The effect of particle sizes and solids concentration on the rheology of silica sand based suspensions

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Synopsis

The effect of high solids concentration and coarse particle sizes (d_{50} ranges 90–300 μm) on the viscosity of a suspension of water and silica sand was investigated. The experiments were designed to include conditions that have been tested by other authors and those encountered in the operation of tumbling mills.

The rheological data was measured using a tube rheometer with operating pseudo shear rates of up to 1500 s^{-1}. The rheograms obtained depicted dilatant behaviour. The Otswald-de Waele, Bingham, Herschel-Buckley and Casson models were fitted to the experimental data. The Herschel-Buckely model, which approximates the yield point and gives an indication of the shear thinning or shear thickening behaviour of the suspension, was found to provide the best description of the flow curves for all slurries. The Herschel-Buckely model was then used to determine the apparent viscosity of all the tests.

The apparent viscosity and yield stress increased with solids concentration and particle size at the different pseudo shear rates. The increase in slurry viscosity with solids concentration was attributed to increased frequency of particle-particle interactions. The increase in slurry viscosity with particle size was attributed to increased inertial effects. At low shear rates the sand slurry exhibited shear thinning behaviour for all solids concentrations. At higher shear rates above 1000 s^{-1} the high solids concentration suspensions exhibited a transition from shear thinning to a shear thickening behaviour.

Introduction

Comminution using tumbling mills involves two sub processes, namely: transport and breakage. In order to develop a mechanistic model for the transport sub process, viscosity measurements at high solids concentration and relatively coarse particles are required. The models used to describe slurry transport for grinding applications are either empirical or semi-empirical. The model developed by Morrel and Stephenson from a combination of laboratory, pilot and plant-scale studies on high aspect ratio tumbling mills is widely used for design and optimization studies (Morrell and Stephenson, 2005). However, for low aspect mills, South African style mills slurry pooling and other problems related to transport and discharge are encountered and the model developed by Morrell and Stephenson cannot accurately predict the performance of these mills because most of the design variables are outside its window of application.

A mechanistic model is required for use in design and optimization studies that can be applied to different aspect ratio mills. Slurry viscosity is one of the most important inputs in the slurry transport and discharge mechanistic model for tumbling mills (Shi, 1994). The mechanistic model being developed has a similar structure to the Ergun Equation which is used to describe flow in static beds. This study was initiated to investigate the effect of particle size and solids concentration on viscosity, which is a significant input in the viscous energy loss term of the transport model.

There is generally a lack of knowledge concerning the flow properties of processes involving suspensions with fast settling solids. It has been noted that there are significant discrepancies in existing literature concerning the effects of key parameters such as high solids concentration and coarse particle sizes on slurry viscosity. This is because viscous properties are difficult to measure in an unstable system of settling particles (Clarke, 1967). Tumbling mills are usually operated at solids concentrations ranging between 40 and 60% solids by volume. However, most of the measurements to characterize the rheology for slurries were performed for slurries with a maximum solids concentration of 30% by volume. This is most probably influenced by the limitations imposed by the equipment used in obtaining the rheological measurements and the properties of the slurry being tested. The work in this study extended the experiments to include slurries with solids concentrations in the tumbling mills operations range.

Typical measurements for rheology characterization of slurries are performed using very fine particles mixed with water. In this study

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the effect of particle size was evaluated on three different particle size fractions of silica sand with d_{50} values of 90, 180 and 300 microns. These slurries with relatively coarse particles are a representation of what is typically found in the primary mill discharge.

Literature review

The effect of solids concentration on slurry viscosity

Physical particle interactions and the operating shear rate ranges are the two main factors that affect the solids concentration when one determines slurry viscosities. The viscosity of a suspension will increase with solids concentration. This phenomenon is attributed to the physical particle interactions that occur when a solid is dispersed in a liquid. According to Cheng (Cheng, 1980) there are three main categories of these physical interactions:

- Interparticle attraction promotes the formation of flocs and aggregates. This phenomenon occurs mostly in fine particle suspensions
- Hydrodynamic interactions give rise to viscous dissipation in the liquid
- Particle-particle contact brings into play frictional interactions.

At low to medium solids concentration the effect of hydrodynamic interactions dominate, whereas at low solids concentration viscosity appears to increase linearly with increasing solids concentration. However, Rutgers observed that after a certain solids concentration, the viscosity of the slurry increases significantly with small increments of the concentration (1962). According to Cheng, from medium to high solids concentration, particle frictional contact dominates and at very high solids concentration the particle effect predominates over the hydrodynamic effects (1980).

The effect of operating shear rate ranges on slurry viscosity

According to Cheng (1980), as the concentration increases from medium to high, non- Newtonian behaviour is exhibited. The transition from Newtonian to non-Newtonian is not only dependent on concentration but also on shear rate. The shear rate at which the non-Newtonian behaviour starts decreases as concentration increases. As the shear rate increases, the suspension first becomes shear thinning and then shear thickening. Ferreira and Olhero (2003) noted similar observations in studies with silica sand-water suspensions. The experiments were conducted using particle sizes ranging from 2.2 to 19 microns with a solids concentration up to 46% by volume. The rheological measurements were conducted using a rotational controlled stress rheometer. They observed that the shear thinning behaviour occurred at low shear rates. This was followed by a shear thickening behaviour at intermediate shear rates and a trend to Newtonian behaviour at the highest shear rate region (Figure 1). The transition from shear thinning to shear thickening at higher shear rates for narrow particle size distributions was attributed to particle rearrangements and the increasing average distances between layers of particles. Under these conditions the capillary forces oppose to the flow and the suspension thickens.

The effect of particle size on slurry viscosity

Kawatra and Eisele observed that at a constant solids concentration, a reduction in particle size will result in an increase in slurry viscosity (1998). This was attributed to increased surface area, which binds up water molecules and thus increases the effective solids concentration. This is in contradiction with the work of Clarke, (1967) and De Bruijn, (1951) as reported by Thomas, (1965), which revealed that slurry viscosity increases with particle diameter. They attributed this to inertial effects, which resulted in additional energy dissipation. Clarke conducted his investigations on silica sand suspensions in water having particle sizes of up to 211 microns and concentrations of up to 50% by volume. A rotational viscometer was used to determine the rheological measurements of the suspensions.

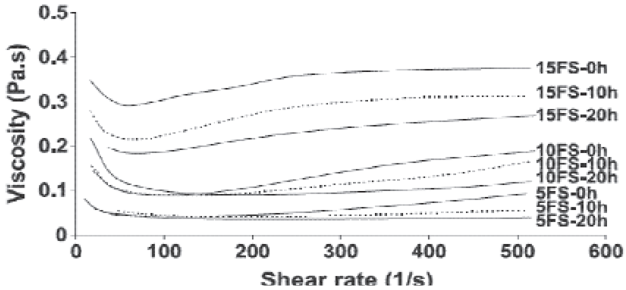

Figure 1—Equilibrium viscosity curves for the suspensions of the sand mean particle size of 2 μm with different amounts of fine silica particle size (FS) with mean size of 0.07 μm (total solid loading 46 vol.%) and different milling times

Experimental work

Test materials and particle characterization

Silica sand with different particle size distributions, known by their commercial names as Sand 2 and Sand 55, were used. A portion of Sand 2 was milled to produce a third sample with a finer distribution. The particle size distribution curves of these sands are shown in Figure 2. Three distinct particle size distributions were used and they all appear to be narrow.

The experimental set-up

The experimental set-up used to conduct the viscosity measurements on the different sand-water suspensions consisted of a 45ℓ sump, Weir Enviro Tech variable speed pump of up to 50 Hz and maximum power of 50 kW. An

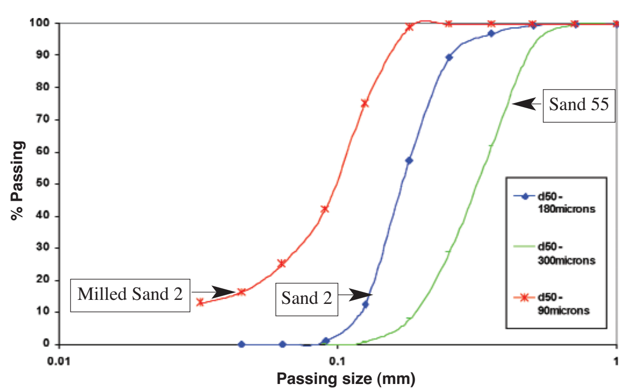

Figure 2—Particle size distribution curves for the different sands

Table I

Tube rheometer dimensions

Active volume (ℓ)	5.72
Internal tube diameter (m)	9.90 x 10-3
Measuring tube length (m)	9.64
Pipe roughness	30

agitator was used to ensure that the material in the sump was well mixed. A Paterson and Cooke Consultants computerized tube rheometer was used. The tube rheometer works on a principle of measuring the pressure drop across a measured length of tube. The pressure drop determines the shear stress. The slurry flow rate in the tube determines the pseudo shear rates. These results are used to plot a pseudo rheogram (Kahn, 2005). Table I shows the dimensions of the tube rheometer.

At solids concentrations above 40% by volume shear rates above 1000 s^{-1} could not be obtained. Settling rates increased more rapidly for the coarser particle size of d_{50} - 300 microns. Due to the above-mentioned limitations solids concentrations above 50% by volume could not be tested. Details of the experimental programme are given in Table II Temperature and surface chemistry effects were minimized by maintaining similar experimental conditions, and temperature readings were logged continuously. The surface chemistry of the sand-water suspension was kept constant by adding 2g/ℓ of sodium chloride for all tests. The effect of particle shape was not considered due to the difficulties associated with quantifying particle shape. To ensure that the shapes for the two sand types used in the experimental work were not very different, a qualitative comparison was performed using an electron microscope. The shapes for particles from the three samples were found to be predominantly spherical. The solids concentration was varied to cover the range 10 to 50 per cent by volume.

Table II

Experimental programme for the viscosity tests

Particle size	Solids concentration by volume (%)	Solids concentration by mass (%)	Solids density (g/cm³)
d_{50} – 90 microns	8	23	2.65
	20	40	2.65
d_{50} – 180 microns	30	53	2.65
	40	64	2.65
d_{50} – 300 microns	50	73	2.65

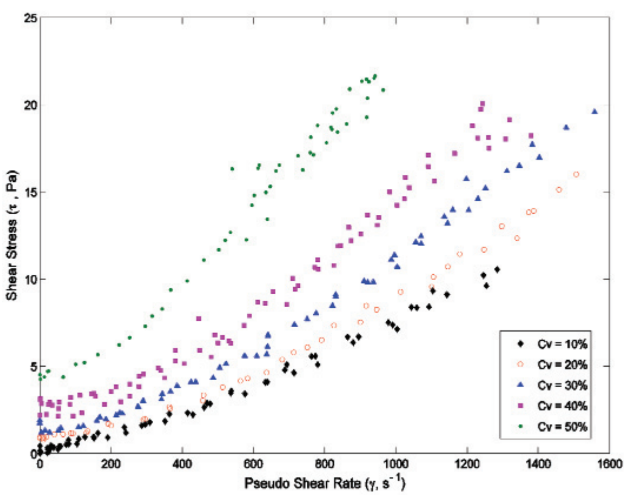

Figure 3—Rheograms for the d_{50} – 90 µm sand at different solids concentrations

Results and discussion

For each test condition the tube rheometer provided the variation of shear stress with shear rate. Figure 3 to Figure 5 show the rheograms obtained for the three sand size distributions at solids concentrations ranging from 8 to 50% by volume. The flow curves appear to be non-Newtonian in all cases and can be described as yield dilatants. It was also observed that the yield stress increased with increase in the solids concentrations for all cases. Klimpel also observed dilatant behaviour at low solids concentrations but his results appeared to be pseudoplastic behaviour at solids concentrations greater than 45% solids by volume (Klimpel, 1984).

The rheological data was then fitted to the Otswald-de Waele, Bingham, Herschel-Buckley and Casson models to identify the rheological model that best describes the data.

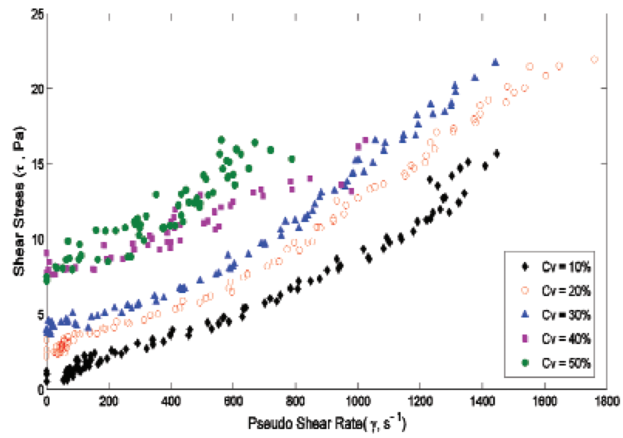

Figure 4—Rheograms for the d_{50} – 180 µm sand at different solids concentrations

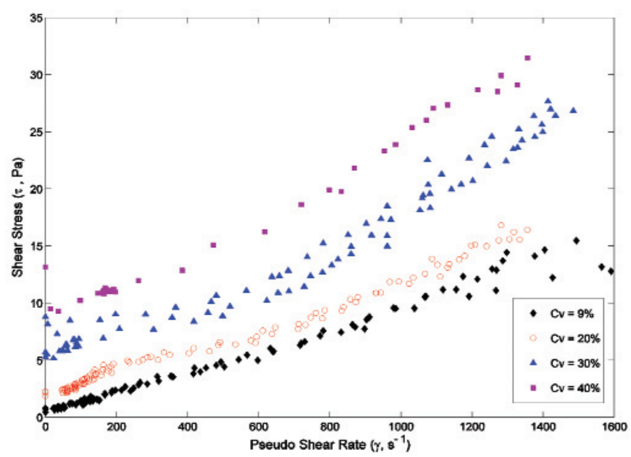

Figure 5—Rheograms for the d_{50} – 300 µm sand at different solids concentrations

Model fitting

In an attempt to find the model that best describes the data, four models from the literature, namely the Otswald-de Waele, Bingham, Herschel-Buckley, and Casson models were fitted to the data. It should be noted that all the models, used in this work have the yield stress and power parameters, and inspection of the experimental data appears to suggest that these would be the most suitable models. Figure 6 to Figure 9 show the fitting results from the four models on the same data-set. It can be seen that all the models fitted the general trend well but the Herschel-Buckley model matched the experimental yield stress closely. All the models had high R^2 values ranging from 0.95 to 0.99 and the highest value was from the Herschel-Buckley fit. The same procedure was applied to the data from all the tests and similar results were obtained; the Herschel-Buckley model appears to fit the data better than the other models tested.

Figure 10 and Figure 11 show a comparison of the Herschel-Buckley fit for all three size fractions at solids concentrations of 20% and 40%, respectively. Reasonably high R^2 values were obtained in all cases. The corresponding

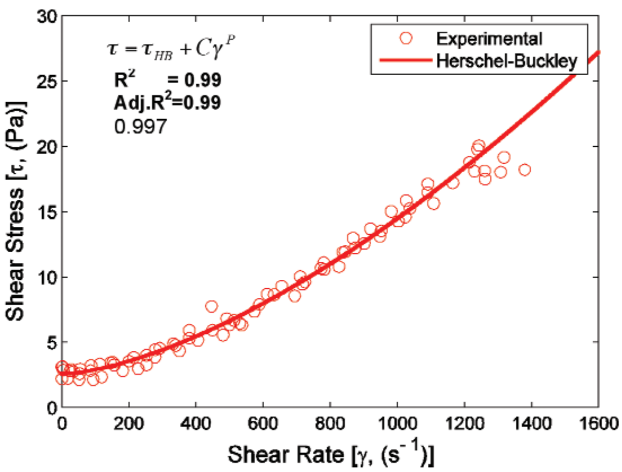

Figure 6—Herschel-Buckley model fit to d_{50} - 90 microns sand at Cv = 40% (Model parameters: τ_{HB}= 2.574, C= 2.66*10^{-4} and P = 1.55)

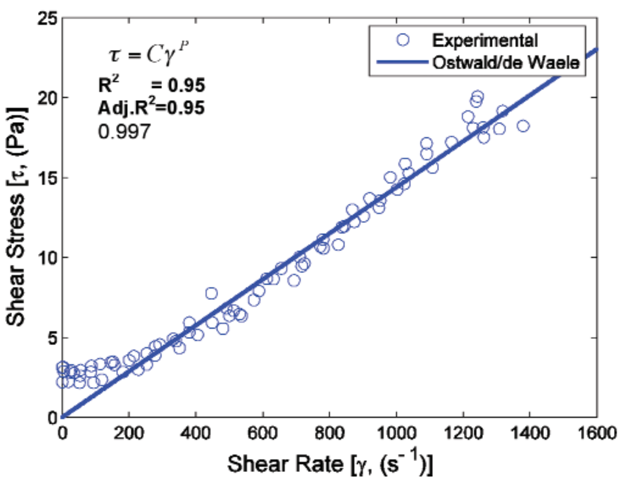

Figure 7—Ostwald-de Wael model fit to d_{50} - 90 microns sand at Cv = 40% (Model parameters: C= 0.0142 and P = 1.002)

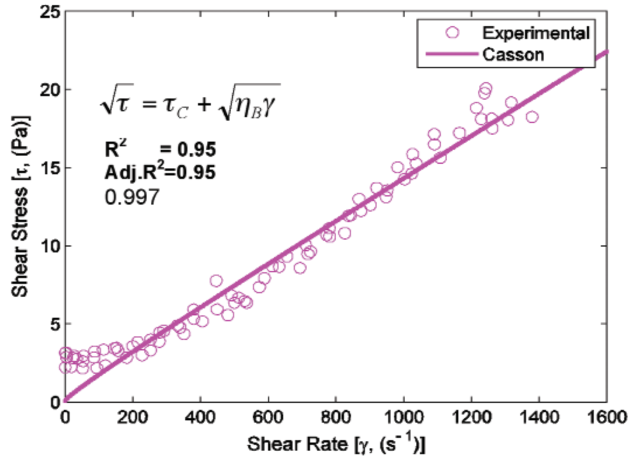

Figure 8—Casson model fit to d_{50} - 90 microns sand at Cv = 40% (Model parameters: τ_C= 0.183 and η_B= 0.013)

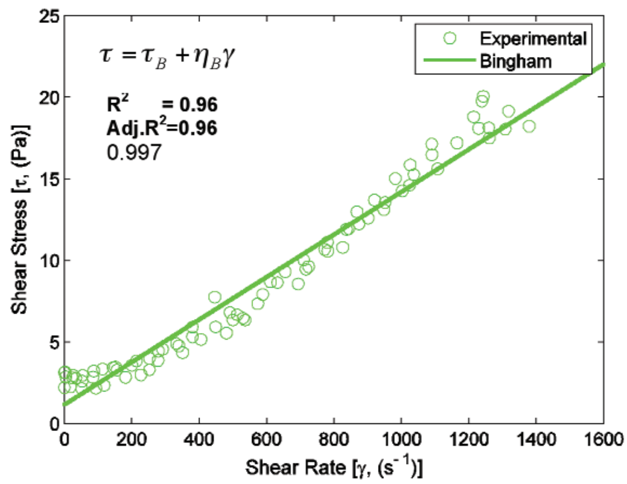

Figure 9—Bingham model fit to d_{50} - 90 microns sand at Cv = 40% (Model parameters: τ_B= 1.147 and η_B= 0.013)

Herschel-Buckley model parameters obtained for each of the fits are given in Table III and Table IV, respectively. The power index (P) is greater than 1 in all cases, suggesting that the flow curves exhibit dilatant behaviour. It was observed from Table III and Table IV that the yield stress parameter τ_{HB} increased with particle size.

The shear stress values obtained from the Herschel-Buckley model were then used to calculate apparent viscosity using Equation [1] The effect of solids concentration and particle size on apparent viscosity were then analysed.

$$\eta = \frac{\tau}{\gamma} \qquad [1]$$

where η is apparent viscosity, τ is the shear stress and γ, the shear rate.

Effect of solids concentration on viscosity

Figure 12 to Figure 14 show the variation of apparent viscosity with shear rate at different solids concentration for the three size fractions. It can be seen that the apparent viscosity increased with increase in solids concentration. The exception to this trend was the test for the 40% solids concentration for the size fractions with d_{50} of 300 μm where

Table III

Herschel Buckley parameters for tests performed at Cv = 20%

	$\tau_{HB} \pm \Delta\tau_{HB}$ (Pa)	C ± ΔC (Pa.s)	P ± ΔP
d_{50} - 90 μm	0.918 ± 0.1	(18.4 ± 5)*10-5	1.545 ± 0.04
d_{50} - 180 μm	2.810 ± 0.2	(40.1 ± 1)*10-5	1.458 ± 0.05
d_{50} - 300 μm	5.441 ± 0.2	(4.27 ± 4)*10-5	1.780 ± 0.1

Table IV

Herschel Buckley parameters for tests performed at Cv = 40%

	$\tau_{HB} \pm \Delta\tau_{HB}$ (Pa)	C ± ΔC (Pa.s)	P ± ΔP
d_{50} - 90 μm	2.574 ± 0.3	(26.6 ± 20)*10-5	1.550 ± 0.08
d_{50} - 180 μm	7.838 ± 0.4	(122 ± 170)*10-5	1.274 ± 0.2
d_{50} - 300 μm	9.659± 0.3	(104± 70)*10-5	1.382 ± 0.1

the curve approaches the 30% solids concentration curve at shear rates above 600 s^{-1}. This result could be erroneous because difficulties were encountered in using the rheometer at high solids concentration. Due to these problems the maximum shear rate that could be measured for the d_{50} of180 μm sand was 500 s^{-1} for Cv = 40% and Cv = 50%.

Similar to the results obtained by Cheng, the increase in apparent viscosity with increase in solids concentration can be attributed to increased particle-particle interactions in the fluid (Cheng, 1980). The study by Cheng suggests that the relatively coarse particles have negligible interparticle attraction, which promotes the formation of flocs, aggregates, agglomerates and structure.

Thus at low to medium solids concentrations, the effect of hydrodynamic interactions prevails and they give rise to the viscous dissipation of the liquid. As a result, the viscosity increases with increase in concentration. This is also observed at high solids concentrations but the particle-particle contact effect predominates over the hydrodynamic effect. This particle-particle contact brings into play frictional interactions. The frictional interactions per unit volume increase with solids concentration, thereby increasing the

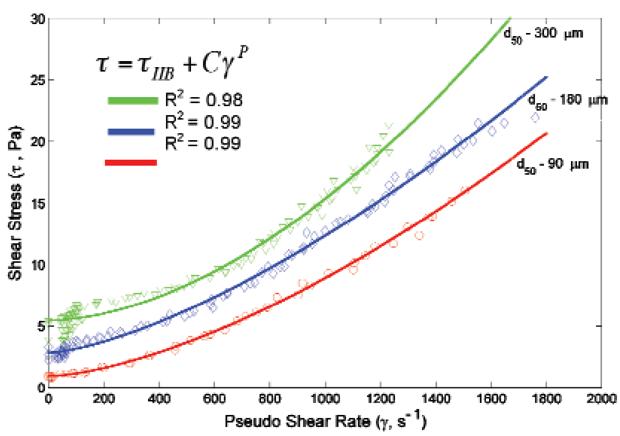

Figure 10—Comparison of Herschel-Buckley fit for the three different size fractions at Cv = 20%

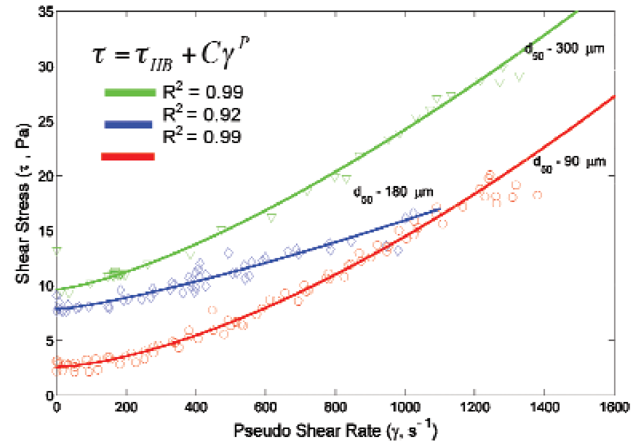

Figure 11—Comparison of Herschel-Buckley fit for the three different size fractions at Cv = 40%

viscosity of slurry. Cheng did not quantify what low, medium or high solids concentration was, thus the observed phenomenon could be due to either hydrodynamic interactions or particle-particle contact.

Shear thinning behaviour was observed in the low shear rate region of the curve and the curves appear to depict slight shear thickening behaviour at higher shear rates. Similar results were obtained by Olhero and Ferreira from experiments performed on fine silica powder of average size 19 μm (Olhero and Ferreira, 2003).

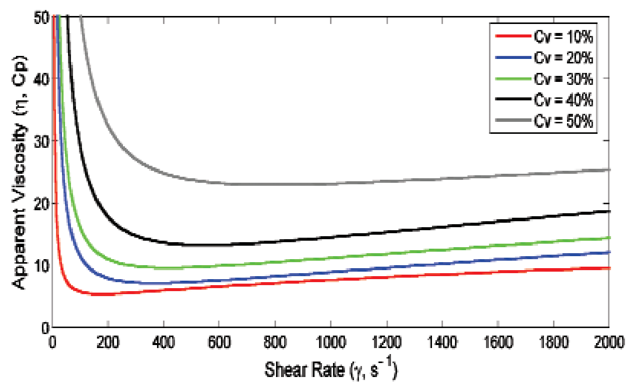

Figure 12—Apparent viscosity of d_{50} - 90 microns sand as function of shear rate

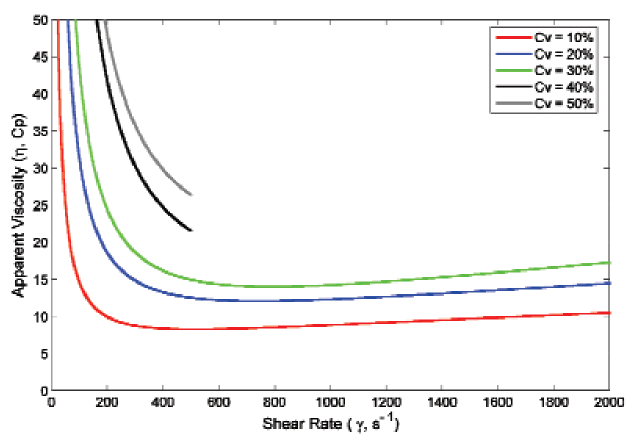

Figure 13—Apparent viscosity of d_{50} - 180 microns sand as a function of shear rate

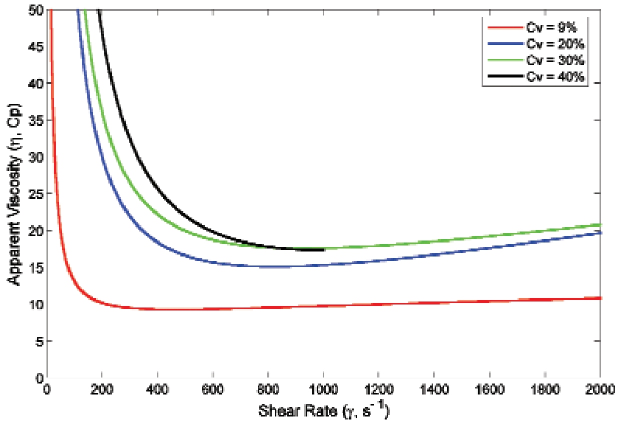

Figure 14—Apparent viscosity of d_{50} - 300 microns sand as a function of shear rate

Effect of particle size on apparent viscosity

Figure 15 to Figure 17 show that viscosity increases with particle size at a fixed concentration for any given shear rate. This is similar to what was observed by Clarke (1967). The reason suggested for this increase was that particles of greater size possess greater inertia such that on interaction, the particles are momentarily retarded and then accelerated. In both these stages their inertia affects the amount of energy required. This dissipation of energy is what may appear as extra 'viscosity'.

Relationship between yield stress and particle size and concentration

Yield stress is used to characterize slurry rheology and it represents the threshold amount of stress to initiate flow. Figure 18 shows the effect of particle size and concentration on the yield stress. For a given particle size, as solids concentration increases, the yield stress increases. The trends in Figure 18 indicate that the yield stress for the larger particle size is higher than that of smaller particle sizes at a fixed solids concentration. Slurries with higher yield stresses require a higher initial input energy before the fluid starts to flow. For the smallest sized fraction at 10% solids concentration the yield stress is close to zero but increases to as high as 4 Pa at 50% solids concentration. For coarser particle sizes tested the yield stress is almost 10 Pa at 40% solids concentration.

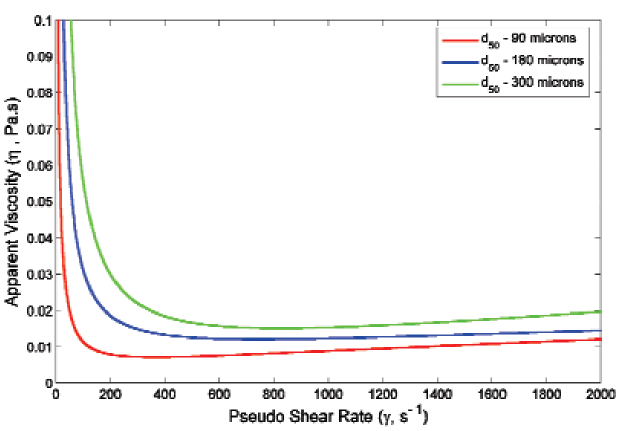

Figure 15—Effect of particle size on viscosity at Cv = 20%

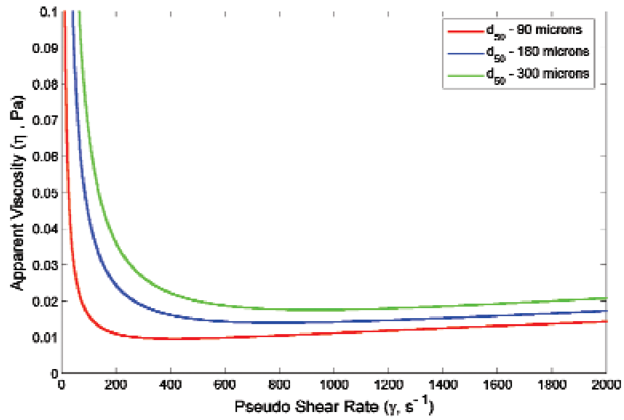

Figure 16—Effect of particle size on viscosity at Cv = 30%

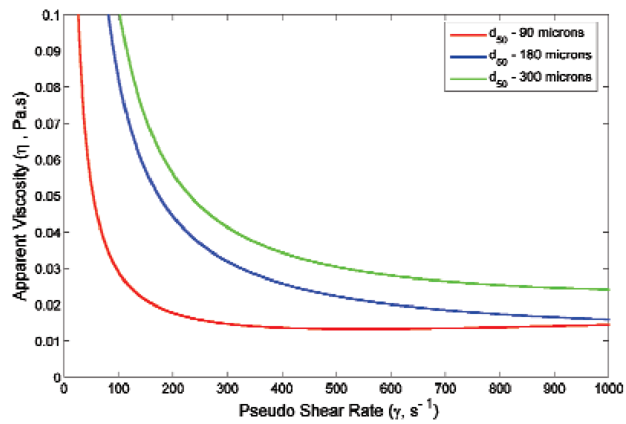

Figure 17—Effect of particle size on viscosity at Cv = 40%

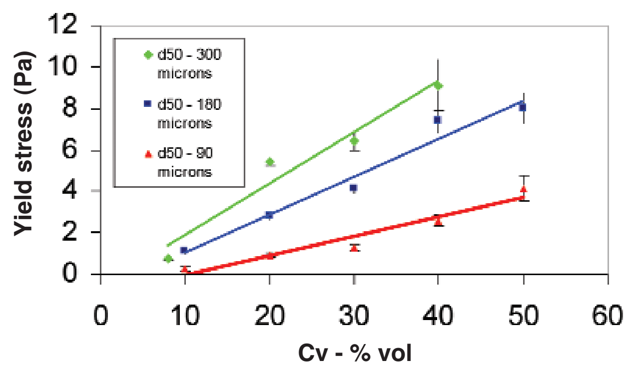

Figure 18—Effect of particle size and solids concentration on yield stress

Conclusions

Tests were performed to characterize the rheology of the silica-sand water suspension at conditions that are encountered in tumbling mills. The flow curves appear to be non-Newtonian and exhibit shear thickening behaviour in all cases and can be described as yield dilatants. Of the models tested, the Herschel-Buckley model was found to give the best description of the flow curves.

Shear thinning behaviour was observed in the low shear rate region of the curve, and the curves appear to depict slight shear thickening behaviour at higher shear rates.

The apparent viscosity increased with increase in particle size and solids concentration. It was also observed that the yield stress increased with increase in particle size and solids concentrations for all cases.

Acknowledgements

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