Investigating continuous time open pit dynamics

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Synopsis

Current mine production planning, scheduling, and allocation of resources are based on mathematical programming models. In practice, the optimized solution cannot be attained without examining all possible combinations and permutations of the extraction sequence. Operations research methods have limited applications in large-scale surface mining operations because the number of variables becomes too large. The primary objective of this study is to develop and implement a hybrid simulation framework for the open pit scheduling problem. The paper investigates the dynamics of open pit geometry and the subsequent material movement as a continuous system described by time-dependent differential equations. The continuous open pit simulator (COPS) implemented in MATLAB, based on modified elliptical frustum is used to model the evolution of open pit geometry in time and space. Discrete open pit simulator (DOPS) mimics the periodic expansion of the open pit layouts. Function approximation of the discrete simulated push-backs provides the means to convert the set of partial differential equations (PDEs), capturing the dynamics of open pit layouts, to a system of ordinary differential equations (ODEs). Numerical integration with the Runge-Kutta scheme yields the trajectory of the pit geometry over time with the respective volume of materials and the net present value (NPV) of the mining operation. A case study of an iron ore mine with 114 000 blocks was carried out to verify and validate the model. The optimized pit limit was designed using Lerchs-Grossman's algorithm. The best-case annual schedule, generated by the shells node in Whittle Four-X yielded an NPV of $449 million over a 21-year mine life at a discount rate of 10% per annum. DOPS best scenario out of 2 500 simulation iterations resulted in an NPV of $443 million and COPS yielded an NPV of $440 million over the same time span. The hybrid simulation model is the basis for future research using reinforcement learning based on goal-directed intelligent agents.

Introduction

The final pit limits optimization algorithms conventionally search for an ultimate contour, which maximizes the total sum of the profits of all the blocks in the contour. The optimal final pit limits is an important key for long-term strategic planning. Current algorithms assume that this contour is dug at once without considering the time aspect of the problem. The planning of an open pit mine considers the temporal nature of the exploitation to determine the sequence of block extraction in order to maximize the generated income throughout the entire planning period. Mine planning as an economic exercise is constrained by certain geological, operating, technological, and local field constraints. The mine planning models usually define a discrete finite planning horizon. These models usually attempt to maximize the discounted present value of profit[1–6], or to optimize the plant feeding conditions[7–10]. Heuristic methods, economic parametric analysis, operations research, and genetic algorithms have been used to formulate periodic open pit planning problems. Open pit design, optimization, and subsequent materials scheduling are governed by stochastic dynamic processes. Thus, current algorithms are limited in their abilities to address the problems arising from these random and dynamic field processes[11,12]. In practice, the optimized mining schedule cannot be attained without examining all possible combinations and permutations of the extraction sequence. Therefore, to solve large industrial problems with efficiency, it is crucial to deal with the limitations of computing resources, time and space.

The main purpose of this study is to model the dynamics of open pit geometry and the materials movement as a continuous system described by time-dependent differential equations. The simulator uses a geometrical open pit model based on modified elliptical frustum[13] to capture the changes of the open pit geometry and the subsequent materials' movement. A set of partial differential equations (PDEs) captures the time-related behaviour of the open pit mining systems. Merely the specification of the PDEs does not allow a unique solution to the problem,

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because an indefinite integral must be integrated. Therefore, there is a need for additional information and auxiliary conditions to obtain a unique solution. The discrete stochastic simulation model[14] is used to yield the additional and auxiliary conditions needed to convert the set of PDEs to a set of ordinary differential equations (ODEs).

The discrete open pit production simulator[13] (DOPS) mimics the periodic expansion of the open pit layouts. The interaction of the economic expansion model (EPEM) with the geological and economic block model returns the respective amount of ore, waste, stockpile materials, and the net present value (NPV) of the venture. The simulation is run under different scenarios with a sufficient number of iterations to yield the sequence of extraction, which maximizes the NPV, subject to all the underlying constraints. Function approximation of the discrete simulated open pit push-backs provides the framework to reduce the number of independent variables, and convert the set of PDEs to a system of ODEs. Numerical integration with the Runge-Kutta[15] scheme yields the trajectory of the pit geometry over time with the respective volume of materials transferred. Interaction of the continuous open pit simulator (COPS) with EPEM returns the cash flow of the mining operation throughout the mine life.

This paper discusses the theoretical framework of the study based on open pit geometrical model, continuous time open pit dynamics, and the economic pit expansion model. It contains COPS models, its application to an iron ore mine, and comparative analysis of COPS vs. DOPS schedule. The NPV of the best-case schedule, generated by parametric analysis using Whittle Four-X[16] software, is compared to the results of DOPS.

Geometrical model of an open pit layout

Ore and waste extraction from an open pit mine takes place on different elevations. The pit expands horizontally and vertically towards the final pit limits. The main long-term objective is to meet quantity and quality targets of production in order to maximize the market value of the mining venture. There is therefore a need for models that capture the evolution of the open pit geometry as the pit expands through time and space. Frimpong *et al.*[11] provided a basis for using the solid geometry of an elliptical frustum to model the open pit expansion process. The assumption underlying the elliptical frustum causes a considerable error in volume computations. To reduce this error, a more reliable, modified elliptical frustum model was presented by Askari-Nasab *et al.*[13]. The modified geometry consists of four quadrants of elliptical frustums, which are appended along the major and minor axes of the top ellipsoid. Figure 1 illustrates the modified elliptical frustum, which is defined by two ellipsoids with areas A_1 and A_2 separated by a vertical distance h. This model divides the open pit into four sections, north-west, north-east, south-west, and south-east. Each area is defined by the major and minor axes of the respective top ellipsoid. The overall stable pit slope θ is defined for each region according to the rock slope stability and geomechanical studies.

The volume of material in each area is given by a quarter of the volume of Equation [4]. Table I illustrates variables defining open pit quadrants. The areas of the top and bottom ellipsoids, defining the frustum, are given by Equations [1], [2], and [3].

$$A_1 = A_{NW} + A_{NE} + A_{SW} + A_{SE} \quad [1]$$

$$A_1 = \frac{\pi}{4} \times \left(a_W b_N + a_E b_N + a_W b_S + a_E b_S \right) \quad [2]$$

$$A_2 = \frac{\pi}{4} \times \begin{bmatrix} (a_W - \frac{h}{\tan\theta_{NW}})(b_N - \frac{h}{\tan\theta_{NW}}) + (a_E - \frac{h}{\tan\theta_{NE}}) \\ (b_N - \frac{h}{\tan\theta_{NE}}) + a_W - \frac{h}{\tan\theta_{SW}}(b_S - \frac{h}{\tan\theta_{SW}}) + \\ (a_E - \frac{h}{\tan\theta_{SE}})(b_S - \frac{h}{\tan\theta_{SE}}) \end{bmatrix} \quad [3]$$

The total volume of material in the frustum is given by Equation [4]:

$$V = \frac{1}{3} \times \left[A_1 + A_2 + \left(A_1 \times A_2 \right)^{1/2} \right] \times h \quad [4]$$

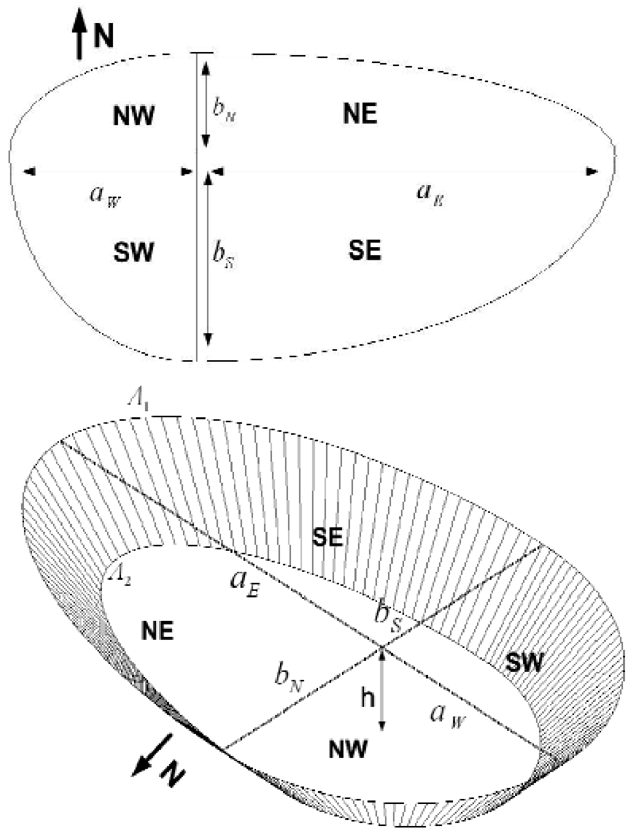

Figure 1—Modified geometrical frustum model

Table I

Frustum variables in each region of open pit

Region	Major axis (m)	Minor axis (m)	Slope (degree)	Area (m²)
NW	a_W	b_N	θ_{NW}	A_{NW}
NE	a_E	b_N	θ_{NE}	A_{NE}
SW	a_W	b_S	θ_{SW}	A_{SW}
SE	a_E	b_S	θ_{SE}	A_{SE}

Continuous modelling of open pit dynamics

The mechanics of open pit layout expansion depends on the production rate, which is a function of several variables. These variables include: (i) volume of proven reserve; (ii) reserve geometry; (iii) proposed mine life; (iv) capacity of loading and hauling fleet; (v) number of active loading equipments; (vi) number of active working faces; (vii) milling and processing plant capacity; (viii) mill head grade; and (ix) site and working conditions. A uniform feed is required to maintain the plant's optimal capacity during the mine life. Continuous modelling of open pit dynamics involves characterization of the changes made in the geometry of the open pit and the volume of materials moved by a set of differential equations. In general, the model does not have analytic solutions, and requires using approximate solution methods such as finite elements, finite differences, or boundary elements. Discretization of the open pit dynamics concerns the process of transferring the continuous models and equations into discrete counterparts. Typically discretization involves splitting the region of interest into a set of small elements, producing a discrete approximation of the differential equations in each element, and then solving all of the discrete approximations simultaneously. The total volume of material transferred throughout the pit expansion process given by Equation [4] can be represented as the differential changes in volume of material by Equation [5]. The differential changes in volume, ΔV, is a function of the fleet size, F_S; fleet capacity, F_C; availability, A; utilization, U; cycle time, C_T; the loose density of materials, γ_l; and the elliptical frustum parameters, ϕ_2.

$$\Delta V = V_{i+1} - V_i = \psi(\phi_1(F_S, F_C, A, U, C_T, \gamma_l); \phi_2(a_W, a_E, b_N, b_S, h, \theta_{NW}, \theta_{NE}, \theta_{SW}, \theta_{SE})) \qquad [5]$$

The geometrical model can capture the push-back outlines of most of the open pits with a good approximation. The modified elliptical frustum will have some limitations in calculating the volume of materials in very irregular shaped open pits. Current research is focusing on developing simulation models of dynamics of each bench separately. The results are expected be able to overcome the current error for very irregular shape open pits.

DOPS[13], based on Monte Carlo simulation methods, returns a series of equally probable simulated realizations of the long-term mine plan. One of these realizations would be chosen as a guide for the long-term plan, which best satisfies the objectives of the management. After the approval of the strategic plan, the next step is to break it into operating and achievable targets within the framework of a tactical plan. COPS generates a schedule based on the same conditions underlying the strategic plan. It defines periodic targets within a shorter time frame. The amount of change in volume given by Equation [5] depends on the production planning and scheduling requirements. At this stage the goal is to design a production process to make infinitesimal changes in Equation [5] with respect to changes in parameters of function ϕ_2. The result will be sequential $\Delta V's$ with respect to any changes in production time, Δt. Overall slopes in different regions θ_{NW}, θ_{NE}, θ_{SW}, θ_{SE}, are assumed to be constant over each region. The changes in parameters of function $\theta_1(F_S, F_C, A, U, C_T, \gamma_l)$ will have a direct effect on ΔV, which are defined in terms of Δa_W, Δa_E, Δb_N, Δb_S and Δh. So V will be a function of five variables a_W, a_E, b_N, b_S, and h. V, (a_W, a_E, b_N, b_S, h) is assumed as an analytic function, so all its derivatives would exist. As a result, V can be represented by its Taylor series expansion in five variables. The generalized form of Taylor series expansion for a real function in n variables[17] is given by:

$$f(x_1 + a_1, \ldots, x_n + a_n) = \sum_{j=0}^{\infty} \left\{ \frac{1}{j!} \left(\sum_{k=1}^{n} a_k \frac{\partial}{\partial x'_k} \right)^j f(x'_1, \ldots, x'_n) \right\}_{x'_1 = x_1, \ldots, x'_n = x_n} \qquad [6]$$

Accordingly, the Taylor series expansion of function V (a_W, a_E, b_N, b_S, h) in five variables is given by Equation [7]:

$$V(a_W + \Delta a_W, a_E + \Delta a_E, b_N + \Delta b_N, b_S + \Delta b_S, h + \Delta h) = \sum_{j=0}^{\infty} \left\{ \frac{1}{j!} \left(\Delta a_W \frac{\partial}{\partial x'_1} + \Delta a_E \frac{\partial}{\partial x'_2} + \Delta b_N \frac{\partial}{\partial x'_3} + \Delta b_S \frac{\partial}{\partial x'_4} + \Delta h \frac{\partial}{\partial x'_5} \right)^j f(x'_1, x'_2, x'_3, x'_4, x'_5) \right\}_{x'_1 = a_W, x'_2 = a_E, x'_3 = b_N, x'_4 = b_S, x'_5 = h} \qquad [7]$$

Using the second order approximation in Equation [7] yields to Equation [8]:

$$\begin{aligned} V\begin{pmatrix} a_W + \Delta a_W, a_E + \Delta a_E, b_N + \\ \Delta b_N, b_S + \Delta b_S, h + \Delta h \end{pmatrix} &= V(a_W, a_E, b_N, b_S, h) + \\ \frac{\partial V}{\partial a_W}\Delta a_W + \frac{\partial V}{\partial a_E}\Delta a_E + \frac{\partial V}{\partial b_N}\Delta b_N + \frac{\partial V}{\partial b_s}\Delta b_S + \frac{\partial V}{\partial h}\Delta h + \\ \frac{1}{2}\frac{\partial^2 V}{\partial a_W^2}(\Delta a_W)^2 + \frac{1}{2}\frac{\partial^2 V}{\partial a_E^2}(\Delta a_E)^2 + \frac{1}{2}\frac{\partial^2 V}{\partial b_N^2}(\Delta b_N)^2 + \\ \frac{1}{2}\frac{\partial^2 V}{\partial b_S^2}(\Delta b_S)^2 + \frac{1}{2}\frac{\partial^2 V}{\partial h^2}(\Delta h)^2 + \frac{\partial^2 V}{\partial a_E \partial a_W}\Delta a_E \Delta a_W + \\ \frac{\partial^2 V}{\partial a_E \partial b_N}\Delta a_E \Delta b_N + \frac{\partial^2 V}{\partial a_E \partial b_S}\Delta a_E \Delta b_S + \frac{\partial^2 V}{\partial a_E \partial h}\Delta a_E \Delta h + \\ \frac{\partial^2 V}{\partial a_W \partial b_N}\Delta a_W \Delta b_N + \frac{\partial^2 V}{\partial a_W \partial b_S}\Delta a_W \Delta b_S + \frac{\partial^2 V}{\partial a_W \partial h}\Delta a_W \Delta h + \\ \frac{\partial^2 V}{\partial b_N \partial b_S}\Delta b_N \Delta b_S + \frac{\partial^2 V}{\partial b_N \partial h}\Delta b_N \Delta h + \frac{\partial^2 V}{\partial b_S \partial h}\Delta b_S \Delta h + O(h^3) \end{aligned} \qquad [8]$$

According to Equation [8], the changes in volume of the open pit geometry at any specific period of time, Δt, could be captured as a set of partial differential equations. The boundary conditions underlying Equation [8] are the initial box cut and the final pit limits geometry. The volume of open pit is a function given by Equation [4]. The first order, second order, and cross term partial derivatives in Equation [8] are derived from Equation [4]. There is a need to represent Δa_W, Δa_E, Δb_N, Δb_S, and Δh as functions of time. Functional approximation of DOPS simulation results is used to yield the additional conditions needed to convert the set of PDEs to a system of ODEs. DOPS returns the feasible mining increments Δa_W, Δa_E, Δb_S, Δb_N, and Δh at the end of each period of production. DOPS simulates tabular data for the increments over the mine life. Trend analysis and curve fitting are used to approximate functions for increments Δa_W,

Δa_E, Δb_S, Δb_N, and Δh over the mine life. The increments are represented as functions of time. Substitution of the approximated incremental functions of Δa_W, Δa_E, Δb_S, Δb_N, and Δh, in Equation [8], and replacement of the partial derivatives derived from Equation [4] in Equation [8], converts the set of PDEs into a system of ODEs with time as the only independent variable. Thus, the changes in the volume of the frustum as well as the changes in depth, major, and minor axes of the frustum are represented as functions of time given by Equations [9] to [14].

$$\frac{d\underline{V}}{dt} = f_1(\underline{V}, t);\ V(t_0) = V_0,\ V(t_n) = V_{pit\_limit} \quad [9]$$

$$\frac{d\underline{a_W}}{dt} = f_2(\underline{a_W}, t);\ a_W(t_0) = a_{W0},\ a_W(t_n) = a_{W-\max} \quad [10]$$

$$\frac{d\underline{a_E}}{dt} = f_3(\underline{a_E}, t);\ a_E(t_0) = a_{E0},\ a_E(t_n) = a_{E-\max} \quad [11]$$

$$\frac{d\underline{b_S}}{dt} = f_4(\underline{b_S}, t);\ b_S(t_0) = b_{S0},\ b_S(t_n) = a_{S-\max} \quad [12]$$

$$\frac{d\underline{b_N}}{dt} = f_5(\underline{b_N}, t);\ b_N(t_0) = b_{N0},\ b_N(t_n) = b_{N-\max} \quad [13]$$

$$\frac{d\underline{h}}{dt} = f_6(\underline{h}, t);\ h(t_0) = h,\ h(t_n) = h_{\max} \quad [14]$$

The set of Equations [9] to [14] with the respective boundary conditions are represented as Equations [15] to [17].

$$\frac{d\underline{U}}{dt} = \underline{\underline{A}}.\underline{U} + \underline{B} \quad [15]$$

Where:

$$\frac{d\underline{U}}{dt} = \begin{bmatrix} \frac{d\underline{V}}{dt} \\ \frac{d\underline{a_W}}{dt} \\ \frac{d\underline{a_E}}{dt} \\ \frac{d\underline{b_S}}{dt} \\ \frac{d\underline{b_N}}{dt} \\ \frac{d\underline{h}}{dt} \end{bmatrix} \quad [16]$$

$$\underline{U} = \begin{bmatrix} \underline{V} \\ \underline{a_W} \\ \underline{a_E} \\ \underline{b_N} \\ \underline{b_S} \\ \underline{h} \end{bmatrix} \quad [17]$$

The numerical integration of the system of ODEs specified by Equation [15] captures the behaviour of pit shell expansion over time. The results are a practical guide for the short-term production planning. The most commonly used family of numerical integration schemes, Runge-Kutta[15], is used by COPS to capture the dynamics of the continuous-time system described by the set of ODEs given by Equation [15]. The hybrid simulation model above is a combination of discrete and continuous-time mathematical models, numerical solutions, and analytic techniques to capture the dynamics of open pit and material movements in open pit environment.

Economic pit expansion model

A useful planning model must be able to relate the dynamics of the open pit with the geological and economic block model. Such a model must yield grade of ore, stockpile, and contaminant materials. It must also provide the amount of ore and waste moved on a bench-by-bench basis, as well as, the economics of the push-back design for each period of the mining operations. EPEM places the modified open pit geometrical model on the economic block model, and it returns the pit monetary value, average grade of ore, waste, and stockpile material at any desired period of production. Figure 2 illustrates how EPEM fits the open pit geometrical model on the economic block model. The centre of the top

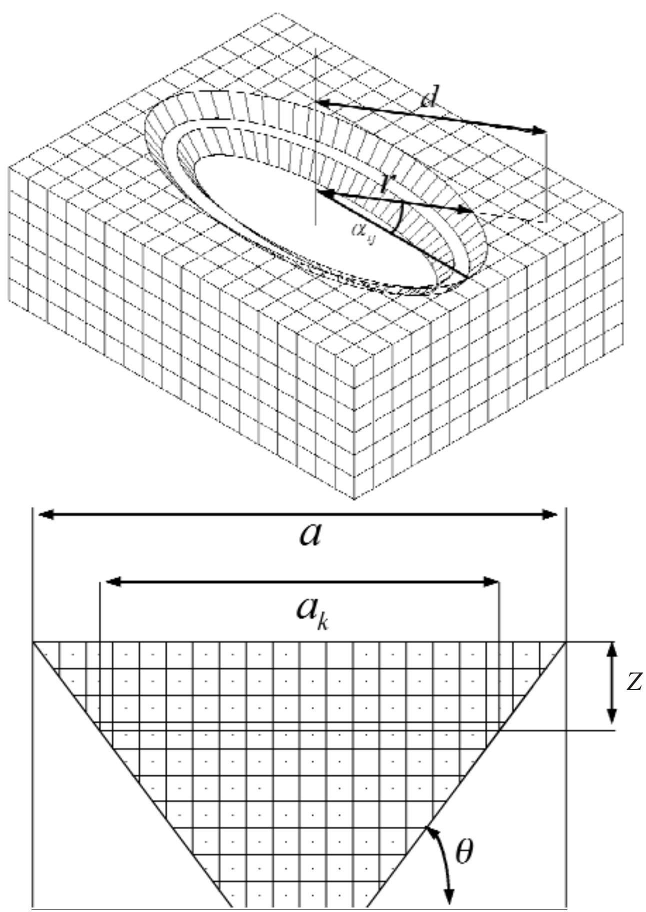

Figure 2—Economic pit expansion model

ellipse is located on the excavation starting point. The procedure starts searching the economic block model level by level. In each level, the distance between the centre of the ellipse and the centre of the current block denoted by Equation [19] is compared to the length of the radii of the ellipse given by Equation [24]. Afterward, a decision is made as to whether or not the block is inside the frustum. In the following formulas and Figure 2: $C(x_0, y_0)$ = centre of the ellipse; (x_0, y_0) = starting point of extraction; α_{ij} = the angle between the centre of the ellipse and the block (i,j) denoted by Equation [18]; k=0,1,...,n-1, number of levels in the block model; h = bench height; d = the distance between the centre of each block and the centre of the ellipse denoted by Equation [19], r = the distance between the centre of the ellipse to the perimeter in the direction of d given by Equation [24].

$$\alpha_{ij} = \arctan\left(\frac{|y_{ij} - y_0|}{|x_{ij} - x_0|}\right) \quad [18]$$

$$d = \left[\left(x_{ij} - x_0\right)^2 + \left(y_{ij} - y_0\right)^2\right]^{1/2} \quad [19]$$

The equation of the ellipse is as follows:

$$\frac{\left(x - x_0\right)^2}{a^2} + \frac{\left(y - y_0\right)^2}{b^2} = 1 \quad [20]$$

In polar coordinates, the angle α_{ij} in Equation [18] is called the eccentric angle. Substituting the polar equations into Cartesian coordinates in Equation [20] and solving for r will yield in Equation [24].

$$z = k \times h + \left(h/2\right) \quad [21]$$

$$a_k = a - \frac{z}{\tan\theta} \quad [22]$$

$$b_k = b - \frac{z}{\tan\theta} \quad [23]$$

$$r = \left(\frac{b_k^2 . a_k^2}{b_k^2 . \cos^2\alpha_{ij} + a_k^2 . \sin^2\alpha_{ij}}\right) \quad [24]$$

The procedure compares r from Equation [24] with d from Equation [19] to decide if the block is in the current pit or not. The routine finally returns the volume of ore, waste, stockpile material, and their respective grades, and the cash flows of the simulated push-backs.

Continuous open pit simulator

COPS is a hybrid simulation model implemented in MATLAB, based on the modified open pit geometrical model, the system of PDEs capturing the continuous-time open pit dynamics, and the economic pit expansion search scheme.

The specification of the continuous-time open pit dynamics as a system of PDEs with the respective boundary conditions given by Equations [9] to [14] does not allow a unique solution to the long-term scheduling problem. There are possibly infinitely many solutions to the set of PDEs, or in other words there are many pushback designs that can deplete the orebody from the initial box cut to the final pit limits. Whereas general ODEs have solutions that are families with each solution characterized by the values of some parameters, for PDEs the solutions often are parameterized by functions. Informally put, this means that the set of solutions is much larger. Therefore, there is a need for additional information and conditions to obtain a unique solution to the continuous-time open pit expansion model, which satisfies the management objectives over time.

DOPS is used to yield the additional and auxiliary conditions needed to convert the set of PDEs to a set of ODEs. Discrete simulation is used to capture the open pit layout evolution as a result of material movement throughout the mine life. DOPS simulates the material movement in open pit mining process and records net pit value at the end of every period of pit expansion. The simulation is run under different scenarios with sufficient realizations to yield the sequence of extraction, which maximizes the NPV subject to all the underlying constraints. Figure 3 demonstrates the DOPS flow chart. The simulation generates probability distribution functions for incremental push-backs of Δa_E, Δa_W, Δb_N, Δb_S, and Δh. Using Monte Carlo methods, the simulator samples from incremental push-back distributions and interacts with EPEM. Accordingly it records the discrete changes in the open pit geometry, which maximizes the NPV among all the simulation realizations. The DOPS output is the input for COPS, where the set of PDEs is converted to a set of ODEs and is solved by Runge-Kutta integration scheme. The best-case outcome of the discrete simulation is used as the input for the continuous simulation model. The best curve fit is used to approximate functions of the incremental push-backs Δa_E, Δa_W, Δb_N, Δb_S, and Δh generated by DOPS. Numerical integration of the system of ODEs generates the continuous trajectory of changes in the open pit geometry, with its respective volume of ore, waste, stockpile material and the NPV of the venture. The first step in investigating the open pit dynamics with continuous-time systems characterized by the set of ODEs given by Equation [15] is integration to obtain trajectories. Since most ordinary differential equations are not soluble analytically, numerical integration is the choice to obtain information about the trajectory. Many different methods have been proposed and used to solve accurately various types of ordinary differential equations. However, there are a handful of methods known and used universally (i.e., Runge-Kutta, Adams-Bashforth-Moulton and Backward Differentiation Formulae methods). All these methods discretize the differential system to produce a difference equation or map. The numerical methods have the same aim that the dynamics of the map should correspond closely to the dynamics of the differential equation[15]. MATLAB uses explicit Runge-Kutta codes in its ode45 suite, which are used in integrating Equation [15] in COPS. The solution of the continuous system returns the trajectories of changes in major and minor axes of the frustum as well as the volume of materials transferred. Then, the solution is passed to the EPEM. The scheme returns the blocks in the push-back design contour, net present value of the simulated schedule, and the volume of ore and waste.

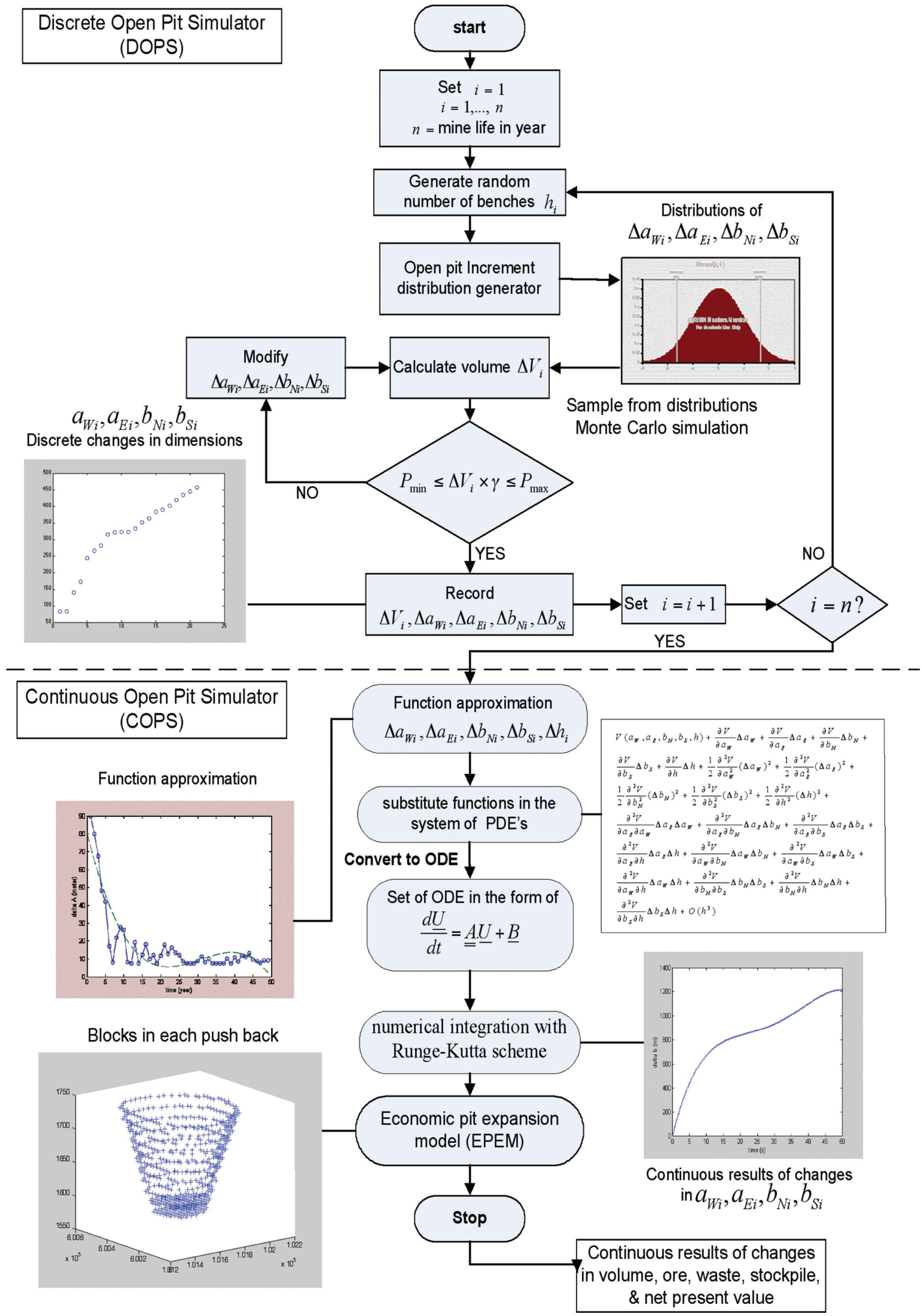

Figure 3—DOPS and COPS flowchart

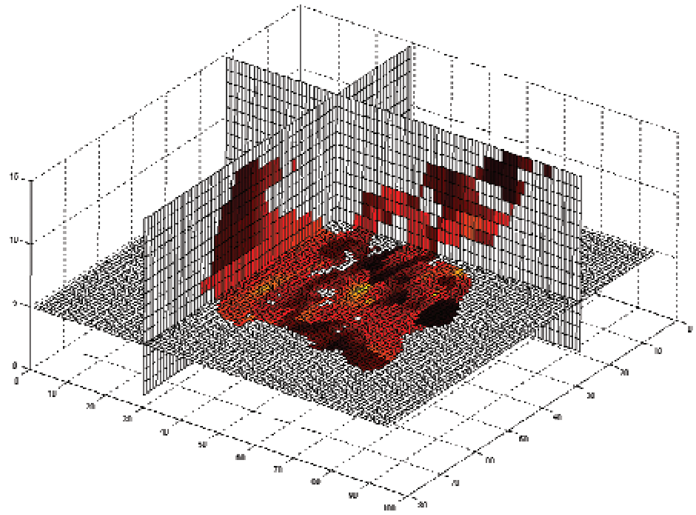

Figure 4—3D view of the deposit

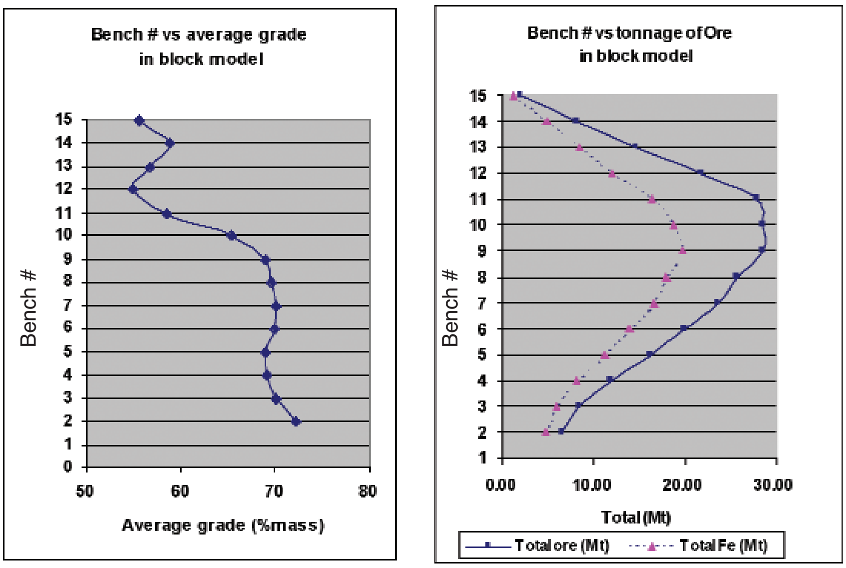

Figure 5—Tonnage of ore - bench report

Numerical application of COPS

A case study of an iron ore deposit is carried out to verify and validate the models. The case study is under the following assumptions: (i) no stockpiles or materials re-handling was considered; (ii) blending of materials was not considered; (iii) the mill head grade and the annual mill feed were not set as a rigid requirements.

The final pit limits are determined using the LG algorithm[18], using Whittle Four-X[16] software. The final pit limits geometry captured by the elliptical frustum model with parameters, a_W, a_E, b_N, b_S, and h, are the inputs into DOPS for simulation of the practical push-backs. The NPV of the best-case nested pits, designed with Whittle Four-X is compared to the results of the best-case DOPS simulation.

The iron ore deposit is explored with 159 exploration drill holes and 113 infill drill holes totalling 6 000 m of drilling. Three types of ore, top magnetite; oxide; and bottom magnetite are classified in the deposit. Processing plant is based on magnetic separators so the main criterion to send material from mine to the concentrator is weight recovery. Kriging is used to build the geological block model[19,20]. The small blocks represent a volume of rock equal to 20 m x 10 m x 15 m. The model contains 114 000 blocks that make a model framework with dimensions of 95 x 80 x 15. Figure 4 illustrates a multi cross-section of the deposit along sections 100100-east, 600245-north, and elevation of 1 590 m.

The block model contains almost 243 million tons of indicated resource of iron ore with the average grade of 63%. Figure 5 illustrates the average grade, total amount of ore, and iron ore concentrate on a bench-by-bench basis. Slope stability and geo-mechanical studies recommended a 43 degree overall slope in all regions. The economic block model is based on: (i) mining cost = \$2/ton; (ii) processing cost = \$2/ton; (iii) selling price = \$14/ton (Fe); (iv) maximum mining capacity = 20 Mt/year; (v) maximum milling capacity = 15 Mt/year; (vi) density of ore and waste = 4.2 ton/m^3; (vii) annual discount rate = 10%.

The dimensions of the final pit limits captured by modified elliptical frustum and designed with Whittle Four-X are defined as: a_W = 1 040m; a_E = 440 m; b_N = 260 m; b_S = 360 m; and h = 120 m. DOPS was used to run the simulation with 2 500 iterations with different scenarios of mining starting points. The best-case scenario had a starting point at 101600-east and 600340-north, which is located inside the smallest pit generated by nested pits in Whittle Four-X software. Figure 6 demonstrates the push-backs of the best-case schedule simulated by DOPS for years 16 to 18. The optimized final pit limits shows the total amount of 343 million tonnes of material consisting of 201 million tons of ore and 142 million tons of waste. The best-case long-term schedule designed with parametric analysis in Whittle Four-X shows an NPV of $449 million for a 21-year mine life. The results of the DOPS schedule under the same circumstances with equal mine life, maximum annual production, and milling capacity demonstrates an NPV of $443 million.

The best-case simulation result of DOPS, with the highest NPV is used as the input for COPS. The annual incremental push-backs generated by DOPS represents the discrete changes of Δa_W, Δa_E, Δb_N, Δb_S, and Δh. COPS requires the increments to be represented as functions of time. This feature facilitates capturing the dynamics of open pit expansion as a set of continuous time-dependent, differential equations. Function approximation of DOPS simulation results, is used to yield the additional conditions needed to convert the set of PDEs to a system of ODEs. Trend analysis and curve fitting are used to approximate functions for Δa_E, Δa_W, Δb_N, Δb_S, and Δh, and increments over the mine life. To obtain reliable results, goodness of fit statistics for all the approximations is evaluated by residuals. Residuals are the differences between the response data and the fit to the response data at each predictor value. The sum of squares due to error (SSE)[21] is evaluated, as well, to obtain the best fit.

Figure 7 illustrates the residuals of exponential, Gaussian, and polynomial function approximations on the data along axis, b_S. The Gaussian function with an SSE = 285.72 and rsquare = 0.99399 is the best fit among all the approximations.

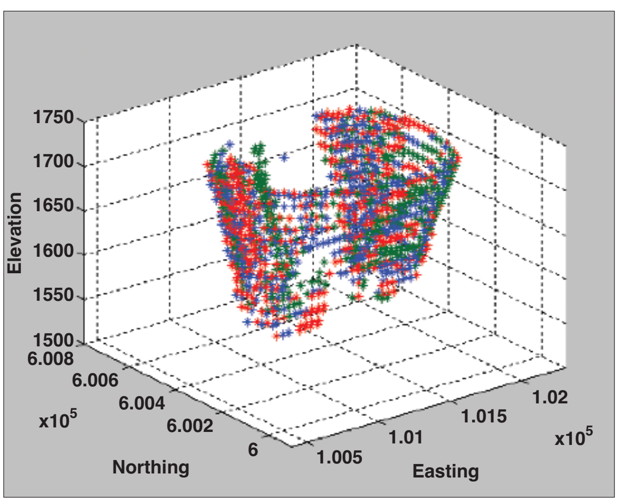

Figure 6—Push-backs years 16 to 18 (m)

Figure 8 demonstrates the Gaussian fit on the incremental changes along b_S with the error prediction bounds. The best fit analysis recommends Gaussian functions for Δa_W, Δa_E, and Δb_S, and exponential functions for Δb_N and Δh. The substitution of the approximated functions in Equation [8] converts the set of PDEs to a system of ODEs. The COPS uses MATLAB's standard solver for ordinary differential equations ode45 suite. This function implements the explicit Runge-Kutta[15] method with variable time step for efficient computation. Numerical integration with the Runge-Kutta scheme yields the trajectory of changes in open pit geometry and the volume of material transferred over mine life. The volume of rock in the final pit limits is equal to 8.2056 x 10^7 m^3, but the solution to the differential equations demonstrates a volume of 8.3203 x 10^7 m^3 over the mine life (Figure 9).

The linear format of Figure 9 demonstrates that the solution of the differential equations produces a schedule with a constant annual rock excavation rate. Figures 10 to 13 illustrate the discrete open pit geometry push-backs

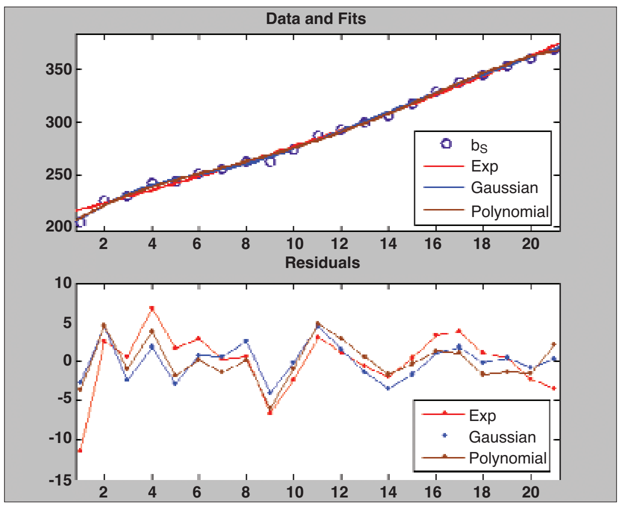

Figure 7—b_S function approximation residuals

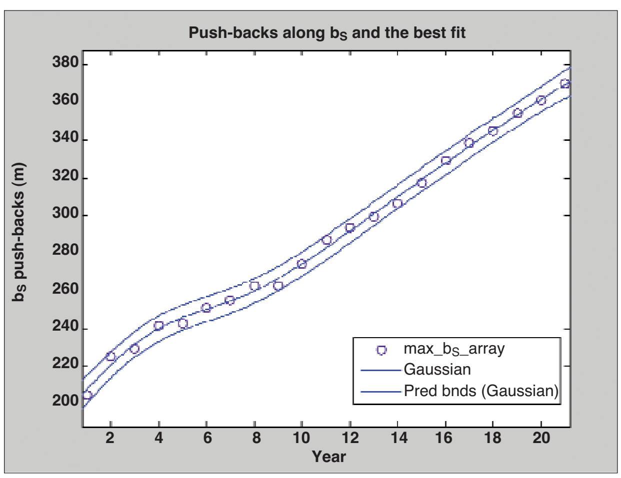

Figure 8—b_S best fit and the error prediction bounds

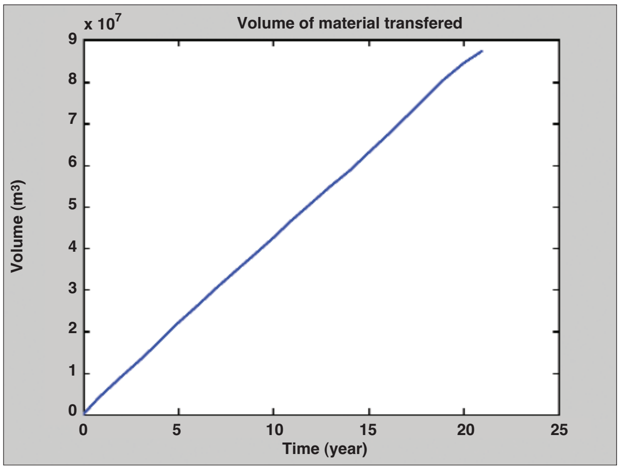

Figure 9—Volume of rock excavated over mine life

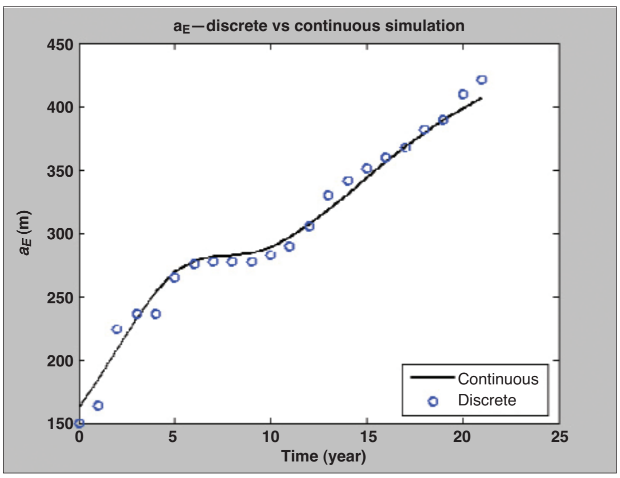

Figure 10—Changes in a_E, COPS vs. DOPS

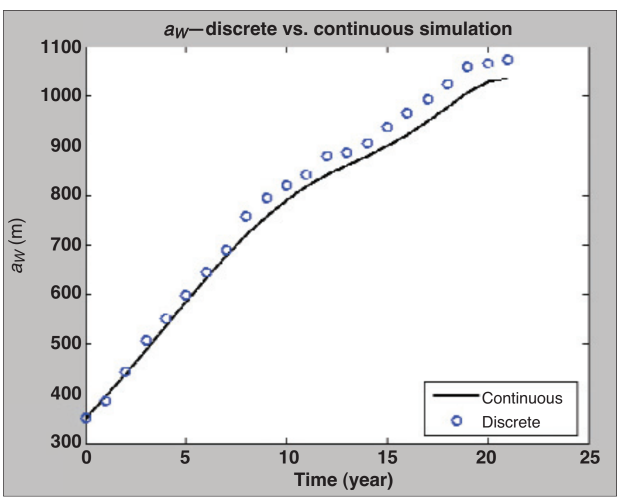

Figure 11—Changes in a_W, COPS vs. DOPS

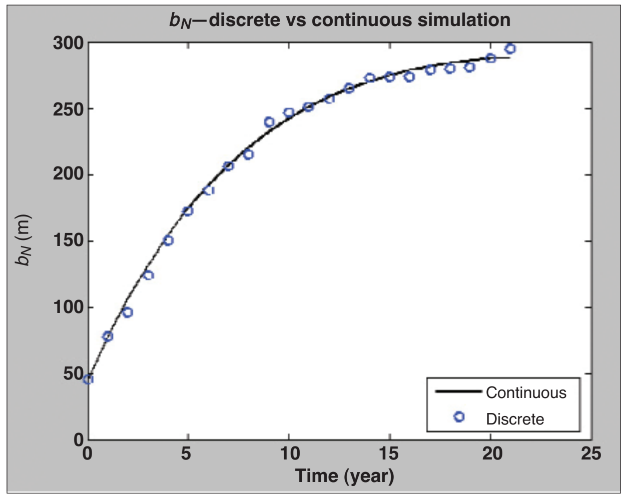

Figure 12—Changes in b_N, COPS vs. DOPS

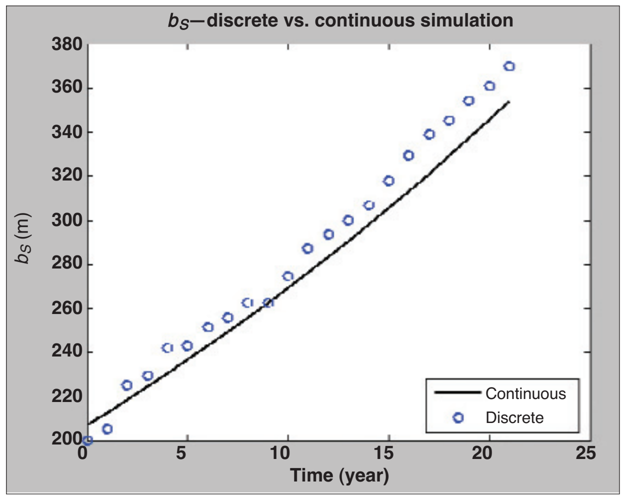

Figure 13—Changes in b_S, COPS vs. DOPS

simulated by DOPS compared to the continuous solution of the differential equations capturing the dynamics of the open pit over the mine life. The interaction of the continuous changes in geometry of the open pit with EPEM returns the volume of ore, waste and the net present value of the mining operation. The best-case annual schedule out of 2 500 simulation iterations, generated by DOPS yielded an NPV of $443.6 million over a 21-year mine life at a discount rate of 10% per annum. COPS schedule resulted in an NPV of $440.2 million over the same time span. Figure 14 illustrates the comparative analysis of DOPS vs. COPS results.

Conclusions

A continuous-time open pit production simulator, COPS, is developed and implemented in MATLAB. The open pit geometry is captured using modified elliptical frustum. Discrete open pit production simulator, DOPS, mimics the stochastic dynamic expansion of an open pit using discrete incremental push-backs in different directions. The interactions of economic pit expansion model, EPEM, with

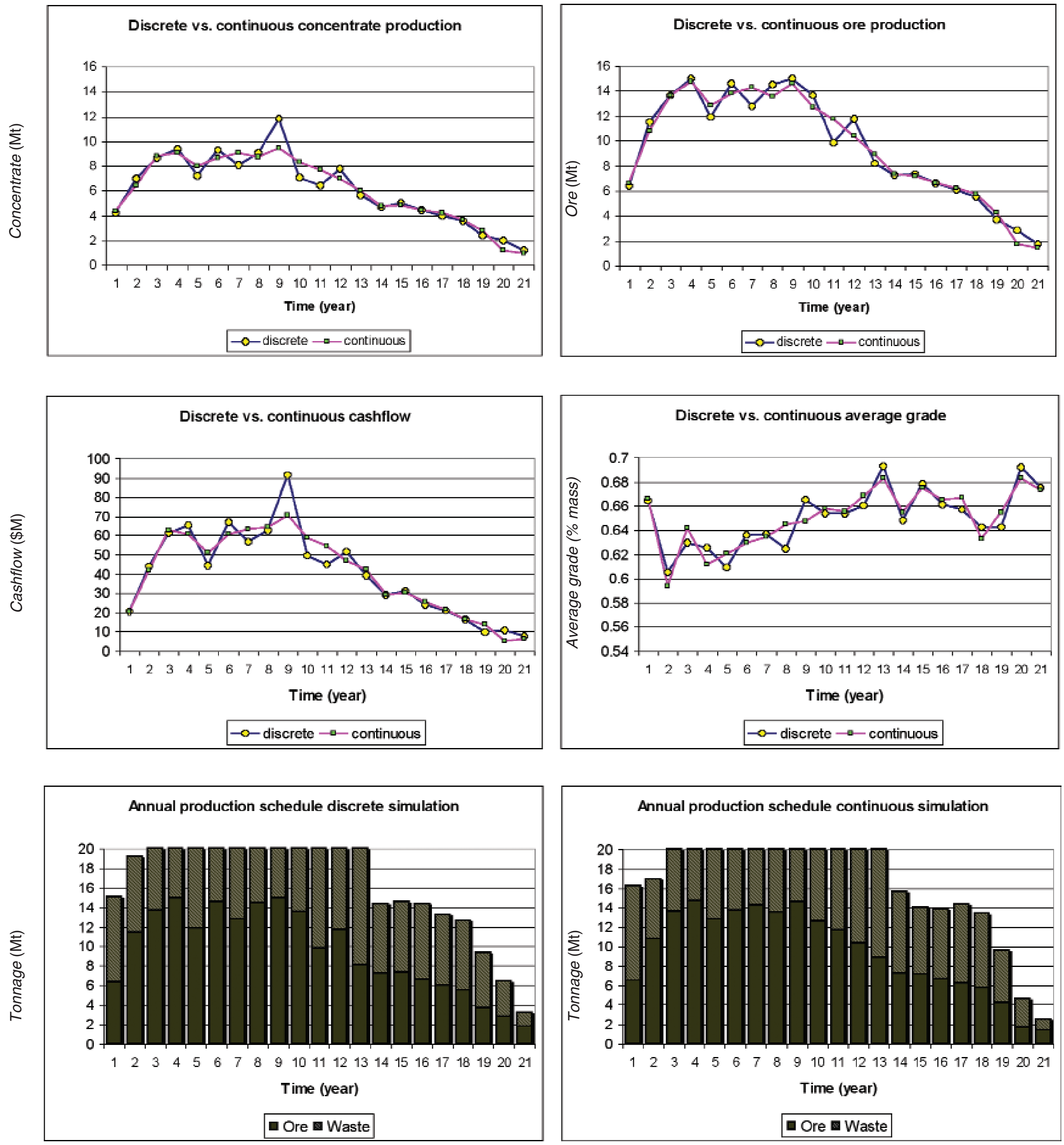

Figure 14—Discrete and continuous simulation results

DOPS returns the pit's NPV following the simulated schedule. The simulation is run for sufficient iterations to find the practical sequence of extraction among all realizations, which results in the highest NPV. COPS models the dynamics of open pit geometry and the subsequent materials movement as a continuous system described by time-dependent partial differential equations. Function approximation of the discrete simulated push-backs generated by DOPS, provides the means to convert the set of PDEs to a system of ODEs. Numerical integration with Runge-Kutta scheme yields the trajectory of the pit geometry over time with the respective volume of materials transferred and the NPV of the mining operation.

A case study of an iron ore deposit with 114 000 blocks was carried out to verify and validate the model. The final pit limits were determined using the LG algorithm. Comparative analysis of the results of the extraction sequence generated by parametric analysis using Whittle Four-X software, DOPS, and COPS resulted in the following conclusions: (i) the optimized final pit limits show the total amount of 343 million tons of material consisting of 201 million tons of ore and 142 million tons of waste; (ii) Whittle Four-X yielded an NPV of $449 million over a 21-year of mine life at a discount rate of 10% per annum; (iii) DOPS yielded an NPV of $443 million after 2 500 simulation iterations, and COPS generated an NPV of $440 million under the same circumstances over the same mine life. Although optimality is not guaranteed with the parametric analysis in Whittle 4X, it is a strong tool for identifying high grade ore clusters in the model. The hybrid simulation model provided is the basis for future research using reinforcement learning by a goal-directed intelligent agent interacting with open pit stochastic environment. The intelligent agent framework, in conjunction with the presented stochastic simulation models, provides a powerful tool for optimizing the scheduling process, while addressing the random field and dynamic processes in open pit mine planning.

References

1. CHANDA, E.K. and WILKE, F.L. An EPD model of open pit short term production scheduling optimization for stratiform orebodies. *Proceedings of 23rd APCOM Symposium,* SME (ed.), 1992. pp. 759–768.

2. ELVELI, B. Open pit mine design and extraction sequencing by use OR and AI concepts. *International Journal of Surface Mining. Reclamation and Environment*. 1995. vol. 9, pp. 149–153.

3. ERARSLAN, K. AND CELEBI, N. A simulative model for optimum open pit design. *The Canadian Mining and Metallurgical Bulletin*. 2001. vol. 94, pp. 59–68.

4. HALATCHEV, R.A. A model of discounted profit variation of open pit production sequencing optimization. *Proceedings of Application of Computers and Operations Research in the Mineral Industry*, Tucson, Arizona. Taylor & Francis Group(ed.), London, 2005. pp. 315–323.

5. ONUR, A.H. AND DOWD, P.A. Open pit optimization—part 2: production scheduling and inclusion of roadways. *Transactions of the Institution of Mining and Metallurgy*. 1993. vol. 102, pp. A105–A113.

6. TOLWINSKI, B. and UNDERWOOD, R. An algorithm to estimate the optimal evolution of an open pit mine. *Proceedings of 23rd APCOM Symposium, University of Arizona*. SME (ed.), Littleton, Colorado, 1992. pp. 399–409.

7. DOWD, P.A. and ELVAN, L. Dynamic programming applied to grade control in sub-level open stopping. *Trans. IMM*. 1987. vol. 96, pp. A171–A178.

8. CHANDA, E.K. and DAGDELEN, K. Optimal blending of mine production using goal programming and interactive graphics system. *International Journal of Surface Mining Reclamation and Environment*. 1995. vol. 9, pp. 203–208.

9. YOUDI, Z., QINGZIANG, C., and LIXIN, W. Combined approach for surface mine short-term planning optimization. *Proceedings of 23rd APCOM Symposium*, SME (ed.), Colorado, 1992. pp. 499–506.

10. MANN, C. and WILKE, F.L. Open pit short term mine planning for grade control—a combination of CAD techniques and linear programming. *Proceedings of 23rd APCOM Symposium,* SME (ed.), Colorado, 1992. pp. 487–497.

11. FRIMPONG, S., ASA, E., and SZYMANSKI, J. MULSOPS: multivariate optimized pit shells simulator for tactical mine planning. *International Journal of Surface Mining, Reclamation & Environment*. 1998. vol. 12, pp. 163–169.

12. FRIMPONG, S., ASA, E., and SUGLO, R.S. Numerical simulation of surface mine production system using pit shell simulator. *Mineral Resources Engineering*. 2001. vol. 10, pp. 185–203.

13. ASKARI-NASAB, H. and SZYMANKSI, J. Modelling open pit dynamics using Monte Carlo simulation. *Proceedings of Computer Applications in the Minerals Industry (CAMI), Banff, Alberta, Canada*. On CD-ROM, The Reading Matrix Inc., CA, USA, 2005. pp. 21–32.

14. ASKARI-NASAB, H., AWUAH-OFFEI, K., and FRIMPONG, S. Stochastic simulation of open pit pushbacks with a production simulator. *Proceedings of CIM Mining Industry Conference and Exhibition, Edmonton, Alberta, Canada*. 2004. pp. on CD-ROM

15. CARTWRIGHT, J.H.E. and PIRO, O. The dynamics of Runge-Kutta methods. *Int. J. Bifurcations Chaos*. 1992. vol. 2, pp. 427–449

16. WHITTLE PROGRAMMING PTY, LTD. Whittle strategic mine planning software, Gemcom Software International Inc., 1998–2004.

17. ABRAMOWITZ, M. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. Stegun, I.A.E. (eds). New York: Dover, 1972. 880.

18. LERCHS, H. and GROSSMANN, I.F. Optimum design of open-pit mines. *The Canadian Mining and Metallurgical Bulletin*, *Transactions.* 1965. vol. LXVIII, pp. 17–24.

19. DEUTSCH, C.V. and JOURNEL, A.G. (eds). GSLIB geostatistical software library and user's guide. *Applied geostatistics series*. New York, Oxford University Press, 1998.

20. KRIGE, D.G. A statistical approach to some basic mine valuation and allied problems at the Witwatersrand, Masters thesis, University of Witwatersrand, South Africa, 1951.

21. MARQUARDT, D. An algorithm for least squares estimation of nonlinear parameters. *SIAM J. Appl. Math*. 1963. vol. 11, pp. 431–441. ◆