Incorporation of rehabilitation cost into the optimum cut-off grade determination

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Synopsis

Determination of the optimum cut-off grades is one of the most important aspects of mine production planning. A cut-off is a grade below which we choose not to process material. This material is treated as waste and dumped. Dumping waste is accompanied by the rehabilitation cost which will affect the overall cost of final production and also the optimum cut-off grade. Rehabilitation cost is the cost per ton of rehabilitating material of a particular type of rock after it has been dumped as waste. One of the most popular algorithms for determination of the optimum cut-off grade is Lane's method. Lane formulated the cut-off grade optimization, but he did not consider rehabilitation cost during the optimization process. This cost item should be evaluated first, and then considered during the cut-off grade optimization process. In this paper the rehabilitation cost is inserted directly into the cut-off grade optimization process using Lane's theory. The cut-off grades obtained using the suggested method will be more realistic than ones using the original form of Lane's formulations.

Introduction

The production planning problem is related to the criterion that is used to optimize the open pit design. Ideally, the criterion should be the maximization of the NPV of the pit, but unfortunately, after four decades of continuing efforts, this goal could not be achieved. The reason for this problem has been simply paraphrased by Whittle[1].

The pit outline with the highest value cannot be determined until the block values are known. The block values are not known until the mining sequence is determined; and the mining sequence cannot be determined unless the pit outline is available.

This is a large-scale mathematical optimization problem, which could not be solved currently using commercial packages. The most common approach to the problem is dividing it into sub-problems as shown by Dagdelen (Figure 1)[2].

The approach starts with the assumptions about initial production capacities in the mining system and estimates for the related costs and commodity prices. Then, using the economic block values, each positive block is further checked to see whether its value can pay for the removal of overlaying waste blocks. This analysis is based on the breakeven cut-off grade that checks if undiscounted profits obtained from a given ore block can pay for the undiscounted cost of mining of waste blocks. Then, the ultimate pit limit is determined using either a graph theory based algorithm[3,4] or a network flow one[5,6] with the object of maximum (undiscounted) cash flow. Within the ultimate pit, pushbacks are design so that the deposit is divided into nested pits going from the smallest pit with the highest value per ton of ore to the largest pit with the lowest value per ton of ore. These pushbacks act as a guide during the schedule of yearly based production planning. Before determining the extraction scheduling, the cut-off grade strategy should be determined to discriminate between ore and waste during the scheduling process. One of the best definitions for cut-off grade in one that proposed by Taylor[7]. His definition can be stated as:

'Cut-off grade is the grade that is used to separate two courses of action, e.g. to mine or to leave, to mill or to dump…'

Cut-off grade optimization is important because:

- Cut-off grade optimization can improve both long-term and short-term cash-flows
- Cut-off grade optimization is used for the simulation of mining/ processing/ stockpiling configurations to determine which configuration yields the maximum economic benefit.

Before 1964 breakeven cut-off grade criteria had been used to discriminate between

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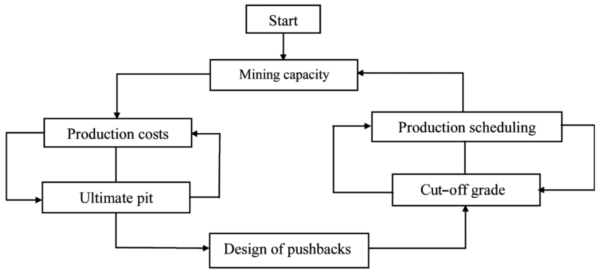

Figure 1—Open pit long-term production planning variables interacting in a circular fashion[2]

ore and waste. This method is not optimum and does not lead to NPV maximization.

Lane[8] proposed an algorithm to determine cut-off grades that maximize the profit/net present value of a project subjected to mine, mill and refinery capacity constraints. This strategy was based on raising the cut-off grade above the breakeven cut-off grade. This results in increasing the average grade of material that is going to be sent to the mill; consequently, the amount of produced product and cash flow per year will increase. Now a question arises: How much can the cut-off grade be increased? Increasing the cut-off grade, on the other hand, will result in increasing the amount of waste material that needs to be disposed. This waste material involves an extra rehabilitation cost. This cost item can itself decrease the NPV of a mining project. Therefore, in optimizing the cut-off grade, a trade-off between increasing the average mill grade and increasing the rehabilitation cost should be performed.

In this paper the effects of the rehabilitation cost on the economic cut-off grade is surveyed. Also, in order to show this effect, an example is discussed.

Cut-off grade optimization

The objective of cut-off grade optimization is to determine the long-term ore/waste discrimination strategy that will maximize profits. In large open pit mines, there are typically three stages of operation. First is the mining stage in which various grades are mined up to some capacity. Material with a grade bellow cut-off is sent for disposal in the waste area and the amount above cut-off is sent to the treatment stage, where ore is milled and concentrated with a capacity. In the third stage, concentrate is smelted and refined in order to produce saleable product. Each stage has its own capacity constraints and costs. Definitions of the maximum capacities, unit costs and quantities of the model are presented below:

- **Maximum capacity**
  M: Maximum mining capacity in terms of tons per year.
  C: Maximum concentrator capacity in terms of tons per year.
  R: Maximum refinery and/or marketing capacity in terms of lbs per year.
- **Costs**
  m: Mining cost in terms of \$ per ton of material removed.
  c: Concentrating cost in terms of \$ per ton of material milled.
  r: All costs incurred at the smelting, refinery and selling stage. These costs are reported in terms of \$ per unit product.
  h: The rehabilitation cost which is the cost per ton of rehabilitating material of a particular type of rock after it has been dumped as waste. It also includes some haulage if waste is trucked further than ore.
  f: Fixed costs over the production period (for example, one year)
- **Selling price** (s): in terms of selling price per unit of product.
- **Recovery** (y): The percentage of mineral that can be recovered in final product.
- **Quantities**: Q_m is the amount of material to be mined, Q_c is the amount of ore sent to the concentrator and Q_r is the amount of product produced over the production period of T. Therefore $Q_m - Q_c$ is the quantity of material that is sent to the waste dump.

According to the above definitions, the following equation can be used to calculate the profit:

$$P = sQ_r - [mQ_m + cQ_c + rQ_r + fT + (Q_m - Q_c)h] \quad [1]$$

Combining terms yields:

$$P = (s - r)Q_r - (m + h)Q_m - (c - h)Q_c - fT \quad [2]$$

The economic cut-off grade may be limited by mining, processing and/or marketing capacities; consequently, six cases arise depending upon which of the constraints is the limiting factor:

- If the mining capacity is the governing constraint, then the time needed to mine material Q_m is given by:

$$T = \frac{Q_m}{M} \quad [3]$$

Substituting Equation [3] into Equation [2] yields:

$$P = (s - r)Q_r - (m + h + f/M)Q_m - (c - h)Q_c \quad [4]$$

The amount of product (Q_r) is related to the quantity of ore sent to the concentrator (Q_c) by the following relation:

$$Q_r = \bar{g}.y.Q_c \quad [5]$$

Where $\bar{g}$ is the average grade of material sent to the mill. Combining Equations [4] and [5] yields:

$$P = [(s - r).\bar{g}.y - (c - h)]Q_c - (m + h + f/M)Q_m \quad [6]$$

In order to find the grade that maximizes the profit under the mining capacity constraint, the derivative of Equation [6] must be taken with respect to grade (g):

$$\frac{dP}{dg} = [(s - r).\bar{g}.y - (c - h)]\frac{dQ_c}{dg} - (m + h + f/M)\frac{dQ_m}{dg} \quad [7]$$

Because the amount of material to be mined is independent of the grade, we have:

$$\frac{dQ_m}{dg} = 0 \quad [8]$$

Therefore, Equation [7] becomes:

$$\frac{dP}{dg} = [(s - r).\bar{g}.y - (c - h)]\frac{dQ_c}{dg} \quad [9]$$

The lowest acceptable value of g is that which makes:

$$\frac{dP}{dg} = 0 \quad [10]$$

Thus, the cut-off grade based on mining constraint (g_m) is the value of g which makes:

$$[(s-r).\bar{g}.y-(c-h)] = 0 \quad [11]$$

Thus:

$$g_m = \bar{g} = \frac{c-h}{(s-r).y} \quad [12]$$

- If the concentrating rate is the governing constraint, then the time required to process a Q_c of material is:

$$T = \frac{Q_c}{C} \quad [13]$$

Hence, the profit function can be written as:

$$P = (s-r)Q_r - mQ_m - (c+{}^{f}\!/_{C})Q_c - (Q_m - Q_c)h \quad [14]$$

Following the same procedure that is carried out in the previous case, the cut-off grade (g_c) when the concentrator is the limiting constraint is:

$$g_c = \bar{g} = \frac{c+{}^{f}\!/_{C}-h}{(s-r).y} \quad [15]$$

- If the refining rate is the governing constraint, then the time is controlled by the refinery capacity; therefore:

$$T = \frac{Q_r}{R} \quad [16]$$

Hence, the profit function can be written as:

$$P = \left[(s-r)-{}^{f}\!/_{R}\right]Q_r - mQ_m - cQ_c - (Q_m - Q_c)h \quad [17]$$

As before, the cut-off grade when the refinery capacity is the limiting constraint will be calculated by the following equation:

$$g_r = \bar{g} = \frac{c-h}{\left[(s-r)-{}^{f}\!/_{R}\right].y} \quad [18]$$

- If both mining and concentrating capacities are the limiting factors, Equations [4] and [14] are equal, ie.:

$$(s-r)Q_r - (m+h+{}^{f}\!/_{M})Q_m - (c-h)Q_c = (s-r)Q_r$$
$$-mQ_m - (c-{}^{f}\!/_{C})Q_c - (Q_m - Q_c)h \quad [19]$$

Hence, Equation [19] becomes:

$$\frac{Q_m}{M} = \frac{Q_c}{C} \quad [20]$$

Therefore, the balancing cut-off grade between mine and concentrator (g_{mc}) is the cut-off grade that results in satisfaction of Equation [20].

- If both mining and refining capacities are the limiting factors. In this case Equations [4] and [16] are equal. This gives:

$$\frac{Q_m}{M} = \frac{Q_r}{R} \quad [21]$$

As before, the balancing cut-off grade between mine and refinery (g_{mr}) is the grade that results in satisfying Equation [21].

- If both refining and concentrator capacities are the limiting factor. In this case Equations [14] and [16] are equal. This gives:

$$\frac{Q_c}{C} = \frac{Q_r}{R} \quad [22]$$

The balancing cut-off grade between concentrator and refinery (g_{cr}) is the cut-off grade that satisfies Equation [22].

Until now, six possible cut-off grades are achieved. Three (g_m, g_c, g_r) are based on capacities, costs and price; the other three (g_{mr}, g_{cr}, g_{mc}) are based only on the grade distribution of the mined material and capacities.

In order to find the optimum cut-off grade among these six cut-offs, Lane's method can be applied. This method will be explained later through an example.

Example

In this section we follow an example to show the effect of rehabilitation cost on the optimum cut-off grade.

In an open pit mine there is 1 200 tons of material within the pit outline. The grade distribution of this material is shown in Table I.

The associated costs, price, capacities and recovery are:

- **Maximum capacities**
  M = 100 tons/year
  C = 50 tons/year
  R = 40 lbs/year
- **Costs**
  m = $1/ton
  c = $2/ton
  r = $5/lb
  h = $0.5/ton
  f = $300/year
- **Selling price** (s) is equal to $25/lb
- **Recovery** (y) is equal to 1.0 (100% recovery is assumed).

Substituting the above data into Equations [12], [15] and [18] yields:

$$g_m = \bar{g} = \frac{2-0.5}{(25-5).1} = 0.075 \text{ lbs/ton}$$

Table I

Initial material inventory for the example

Grade (lbs/ton)	Quantity (tons)
0.0–0.1	130
0.1–0.2	145
0.2–0.3	115
0.3–0.4	140
0.4–0.5	110
0.5–0.6	170
0.6–0.7	110
0.7–0.8	90
0.8–0.9	125
0.9–1.0	65
Total	1 200

Table II

Average grade, quantities (Q_m, Q_c and Q_r) and the total profits as a function of cut-off grade considering rehabilitation cost

Cut-off (lbs/ton)	Average grade (lbs/ton)	Q_m (tons)	Q_c (tons)	Q_r (lbs)	P_m ($)	P_c ($)	P_r ($)
0	0.463	1200	1200	556	3920	320	3350
0.1	0.514	1200	1070	549.5	3985	1165	3463.75
0.2	0.571	1200	925	527.75	3767.5	1817.5	3409.375
0.3	0.616	1200	810	499	3365	2105	3222.5
0.4	0.672	1200	670	450	2595	2175	2820
0.5	0.715	1200	560	400.5	1770	2010	2366.25
0.6	0.787	1200	390	307	155	1415	1452.5
0.7	0.841	1200	280	235.5	-1110	810	723.75
0.8	0.884	1200	190	168	-2325	135	15
0.9	0.950	1200	65	61.75	-4262.5	-1052.5	-1125.63

Note: P_m, P_c and P_r are total profit when mining, concentrating and refining rates are limiting factors respectively.

$$g_c = \bar{g} = \frac{2 + 300/50 - 0.5}{(25-5).1} = 0.375 \text{ lbs/ton}$$

$$g_r = \bar{g} = \frac{2 - 0.5}{\left[(25-5) - 300/40\right] \times 1} = 0.12 \text{ lbs/ton}$$

The average grade, quantities (Q_m, Q_c and Q_r) and the total profits as a function of cut-off grade under different constraints are given in Table II.

The total profit as a function of cut-off grade under different constraints is shown in Figure 2.

Now three other cut-off grades (by balancing operation) should be obtained. By substituting data into Equation [20] we have:

$$\frac{Q_m}{100} = \frac{Q_c}{50} \quad \Rightarrow \quad Q_m = 2Q_c$$

Therefore, the balancing cut-off grade by balancing the mine and mill operation is one that results in $Q_m = 2Q_c$.

According to the Table II the corresponding cut-off is something between 0.3% and 0.4%. Interpolating one finds that the cut-off grade is equal to 0.47 lbs/ton. The balancing cut-off between mine and refinery and also refinery and concentrator can be achieved similarly by using Equations [21] and [22] and also from Table II. The three balancing cut-offs are:

$g_{mc} = 0.47$lbs/ton

$g_{cr} = 0.6$lbs/ton

$g_{mr} = 0.345$lbs/ton

Now six cut-offs are candidate for the overall optimum cut-off grade. All these cut-offs are shown in Figure 1. First, the local optimums for each pair of operations are obtained using the following rules[8]:

$$G_{mc} = \begin{cases} g_m & \text{if } g_{mc} \le g_m \\ g_c & \text{if } g_{mc} \ge g_c \\ g_{mc} & \text{otherwise} \end{cases} \qquad [23]$$

$$G_{cr} = \begin{cases} g_r & \text{if } g_{cr} \le g_r \\ g_c & \text{if } g_{cr} \ge g_c \\ g_{cr} & \text{otherwise} \end{cases} \qquad [24]$$

$$G_{mr} = \begin{cases} g_m & \text{if } g_{mr} \le g_m \\ g_r & \text{if } g_{mr} \ge g_r \\ g_{mr} & \text{otherwise} \end{cases} \qquad [25]$$

According to Equations [23] to [25] we have:

$G_{mc} = 0.375$

$G_{cr} = 0.375$

$G_{mc} = 0.12$

The overall optimum cut-off grade is the middle value of G_{mc}, G_{mr} and G_{cr}. Therefore, the optimum cut-off grade is:

$g_{op} = g_c = 0.375$lbs/ton

From the Table II, the average grade of material sent to the mill for a cut-off grade of 0.375 lbs/ton can be obtained

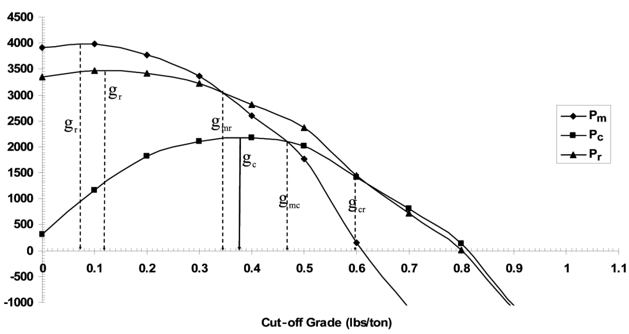

Figure 2—Total profit as a function of cut-off grade under different constraints considering rehabilitation cost

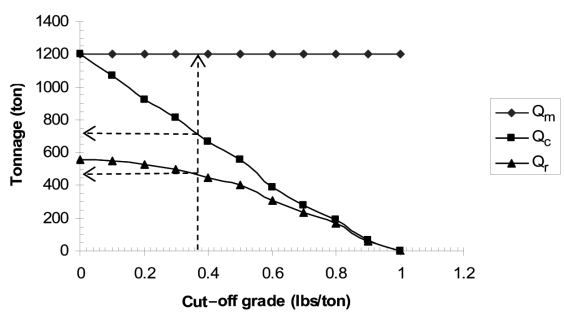

Figure 3—Total amount of Q_m, Q_c and Q_r versus cut-off grade

as:

$g_{av} = 0.66$ lbs/ton

In order to calculate the quantities, the total amount of Q_m, Q_c and Q_r versus cut-off grade is plotted. This is shown in Figure 3.

Hence, according to the Figure 2 the quantities are:

$Q_m = 1200$ tons

$Q_c = 715$ tons

$Q_r = 472$ lbs

And the amount of waste that should be dumped and rehabilitated is equal to:

$Q_m – Q_c = 1\,200–715 = 485$ tons

According to the above quantities the total life of mine is equal to:

$$T = \frac{715}{50} = 14.3 \text{ years}$$

The total profit is equal to:

$$P = (25-5)\times 472 - (1\times 1200) - (2 + 300/50)\times 715 - (1200-715) \times 0.5 = \$2277.5$$

and the yearly profit is equal to:

$$p_y = \frac{2277.5}{14.3} = \$159.27$$

Now assuming an interest rate of 12%, the net present value of these profits is:

$$NPV = 159.27 * \frac{1.12^{14.3} - 1}{0.12 \times (1.12)^{14.3}} = \$1064.75$$

Now, we resolve the above example without considering the rehabilitation cost (h=0). The average grade, quantities (Q_m, Q_c and Q_r) and the total profits as a function of cut-off grade under different constraints are given in Table III.

The total profit as a function of cut-off grade under different constraints is shown in Figure 4.

The results are summarized as bellow:

- Optimum cut-off grade =0.4 lbs/ton
- Average grade of material sent to the mill = 0.672 lbs/ton
- $Q_m = 1\,200$ tons
- $Q_c = 670$ tons
- $Q_r = 450$ lbs
- Total amount of waste that should be dumped and rehabilitated can be achieved as: $Q_m – Q_c = 1\,200–670= 530$ tons.
- Mine life is 13.4 years and according to Table III the total profit is equal to 2440. But this is not the real profit, because the rehabilitation cost should be calculated for 530 tons of waste material for the life of mine. Hence the total profit is equal to: 2440–(1200–670)*0.5 = $2175
Hence, the yearly profit is equal to:

$$p_y = \frac{2175}{13.4} = \$162.31$$

Now assuming an interest rate of 12%, the net present value of these profits is:

$$NPV = 162.31 \times \frac{1.12^{13.4} - 1}{0.12 \times (1.12)^{13.4}} = \$1056.3$$

As can be seen in the above example, considering the rehabilitation cost during the cut-off grade optimization process results in decreasing the amount of waste sent to the dump. This is as a result of cut-off grade reduction. If the rehabilitation cost is incurred during optimization of the cut-off grade, this can make it more economic to process material at a lower cut-off grade in order to decrease the amount of disposed material; therefore, inserting rehabilitation cost into the optimization process will force the model to decrease the cut-off grade as much as possible (in the light of economic and capacity considerations) in order to decrease the amount

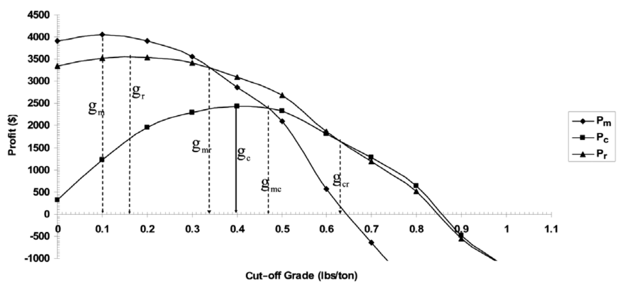

Figure 4—Total profit as a function of cut-off grade under different constraints without the cost of rehabilitation

Table III

Average grade, quantities (Q_m, Q_c and Q_r) and the total profits as a function of cut-off grade without considering rehabilitation cost

Cut-off (lbs/ton)	Average grade (lbs/ton)	Q_m (tons)	Q_c (tons)	Q_r (lbs)	P_m ($)	P_c ($)	P_r ($)
0	0.463	1200	1200	556	3920	320	3350
0.1	0.514	1200	1070	549.5	4050	1230	3528.75
0.2	0.571	1200	925	527.75	3905	1955	3546.875
0.3	0.616	1200	810	499	3560	2300	3417.5
0.4	0.672	1200	670	450	2860	2440	3085
0.5	0.715	1200	560	400.5	2090	2330	2686.25
0.6	0.787	1200	390	307	560	1820	1857.5
0.7	0.841	1200	280	235.5	-650	1270	1183.75
0.8	0.884	1200	190	168	-1820	640	520
0.9	0.950	1200	65	61.75	-3695	-485	-558.125

of material that is sent to the waste dump. Consequently, the total amount of waste that should be dumped and rehabilitated during the life of mine is decreased. As a result, the NPV of the project will increase (in the above example it is increased from $1056.31 to $1064.75).

Net present value maximization

In the previous section, the optimum cut-off grade is obtained with the objective of maximizing the profit. Nowadays, most mining companies attempt to maximize the net present value (NPV) rather than profit. The objective function of the problem with regard to NPV maximization is proposed by Lane[8]:

$$v = (s-r)Q_r - mQ_m - cQ_c - T(f+Vd) \qquad [26]$$

where:

v: the difference between the present values of the remaining reserves at time t=0 and t=T.

V: present value at time t=0.

d: the discount rate.

Now considering the rehabilitation cost into the optimization process results in converting Equation [26] as follows:

$$v = (s-r)Q_r - mQ_m - cQ_c - h(Q_m - Q_c) - T(f+Vd) \qquad [27]$$

thus,

$$v = (s-r)Q_r - (m+h)Q_m - (c-h)Q_c - T(f+Vd) \qquad [28]$$

The above function should be maximized. As in the profit maximization case, we first take the derivative of v with respect to grade and then set the derivative equal to zero. Finally, the resulting equation is solved for the appropriate cut-off grades subject to mining, concentrating and refining constraints. This results in:

$$g_m = \bar{g} = \frac{c-h}{(s-r).y} \qquad [29]$$

$$g_c = \bar{g} = \frac{c + \dfrac{f+dV}{C} - h}{(s-r).y} \qquad [30]$$

$$g_r = \bar{g} = \frac{c-h}{\left[(s-r) - \dfrac{f+dV}{R}\right].y} \qquad [31]$$

In Equations [29] and [30] V is unknown value, because it depends on the cut-off grade. Thus, the iterative process must be used in order to find the best cut-offs. This iterative process is started by V=0 and continued by substituting the new value of V in the Equations [30] and [31]) until the $V_n - V_{n-1}$=0.

Results and discussion

In Lane's algorithm, rehabilitation cost has not been considered, so it can be said that the obtained cut-offs are not real ones. In this paper this cost item is inserted into the optimization process and a new mathematical model developed based on Lane's method. Results of the application of this method show that considering rehabilitation cost can decrease cut-off grades. Considering rehabilitation cost during the cut-off grade optimization process results in decreasing the amount of waste sent to the dump. This is as a result of cut-off grade reduction. If the rehabilitation cost is incurred during optimization of cut-off grade, this can make it more economic to process material at a lower cut-off grade in order to decrease the amount of disposed material; therefore, inserting the rehabilitation cost into the optimization process will force the model to decrease the cut-off grade as much as possible in order to decrease the amount of material that is sent to the waste dump. Consequently, the total amount of waste that should be dumped and rehabilitated during the life of mine is decreased.

This approach can be applied to both profit maximization and net present maximization versions of Lane's algorithm.

Conclusions

Cut-off grade optimization is one of the major steps in open pit mine planning and design. Cut-off grade determines the destination of mined material: material above the cut-off is sent to concentrator and material below it is sent to the waste dump. Lane's method is the most widespread algorithm for determination of the cut-off grade. In this algorithm rehabilitation cost has not been considered, so it can be said that the obtained cut-offs are not real ones. In this paper this cost item is inserted into the optimization process and a new mathematical model is developed based on Lane's method. Results of the application of this method show that considering rehabilitation cost can decrease the cut-off grade. As a result, the amount of ore that is sent to concentrator is increased and consequently, the amount of material that should be sent to waste dump is decreased. Hence the total amount of rehabilitation cost during and after ore extraction is decreased and the total achievable NPV of the project will increase.

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