Breakage mechanisms and an encouraging correlation between the Bond parameters and the friability value

by H.T. Ozkahraman*

Synopsis

It is important to know the breakage mechanism in materials since this knowledge influences the results of subsequent grinding operations. There are two distinct failure mechanisms in breakage: one is tensile micro crack generation at low stresses, which leads to macroscopic failure by disintegration, and the other is formation of shear zones under heavier dynamic impact forces, which generates more fines as seen in crush zones in blasting. Tensile fracturing simply breaks the material into fragments. It is seen as the disintegration of the specimen into two or more separate fragments. This happens under the absence of lateral stresses and the material is free to expand. On the other hand, compressive-shear breakage produces finer fragments due to shear stresses. The first mechanism is observed in laboratory tensile and bending strength tests and the second mechanism is observed both in laboratory brittleness tests and *in situ* blasting operations under dynamic impact forces.

The friability of rocks and ores can be determined by a brittleness test. A test apparatus to determine the friability value has been designed to suit limestone strength characteristics used in cement production. The friability and stored strain energy values of barite, marble, limestone and bauxite have been determined and compared with the corresponding Bond work index (W_i) and grindability index (G) of these materials. The physico-mechanical properties of the tested materials have also been determined to investigate their effect on friability and grinding. The relationships obtained between the indices were in surprisingly good agreement, with high correlations (0.99 and 0.97). The Bond work index and grindability index can therefore be estimated from the friability value, which can be determined more rapidly than the Bond test. But for certain rock types such as andesites the relationships does not hold.

Key words: comminution, tensile stress, shear forces, brittleness test, strain energy.

Introduction

Although brittleness is defined as the lack of ductility, it is a material condition characterized by its reduced ability to carry load as the strain increases. In a broad sense brittleness encompasses the whole failure process (Hajiabdolmajid *et al.*[5]). Natural heterogeneities and micro cracks promote non-homogeneous distributions of higher stresses than tensile strength, causing local failures and progressively form the failure plane. High stresses are generated due to stress concentrations arising around pores and flaws.

Testing of rock has provided insights into how fractures are developed in relation to the orientation of principal stresses. Extension fractures will develop at a right angle to the minimum principal compressive stress direction σ_3, and will contain the orientation of the maximum principal compressive stress σ_1 as shown in Figure 1, (Herget[6]).

A rock material contains a large number of randomly orientated zones of potential failure in the form of grain boundaries. These boundaries contain a number of open flaws. Griffith[4] postulated that these flaws are approximately elliptical in shape. It can be shown that very high tensile stresses occur on the boundary of a suitably orientated elliptical opening, even under compressive stress conditions, and it is assumed that fracture initiates from the boundary of an open flaw when the tensile stress on this boundary exceeds the local tensile strength of the material. Hoek and Bieniawski[7] studied this fracture initiation and propagation in rock under compression, where the loading conditions were quasi-static.

On the contrary, under high dynamic impact forces, materials undergo various stages. Each stage has a different stress field acting upon it. Also at each stage more micro cracks and flaws are formed. These are in different shape, length and location, some inside grains, some along the grain boundaries, and some extend along several grains. Also, existing cracks propagate by dislocations along slip planes.

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Campbell *et al.*[2], used high speed image and stress analysis for the evaluation of tensile based breakage tests and confirmed that the failure mechanisms in three point bend and ring-loaded strength tests on rock samples were inherently tensile in nature, as crack propagation was observed to originate from areas of induced tension.

Nielsen and Malvik[10] showed that grindability is enhanced by blast-induced micro cracks. The reductions of the crushing and grinding resistance is caused by a large number of micro cracks which are generated by the shock waves emitted throughout the rock mass by the detonating explosive in the drill hole. Michaux and Djordjevic[9], demonstrated that a relatively large increase in applied explosive energy would produce a weaker ore, which in turn would result in a higher throughput through the grinding circuit. Blasting is probably the most economic method of fragmenting rock.

Rock fails when stresses exceed strength. Failure in rock occurs by development of fracture or slip surfaces. Previous studies show that the type of fractures and their orientation depend on the distribution of stresses across a specimen, the type of material, and whether stresses are applied in tension or compression. Therefore the breakage mechanism in materials must be well understood to find an explanation as to what is needed in subsequent comminution operations.

Breakage mechanism

Like any other material, rock fails when stresses exceed strength. Rock materials are composed of crystals which contain many flaws and defects called micro fractures. They are found in mineral grains or on grain boundaries. There are two breakage mechanisms: tensile fracturing and compressive-shear failure depending on the level of applied stresses. Tensile fracturing occurs under low stresses. This is logical as tensile strength of rocks is 10 to 12 times lower than compressive strength. Nielsen and Malvik[10], experimentally confirmed the Griffith theory. Failure in rock occurs by development of micro cracks which weaken the material. Hajiebdolmajid *et al.*[5] examined brittle fracturing, and found that rock type, grain size, presence of voids (pre-existing cracks and pores), and the presence of flaky, soft and altered minerals influence the rock brittleness.

In order to understand breakage mechanism under external forces one must look into triaxial loading conditions at a confining pressure (stress). Fracture development during failure of a rock sample under triaxial compressive stresses is shown in Figure 1 (Herget[6]). The triaxial testing is designed to determine an upper limit to how much shear stress a solid material can support. The intact rock material is contained in a cylindrical latex sleeve with a flat, circular platens closing off the top and bottom ends. This cylinder is placed into a bath of oil to provide pressure along the sides of the cylinder. These horizontal pressures (stresses) applied in the horizontal directions (along the sides of the cylinder) are called minor principal stresses ($\sigma_2 = \sigma_3$), Hoek and Bieniawski[7]. The top platen can then be mechanically driven up or down along the axis of the cylinder to squeeze the material with σ_1 (major principal stress). The stress on the platens is increased until the material in the cylinder fails and forms sliding regions within itself, known as shear bands. There is no movement along the failure planes until the axial load reaches a certain value, which is dependent upon the material properties. From the triaxial test data, it is possible to extract fundamental material parameters about the rock sample, including its angle of internal friction, apparent cohesion, and dilatancy angle. These parameters are then used in computer models to predict how the material will behave in a larger-scale engineering applications. Under triaxial loading conditions at a confining pressure applied laterally, shear failure planes are formed and sliding takes place on these planes. Material strength increases as lateral stress level σ_3 increases. Therefore shear stress causing sliding failure must overcome the increasing strength of the material. Thus displacement on a shear plane under high normal stresses could cause crushing of grains and asperities. Shear (τ) stresses calculated from principal stresses acting (σ_1, $\sigma_2 = \sigma_3$) at failure and plotted against increasing σ_3 from triaxial test data. A linear failure envelope is called the Mohr failure envelope, which is drawn tangent to Mohr circles, is given by Equation [1] and (Herget[6]):

$$\tau = \sigma_n \cdot Tan\phi + c \qquad [1]$$

According to elastic theory the stresses at failure are (Mohr's circle):

$$\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{1}{2}(\sigma_1 + \sigma_3)Cos2\theta \qquad [2]$$

Mohr circle's formula Equation [4] is derived from Equations [2] and [3]

$$\tau = \frac{1}{2}(\sigma_1 - \sigma_3)\mathrm{Sin}2\theta \qquad [3]$$

$$\left[\sigma_n - \left(\frac{\sigma_1 + \sigma_3}{2}\right)\right]^2 + \tau = \left(\frac{\sigma_1 + \sigma_3}{2}\right)^2 \qquad [4]$$

Figure 2 shows that at failure, shear and normal stresses (σ_n) acting along failure plane. It can be seen that shear stresses (τ) are very sensitive to the level of lateral stress σ_3,

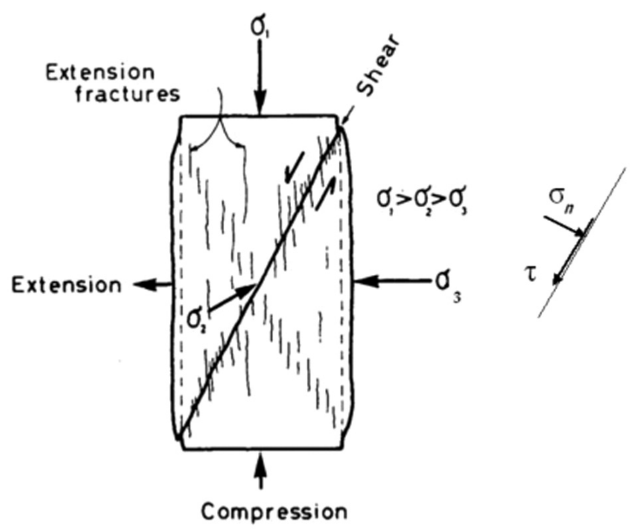

Figure 1—Extensions fractures and shear failure of a rock sample under triaxial loading

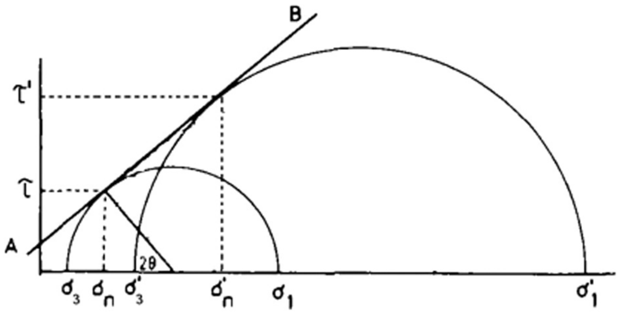

Figure 2—Mohr failure envelope which is drawn tangentially to Mohr circles. Increase in lateral stress σ_3, increases shear stresses causing failure along shear plane at a higher rate than σ_3

and increase at a higher rate than σ_3. Therefore the compressive–shear failure mechanism requires high applied stress levels. Most hard rocks like limestone exhibit brittle behaviour and under high stress levels produce finer fragmentation along shear bands. The higher the magnitude of stresses, the finer the fragmentation becomes, and grinding occurs with increasing resistance to movement under frictional forces along these slip lines inside the materials. The easiest way of generating high stresses is by dynamic impact forces. Disintegration of the material into fragments under low or no confinement (uniaxial stress field) is in contrast to this shear movement under applied triaxial compressive stresses.

Grindability enhancement in blasting

Crushed and fractured zones around a blast hole are shown in Figure 3. The extension length of the crushed zone size is two diameters in length and the similar zones, which are produced by two simultaneously fired adjacent blast holes, are shown in Figure 4. This phenomenon has been observed in *in situ* blasting by the author. Simultaneously blasting two adjacent holes produces enhanced crushing in the middle zone between the holes due to shock waves travelling at very high velocities and reinforcing each other. Fractures due to blasting are produced by two mechanisms.

One mechanism is related to the compressive–shear failure of the rock (mainly of the rock matrix) close to the blast holes and between the blast holes where shock waves collide with each other. The second mechanism is the tensile failure of the rock mass. Fines in a blast are generated predominantly by the crushing of rock around the blast hole due to compressive–shear failure and at the mid distance between blast holes by the impacting shock waves (blast holes must be fired simultaneously and crushing dynamic forces is maximum at the mid point between blast holes). The coarse fragments are generated predominantly by tensile failure beyond the crushing zone (Figures 3 and 4).

Materials tested

Goltas limestone, Ilmen barite, Seydisehir bauxite, Mugla marble and Isparta andesite were tested in the study. Their physico-mechanical properties were first determined. These properties were unit weight, uniaxial compressive strength, tensile strength, and point load index. The stored strain energy of the marble and the limestone at failure was calculated (see Appendix 1). The physico-mechanical properties, grindability indices, work indices, friability values, and strain energies at failure were compared. Test results are given in Table I.

Standard Bond tests

The standard Bond grindability test is a closed-cycle dry grinding and screening process, which is carried out until steady state conditions are obtained (Bond[1]; Yap *et al.*[13] and Magdalinovic[8]).

A feed sample of 700 cm^3 of material is used in the Bond grinding tests. At the end of each grinding cycle, the entire product is discharged from the mill and is screened on a test sieve (P_i). The oversize fraction is returned to the mill for the second run together with fresh feed to make up the original weight corresponding to 700 cm^3. The weight of product per unit of mill revolution, called the ore grindability of the cycle,

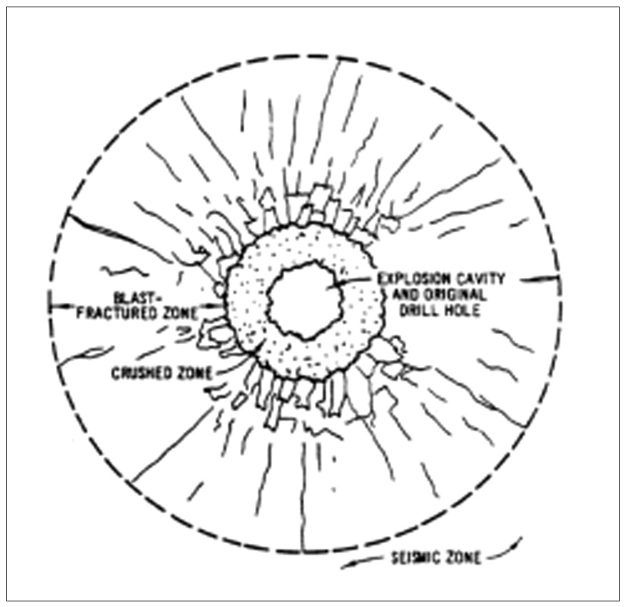

Figure 3—Fines in a blast are generated predominantly by the crushing of rock around the blast hole due to compressive–shear failure close to the blast holes, fractured zone created by the tensile failure of the rock mass (Engineer Manual No.1110-2-38003)

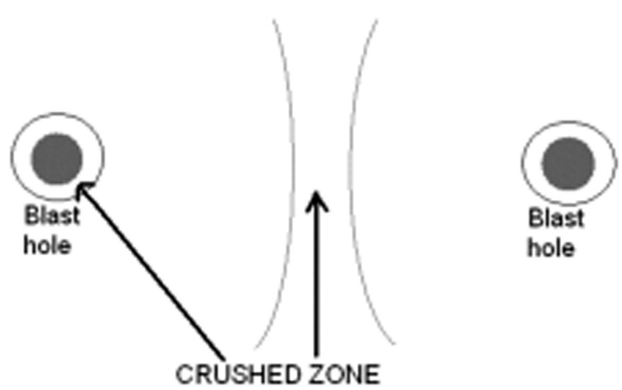

Figure 4—Compressive–shear failure zone of the rock is around the blastholes and the mid distance between the blast holes, where shock waves collide with each other

Table I

Physico-mechanical properties and friability values (S_7), Bond work indices (W_i) and grindability indices (G) of tested materials

	Barite $BaSO_4$	Marble $CaCO_3$	Limestone CaO,MgO,SiO_2,Fe_2O_3	Bauxite (Al,Si, Fe, Ti, Ca, Mg)O
Unit weight (g/cm3) ± 5%	4.40	2.65	2.67	2.98
Uniaxial compressive strength (MPa) ± 7%	56.88	60.2	54.95	90.56
Tensile strength (MPa) ± 7%*	3.25	4.90	4.62	5.18
Point load index (N/mm2) ± 9%	2.60	3.92	2.82	4.14
G (g/rev) ± 10%	6.19	2.98	1.11	0.95
W_i(kWh/sh ton) ± 12%	5.30	10.69	22.67	25.66
Brittleness index, S_7, ± 6%	288.9	124.3	54.0	34.5
Elastic modulus (static, GPa), ± 20%	-	70	37	-
Stored strain energy (kJ/m3)	-	25.9	40.8	-

*Standard deviation on average

is then calculated and is used to estimate the number of revolutions required for the second run, equivalent to a circulating load of 250%. The process is continued until a constant value of the grindability is achieved, which is the equilibrium condition. The average value of the last three cycles is taken as the standard Bond grindability (G), which is the net gram of undersize, produced per mill revolution. Work index, W_i is given by the following empirical Equation [5]:

$$W_i = \frac{44.5}{p^{0.23} \cdot G^{0.82} \left(\frac{10}{\sqrt{P_{80}}} - \frac{10}{\sqrt{F_{80}}} \right)} \quad [5]$$

Brittleness test for the determination of the friability value

The friability value S_7 was measured by the brittleness test (Figure 5), which gives a value for rock resistance to crushing due to repeated weight-drop (7) impacts. The brittleness test apparatus is designed to determine grindability characteristics of Tauros limestones, which are used in cement production (Ozkahraman and Sirin[11]). The dimensions of the mortar, the amount of drop weight, and the height are all chosen in order to suit the compressive and tensile strength of the limestone tested and from the energy of the impact load.

The energy for each impact load in a brittleness test is:

$$E = m \cdot g \cdot h$$
$$E = 14\text{kg} \cdot 9.81 \frac{\text{m}}{\text{s}^2} \cdot 0.25\text{m} = 34.3\text{joule} \quad [6]$$

Dynamic stresses generated by impact loads are given by Popov[12]. Stresses generated by this energy level in the brittleness test are:

$$\sigma_{\max} = \sigma \left(1 + \sqrt{1 + \frac{2hAE}{WL}} \right) \quad [7]$$

$$X = 1 + \sqrt{1 + \frac{2hAE}{WL}} \quad [8]$$

The derivations of these Equations [7] and [8] are given in Appendix 2. The tested rock sample of a 500 g aggregate in the fraction -16.0+11.2 mm was inserted in to the mortar. The friability value S_7 equals the percentage of undersized material, which passes through the 11.2 mm mesh after the aggregate test crushing in the mortar by seven weight-drops, the mean value for a minimum of three to four parallel tests being chosen as the rock sample S_7 value. A weight of 14 kg was dropped from a height of 25 cm, 7 times. S_7 is the percentage of undersized material, which passes through the 11.2 mm mesh. The results of brittleness test are given in Table I. In the brittleness test the number of weight drops is dependent on the strength of the materials tested, more weight drops being required for harder rocks. A 14 kg weight must be dropped 7 times from a height of 25 cm in order to produce a product of 55% of total feed of limestone. Therefore a friability value S_7 is equal to 55% for limestone that means after 7 weight drops 55% of feed passed a -11.2 mm sieve. The apparatus is designed for limestone therefore limestone and bauxite produced less than 100% of feed after 7 drops. But for weak ores like marble one weight drop produced 17.76 % therefore 7 drops produces 7 x 17.76 = 124.3 %. And in the case of barite, 7 x 41.27= 288.9%. Therefore for weak ores if too much breaks with one drop, one can then extrapolate to get over 100% breakage.

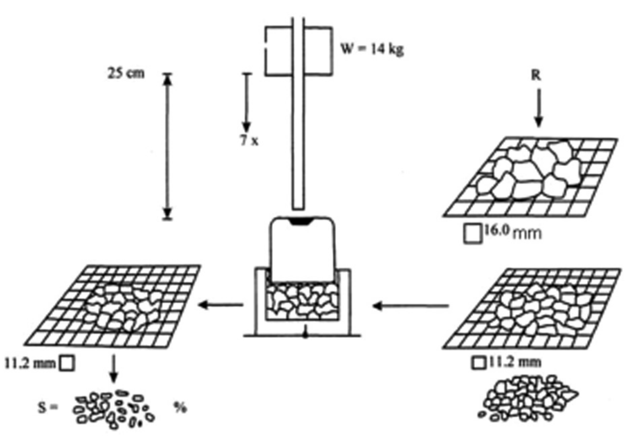

Figure 5—Brittleness test set-up

Results and discussion

The results are given in Table I. The stored strain energies of marble and limestone are calculated from their corresponding static elastic modulus. Elastic modulus of barite and bauxite could not be determined due to non-availability of their intact samples. The strongest material was bauxite ore. The compressive, tensile and point load strength of bauxite were the highest, as was the work index, indicating that the higher the strength the higher the work index. This means that more energy is needed to grind it. Highest strength gives the lowest friability value and grindability index. Compressive, tensile and point load strengths of marble and limestone were similar, marble being pure calcium carbonate compared to limestone which contains SiO_2, MgO, and Fe_2O_3. The high work index of limestone could be due to its strong interlocking matrix and micro-crystalline structure, whereas marble has larger crystals. By the same reasoning, the stored strain energy (at failure) of limestone is 1.58 times higher than marble (40.9/25.9 = 1.58). Similarly, the work index of limestone is 2.12 times higher than marble, (22.67/10.69 = 2.12).

The friability value (S_7), Bond work index (W_i) and grindability Index (G) are compared with each other and the relationships are shown in Figure 6.

A linear relationship between grindability index, G, and friability value was found as given in Equation [9]. The correlation coefficient is very high, at 0.99.

$$G = 0.171 + 0.021(S_7) \qquad [9]$$

The relationship between friability value (S_7) and Bond Work Index (W_i) also has a high correlation of 0.97 and this relationship is given in Equation [10]. The lower correlation coefficient than G might be due to 'the calculation of W_i from G introduces P80 and F80, therefore the correlation has more scatter'.

$$W_i = 61.839 - 10.158\mathrm{Ln}(S_7) \qquad [10]$$

In Table II, Bond parameters G, W_i and the Brittleness index S_7 of Traki andesite of Isparta is given. When grindability value G = 1.39 g/rev is inserted in Equation [9], the brittleness value S_7 = 58% is obtained. But this predicted value is 3.5 times the actual value obtained from friability test, which is 16.5%. Similarly the work index predicted from Equation [10] is 33.36, instead of the actual value obtained from Bond which is 12. These discrepancies might be the result of a high porosity value of 5.2 for Traki andesite. High stress concentration factors were produced around pores and the breakage mechanism is influenced from these high induced stresses around pores. Therefore the relationships do not hold for volcanic rocks such as andesite with a high porosity value.

Table II

Bond parameters G, W_i and the brittleness index S_7 of Traki andesite of Isparta

	Traki andesite volcanic rock, SiO_2,Al_2O_3, (K,Na,Ca)O
Unit weight (g/cm³) ± 1%	2.35
Uniaxial compressive strength (MPa) ± 2%	40
Tensile strength (MPa) ± 3%*	2.8
Point load index(N/mm²) ± 5%	2.5
G (g/rev) ± 1%	1.39
W_i (kWh/sh ton) ± 2%	12
Brittleness index, S_7, ± 2%	16.5
Real Pporosity, ± 2%	5.2

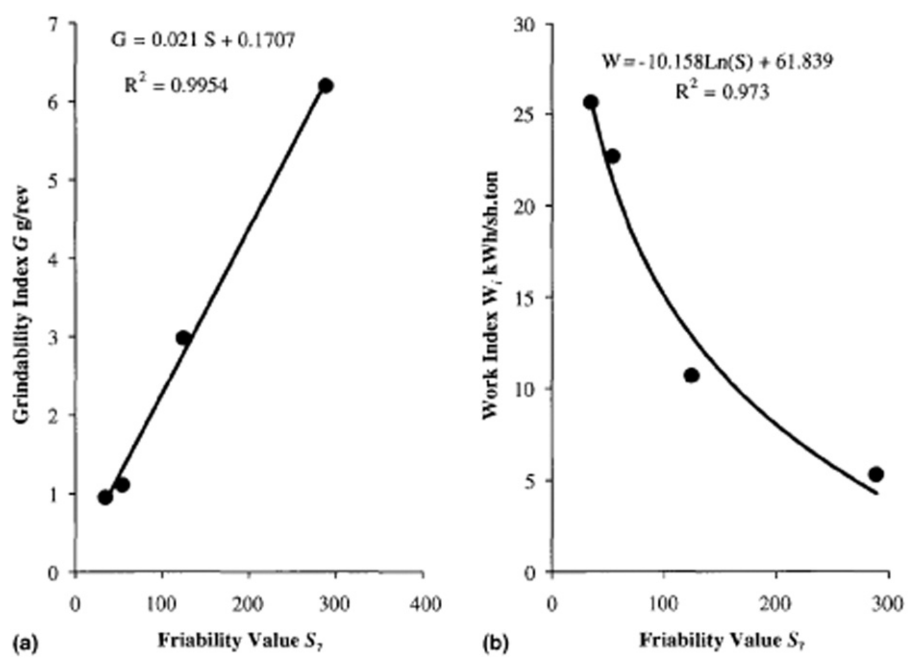

Figure 6—The relationship between (a) grindability index G, (b) work index W_i and friability value S_7

Conclusion

The literature survey showed that crack propagation is originated from areas of induced tension. Also tensile fracturing simply breaks and disintegrates the material into fragments. This happens under the absence of lateral stresses and the material is free to expand. On the other hand, compressive-shear breakage produces finer fragments due to higher shear stresses which are produced under impact forces. Studying crushed and fractured zones generated around blast holes showed that fines in a blast are generated predominantly by the crushing of rock around the blast hole due to compressive–shear failure and between the blast holes, where shock waves collide with each other. On the other hand, fractured zones are created by the tensile failure of the rock mass. Blasting practices in mines showed that underground blasting produces finer fragments than surface blasting. This is due to the higher stress levels due to confinement. Therefore it can be postulated that above a certain limit of surrounding compressive stress level (triaxial), fragmentation gets finer in breakage. The author conducted blasting trials and found that the damaged rocks as a result of blasting contained micro cracks which enhance the subsequent grinding. Blasts are usually designed to fracture the *in situ* rock mass and prepare it for transportation. Blasts in mines should also be designed to produce a well fragmented rock to optimize crushing and grinding performance.

The work index of limestone was higher than marble, due to its strong interlocking matrix and micro-crystalline structure, compared to marble with larger crystals. For similar reasons 'strain energy' (at failure) was higher for limestone than marble and is shown in Appendix 1.

Dynamic impact loads produce much higher stresses than static stresses due to the dynamic load factor, as demonstrated by calculation in Appendix 2. Very high correlations, of 0.99 and 0.97, were found between the friability value and grindability and brittleness indices. The grinding characteristics of ores can therefore be studied by the brittleness test, designed to suit material properties and strength.

The Bond method of grindability determination is time consuming and a constant value of ore grindability is achieved only after several grinding periods. In contrast, the brittleness test is rapid and the friability value is obtained in a much shorter time.

The tensile strength of ores was found to correlate better with work index than with compressive strength and point load index. Many factors, including porosity, influence the grindability of ores and it is difficult to find analytical relationships between them, as in the case of andesite. The relationships obtained do not hold for volcanic rocks such as andesite with high porosity value.

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Appendix 1

The importance of material strength at breakage

If the normal stress σ, reaches the failure strength, σ_c, of a rock, then the stored strain energy at failure is per unit volume is given by the equation below (Herget[6]):

$$W = \frac{1}{2E}\sigma_c^2$$

Therefore stored strain energy in failure is proportional to the square of strength, σ_c.

For example, the uniaxial compressive strength of limestone is σ_c = 55 MPa and its elastic modulus is E = 37 GPa then,

$$W = \frac{\left(55\ \mathrm{Mn/m^2}\right)^2 \times 1\mathrm{m}}{2 \times 37000\ \mathrm{Mn/m^2} \times 1\mathrm{m}} =$$

$$\frac{40.9\,\mathrm{kN\cdot m}}{\mathrm{m^3}} = 40.9\,\mathrm{kJ}(\text{kilo joule})$$

Marble:

$\sigma_c = 60.2$ MPa, and E = 70 GPa,

$$W = \frac{(60.2\,\mathrm{MPa})^2 \times \mathrm{m}}{2 \times 70000\,\mathrm{MPa} \times \mathrm{m}} = \frac{25.9\,\mathrm{kN\cdot m}}{\mathrm{m^3}} = 25.9\mathrm{kj}$$

The stored energy of limestone at failure is 1.58 times higher than marble (40.9/25.9 = 1.58).

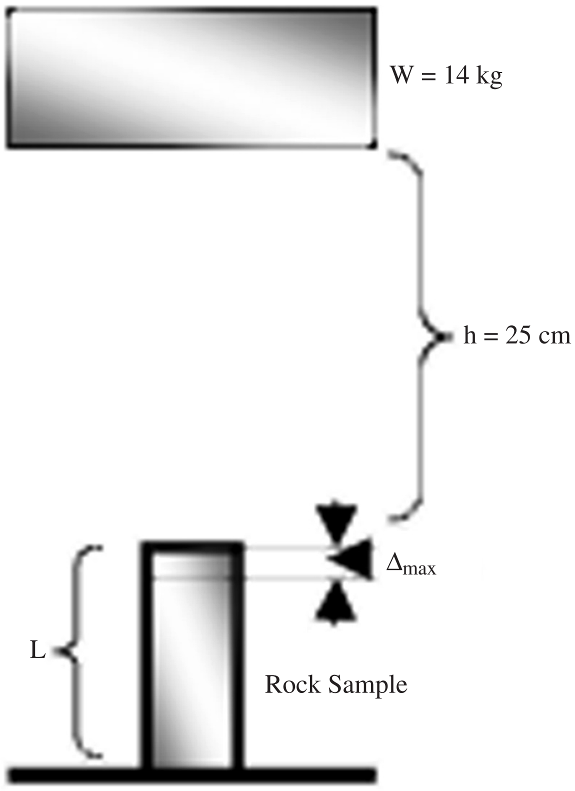

Figure 7—Elastic behaviour under an impact force

Appendix 2

Dynamic stresses generated by impact loads in brittleness test

Supposing a weight W, dropped vertically from a height h, on to a rock sample as shown in Figure7. $_{max}$ is the maximum strain created in the rock sample of length L. And the corresponding stress in the rock is σ_{max}.

Let P be the equivalent static or gradually applied load which would produce the same extension $_{max}$. Then the strain energy in the rock is, $\frac{P{\cdot}\Delta_{max}}{2}$

Also, loss of potential energy by falling weight = Gain of strain energy of rock

$$W\left(h+\Delta_{max}\right)=\frac{P\cdot\Delta_{max}}{2} \qquad [1]$$

Applying Hook's law, $_{max}$ $=PL/AE$, a quadratic in P is obtained,

$$W\left(h+\frac{PL}{AE}\right)=\frac{1}{2}\left(\frac{P^2L}{AE}\right) \qquad [2]$$

Rearranging, and multiplying through by AE/L,

$$\frac{P^2}{2}-WP-\frac{WhAE}{L}=0$$

Solving,

$$P=W+\sqrt{W^2}+\frac{2WhAE}{L}$$

$$P=W\left(1+\sqrt{1+\frac{2hAE}{WL}}\right) \qquad [3]$$

By dividing both side of the Equation [3] byh area A:

Maximum stress produced (dynamic) is $\sigma_{max} = P/A$ and $\sigma = W/A$ is the static stress gradually applied.

$$\sigma_{max}=\sigma\left(1+\sqrt{\frac{2hAE}{WL}}\right) \qquad [4]$$

$$X=1+\sqrt{1+\frac{2hAE}{WL}} \qquad [5]$$

When $h = 0$, for a suddenly applied load gives a value $\sigma_{max} = 2\sigma$. The stress produced by a suddenly applied load is twice the static stress. In the case of the brittleness test, the weight (14 kg) is dropped from a height of 25 cm. Then dynamic load factor is $X = 2260$ (see below for details). This means dynamic stress is 2260 times the static stress produced by the falling weight.

In the calculation of load factor, X given by Equation [5], the values below are used:

$H = 25$ cm.

$A = 10$ cm2 (10 cm2 is chosen arbitrarily)

$E = 70\ 000$ MPa (714 000 kg/cm2)

$W = 14$ kg

$L = 5$ cm. (the height of rock specimen)

Then $X = 2260$. ◆