k!}\ r^{2k}$$