A new mathematical programming model for long-term production scheduling considering geological uncertainty

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Synopsis

Determination of the optimum production schedules over the life of a mine is a critical mechanism in open pit mine planning procedures. Long-term production scheduling is used to maximize the net present value of the project under technical, financial, and environmental constraints. Mathematical programming models are well suited for optimizing long-term production schedules of open pit mines. There are two approaches to solving long-term production problems: deterministic- and uncertainty- based approaches. Deterministic-based models are unable to deal with grade and geological uncertainties, which are two important sources of risk in mining industries. This may lead to discrepancies between actual production obtained by these algorithms and planning expectations. In this paper, a new binary integer programming model was developed for long-term production scheduling that incorporates geological uncertainty within the orebody. Then, traditional and uncertainty-based models are applied to an iron ore deposit. Results showed that the uncertainty-based approach yields more practical schedules than traditional approaches in terms of production targets.

Keywords

open pit mine, long-term production scheduling, integer programming, geological uncertainty.

Introduction

Long-term production scheduling involves the determination of sequences of ore and waste blocks to give the maximum net present value (NPV). This problem is subject to many aspects being satisfied, such as grade constraints, tonnage requirement for the plants, wall slope restriction, equipment capacities, etc. Long-term production scheduling determines the distribution of cash flow over life of mines. The NPV of each project depends on the grade and tonnage of the deposit, economic issues (operating and capital costs, and commodity price), and technical mining specifications (slope constraints, excavation capacities, etc.) In reality, some of the abovementioned issues can vary within certain limits, and the planner should make his decision on the production plan before knowing the exact values of the data. Among these uncertainties, those that are related to the orebody model lead to considerable deviation from production targets during the extraction process. Orebody model uncertainty has two important components[1]:

- Geological uncertainty (tonnage uncertainty), which reflects the uncertainty related to the ore/waste contacts
- Grade uncertainty, which reflects the error associated with the ore block grade estimation.

The geological and grade uncertainty cannot be eliminated; therefore, the best solution is to quantify uncertainties, reduce these uncertainties as far as investment permits, and finally manage the associated risk during the scheduling procedure. The latter can be achieved by explicit incorporation of uncertainty into the pit design and production scheduling process.

Many researchers have incorporated grade uncertainty into the optimization process during production scheduling[2–8]. However, geological uncertainty has not been considered yet during the scheduling process. In this paper, a binary integer programming formulation is developed for long-term production scheduling based on geological uncertainty to present an optimal solution under geological uncertainty. This formulation considers a probability index (PI) that reflects the probability that a block is an ore block. This index can be determined by the use of Indicator Kriging method. This model is then applied to an iron ore deposit.

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Deterministic model for long-term production scheduling

Here the traditional integer programming formulation for long-term production scheduling is presented. Let:

T = maximum number of scheduling periods
t = period in a scheduling span T
ijk = indices correspond to the row, column, and level of blocks in the model
Γ = the set of blocks
y_{ijkt} = a binary variable that is equal to 1 if block ijk is mined in period t, otherwise 0
c_{ijk} = the net value of block ijk
d_1 = economic rate of interest
d_2 = the risk discount rate
Z_{ijk} = average grade of a block ijk
OT_{ijk} = ore tonnage in the block ijk
WT_{ijk} = tonnage of waste material within the block ijk
PU_t = maximum processing capacity in any period, t
PL_t = minimum amount of ore that should be processed in period t
MU_t = maximum available equipment capacity for each period t
ML_t = minimum amount of material (waste and ore) that should be removed in any period t
Z^t_{max} = maximum average grade of ore sent to the mill in any period t
Z^t_{min} = minimum average grade of ore sent to the mill in any period t
x = index for 9 overlaying blocks.

Then, the deterministic IP formulation's for long-term production scheduling can be written as follows:

$$Max\ F_1 = \sum_{ijk\in\Gamma}\sum_{t=1}^{T}\frac{1}{(1+d_1)^t}\cdot c_{ijk}\cdot y_{ijkt} \quad [1]$$

Subject to:

$$\sum_{ijk\in\Gamma}\left(Z_{ijk}-Z^t_{\max}\right)\times OT_{ijk}\times y_{ijkt}\le 0 \qquad t=1,2,\ldots,T \quad [2]$$

$$\sum_{ijk\in\Gamma}\left(Z_{ijk}-Z^t_{\min}\right)\times OT_{ijk}\times y_{ijkt}\ge 0 \qquad t=1,2,\ldots,T \quad [3]$$

$$\sum_{ijk\in\Gamma}\left(OT_{ijk}\times y_{ijkt}\right)\le PU_t \qquad t=1,2,\ldots,T \quad [4]$$

$$\sum_{ijk\in\Gamma}\left(OT_{ijk}\times y_{ijkt}\right)\ge PL_t \qquad t=1,2,\ldots,T \quad [5]$$

$$\sum_{ijk\in\Gamma}\left(OT_{ijk}+WT_{ijk}\right)\times y_{ijkt}\le MU_t \qquad t=1,2,\ldots,T \quad [6]$$

$$\sum_{ijk\in\Gamma}\left(OT_{ijk}+WT_{ijk}\right)\times y_{ijkt}\ge MU_t \qquad t=1,2,\ldots,T \quad [7]$$

$$9y_{ijkt}-\sum_{r=1}^{t}(y_{i-1,j-1,k+1,r}+y_{i-1,j,k+1,r}+y_{i-1,j+1,k+1,r}+ y_{i,j-1,k+1,r}+y_{i,j,k+1,r}+y_{i,j+1,k+1,r}+y_{i+1,j-1,k+1,r}+ y_{i+1,j,k+1,r}+y_{i+1,j+1,k+1,r})\le 0 \qquad t=1,2,\ldots,T \quad [8]$$

$$\sum_{t=1}^{T}y_{ijkt}\le 1 \quad [9]$$

Objective function [1] will maximize the NPV of a mining project. Constraints [2] and [3] that are grade blending constraints and control the feed quality sent to the mill. Constraints [4] and [5] are processing capacity constraints and ensure that milling capacities hold. Constraint [6] ensures that the equipment capacities hold. Constraint [7] can control the stripping ratio of ore. Constraints [8] are the wall slope restriction and constraint [9] ensures that the entire block in the block model should be removed in one period only.

In order to incorporate the geological uncertainty into this model, this kind of uncertainty should first, be quantified. In the next section, the Indicator Kriging method is introduced as a powerful tool to quantify the geological uncertainty.

Quantification of geological uncertainty by the use of Indicator Kriging

Indicator Kriging (IK) as a non-parametric technique in resource estimation is over fifteen years old. IK was introduced by Journel in 1983[9], and since then, despite the relative difficulty in its application, it has grown to become one of the most widely-applied grade estimation techniques in the minerals industry. Its appeal lies in the fact that it makes no assumptions about the distribution underlying the sample data, and, indeed, it can handle moderate mixing of diverse sample populations. However, despite the elegant and simple theoretical basis for IK, there are many practical implementation issues that affect its application and require serious consideration. These include aspects of order relations and their correction, the change of support, issues associated with highly skewed data, and the treatment of the extremes of the sample distribution when deriving estimates[10]. It is the main non-linear geostatistical technique used today in the minerals industry.

The essence of the indicator approach is the binomial coding of data into either 1 or 0 depending upon its relationship with a cut-off value, Z_c. For a given value $Z(x)$, $i_k(x) = 1$ if $Z(x) \ge Z_c$ and $i_k(x) = 0$ otherwise[11]. This is a non-linear transformation of the data value into either a 1 or a 0. Values that are much greater than a given cut-off, Z_c, will receive the same indicator value as those values which are only slightly greater than that cut-off. Thus indicator transformation of data is an effective way of limiting the effect of very high values. It should be noted that this transformation can be also performed according to the presence or absence of a rock type (direct binary data requiring no transform), or a series of lithological or facies codes, or mineral sands hardness values. Simple or Ordinary Kriging of a set of indicator-transformed values will provide a resultant value between 0 and 1 for each point estimate, which can be interpreted either as[12]:

- *Proportions*—the proportion of the block above the specified cut-off on data support
- *Probabilities*—the probability that the grade is above the specified indicator. This probability is considered as the Probability Index (PI_{ijk}) for block ijk that is used in

the optimization process in this paper. The blocks with those higher of probability have less risk than with the lower probability.

In the next section, this probability index is incorporated into the traditional deterministic model in order to obtain an uncertainty-based mathematical model for long-term production scheduling.

Production scheduling under geological uncertainty

In this section, an integer programming-based model is developed in order to taking account the geological uncertainty. In this approach, a probability is assigned to each block (PI_{ik}) which represents the probability that is produced from ik for each block in the block model. Now, we should set our objective function in a way that the blocks with higher certainty are mined in earlier production periods, leaving uncertain blocks for later periods, when additional information usually becomes available. Therefore, another objective function is added to the objective function of traditional model in the form of:

$$MaxF_2 = \sum_{ijk\in\Gamma}\sum_{t=1}^{T}\frac{1}{(1+d_2)^t}\cdot PI_{ijk}\cdot y_{ijkt} \quad [10]$$

where d_2 is the risk discount rate, which is used to discriminate the block extraction preference between time periods. If a higher rate is used, the differences in the probabilities between different periods are expected to be higher.

It can be seen that we have two objective functions [1] and [10] that should be optimized simultaneously. The process of optimizing systematically and simultaneously a collection of objective function is called multi-objective optimization. There are several approaches to solve multi-objective optimization problems[13], but the most common one is the weighted sum method. If we have M objective function $F_l(x)$, $l = 1,2,\ldots,M$, then the utility function (utility function is an amalgamation of the individual functions and is a mathematical expression that attempts to model the decision-maker's preferences) can be expressed as:

$$U = \sum_{l=1}^{M} W_l F_l(x) \quad [11]$$

Here, W_l are weights typically set by the decision-maker. These weights reflect the relative importance of each objective. There are several ways to select W_l which can be seen in Marler and Arora[13]. If all of the weights are positive, the maximum of [11] is Pareto-optimal[14]; i.e. maximizing [11] is sufficient for Pareto optimality. In addition, as it is clear, objective functions should be transformed such that they are dimensionless. One of the robust approaches to transform objective functions, regardless of their original range, is given as follows[13]:

$$F_l^{trans} = \frac{F_l(x)}{F_l^o} \quad [12]$$

where

F_l^{trans} : transformed objective function with a lower limit of unity

F_l^o : utopia point (point that optimizes the objective function within the feasible design space).

In this model we set $W_1 = W_2 = 1$, therefore the final objective function of this model can be written as:

$$\text{Max.}\, U = F_1^{trans} + F_2^{trans} \quad [13]$$

Here:

$$F_1^{trans} = \frac{\sum_{ijk\in\Gamma}\sum_{t=1}^{t}\frac{c_{ijk}\cdot y_{ijkt}}{(1+d_1)^t}}{F_1^0} \quad [14]$$

and,

$$F_2^{trans} = \frac{\sum_{ijk\in\Gamma}\sum_{t=1}^{T}\frac{PI_{ijk}\cdot y_{ijkt}}{(1+d_2)^t}}{F_1^0} \quad [15]$$

Therefore Equation [13] can be rewritten as:

$$\text{Max.}\, U = \frac{\sum_{ijk\in\Gamma}\sum_{t=1}^{T}\frac{c_{ijk}\cdot y_{ijkt}}{(1+d_1)^t}}{F_1^0} + \frac{\sum_{ijk\in\Gamma}\sum_{t=1}^{T}\frac{PI_{ijk}\cdot y_{ijkt}}{(1+d_2)^t}}{F_2^0} \quad [16]$$

This objective function is subject to the constraints [2] to [9].

Application of deterministic and uncertainty based model in an iron ore deposit

A case study has been conducted on a central Iranian iron orebody to compare the results of the suggested algorithm with the deterministic one. The annual ore production rate of this deposit is 7 Mt of iron ore. The deposit contains about 317 Mt of ore with an average grade of 53% Fe and 1% P.

The block model contains 17 921 blocks with these dimensions 25 m × 25 m × 12.5 m. In order to determine the PI of each block, Indicator Kriging analysis was performed using Surpac 6.1.2 software. Cut-off grade is set at 25% Fe[15].

Then the optimal pit limit has been determined using Lerches and Grossman method[15] by the aid of Surpac[16]. This pit contains 232 Mt of ore with the stripping ratio of 1.33 and Fe and P average grades of 56.2% and .086%, respectively. Because there are a large number of blocks within the ultimate pit limit, the pit can be divided into a series of sub-pits commonly called push-backs, cut-backs, or phases. These push-backs are designed with haul road access and act as a guide during the yearly scheduling process. Five push-backs are designed and the blocks within the first and second push-backs were submitted to the deterministic (optimization) formulation in Equation [1] and constraints in Equations [2] through [9]) and uncertainty-based algorithm (optimization) formulation in Equation [16] and constraints in Equations [2] through [9]. This push-back, which contains 6770 blocks (4100 waste blocks and 2670 ore blocks) is depicted in Figure 1. Production schedule was fulfilled for a 5-year mine life. This study considers that ore material sent to the processing plant during each production period should have an average Fe grade of more than 52% and average P grade of less than 1%. d_1 and d_2 are assumed to be equal to 10%. The deterministic and suggested models were then solved by using the Risk Solver Platform V9.6

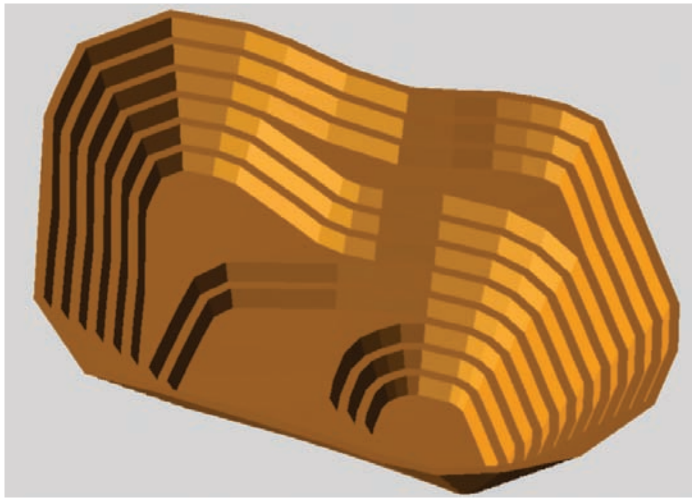

Figure 1—A 3D view of the second pushback in the iron ore mine[17]

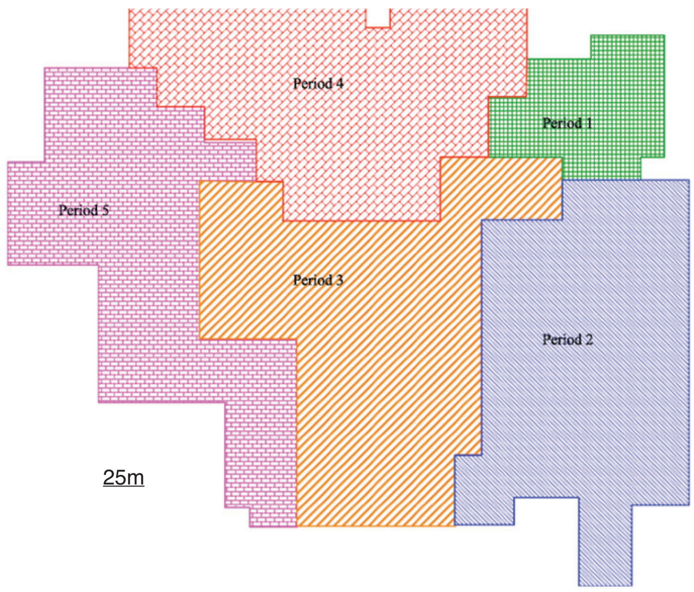

Figure 2—Plan showing production scheduling results incorporating geological uncertainty

Table I

Summary results of production scheduling using the deterministic model (DM) and uncertainty-based model (UM)

Periods (years)	Fe (%)		P (%)		Ore (106 tons)		CI	
	DM	UM	DM	UM	DM	UM	DM	UM
1	55	53.7	0.9	1	6.95	7	356	467
2	54	52.9	0.88	0.95	7	7.1	295	402
3	55	52	0.75	0.7	7.3	6.85	489	355
4	53	55	0.74	0.71	7.12	7.25	445	330
5	52.6	53.2	0.65	0.60	6.94	6.5	270	301

software[18]. Figure 2 shows the scheduling pattern generated from the proposed uncertainty-based model in the form of a bench plan. In order to compare the performance of these two algorithms, the number of blocks with the probability of more than 0.5 (PI>50%) is calculated in each period. This number is called the confidence index (CI). A low CI reflects high risk in achieving planned ore production, and a high CI a low risk.

Table I shows the summary results of production scheduling using traditional and uncertainty-based approaches.

As shown in Table I, the produced schedule using the uncertainty-based model has the highest CI in the first year (467), a lower CI in the second year (402) and the lowest in the last year (301). This means that the uncertainty-based model seeks the more certain areas of deposit in the first periods of operation and leaves less certain areas for the later periods, when additional information usually becomes available. Therefore, the probability of deviation from ore production target becomes lower by the use of the suggested model. The NPV (not mentioned in Table I) of the schedule generated by the traditional model is 9% higher than that of the suggested model, because the suggested model tends to maximize NPV and minimize the geological risk. This leads to some high-grade blocks are left for the latter periods. Therefore, it generates a realistic NPV, which is the best under conditions of uncertain geology. However, increasing the NPV will generally increase the risk of not meeting production targets.

Conclusion

In this paper, a geological uncertainty-based model is presented for open pit long-term production scheduling. The suggested integer programming model generates schedules that explicitly maximize the NPV of a project and reduce geological risk during the early production periods. Geological uncertainty was quantified by the use of the Indicator Kriging method. Using this method, a probability index can be obtained that reflects the probability that a block is an ore block. This index is then incorporated into the mathematical formulation for long-term production scheduling. The proposed model was applied on an iron ore deposit. Comparing the results from the proposed model with those from the deterministic one shows that in the proposed model, the total number of blocks with higher confidence in the first periods of exploitation is greater than that of the deterministic model while all the production constraints are satisfied. This can result in decreasing the risk of not meeting production targets during the first period of the mining process.

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