



Nugget effect, artificial or natural?

by P.C. Carrasco*

Synopsis

Knowing the origin of the relevant and irrelevant components of the nugget effect is essential to optimize the process under study. This paper describes the causes of the nugget effect and, its dependency on the sample support, sampling density, and on the QA/QC procedures. Several examples of natural and 'human' nugget effect are given. A method to estimate the magnitude of the relevant and irrelevant components is suggested. Finally, the economic consequences of misunderstanding the nugget effect are analysed.

Introduction

In practice is very important to estimate the origin of the behaviour at the origin of the variogram function. The understanding of the nature of the nugget effect can provide important information on the phenomenon under study. It also has profound implications in ore resource estimation strategies, in the selection of a mining method, and in the process control strategies.

Causes of the nugget effect

The causes of the nugget can be:

- A microstructure or 'process noise', namely a component of the phenomenon with a range shorter than the sample support (true nugget effect).^{1,2,3,8}
- A structure with a range shorter than the sampling interval
- Measurement errors. Sampling and/or assaying errors can create an artificial nugget effect or the so called 'human nugget effect'.².

The components of the variogram function if sampling and/or assaying errors exist are (see Appendix 1):

$$\gamma_{Z^*}(h) = \gamma_Z(h) + \gamma_e(h) + 2\gamma_{Z^*,e}(h) \quad [1]$$

Z = True value
 Z^* = Estimated value
 e = Random error
 γ = Semivariogram

h = Distance or time

$\gamma_{Z^*,e}$ = Cross variogram between Z^* and e

- If the errors have no temporal or spatial correlation
- If the grade and the errors are not temporally or spatially correlated.

$$\gamma_{Z^*}(h) = \gamma_Z(h) + cte_1 + 0 \quad [2]$$

$$\gamma_{Z^*}(h) = \gamma_Z(h) + C_{os} \quad [3]$$

Therefore the sampling and/or the assaying error variance add to the true variogram function as a constant.

Observation scale

The observation scale—sampling density and sample support—are very important in understanding the nature and magnitude of the nugget effect.

As an example consider Figure 1.

When the sample support is smaller than the range of the phenomenon under study, in this case the average size of the red spots, and the sampling pattern is dense enough to intersect the nuggets, the variogram function is spherical with zero nugget effect. In contrast, if the range is shorter than the sample support and the sampling pattern is dense, the variogram function would be flat.

According to the central limit theorem there are a very strong relation between the magnitude of the nugget effect and the sample support (see Appendix 2)^{1,2,3,8}

$$C_{0v} = C_{0v} \times \left(\frac{v}{V} \right) \quad [4]$$

V, v = Sample supports

C_{0v} = Nugget effect at sample support v

C_{0V} = Nugget effect at sample support V

* Technical Group, Codelco, Chile.

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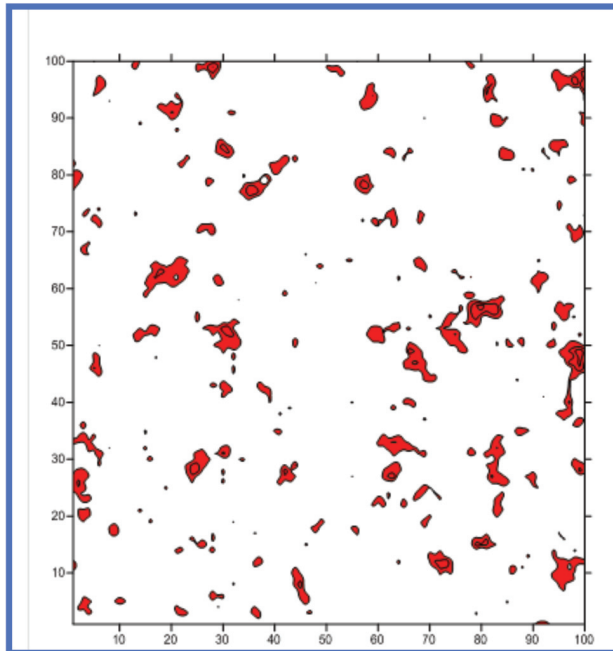


Figure 1—Short-range nugget effect. Stationary gaussian random function

Equation [4] is very important when sampling ore deposits by different types of drilling techniques.

As shown by Figure 2 the support differences between small diameter diamond drill holes and reverse circulation or blast holes can be huge.

Sometimes small diameter diamond drill holes are useless in the assessment of natural variability. Table I shows an experiment done in an Australian gold deposit.

After studying the AQ/QC procedures for sampling, sample preparation, and assaying, the conclusions were:

- Total sampling and sample preparation errors are low
- Assaying error is low.

Then, Table I suggests

- When kriging the same ore zone, blast holes' gold mean grade and variance are bigger than gold mean grade and variance given by the diamond drill hole samples. The variance difference is a contradiction because the sample support of the blast holes samples is much bigger than the support of the diamond drill samples.
- The sampling grid of the blast holes is very dense compared with the diamond drill sampling grid. A possible explanation could be the so-called information effect^{1,2,3,8}. In order to test the hypothesis, a near neighbour comparison between the diamond drill and the blast holes was done to equalize the sampling density.
- The contradiction stays intact. Actually the variance difference is higher than before, and therefore the information effect (see Appendix 3) is not the cause of the difference. The difference in gold mean grade stays as well. Maybe a more realistic hypothesis is the incapacity of the diamond drill samples to detect short-range variability gold structures (nuggets) due to their small support.

- From the theoretical viewpoint the hypothesis is possible. The spatial distribution of gold particles of different size was simulated according to a Poisson process. Figure 3 shows the probability of intersecting no gold particles as a function of the sample support.

The simulation suggests:

- As the sample support increases, the probability of selecting gold nuggets increases.
- As the size of the nuggets increases and the number of nuggets decreases, the probability of missing the gold nuggets increases in a significant way. In this case small sample supports do not have a chance to represent the natural variability properly.
- In this case even the selection of more abundant small particles is difficult when the sample support is small.

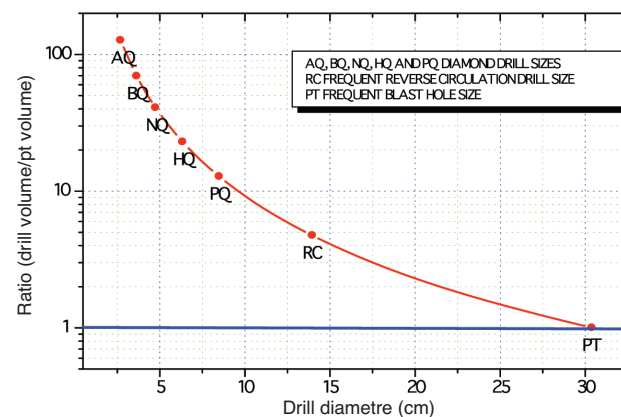


Figure 2—Drill diameter vs. sample support

Table I

Sample support and assessment of gold variability

Method and support	Mean (g/t)	Variance (g/t) ²
Kriging BH	1.27	0.77
Kriging DDH	0.94	0.41
Near neighbour (<2 m) BH	1.37	15.5
Near neighbour (<2 m) DDH	1.04	3.5

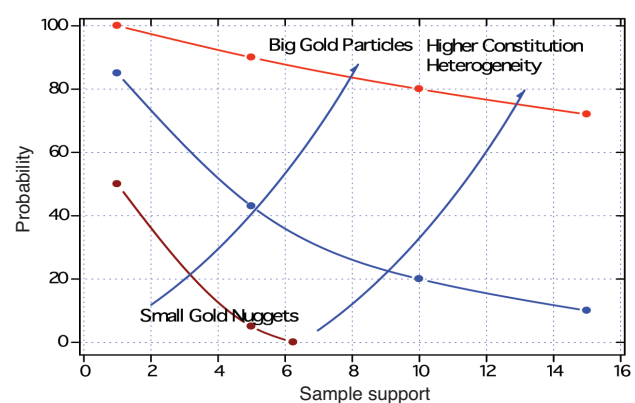


Figure 3—Probability of selecting zero gold particles vs. sample support

Nugget effect, artificial or natural?

Some examples of natural nugget effect

A fundamental step in the understanding of the nature of the nugget effect is a good knowledge of the phenomenon under study. Figure 4 shows a pure natural nugget effect.

In this case QA/QC for sampling, sample preparation, and assaying were excellent, therefore the contribution of sampling plus assaying errors to the nugget effect is not important. The explanation for the natural component of the nugget effect is that the gold rhodochrosite veins have been under high stress. As a consequence they have been broken into small pieces all over the space, losing the original spatial correlation.

Figure 5 shows a natural nugget effect for the molybdenum grade in a Chilean porphyry deposit.

In this case the origin of the nugget effect is the existence of a set of very high molybdenum blue veins. In between the veins the molybdenum grade is low.

Some examples of 'human' nugget effect

When the sampling procedures are divorced from the TOS^{4,5,6,7} and the analytical practices are away from the principles of analytical chemistry, irrelevant variability is added to the process under study. (See Appendix 1.) Figure 6 shows the variograms for two sectors of El Hueso gold mine in Chile.

In this case there are three facts which allow understanding of the origin of the pure nugget effect:

- Coarse gold particles are very scarce in the deposit. The gold is disseminated as in Carlin-type ore deposits; therefore the constitution heterogeneity is low.
- The sampling protocol for the blast holes was very bad. The sample was selected by hand, taking one single 250 gram increment at a top size of 1.3 cm. This incorrect sampling procedure generated fundamental, delimitation, and extraction errors of great magnitude.
- The diamond drill variograms did not show such a big nugget component.

Figure 7 shows the variogram for the silica grade of the slag of a Chilean smelter.

Because the sampling of the hot slag is difficult, it is logical to assign the origin of the pure nugget effect to the incorrect sampling procedures. Small sample, non-constant support, and the presence of some copper nuggets can play a role as well.

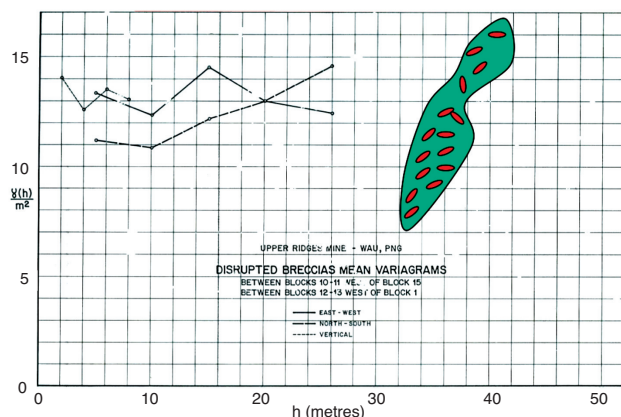


Figure 4—Relative gold variograms. Natural nugget effect. Shattered gold veins. Papua New Guinea

A suggested method to estimate the human component of the nugget effect

The main steps of the method are:

- Duplicate the sampling and assaying procedures. Duplicates must represent the spatial/temporal variability of the phenomenon under study. When on studies nature, the geological controls must be fully

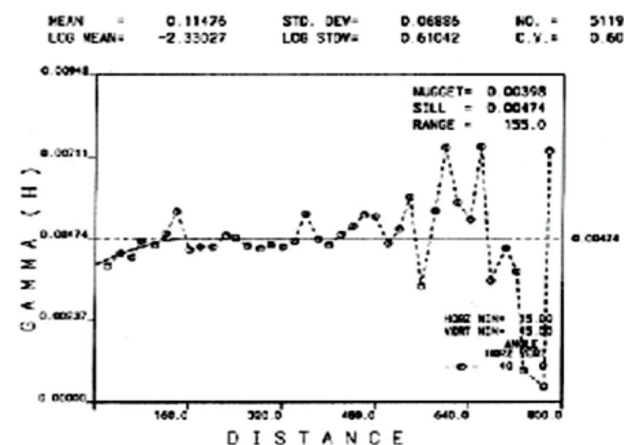


Figure 5—High grade Mo veins. Chuquicamata Mine

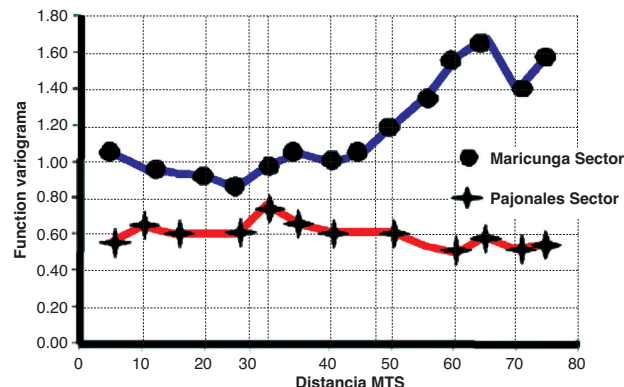


Figure 6—'Human' nugget effect at El Hueso Gold Mine. Blast hole samples

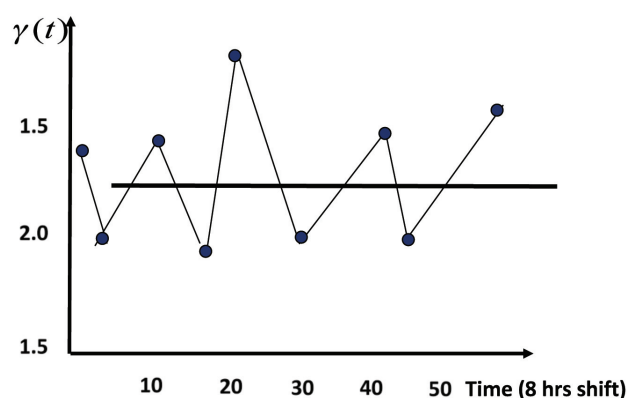


Figure 7—SiO₂ Variogram. Hot smelter slag sampling procedures

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- respected. When on studies metallurgical processes' the nature of the feed and the special characteristics of the process must be taken into account.
- Record the spatial and/or temporal location of the duplicates.
 - Estimate the variogram of the original value.
 - Estimate the variogram of the differences.
 - Estimate the cross variogram between the differences and the original values.
 - Test the hypothesis of spatial/temporal independence of the differences and the independence between the differences and the original values.
 - Estimate the contribution of sampling and assaying errors to the nugget component by the ratio of the sill of the differences variogram to the sill of the original values variogram.

Figure 8 shows an example based on blast hole samples at the Andina Chilean porphyry mine.

- Figure 8 suggests:
- The differences between the original grade and the duplicate grade are spatially independent.
 - The copper grade is spatially independent of the duplicate grade.
 - The 'human' component of the nugget effect is small. It is 22% of the total nugget effect and just 6% of the total variability. Then the geological nature of the deposit accounts for most of the nugget component.
 - As a conclusion, the sampling, sample preparation, sampling for analytical purposes, and chemical analysis are very precise.

Sometimes the results are not so encouraging, particularly when sampling trace elements in cathodes. Table II shows some examples.

Economic consequences of the misunderstanding of the nugget effect origin

From the estimation viewpoint it is essential to understand the nature of the nugget effect. This will enable a correct decision making process, otherwise the decisions could be very misleading and unfortunate.

Figure 9 shows the evolution of the kriging weights as a function of the nugget effect magnitude.

Figure 9 suggests:

- As the nugget effect increases, all the samples assume the same importance, and the arithmetic mean is the optimal estimator.
- Under these circumstances, selective mining is an exercise in futility. All the blocks belonging to the same high nugget stationary geological unit will have a grade very similar to the mean.
- From the economic viewpoint, the only intelligent action will be to clearly define the boundaries of the

Table II
Proportion of 'human' nugget effect

Description	'Human' nugget	Observations
S cathodes	60	Incorrect test portion
O cathodes	100	Incorrect test portion
Cl cathodes	58	Incorrect test portion
Pb cathodes	40	Incorrect test portion
Ag cathodes	20	Incorrect test portion
Cut DDH	40	Incorrect half core selection Incorrect chemical analysis practices

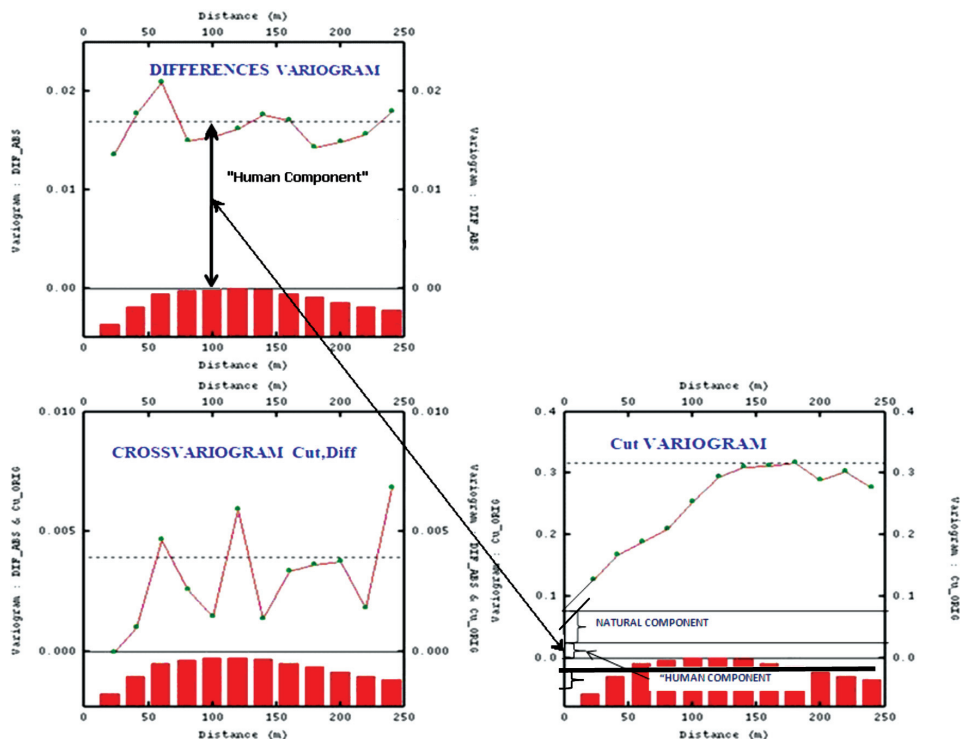


Figure 8—Direct and cross-variograms of cut and duplicates differences

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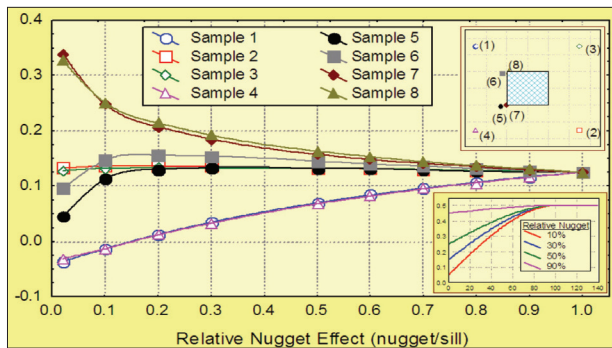


Figure 9—Kriging weights evolution vs. nugget effect

unit and to estimate in a proper way the mean grade, in order to study if bulk mining pays. To design a selective mining method will result in very bad surprises.

If the nugget effect is artificial, then there are many opportunities for improvement. Figures 10 show how the misclassification of ore in a selective mining operation can be minimized by improving the sampling procedures.

In this case the sampling procedures are incorrect, creating irrelevant variability, which inflate the true nugget effect. As a consequence, the kriging estimates deteriorates. As seen in the figure the precision of the estimation is very low as the conditional variance is very high; besides, the estimator is conditionally biased. The misclassification areas are very large. To ignore the existence of the 'human' nugget effect and the bad quality of the estimator will result in significant economic losses. The estimation of the economic net present value of the misclassification was US\$156 million. For further details see reference 9.

Figure 11 shows the consequences of the improvements of the sampling procedures. The results are very good. Now the estimation is by and large more precise and the conditional bias practically disappears; the misclassification areas are also reduced to an acceptable minimum. The estimation of the economic net present value of the misclassification is reduced to only US\$20 million.

Figure 12 illustrates the relevance of the sample support, estimation method, and estimation strategies in a process control case study¹⁰. It is the estimation of the tails grade of a large seam in a Chilean porphyry mine. The sampling procedure is to select samples of small support. The sampling frequency is one sample per shift. The estimation method is polygonal. The nugget component and very short-range variability accounts for 65% of the variability.

Figure 12 suggests:

- The polygonal method, commonly used in process control, is conditionally biased. In other words, it will tend to overestimate the high grades and underestimate the low grades. As a consequence, the process control decisions can be very misleading.
- To increase the sample support and the sampling frequency is very effective and in this particular case very inexpensive. In other words, it would be advisable to install a proper sampling device and to define a sampling frequency as a function of the process control decisions.

Conclusions

- To know the causes of the nugget effect is essential to understand the nature of the phenomenon under study, to optimize the estimation process and, as a consequence, to minimize hidden losses, improving the economic benefit.
- Often, the nugget effect is made up of two components. One represents relevant variability and the second represents irrelevant variability.

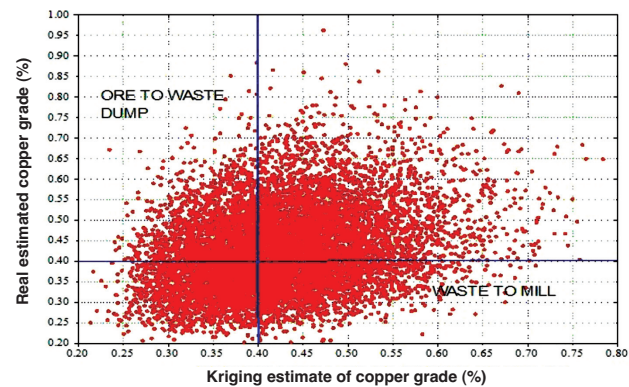


Figure 10—Misclassification of ore/waste. High 'human' nugget effect

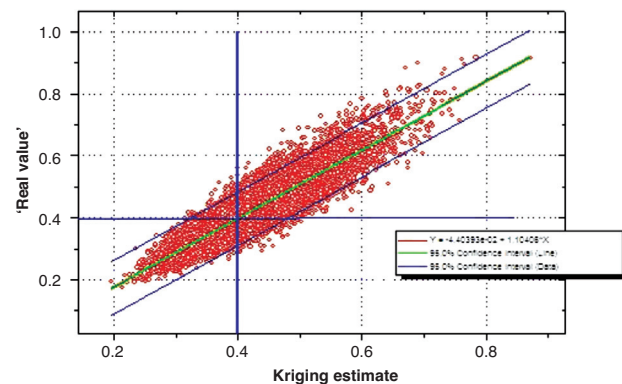


Figure 11—Sample support, estimation method, and sampling strategies

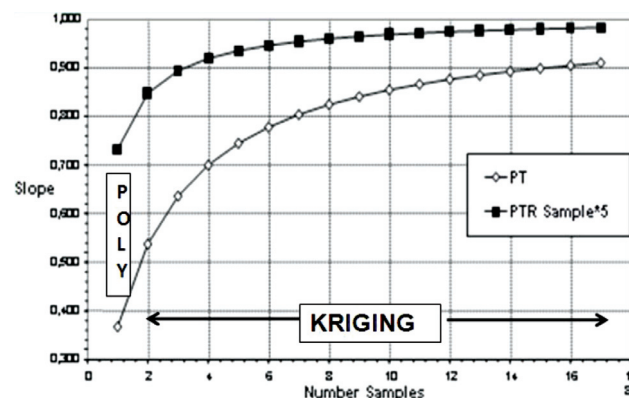


Figure 12—Sample support, estimation method, and sampling strategies

Nugget effect, artificial or natural?

- A relevant nugget effect represents short-range structures which actually belong to the process under study. The irrelevant nugget effect does not belong to the process; it is induced by incorrect sampling, sample preparation, and chemical analysis.
- To optimize the process under study it is essential to know the nature of the phenomenon, the AQ/QC procedures for sampling, sample preparation and chemical analysis, and an estimation of the magnitude of the relevant and irrelevant components.
- The estimation of both components is possible if the errors are spatially independent and, spatially independent of the variable under study, and a duplication of the sampling and chemical analysis procedures is available.
- The magnitude of the nugget effect is very dependent on sample support, sampling density, sampling quality, assaying procedures, and the nature of the phenomenon under study.
- Polygonal estimators, often used in process controls in plants, are generally conditionally biased and imprecise. Estimators taking into account the varying structure of the variables, the geometry of the sampling pattern, the geometry of the estimated support, and the relation between both, is more advisable.

Acknowledgements

The author wishes to thank Mr Fernando Vivanco, Corporate Projects Vice President and Mr. Julio Beniscelli, Manager of Technical Services, Codelco Chile, for their strong support in the implementation of good sampling, assaying and geostatistical practices along the copper business value chain.

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Appendix 1

Variogram function and sampling/assaying errors

$$Z^* = Z + e$$

$$2\gamma_{Z^*} = E[(Z(x+h) + e(x+h) - Z(x) - e(x))^2]$$

$$2\gamma_{Z^*} = E[(Z(x+h) + e(x+h))^2 + (Z(x) + e(x))^2 - 2(Z(x+h) + e(x+h))(Z(x) + e(x))]$$

$$2\gamma_{Z^*} = E[Z^2(x+h) + e^2(x+h) + 2Z(x+h)e(x+h) + Z^2(x) + e^2(x) + 2Z(x)e(x) - 2(Z(x+h)Z(x) + Z(x+h)e(x) + Z(x)e(x) + e(x+h)e(x))]$$

$$2\gamma_{Z^*} = E[(Z(x+h) - Z(x))^2 + (e(x+h) - e(x))^2 + 2(Z(x+h) - Z(x))(e(x+h) - e(x))]$$

$$\gamma_{Z^*}(h) = \gamma_Z(h) + \gamma_e(h) + 2\gamma_{Z,e}(h)$$

If the errors have no temporal or spatial correlation.

If the grade and the errors are not temporal or spatially correlated:

$$\gamma_{Z^*}(h) = \gamma_Z(h) + cte_1 + 0$$

$$\gamma_{Z^*}(h) = \gamma_Z(h) + C_{os}$$

Therefore, under the previous assumptions, sampling + assay errors add like a nugget effect to the true variogram. 'Human' nugget effect.

Note that a sampling bias also contributes to the nugget effect, because there is no such thing as a constant sampling bias.

Legend:

Z^* = Estimated value

h = Distance/time

Z = True value

γ = Semivariogram

$E()$ = Expected value

e = Random error

$\gamma_{Z^*,e}$ = Cross variogram between Z^* and e .

Appendix 2

Sample support and Nugget Effect

$$V > v$$

$$C_{0v} = \frac{A}{v}$$

$$C_{0v} = \frac{A}{V}$$

$$C_{0v} = C_{0v} \frac{v}{V}$$

Nugget effect, artificial or natural?

V, v = Sample supports
 C_{0v} = Nugget effect at sample support v
 C_{0V} = Nugget effect at sample support V
 C_0 = Very small range covariance
 $\bar{C}_0(v, v)$ = Mean covariance between v and v

$$\bar{C}_0(v, v) = \frac{1}{vV} \int_v dx \int_v C_0(x - y) dy$$

As the distance $(x-y)$ is greater than the range:

$$\int_v C_0(x - y) dy = A$$

$$\bar{C}_0(v, v) = C_{0v} = \frac{A}{vV} \int_v dx = \frac{A}{vV} V = \frac{A}{v}$$

Appendix 3

Information effect

The perception of natural variability changes as the drilling density increases. (Graphs 1 and 2).

Notice the increase in variability, the increase in the oxides proportions, and the increase in the complexity of the contacts between both ore facies.

Appendix 4

Variance of the sampling error from duplicate sampling and assaying

$$\begin{aligned}
 Z_o &= Z + e_o \\
 Z_d &= Z + e_d \\
 \text{Var}(Z_o - Z_d) &= \text{Var}((Z + e_o) - (Z + e_d)) \\
 \text{Var}(Z_o - Z_d) &= \text{Var}(e_o - e_d) = \text{Var}(e_o) \\
 &\quad + \text{Var}(e_d) - 2\text{Cov}(e_o, e_d)
 \end{aligned}$$

o = Original value
 d = Duplicate value

As the sampling and assaying procedures are the same, the stochastic nature of the error is the same:

$$\text{Var}(e_o) = \text{Var}(e_d) = \text{Var}(e)$$

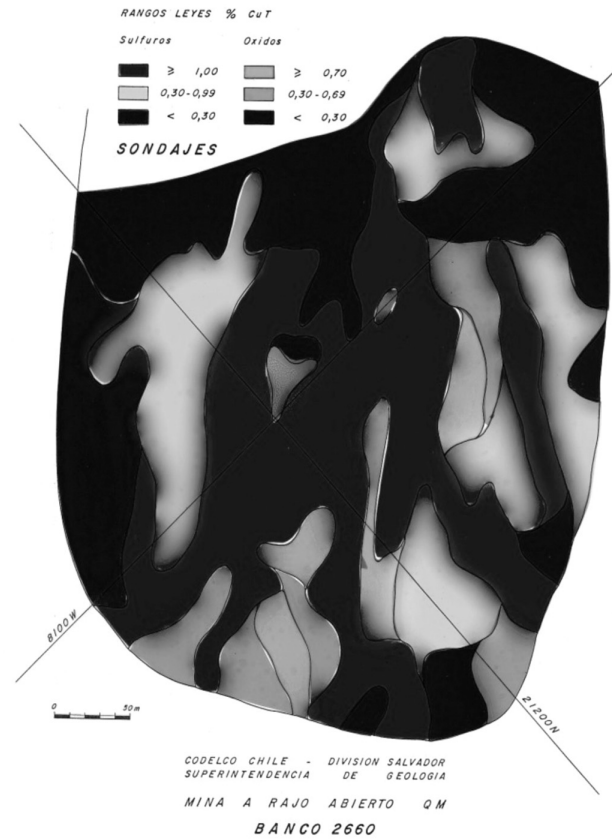
As the duplication process is blind and done in a different time:

$$\text{Cov}(e_o, e_d) = 0$$

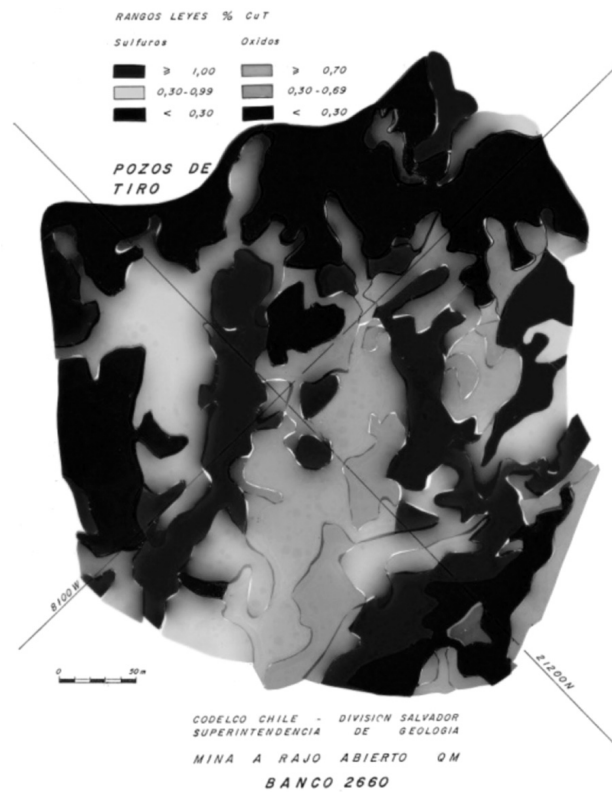
Then:

$$\text{Var}(e) = \frac{\text{Var}(Z_o - Z_d)}{2}$$

Therefore the error plus the assay error variance is the semi variance of the difference between the original value and the duplicate value. In other words, it is the sill of the variogram function of the differences. ♦



Graph 1—Grade spatial variability. DDH Holes at a 100*100 m drilling grid



Graph 2—Cooper grade spatial variability. BH Holes at a 10*5 drilling grid