A new mathematical programming model for production schedule optimization in underground mining operations

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Synopsis

Mixed integer programming (MIP) has been used for optimizing production schedules of mines since the 1960s and is recognized as having significant potential for optimizing production scheduling problems for both surface and underground mining. The major problem in long-term production scheduling for underground orebodies generally relate to the large number of variables needed to formulate a MIP model, which makes it too complex to solve. As the number of variables in the model increase, solution times are known to increase at an exponential rate. In many instances the more extensive use of MIP models has been limited due to excessive solution times.

This paper reviews production schedule optimization studies for underground mining operations. It also presents a classical MIP model for optimized production scheduling of a sublevel stoping operation and proposes a new model formulation to significantly reduce solution times without altering results while maintaining all constraints. A case study is summarized investigating solution times as five stopes are added incrementally to an initial ten stope operation, working up to a fifty stope operation. It shows substantial improvement in the solution time required when using the new formulation technique. This increased efficiency in the solution time of the MIP model allows it to solve much larger underground mine scheduling problems within a reasonable time frame with the potential to substantially increase the net present value (NPV) of these projects. Finally, results from the two models are also compared to that of a manually generated schedule which show the clear advantages of mathematical programming in obtaining optimal solutions.

Keywords

Underground mine optimization, mixed integer programming, long-term scheduling, mathematical programming application.

Introduction

Mining companies face the challenge of scheduling production in their mines in a way that is economically optimal. The scheduling process should provide the company with profit maximization, a high level of equipment utilization, and high quality products in each time period according to demand requirements. Numerous authors have advanced the ability of solving large surface mine scheduling problems over the last fifty years, which has resulted in the development of a number of open pit optimization packages. The underground mine scheduling problem, however, has become more prominent of late and has thus received more attention. Research into this area will become more important as shallow deposits, which are amenable to open pit mining, are increasingly exploited and eventually diminished.

The lack of available software programs to aid the underground production scheduling process has meant that this is still largely carried out manually, often involving extensive and complex spreadsheets. This is no doubt a very time-consuming and tiresome process and whereas a feasible solution may be reached, there is very little chance of obtaining the optimal solution. As such, mathematical programming techniques, which form the basis for many open pit optimization and scheduling packages, can be investigated for similar applications into the underground mine environment.

Mixed integer programming (MIP) is well recognized within industry circles as being able to model and thus find the optimal solution to large, complex, and highly constrained problems. MIP is a combination of linear programming (LP) and integer programming (IP). An LP programming model consists of a linear objective function and a set of linear constraints, without loss of generality, of the following form:

Maximize or (Minimize) \( Z = c_1x_1 + c_2x_2 + c_3x_3 + \ldots + c_nx_n \)
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subject to:
\[ a_1 x_1 + a_2 x_2 + \ldots + a_n x_n \leq b_1 \]
\[ b_2 x_1 + c_2 x_2 + \ldots + c_n x_n \leq b_2 \]
\[ \vdots \]
\[ a_m x_1 + a_m x_2 + \ldots + a_m x_n \leq b_m \]

and a set of non-negativity restrictions: \( x_1, x_2, \ldots, x_n \geq 0 \)

In this model, \( Z \) represents the objective function value which could be maximization of profit or minimization of cost. \( x_i \) are decision variables, whose values determined by the model, \( a_i \) and \( c_i \) are constants whose values are dictated by the nature of the problem and \( b_i \) is the right-hand side constant value. In certain problems, the decision variables must assume integer values. When this restriction is added to the system, the model is called an integer programming (IP) model. In a given model, if some variable values are allowed to be continuous while others must assume integer values; the resulting model is a mixed integer program (MIP).

At present, employing classical MIP models for underground production scheduling requires significant computational effort, resulting in its true potential use being hindered due to excessive solution times. MIP model solution times depend exponentially as the number of variables increase. The worst case scenario in finding the optimal solution involves examining each candidate solution. Therefore solving large problems may become exponentially more complex with increasing problem size. Fortunately in practice all nodes are usually not examined because some branches are fathomed early.

Trout (1995) formulates and attempts to solve a mixed integer programming multi-period production scheduling model for a sublevel stoping copper ore operation located in Mt Isa, Australia. The objective is to schedule stopes for production over a two-year period at four weekly time intervals for the purpose of maximizing net present value. The data set containing 55 stopes within the 1100 orebody had to comply with numerous production, resource, and timing targets and constraints. Although the model produced a solution that improved NPV by 25% over what was realized in practice, the solution time was interrupted prior to proof of optimality at 209 hours with the reported integer solution being obtained after 1.6 hours. As such, the model was not implemented at the mine as it also lacked a number of additional important features.

Nehring (2006) continues on from Trout’s (1995) model by formulating an additional constraint function to limit multiple fillmass exposures which is proposed with (Topal 2003). Trout’s model was implemented over 36 monthly periods and start times further reduced the number of variables. The final model was implemented over 26 monthly periods and comprised 1 440 variables. Written using AMPL code, it was solved in CPLEX on a Sun Ultra 10 machine with 256 MB RAM in less than 100 seconds. Final results indicate an improved deviation from targeted production of 6% down from 10–20% compared to the manual schedule with no constraint violation. The model was ultimately implemented into Kiruna Mine’s mainstream scheduling process (Topal, 2008).

Carlyle and Eaves (2001) use an integer programming model to plan a production schedule for a sub-level stoping operation at Stillwater Mining Company. The model provided near-optimal solutions, for a 10-quarter planning period, to maximize revenue from mining platinum and palladium. However, the authors did not describe any special techniques to expedite solution time or a description of the model in their publication.

Production planning of a deposit containing eleven polymetallic zones with different geological characteristics and thus requiring different mining methods is carried out by McIsaac (2005) using MIP with the objective of maximizing cash flow. Production planning took place over four years using three monthly periods with a minimum daily production capacity set at 500 tonnes. The program is built into Microsoft Excel and solved with Frontline’s Xpress Solver. A total of 1 200 variables were required taking 30 minutes to solve. No mention is made if a feasible solution was reached that could guarantee optimality. It is not stated how many of these variables are integer variables and there is no mention of a previously generated manual production schedule with which to compare results.

This paper seeks to further build and develop the concepts of natural sequence and natural commencement as presented by Little et al. (2008) for the purpose of reducing the number of variables and thus solution time for generating and solving MIP production scheduling models specifically for sublevel stoping operations. As such this paper will...
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Implement these concepts on a much larger scale to a more realistic operating scenario while also extensively implementing predefined production data to again further reduce the number of variables. Furthermore, the proposed model also allows one to control the grade profile of the schedule more realistically by generating new monthly ore production and grade data formulations. This allows greater control over monthly ore grades being fed into the process plant, which would otherwise not have been possible. Two models are presented that highlight the difference that can be achieved by fully utilizing the natural sequence of production phases inherent in the sublevel stoping method. The first model, referred to as the ‘classical model’, is representative of past mathematical scheduling practices which contain a number of inefficiencies and thus require significantly longer solution times. The second model, referred to as the ‘new model’, seeks to take the concepts of natural sequence and natural commencement further. To appreciate the affect of this on the number of variables within the model and its impact on solution time, both models will be applied to an identical fifty-stope operation, starting with an initial ten stopes which is then incrementally increased by five stopes under the same parameters, constraints, and objective. Finally, results from the two models are also compared to that of a manually generated schedule, which shows the clear advantages of mathematical programming in obtaining optimal solutions.

General overview of sublevel stoping operation

The sublevel stoping method (also referred to as open stoping, longhole stoping or blasthole stoping) is one of a number of highly underground mining methods. The main differentiating factor between sublevel stoping and the other methods, including sublevel caving, and block caving is that no caving takes places. For this reason voids that are created as a result of extraction are required to be backfilled. Once consolidated, this fillmass provides the support and confinement to continue mining surrounding stopes. As such, the method generally requires a competent ore and stable host rock needing minimal support. This technique is mostly suited to steeply dipping orebodies where the dip of the footwall exceeds the angle of repose of the broken ore to allow it to freely gravitate to the base of the orebody for collection at the drawpoints (Lawrence, 1998). Figure 1 shows a general layout for the method and what a typical stope within that layout would look like.

The typical mining cycle for each stope follows a number of sequential production phases, as illustrated in Figure 2. Once the main access has been developed to an orebody or a cluster of stopes, an individual stope will generally enter into production beginning with the internal development phase. Internal development for each stope may include excavation of cross-cuts and cut-offs on each of its sublevels including the extraction level. This will be followed by the production drilling phase where the entire stope is drilled out in preparation for the loading and blasting of explosives to break the ore material. The production phase will then begin with initial firing of the drawpoints as well as the winze across all sublevels. The winze is used to create an initial void into which cut-off material is able to then be fired into.

Once this material is drawn out by LHD from the stope drawpoints and larger voids become available, each subsequent production blast is able to become larger, breaking up more and more ore material. As a result ore production from each stope will generally follow a common profile which can be used in predefining production over subsequent months once production is initiated. Once all ore contained within the stope has been extracted, all access drives into the now empty stope are sealed before filling of the stope begins. After filling is completed the new fillmass will generally require some period of time to fully dry out and consolidate.

There are three main scheduling constraints inherent in sublevel stoping, which mainly relate to geotechnical conditions. First, it is crucial once production starts on a stope to progress production through all phases quite rapidly. The reason for this is not to leave the stope open for too long and thus limiting its exposure to intense geotechnical stresses. This decreases the risk of failure, which could potentially propagate through to surrounding stopes and infrastructure. Secondly, all forms of simultaneous adjacent stope production must be avoided in order to prevent excessively large unsupported voids. Thirdly, once a stope moves through the production phase and onto backfilling to ultimately become a fillmass, remaining adjacent stope production must be scheduled so as to prevent simultaneous multiple exposure of the fillmass. This is because the fillmass is generally weak and unable to transfer stresses. For this reason, once production ceases on a stope, it does not simply leave the data-set due to these ongoing interactions which may remain many years after completion of the initial stope. It should also be noted that the objective function for both models seek to maximize net present value (NPV). All activities in this case being scheduled have an associated cash flow. The NPV calculation determined in the optimization process therefore considers only those activities placed in the model for evaluation.
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It should also be noted that the evaluation of constraint costs or shadow prices is not the focus of either the classical or the proposed scheduling model. However, an NPV comparison between scenarios is possible by rerunning the model after each constraint adjustment.

Classical MIP model formulation

The classical model assigns a binary decision variable to signify the beginning of each phase of stope production including development, drilling, production and backfilling for each stope. Once commencement is initiated, predefined data relating to development metres, production, drilling metres, ore tonnage extracted and the backfill tonnage placed for each subsequent period thereafter is recognized and implemented. The NPV calculation determined in the optimization process considers only those activities presented to the model for evaluation. Any other activities that affect revenues in any other way need to be considered outside the model if these are not specified and presented for evaluation. No consideration has yet been given to taxation or depreciation. All subscript notation, sets, parameters, and decision variables used in the construction of the classical MIP model with descriptions are presented in Appendix A.

Proposed new MIP model formulation

In recognizing that each phase of stope production inherently follows the next without any significant time delay, this model takes the concepts of natural sequence and natural commencement further by assigning just a single binary decision variable to signify the start of development which is naturally followed by production drilling, extraction and backfilling. Predefined production data then ensure that subsequent drilling, production, and backfilling activity for each stope are recognized and implemented. It is evident that representing all four stope production phases with a single variable, as opposed to defining a separate variable for each phase, creates significant efficiencies. All subscript notation, sets, parameters, and decision variables used in the construction of the proposed new model MIP model with descriptions are presented in Appendix B.

Implementation of proposed model on a fifty-stope operation

For the purpose of comparison, implementation of both the classical and the newly developed model will take place on a small conceptual underground sublevel stoping operation. Whereas conceptual in nature, production data, parameters, and constraints are reflective of other underground operations of a similar scale and thus justify its use for testing and validating purposes.

The operation itself utilizes a conventional sublevel stoping method (as discussed and illustrated earlier) to extract copper ore from one north-south striking lens dipping at 75 to 80 degrees, reaching depths of 800 metres below surface. Current remaining ore reserves total 7.6 Mt grading 2.8% Cu for 0.2 Mt Cu, which will be exhausted at a rate of 50 000 t/month or 0.6 Mt/annum ore over the operation’s remaining 12-year mine life. Once extracted from stope drawpoints via LHD, all ore is channelled directly to an underground crusher station. All production ore to the crusher is handled by LHD unit with no trucking required and no rehandling required. Once crushed, ore is then hoisted to the surface via the haulage shaft. A plan view of the operation showing all stopes is provided in Figure 3.

As shown, the 50-stope data-set comprises eight stopes (indicated in green) having already completed all phases of production to become a fully consolidated fillmass. Production drilling (indicated by red) is currently taking place in one stope with another two in the extraction phase (indicated by blue), and another stope currently completing the backfill process (indicated by yellow). This leaves 38 stopes available for production. The ore tonnage and grade for these stopes as well as those currently in production are presented in Table I.

In keeping with the natural sequence of activities required to bring each stope into production, the expected length of internal development, production drilling, ore extraction tonnages, and backfill requirements, together with the anticipated time frames to complete each of these activities have been entirely predefined. Table II shows an example of the production profiles of ore tonnage and copper grade expected to be extracted from each of the first ten stopes over each month once their extraction phase is initiated.
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Table I
Ore tonnage and grade for fifty-stope example

<table>
<thead>
<tr>
<th>Stope</th>
<th>Tones (t)</th>
<th>Grade (% Cu)</th>
<th>Tones (t)</th>
<th>Grade (% Cu)</th>
<th>Tones (t)</th>
<th>Grade (% Cu)</th>
</tr>
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<td>R763</td>
<td>187 000</td>
<td>2.8</td>
<td>S465</td>
<td>19 000</td>
<td>2.9</td>
<td>S764</td>
</tr>
<tr>
<td>S163</td>
<td>175 000</td>
<td>3.0</td>
<td>S466</td>
<td>191 000</td>
<td>2.8</td>
<td>S765</td>
</tr>
<tr>
<td>S165</td>
<td>218 000</td>
<td>2.7</td>
<td>S562</td>
<td>200 000</td>
<td>2.6</td>
<td>S766</td>
</tr>
<tr>
<td>S262</td>
<td>201 000</td>
<td>2.9</td>
<td>S563</td>
<td>217 000</td>
<td>2.7</td>
<td>T162</td>
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<tr>
<td>S362</td>
<td>199 000</td>
<td>3.2</td>
<td>S565</td>
<td>180 000</td>
<td>3.0</td>
<td>T163</td>
</tr>
<tr>
<td>S364</td>
<td>183 000</td>
<td>2.8</td>
<td>S566</td>
<td>179 000</td>
<td>2.9</td>
<td>T164</td>
</tr>
<tr>
<td>S462</td>
<td>185 000</td>
<td>2.7</td>
<td>S567</td>
<td>211 000</td>
<td>2.8</td>
<td>T165</td>
</tr>
<tr>
<td>S566</td>
<td>206 000</td>
<td>2.6</td>
<td>S661</td>
<td>198 000</td>
<td>2.7</td>
<td>T166</td>
</tr>
<tr>
<td>S665</td>
<td>196 000</td>
<td>2.9</td>
<td>S662</td>
<td>185 000</td>
<td>2.8</td>
<td>T263</td>
</tr>
<tr>
<td>S663</td>
<td>191 000</td>
<td>2.8</td>
<td>S663</td>
<td>182 000</td>
<td>2.7</td>
<td>T264</td>
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<td>S664</td>
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<td>2.7</td>
<td>S664</td>
<td>185 000</td>
<td>2.8</td>
<td>T265</td>
</tr>
<tr>
<td>S661</td>
<td>185 000</td>
<td>2.7</td>
<td>S665</td>
<td>182 000</td>
<td>2.7</td>
<td>T266</td>
</tr>
<tr>
<td>S667</td>
<td>182 000</td>
<td>2.9</td>
<td>T263</td>
<td>173 000</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>S763</td>
<td>174 000</td>
<td>3.0</td>
<td>T264</td>
<td>184 000</td>
<td>2.8</td>
<td></td>
</tr>
</tbody>
</table>

Table II
Expected monthly ore/grade production rates for first ten stopes

<table>
<thead>
<tr>
<th>Stope</th>
<th>Month 1</th>
<th>Month 2</th>
<th>Month 3</th>
<th>Month 4</th>
<th>Month 5</th>
<th>Month 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tones</td>
<td>Grade</td>
<td>Tones</td>
<td>Grade</td>
<td>Tones</td>
<td>Grade</td>
<td>Tones</td>
</tr>
<tr>
<td>-------</td>
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<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>R763</td>
<td>21 000</td>
<td>2.8</td>
<td>38 000</td>
<td>2.9</td>
<td>43 000</td>
<td>2.7</td>
</tr>
<tr>
<td>S163</td>
<td>19 000</td>
<td>3.2</td>
<td>36 000</td>
<td>3.0</td>
<td>44 000</td>
<td>3.1</td>
</tr>
<tr>
<td>S164</td>
<td>17 000</td>
<td>2.5</td>
<td>37 000</td>
<td>2.6</td>
<td>44 000</td>
<td>2.8</td>
</tr>
<tr>
<td>S262</td>
<td>15 000</td>
<td>2.7</td>
<td>26 000</td>
<td>3.0</td>
<td>40 000</td>
<td>2.7</td>
</tr>
<tr>
<td>S263</td>
<td>22 000</td>
<td>3.3</td>
<td>34 000</td>
<td>3.0</td>
<td>38 000</td>
<td>3.3</td>
</tr>
<tr>
<td>S264</td>
<td>20 000</td>
<td>2.8</td>
<td>35 000</td>
<td>2.9</td>
<td>48 000</td>
<td>2.2</td>
</tr>
<tr>
<td>S362</td>
<td>20 000</td>
<td>2.8</td>
<td>35 000</td>
<td>2.9</td>
<td>45 000</td>
<td>3.1</td>
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<tr>
<td>S363</td>
<td>22 000</td>
<td>3.3</td>
<td>34 000</td>
<td>3.0</td>
<td>40 000</td>
<td>2.7</td>
</tr>
</tbody>
</table>

A full breakdown of data for the extraction profiles of the remaining stopes as well as the advancement metres, drilling metres, and backfill placement profiles for the internal development, production drilling, and backfilling phases of each stope can be provided upon request. It should be noted that internal development in this case refers to all development specific to bringing a particular stope into production. It therefore does not include development activities that are to the benefit of more than one stope. For the purposes of this demonstration it will be assumed that these development activities have been completed. All development activities are given as a length in metres and are stated as a standard equivalent primary horizontal development length. For example, where a length of rehabilitation is required, this is converted into an equivalent primary horizontal length. Similarly, production drilling lengths are also stated as a standard equivalent hole diameter length. It is recognized that whereas internal development activities would result in the production of some ore, it will be assumed that no ore is produced from internal development activities. All periods are quoted in months.

Planning engineers for this 50-stope operation also endeavour to meet mill feed head grade requirements of 2.8% Cu at a deviation of ±15%.

Fleet capacities place an important constraint on an operation that must also be considered. Jumbo and bolting rig fleets at the 50-stope operation limit equivalent primary horizontal development activities to 100 metres per month. Similarly, the production drill rig fleet restricts monthly production drilling activities to 20 000 metres of standard hole diameter. Backfill availability for each month is 90 000t. Monthly primary ore production in this case is limited by the haulage shaft, which has a monthly capacity of 60 000t. After taking into consideration the bucket capacity of each LHD unit as well as all tramming distances, this capacity is expected to be comfortably met by the mine’s LHD fleet. All sequencing restrictions relating to adjacency and single backfill exposure limits inherent in sublevel stope mining, as discussed previously, also need to be rigorously applied.

All fixed and variable development, drilling, production, and backfilling costs associated with each stope have also been predefined. Variable costs, which are normally quoted in $/unit, are therefore incurred throughout the operation of each activity. Fixed costs, however, are incurred at the start of each phase as a one-off cost. Using an average copper price of $4 000/t and an average recovery factor of 90%, revenues for each stope are calculated. Undiscounted values for each stope are then found by subtracting all costs from revenues.

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the revenue. For the purposes of this demonstration, each stope is made available for the start of the internal development phase from the very first period. To ensure compliance with the natural progression of activities inherent to sublevel stoping, all stopes currently in production must remain in production through to completion.

For scheduling purposes, a 10% per annum discount rate is applied. This figure is reflective of the rate currently used by smaller mining companies seeking to start up new operations. A change in economic circumstances resulting in a change in the discount rate would warrant a rerun of the model as this is likely to affect the optimal schedule and resulting NPV.

Implementation of the MIP model

The construction of both the classical and new mixed integer programming (MIP) scheduling models took place with all parameters and constraints implemented. Scheduling was carried out over 48 monthly periods. Both MIP scheduling models were written using a mathematical programming language (AMPL) code and solved using the solver package CPLEX 10.3 (ILOG™) on the same computer. A comparison of characteristics and solution times is contained in Table III for each scheduling model. This table details the implementation of the initial ten stopes through to the final fifty stopes at each five stope interval. Stopes were added according to their alphabetical and then numerical order.

It should be noted that from 30 stopes onwards both models were deemed to have solved the problem once a 5% gap solution was reached. This was to avoid excessive run times as these results can be regarded as optimal. As shown in Table III, solution times for the first 10–15 stopes are similar for both models. These times start diverging when 20 stopes are considered, which records a solution time of 104 and 65 seconds for the classical model and the new model respectively to produce the same objective value. As more stopes are added, the solution time for the classical model rises rapidly while the new model experiences moderate rises in comparison. By the full 50-stope implementation a solution time of 231 063 and 8 416 seconds are recorded for the classical and new model respectively to produce the same objective value. Due to a scheduling horizon of over 48 monthly periods scheduling results are unable to be effectively presented; however, they can be provided on request.

For comparative purposes a manual schedule was carried out on the full fifty-stope operation under the same operational and sequencing conditions. The manual approach initially selects the next available highest cash flow stope. The basis for this process involved ranking each stope in operational and sequencing conditions. The manual approach naturally carried on from one another translated into a significant reduction in solution time from 231 063 seconds for the classical model to just 8 416 seconds for the new model.

A significant reduction in model complexity was also achieved by using these concepts. Combining phases that naturally carry on from one another translated into a reduction in the number of variables, which in turn also translated into a reduction in the number of constraints that were required to reflect the mining process. The replacement of the four separate phase variables with a single variable also eliminated the need for vast amounts of precedence information that had to be put to the model in order to keep programming models produced a superior result, increasing NPV by $6.3 m to $122.1 m, thus representing a 5.4% increase over the manual result. Furthermore, an equivalent of a full working day was spent undertaking the manual evaluation process.

Discussion and conclusion

It is evident that solution times for the mathematical models are directly linked to the number of variables associated with each model. By taking the approach of fully utilizing the concepts of natural commencement and natural sequence and thus defining production by a single variable, the new model consistently achieved a significant reduction in the total number of variables over the classical model. Over a full 50-stope data-set this significant reduction in variables translated into a significant reduction in solution time from 231 063 seconds for the classical model to just 8 416 seconds for the new model.

| Table III |
| Mathematical model comparisons for both models |

| Simple 10 |
| Variables: 5 947 |
| Constraints: 131 103 |
| Objective ($m): 85.9 |
| Solution time (s): 411 |

| Simple 30 |
| Variables: 5 072 |
| Constraints: 167 006 |
| Objective ($m): 91.8 |
| Solution time (s): 1 474 |

| Simple 50 |
| Variables: 5 848 |
| Constraints: 248 838 |
| Objective ($m): 116.5 |
| Solution time (s): 3 318 |

| Simple 40 |
| Variables: 7 084 |
| Constraints: 385 481 |
| Objective ($m): 108.2 |
| Solution time (s): 18 749 |

| Simple 55 |
| Variables: 8 362 |
| Constraints: 527 099 |
| Objective ($m): 115.5 |
| Solution time (s): 134 854 |

| Simple 50 |
| Variables: 9 749 |
| Constraints: 749 523 |
| Objective ($m): 122.1 |
| Solution time (s): 231 063 |

| Simple 25 |
| Variables: 3 946 |
| Constraints: 131 103 |
| Objective ($m): 85.9 |
| Solution time (s): 411 |

| Simple 45 |
| Variables: 8 362 |
| Constraints: 527 099 |
| Objective ($m): 115.5 |
| Solution time (s): 134 854 |

| Simple 50 |
| Variables: 9 749 |
| Constraints: 749 523 |
| Objective ($m): 122.1 |
| Solution time (s): 231 063 |

| Simple 20 |
| Variables: 2 230 |
| Constraints: 107 721 |
| Objective ($m): 70.1 |
| Solution time (s): 104 |

| Simple 35 |
| Variables: 5 848 |
| Constraints: 248 838 |
| Objective ($m): 116.5 |
| Solution time (s): 3 318 |

| Simple 40 |
| Variables: 7 084 |
| Constraints: 385 481 |
| Objective ($m): 108.2 |
| Solution time (s): 18 749 |

| Simple 55 |
| Variables: 8 362 |
| Constraints: 527 099 |
| Objective ($m): 115.5 |
| Solution time (s): 134 854 |

| Simple 50 |
| Variables: 9 749 |
| Constraints: 749 523 |
| Objective ($m): 122.1 |
| Solution time (s): 231 063 |
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Table IV
Fifty stope scheduling process comparison

<table>
<thead>
<tr>
<th></th>
<th>Classical model</th>
<th>New model</th>
<th>Manual process</th>
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<tbody>
<tr>
<td>Objective ($m)$</td>
<td>122.1</td>
<td>122.1</td>
<td>115.8</td>
</tr>
<tr>
<td>Solution time (s)</td>
<td>231 063</td>
<td>8 416</td>
<td>28 800</td>
</tr>
</tbody>
</table>

Production progressing according to the appropriate sequence. This in turn also significantly reduced the number of constraints required by the new model.

Any further addition of stopes to the data-set would again increase solution times for both models; however, the new model would be much better placed to handle this due to its already faster solution time for 50 stopes of 8 416 seconds compared to 231 063 seconds for the classical model. This new model not only allows schedulers to obtain optimal solutions rapidly from stope data-sets such as the one used in this case study, which would have previously been virtually unachievable, it also now allows larger stope data-sets to be incorporated into the scheduling process.

Results obtained by the manual scheduling process of $115.8 \text{ m}$ fell well short of the optimal solution of $122.1 \text{ m}$. This again supports the fact that even if feasible results are able to be obtained from a manually generated schedule, there is no guarantee that these results are anywhere near optimal. From a manual perspective, the further addition of stopes to the scheduling process would have increased the complexity of the problem, adding to the amount of time required to obtain an inevitable sub-optimal result. Only well formulated mathematical programming models with the additional feature of utilizing the natural sequence of activities to reduce the overall number of variables, and in turn solution time, can provide optimal solutions to complex underground scheduling problems within a reasonable and practical time frame.

References


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Appendix A

Classical MIP model formulation sublevel stoping operation

The formulation follows:

Subscript notation

\( t \) schedule period; \( t = 1, 2, 3, \ldots, T \).
\( s \) internal stope development activity identification: \( s = 1, 2, 3, \ldots, S \).
\( d \) stope production drilling activity identification: \( d = 1, 2, 3, \ldots, D \).
\( e \) stope extraction activity identification: \( e = 1, 2, 3, \ldots, E \).
\( f \) stope backfilling activity identification: \( f = 1, 2, 3, \ldots, F \).
\( m \) metal type: \( m = a, b, c, \ldots, M \).
\( b \) backfill type: \( b = a, b, c, \ldots, B \).

Sets

\( \text{adjdd}_s \) Set of all internal stope development activities that are adjacent to and share a boundary with internal stope development activity \( s \).
\( \text{adjddr}_s \) Set of all stope production drilling activities that are adjacent to and share a boundary with internal stope development activity \( s \).
\( \text{adjde}_s \) Set of all stope extraction activities that are adjacent to and share a boundary with internal stope development activity \( s \).
\( \text{adjdf}_s \) Set of all stope backfilling activities that are adjacent to and share a boundary with internal stope development activity \( s \).
\( \text{adjdrd}_s \) Set of all internal stope development activities that are adjacent to and share a boundary with stope production drilling activity \( d \).
\( \text{adjdrd}_s \) Set of all stope production drilling activities that are adjacent to and share a boundary with stope production drilling activity \( d \).
\( \text{adjd}_s \) Set of all stope extraction activities that are adjacent to and share a boundary with stope production drilling activity \( d \).
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adjdf, Set of all stope backfilling activities that are adjacent to and share a boundary with stope production drilling activity d.
adjfd, Set of all internal stope development activities that are adjacent to and share a boundary with stope extraction activity e.
adjede, Set of all stope backfilling activities that are adjacent to and share a boundary with stope extraction activity e.
adjef, Set of all stope backfilling activities that are adjacent to and share a boundary with stope extraction activity e.
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\[ \sum_{t \text{ in period}} w_{a} \leq 1 \quad \forall a \text{ with devlate}_{a} > T \]

\[ \sum_{t \text{ in period}} x_{a} \leq 1 \quad \forall d \text{ with drlate}_{a} > T \]

\[ \sum_{t \text{ in period}} y_{a} \leq 1 \quad \forall e \text{ with elate}_{a} > T \]

\[ \sum_{t \text{ in period}} z_{a} \leq 1 \quad \forall f \text{ with fillate}_{a} > T \]  \[1a\]

\[ \sum_{t \text{ in period}} w_{a} = 1 \quad \forall s \text{ with devlate}_{s} \leq T \]

\[ \sum_{t \text{ in period}} x_{a} = 1 \quad \forall d \text{ with drlate}_{a} \leq T \]

\[ \sum_{t \text{ in period}} y_{a} = 1 \quad \forall e \text{ with elate}_{a} \leq T \]

\[ \sum_{t \text{ in period}} z_{a} = 1 \quad \forall f \text{ with fillate}_{a} \leq T \]  \[2a\]

\[ w_{a} = 1 \quad \forall s \text{ with devearly}_{a} = \text{ devlate}_{a} \]

\[ x_{a} = 1 \quad \forall d \text{ with drearly}_{a} = \text{ drlate}_{a} \]

\[ y_{a} = 1 \quad \forall e \text{ with elearly}_{a} = \text{ elate}_{a} \]

\[ z_{a} = 1 \quad \forall f \text{ with fillearly}_{a} = \text{ fillate}_{a} \]  \[3a\]

\[ r_{i} \times y_{i} \leq \text{ sc}_{i} \quad \forall t \]  \[4a\]

\[ g_{lm} \times r_{i} \times y_{i} \leq 0 \quad \forall m, t \]  \[5a\]

\[ g_{la} \times r_{i} \times y_{i} \leq 0 \quad \forall m, t \]  \[6a\]

\[ d_{i} \times w_{i} \leq \text{ dc}_{i} \quad \forall t \]  \[7a\]

\[ d_{i} \times x_{i} \leq \text{ dc}_{i} \quad \forall t \]  \[8a\]

\[ b_{i} \times z_{i} \leq \text{ ba}_{i} \quad \forall t, f \in \text{ adjdef}_{i} \]  \[9a\]

\[ w_{a} + w_{a} \leq 1 \quad \forall s, t \text{ with adjdd}_{i} \]

\[ x_{a} + x_{a} \leq 1 \quad \forall s, t \text{ with adjdr}_{i} \]

\[ w_{a} + y_{a} \leq 1 \quad \forall s, t \text{ with adjde}_{i} \]

\[ w_{a} + w_{a} \leq 1 \quad \forall s, t \text{ with adjdef}_{i} \]

\[ x_{a} + x_{a} \leq 1 \quad \forall d, t \text{ with adjdr}_{d} \]

\[ y_{a} + y_{a} \leq 1 \quad \forall d, t \text{ with adjde}_{d} \]

\[ x_{a} + z_{a} \leq 1 \quad \forall d, t \text{ with adjdef}_{d} \]

\[ y_{a} + w_{a} \leq 1 \quad \forall e, t \text{ with adjde}_{e} \]

\[ y_{a} + w_{a} \leq 1 \quad \forall e, t \text{ with adjdef}_{e} \]

\[ y_{a} + z_{a} \leq 1 \quad \forall e, t \text{ with adjde}_{e} \]

\[ y_{a} + z_{a} \leq 1 \quad \forall e, t \text{ with adjdef}_{e} \]

\[ z_{a} + w_{a} \leq 1 \quad \forall t, f \text{ with adjdf}_{f} \]

\[ z_{a} + x_{a} \leq 1 \quad \forall t, f \text{ with adjdr}_{f} \]

\[ z_{a} + y_{a} \leq 1 \quad \forall t, f \text{ with adjde}_{f} \]

\[ z_{a} + z_{a} \leq 1 \quad \forall t, f \text{ with adjdef}_{f} \]

\[ \sum_{t \text{ in period}} z_{a} + w_{a} + x_{a} + y_{a} + z_{a} = 2 \quad \forall t, f \text{ with pe}_{f}, e \text{ with pe}_{e}, f \text{ with pd}_{f}, e \text{ with pd}_{e} \]  \[[11a]\]

\[ \sum_{t \text{ in period}} w_{a} + \sum_{t \text{ in period}} x_{a} + \sum_{t \text{ in period}} y_{a} + \sum_{t \text{ in period}} z_{a} \leq 1 \quad \forall t \]  \[[12a]\]

\[ w_{a} + x_{a} + y_{a} + z_{a} = \text{ binary integer} \quad \forall t \]  \[[13a]\]

\[ w_{a} + x_{a} + y_{a} + z_{a} \leq 1 \quad \forall t \]  \[[14a]\]

\[ \sum_{t \text{ in period}} w_{a} - x_{a} \geq 0 \quad \forall t \]

\[ \sum_{t \text{ in period}} x_{a} - y_{a} \geq 0 \quad \forall t \]

\[ \sum_{t \text{ in period}} y_{a} - z_{a} \geq 0 \quad \forall t \]  \[[15a]\]

The objective function of this model seeks to maximize the cash flow of all activities under consideration (when taking the time value of money into account) by determining the optimal schedule within which to carry out all activities required to progress each stope through all four phases of production. Constraint [1a] ensures that commencement of all four stope phases is initiated no more than once during the time horizon if their late start date occurs beyond the maximum time horizon. Constraint [2a] requires all four stope phases to begin at some point during the time horizon if their late start date falls within the time horizon. Constraint [3a] places the stope phases that are currently in progress into the current production schedule, ensuring continuation from previous schedules. Constraint [4a] limits ore production for all stope extraction activities in any period from exceeding the shaft/LHD/truck fleet capacity. Constraint [5a] restricts the combined mill feed ore grade from all stope extraction activities in any period from exceeding a lower grade limit. Constraint [6a] restricts the combined mill feed ore grade from all stope extraction activities in any period from exceeding an upper grade limit. Constraint [7a] limits the amount of equivalent horizontal primary development taking place from all internal stope development activities in any time period from exceeding the combined development fleet capacity. Constraint [8a] ensures that the equivalent length of production drilling taking place from all stope production drilling activities in any period cannot exceed the combined production drill rig fleet capacity. Constraint [9a] ensures that the amount of each backfill type placed across all stope backfilling activities in any period cannot exceed the supply. Constraint [10a] enforces that simultaneous production between all stope phases that share a boundary does not occur. Constraint [11a] enforces ongoing fillmass stability of all stope by limiting simultaneous exposure to just a single side. Constraint [12a] ensures ongoing fillmass stability of all existing fillmasses by limiting simultaneous exposure to just a single side. Constraint [13a] ensures variables maintain integer values. Constraint [14a] ensures that simultaneous progression of activities within each...
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individual each stope is avoided. Constraint \([15a]\) enforces the sequential progression of activities inherent with moving each stope through production.

Appendix B

Proposed new MIP model formulation for sublevel stoping operation

The formulation follows:

Subscript notation

The model is defined in general terms using the following subscript notation.

\(t\) Schedule period: \(t = 1, 2, 3..., T\).

\(s\) Stope identification: \(s = 1, 2, 3..., S\).

\(m\) Metal type: \(m = a, b, c..., M\).

\(b\) Backfill type: \(b = a, b, c..., B\).

Sets

\(adj_s\) Set of all stopes that are adjacent to and share a boundary with stope \(s\).

\(ps\) Pair (2) of stopes that are adjacent to and share a boundary with stope \(s\).

\(badj\) Set of all stopes that are adjacent to and share a boundary with each existing fillmass.

\(t_pb\) Set of time periods that include all periods up to the current period \(t\).

Parameters

\(r_s\) Extraction reserve for each stope \(s\).

\(sct\) Shaft/LHD/truck fleet movement capacity for each period \(t\).

\(early_s\) Earliest start time for stope \(s\).

\(late_s\) Latest start time for stope \(s\).

\(nt\) Present value discount factor applied to period \(t\).

\(cfs\) The undiscounted cash flow generated by each stope \(s\).

\(gums\) Difference between targeted upper ore feed head grade and stope grade for each mineral type \(m\) in each stope \(s\).

\(glms\) Difference between targeted lower ore feed head grade and stope grade for each mineral type \(m\) in each stope \(s\).

\(ds\) Equivalent primary horizontal development length for each stope \(s\).

\(dct\) Total equivalent primary horizontal development fleet capacity for each period \(t\).

\(dr_s\) Equivalent production drill length for each stope \(s\).

\(drct\) Total standard production drill fleet capacity for each period \(t\).

\(bbs\) Backfill requirement of type \(b\) for each stope \(s\).

\(babt\) Total backfill availability of type \(b\) for each period \(t\).

Decision variables

\(w_{st}\) 1 if development from activity \(d\) is scheduled for period \(t\), 0 otherwise.

Objective function

\[
\text{Maximize: } \sum_{s,t} n_s \times cf_s \times w_{st}
\]

Subject to

\[
\sum_{t} w_{st} \leq 1 \quad \forall s \text{ late}_s > T \tag{1b}
\]

\[
\sum_{t} w_{st} = 1 \quad \forall s \text{ early}_s = \text{late}_s \tag{2b}
\]

\[
w_{st} = 1 \quad \forall s \text{ early}_s = \text{late}_s \tag{3b}
\]

\[
\sum_{s} r_s \times w_{st} \leq sc_t \quad \forall t \tag{4b}
\]

\[
\sum_{s} \text{glms}_s \times r_s \times w_{st} \leq 0 \quad \forall m, t \tag{5b}
\]

\[
\sum_{s} \text{gums}_s \times r_s \times w_{st} \leq 0 \quad \forall m, t \tag{6b}
\]

\[
\sum_{s} d_s \times w_{st} \leq dc_t \quad \forall t \tag{7b}
\]

\[
\sum_{s} \text{dr}_s \times w_{st} \leq drct \quad \forall t \tag{8b}
\]

\[
\sum_{s} \text{bbs}_s \times w_{st} \leq babt \quad \forall b, t \tag{9b}
\]

\[
w_{st} + w_{s't} \leq 1 \quad \forall s, t \text{ s'} \in adj_s \tag{10b}
\]

\[
\sum_{s \in \text{ps}} w_{st} + w_{s't} \leq 2 \quad \forall s, t \text{ s'} \in ps \tag{11b}
\]

\[
\sum_{s \in \text{ps}} w_{st} \leq 1 \quad \forall t \tag{12b}
\]

\[
w_{st} = \text{binary integer} \tag{13b}
\]

Once again the objective function seeks to maximize cash flow of all activities under consideration (when taking the time value of money into account) by determining the optimal schedule within which to carry out these activities. The description of all constraints for the new model (Constraints [1b]–[13b]) is reflective of those described in the classical model ([1a]–[15a]).

Constraints for each stope in this case are able to be implemented with a single function, thus avoiding repetition where the variables representing all four phases in the classical model often required four separate functions to achieve the same outcome. Constraints [14a] and [15a] in the classical model, which deal solely with the sequencing of the four phases comprising each stope, means that these are no longer required.◆