Experimental 3-D modelling of surface subsidence affected by underground mining activities

by J. Trčková*

Synopsis

This paper describes the method of modelling rock mechanical problems on experimental models. Based on an actual experiment used as an example, assessment of the surface deformation due to undermining, the extensive possibilities of using a 3-D physical model and application results in practice are demonstrated. A special method of modelling the underground mine progress by using paraffin melting is presented.

Keywords: undermining, subsidence of surface, 3-D experimental model

Introduction

The construction of underground structures causes both qualitative and quantitative changes of rock mass in these structures' surroundings. Surface subsidence is an unusual, unplanned, and frequently discussed problem. The extraction of material from underground mines, without leaving adequate support for the overburden, results in subsidence of surfaces above the mine. The surface area affected by subsidence can be much larger than the mined-out area, as a result of the limiting angle of effect.

The deformation and stress responses of the rock mass can be observed directly *in situ*, even during construction of the underground structure, or in advance, predicted on the basis of numerical or experimental models. Modelling allows us to investigate mechanisms of geotechnical phenomena and predicts stress and deformational changes as well as their manifestation during various stages of underground construction and during simulation of operating conditions. It also influences the surface in the undermined areas.

Experimental modelling of effects occurring within the rock mass altered by mining activities has been used for a very long time (Kuzněcov *et al.*, 1957; Goodman, 1976; Stimpson, 1981). Compared with numerical solutions, the advantage of the 3-D experimental models consists of a more realistic simulation of the process taking place in a rock mass. 3-D experimental models represent a part of the territory under study, reduced to an appropriate scale. These models are mainly used to study problems connected with surface mining and other surface activities, such as the building of foundations, as well as to study the stability of slopes and spoil banks, etc., but very rarely for modelling the progress of underground mining.

The advantage of experimental models consists of a real and complex simulation of the deformational process in the rock mass. The most objective results can be attained by linking experimental and numerical methods (Trčková and Procházka, 2003). The results from experiments can be useful as the input data for numerical modelling (Procházka *et al.*, 2004; Procházka and Trčková, 2005).

In this paper, the values of the surface subsidence and the shape of the subsidence basin as a consequence of the underground mining of a coal seam are determined.

Principles of the modelling

Basic rules of experimental modelling and formulation of the boundary conditions for modelling result from the principles of geometrical and physical similarity, which is inferred for a consideration of dimensional analysis (Kožešník, 1983). To simplify the solved problem, constitutive relevant quantities $v_1, v_2, \ldots, v_n$ can be selected, which possess a decisive influence on the process taking place in the rock material. It is assumed that the influence of the other quantities is smaller. Then, the physical equation involving the function of relevant quantities of various dimensions

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* *Institute of Rock Structures and Mechanics Academy of Sciences of the Czech Republic, Czech Republic.*

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$$F(v_1, v_2, \ldots, v_n) = 0, \qquad [1]$$

describes - in a simplified way, given by a selection of these quantities - the behaviour of the rock material. According to the Buckingham theorem, this dimensional equation for the relationship between reality and model can be reduced to the problem of finding $k < n$ relevant non-dimensional parameters π_i. They are functions of v_i, fulfil the above Equation [1], and are numerically identical for the model and reality. By implementation of non-dimensional parameters π_i, for which the requirement of dimensional homogeneity follows

$$\pi_i = v_1^{x_{1i}} v_2^{x_{2i}} \ldots v_n^{x_{ni}}, (i = 1, 2, \ldots, k)$$

non-dimensional physical equation is obtained

$$F'(\pi_1, \pi_2, \ldots, \pi_k) = 0,$$

in which arguments π_i are dimension-independent. Non-dimensional parameters π_i are defined on the basis of the system of equations formed by relevant variables and corresponding basic central processing units *L* (length), *T* (time), *M* (mass).

The physical model has to obey geometrical similarity. In the whole range of model, the proportionality of dimensions and the identity of angles must be preserved between the model and a modelled object.

The type of modelled geotechnical problem and its extent, the possibility and technique of bringing of forces, the time factor, technical possibilities, and other aspects determine the linear scale of the model. It is the aspect ratio in which length dimensions of the model are proportionately reduced against reality.

If $1/\alpha_l$ is length scale, i.e., the ratio of lengths between the model and reality and a is ratio of bulk densities

$$a = \rho_{model} / \rho_{reality},$$

then it can be defined for

forces $\quad P_{model} = a . (1/\alpha_l)^3 . P_{reality}$
stresses $\quad \sigma_{model} = a . (1/\alpha_l) . \sigma_{reality}$
deformations $\quad \varepsilon_{model} = a . (1/\alpha_l) . \varepsilon_{reality}$.

To determine the time scale of the ratio between time in reality and time in the model, empirical estimation is applied (in terms of the time needed for stress redistribution in the model body caused by pressure changes in the model in comparison to the time the same process took in reality). This time scale is applied for the assessment of the remaining quantities, depending on time. Generally, time scale can be determined as

$$\alpha_t = (v_{model} / v_{reality}) . \alpha_l,$$

where

α_t – time scale
α_l – length scale
v – velocity of deformation.

To simulate the processes taking place in the rock material as perfectly as possible, the rock environment is replaced by equivalent materials in the model; their determinate physical and mechanical properties according to model laws and the scale of the model agree with rock properties and respect the character of failures that simulate those in rock material. The models are constructed from a mixture of various, mostly easily available materials (e.g. sand, bentonite, ballotine, gypsum, mica - vermiculite, composite mortar, cellular concrete and water).

The models are constructed in stands of various dimensions, depending on the problem to be solved and the length scale of the model.

Construction of the model runs with the help of a profile form made according to the dividing line shape of geological beds and the contour of the surface of the modelled area.

General characteristics of the modelled area

The preliminary values of surface subsidence due to undermining have been determined in the area above the safety shaft pillar, where many structures were built on the surface. In the safety pillar of the two shafts, mining has been conducted on the coal seam, which dips towards the north at approximately 6° and which is located at a depth of no more than 500 m below the surface. The thickness of the coal seam is about 8 m. The overlying strata are predominantly composed of sandstones and conglomerates, which are separated by shallower layers of siltstones and clay stones. The main fault of this area, running in a NW-SE direction, is situated to the east of the shaft pillar. Its dip-slip fault reaches up to 117 m. This fault is composed of several different floes (Figure 1). The other tectonic fault runs across the shaft pillar, and its displacement throw is from 8 m on the east to 15 m on the west side.

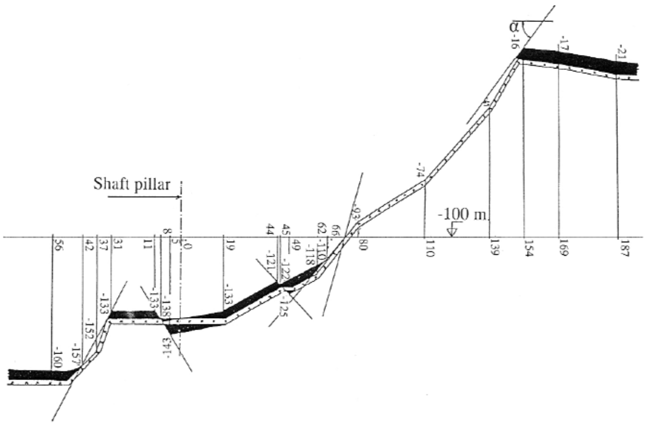

Figure 1—Section normal N-N´ (see Figure 6) in the direction of the main tectonic fault in the modelled space. Figure dimensions are in metres

The angle of draw, which was determined on the basis of long-term levelling measurements, varies within the range from 65° in sites without major tectonic activity, through 58° for heavily faulted areas, down to 55° in sites where multiple undermining of the area took place.

3-D model experiment

Model construction

Based on the excavated coal seam's deposition depth, the necessity to represent a relatively large territory of the expected subsidence basin, in view of the size of the available modelling stand, and the technical possibilities of experimental modelling, the scale of 1:750 was used for the 3-D model from physically equivalent material (Skořepová, 1995).

The 3-D model was constructed in a stand with dimensions 143.5 x 128 x 80 cm (length x width x height) and represented, due to the chosen scale, the territory of 1076 x 960 m of real-world area.

The rock medium was substituted, in the model, by materials whose determining physical and mechanical properties corresponded with the properties of rocks in the modelled area. Emphasis was especially placed on attaining an equal angle of draw. Mixtures of sand of various grain size (from 0.2 to 2.0 mm), plaster, mica and paraffin were used to prepare the model materials.

Model construction was carried out on the basis of geological data, including vertical sections through the modelled area, and also data concerning the topographic plan and hypsography of the surface, as well as mining maps containing areas extracted in the past and data of planned mining in the safety shaft pillar (Figure 2).

The simulation of the mine progress

To simulate the mining progress of the shaft pillar in the experiment, an entirely untraditional method of successive melting of paraffin blocks was used. The blocks' dimensions, on the model, corresponded to those parts of the coal seam which were to be mined. The construction of the model was adapted for this purpose. Bedrock of the mined coal seam, whose physical properties do not particularly affect the deformation process, was made from a plaster block with a 6° dip, corresponding with the general dip of the coal seam in modelled area. Regarding the parts of the shaft pillar coal seam that were subjected to mining, a void cavity, three centimetres deep, was left in the model under these parts (Figure 3). A perforated metal sheet was placed on the surface of the plaster block thus prepared, and heating elements were placed on its lower side, copying the shape of individual mining blocks (Figure 4). To ensure the rigidity of the metal sheet, adjustable braces were installed as well. The paraffin blocks were separated from each other by thermal binding tape and covered with very thin aluminium foil to prevent the modelling material, which substituted the overlying rock, from falling into the cavity under the perforated sheet (Figure 5). The cavities were designed to hold the molten paraffin.

Overlying beds of the coal seam, consisting mainly of sandstone and clay stone layers of varying thickness, were replaced by model materials (mixtures of sand of various grain size, plaster, mica and paraffin), whose relevant physical and mechanical properties corresponded with properties of the modelled area, according to geometrical and physical similarity. The surface of the model was shaped in accordance with the contours of the modelled territory (Figure 6).

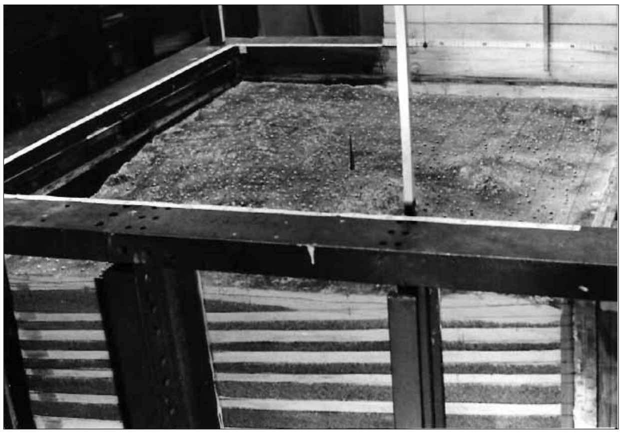

Figure 2—Experimental model prepared for the test

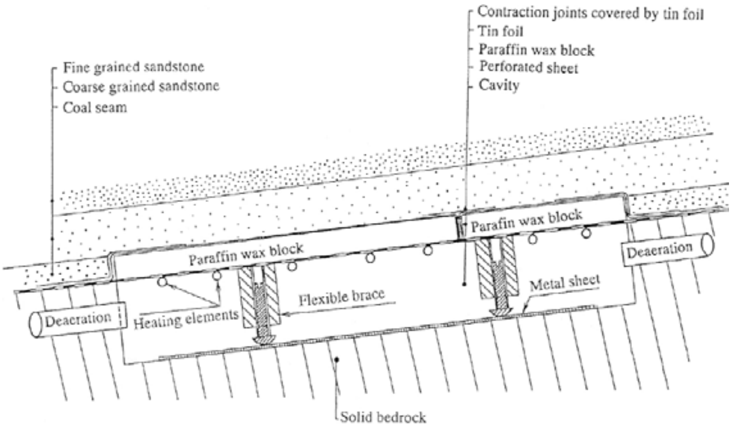

Figure 3—Vertical cross-section of the excavated area in the model

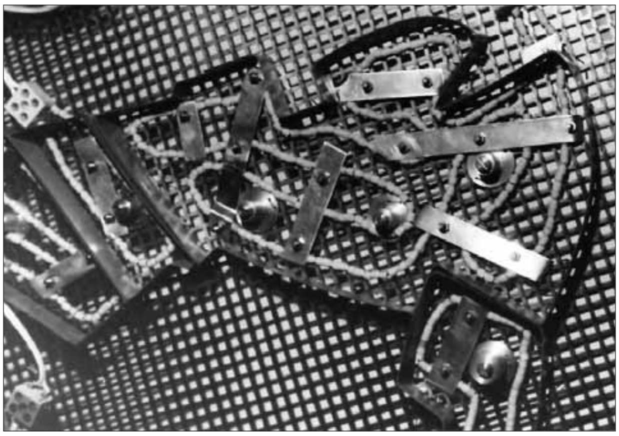

Figure 4—Heating elements (view from below)

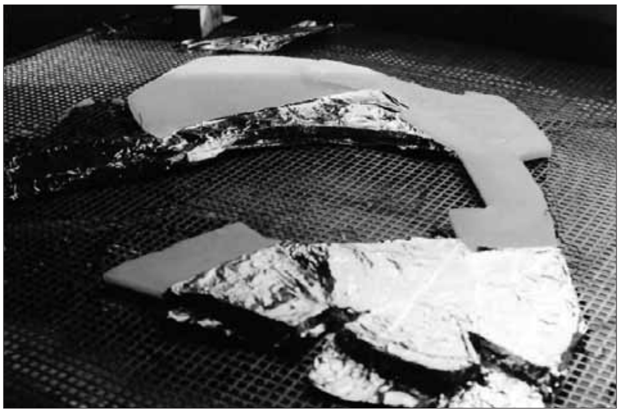

Figure 5—Paraffin block simulating the exploited coal seam within the model

Surface deformation measurement

An analytical transformational stereophotogrammetric method was used to determine surface deformation (Vencovský, 1989). This method enables the digital spatial coordinates of any point, suitably marked on the model surface, to be derived at an arbitrarily chosen moment of the deformation process. For this purpose, exposition of stereo-couples of photographs and analytical processing of the taken coordinates of detailed points on the model surface were carried out. The surface deformations were determined from coordinate differences, obtained for individual deformational conditions, due to the simulation of coal seam extraction.

The accuracy of the stereophotogrammetric method in determining coordinates in a vertical direction and in the direction perpendicular to the axis of sight is 0.1 to 2.2 mm in the model (7.5 to 16.5 cm in reality). The average error in the sight direction is 2 to 2.5 times higher. In order to obtain perfect orientation on the photographs of the model, the model surface was covered by a square net of threads and sprinkled with small balls (Figure 7).

Course of the experiment, results

Before simulation of the coal seam extraction started, basic coupling of photographs of the model surface was carried out. The geometry of the coal seam extraction was very complicated. Therefore, simulation of the coal seam extraction in the model was divided into three phases of the model experiment. Figures 8a, 8b, and 8c show the parts subsequently melted off. In each given stage, the simulation of the extraction was stopped, and the measurement of the deformation was carried out at a time when the deformation process on the model surface would already be terminated. Deformation components d_x, d_y, d_z were found. On Figures 8a, 8b, and 8c, only vertical subsidence d_z is presented graphically in the form of isolines.

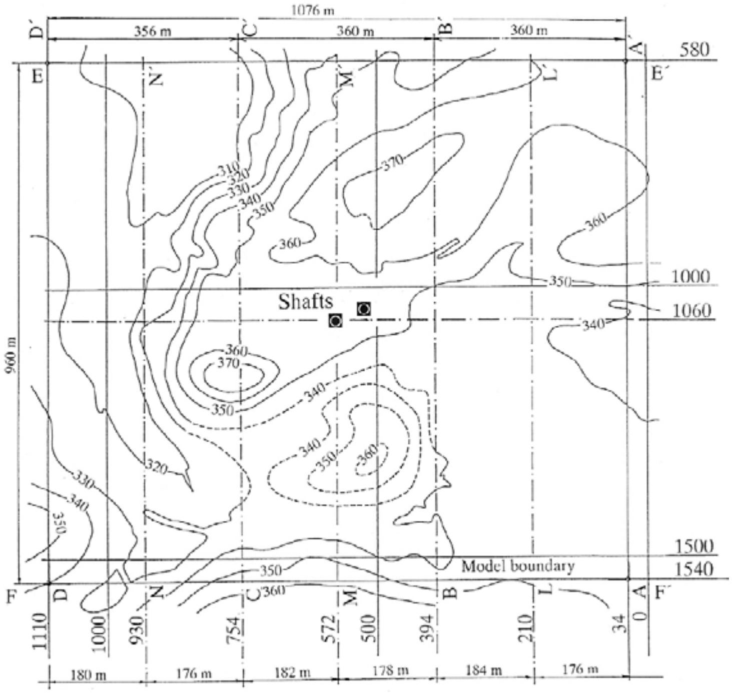

Figure 6—Surface of the modelled area

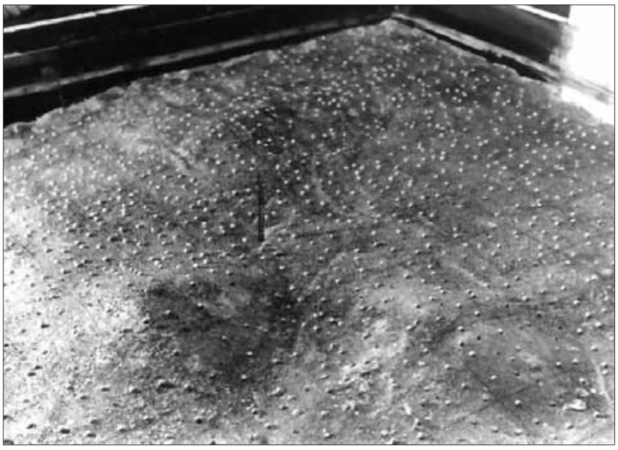

Figure 7—Detail of the model surface

In the first phase, the part demonstrated on Figure 8a was molten off. The second and third phases of the model were carried out in the same way as the first phase (Figures 8b, 8c).

Maximum recorded vertical deformations of the model surface, having taken place in the first phase of the model experiment, reached more than 0.8 mm on the model, which corresponds to 60 cm in real conditions on a scale of 1:750. For the second phase, after the deformation process on the surface died down, the vertical displacement in real condition, amounted to about 135 cm, according to the results of the model experiment (1.8 mm). For conditions when all blocks were extracted, a maximum vertical deformation up to 200 cm on the mine can be ascertained (2.6 mm on the model). Regarding the accuracy of the stereophotogrammetric method used to determine maximum horizontal displacements, the values are established in the direction perpendicular to the

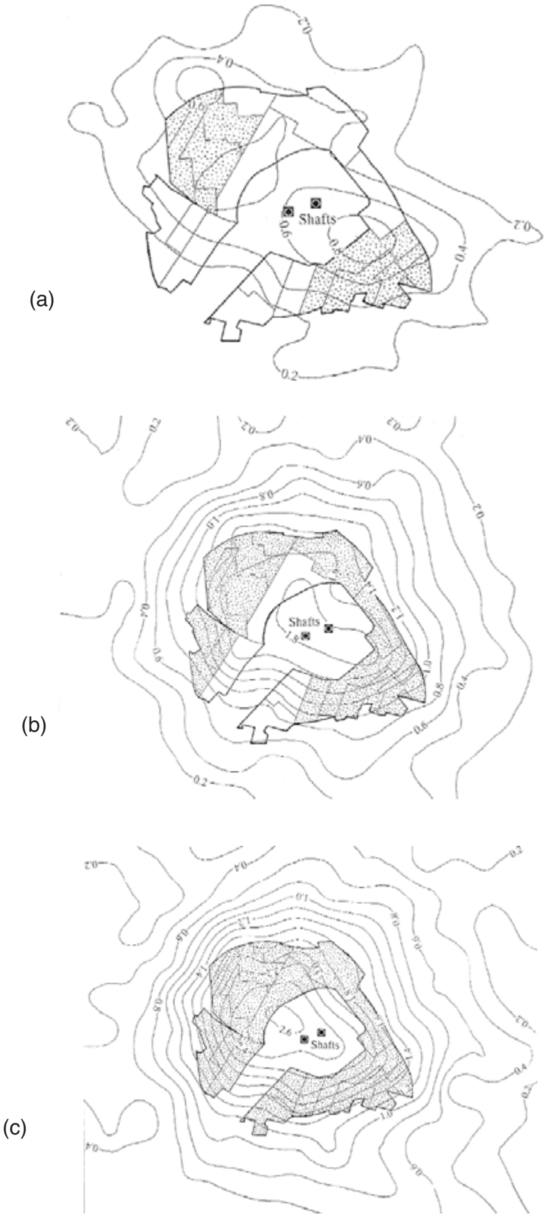

Figure 8—Vertical deformations of the model surface (in millimetres) determined in the first (a), the second (b) and the third (c) phase of the model experiment

axis of sight. As a consequence of mining activities realized in the first phase, maximum horizontal displacements up to 30 cm above the external border of the shaft pillar towards the centre of the subsidence basin were determined, up to 45 cm for the second phase and about 60 cm for the third phase.

Comparison with numerical solution

Results of the measurements on the experimental model were then processed into such a final hardcopy form, which enabled the qualitative and quantitative comparison of these results with the results of numerical prognoses and to derive new information for rendering numerical prognoses more precisely (Vencovský, 1995). The territory represented by the surface of the physical model was virtually identical with that of the numerical model. Therefore it has been possible to carry out comparison of both experimental and numerical models. The numerical model takes into consideration long-lasting mine surveying experience and information, as well as from known theories of undermining effects and is issued from the 'effect function'

$$g(k) = \frac{1}{1 + k^2 \cdot \cot g^2 \mu},$$

where k ($0 \le k \le 1$) is the coefficient of exploitation and μ to be called the critical angle.

The numerical comparison between values of points with identical positions in experimental and numerical models was determined from the mean value of a_x, a_y, a_z of algebraic differences and mean values m_x, m_y, m_z of absolute values of these differences. These parameters were carried out for all three identical states of the experimental and numerical models and for two values of the critical angle (60°, 65°) used in the numerical solution. Differences in the vertical dimensions are shown in Table I. The small differences between measured and computed subsidence are mainly due to the choice of the effect function and critical angle in numerical prognosis.

Conclusion

Prediction of surface subsidence in the undermining areas has been a very important source of information, both to evaluate and to prevent undermining damage arising on the surface and surface structures. Generally, these prognoses have been performed only on a theoretical basis of an analytical character. Therefore, the experimental modelling method used and the results obtained from deformational analysis may be considered quite unique. Compared with theoretical considerations, experimental model results indicate a complex real deformation process as a consequence of underground mining. Prognoses obtained by means of experimental models have an advantage due to the fact that the rock medium is not defined in a numerical way, which must always be considered a very approximate one. Experimental models are always constructed with the ambition to involve all accessible geological and mechanical-physical information about the original rock massif in the structure. In this way, the qualitative and quantitative development of deformation processes can be assumed - this applies to the modelled and the real ones. A key drawback of the numerical solution is that it is always necessary to simplify the problem in order to render it analytically definable. This simplification can bring about inadequate results of numerical methods.

Table I

Vertical dimensions

Phase	$\mu = 60°$		$\mu = 65°$	
	a_z (m)	m_z (m)	a_z (m)	m_z (m)
1	0.19	0.20	0.19	0.22
2	0.10	0.18	0.11	0.21
3	0.07	0.19	0.09	0.25

The presented method of successive de-melting of paraffin blocks, which was used for the simulation of underground mining on the 3-D experimental model, is quite original and very successful. In spite of the small scale, which had to be chosen due to the extent of the modelled territory, the method complied with assumption. Provided models could be constructed in the larger scales, it has been proven that use of experimental models could be adequate for the solution of problems of a narrower extent, namely in areas with complicated geological structure.

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