Slope modification of open pit wall using a genetic algorithm—case study: southern wall of the 6th Golbini Jajarm bauxite mine

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Synopsis

In this paper a genetic algorithm is used in a heavily jointed rock mass in order to investigate the critical circular slip surface and modification of slope surface. This method was applied to the southern wall of the 6th Golbini Jajarm bauxite mine. The mine is the largest bauxite deposit in Iran, located to the northeast of the town of Jajarm in the Khorasan province. Estimated reserve of bauxite in this deposit is about 160 million tonnes. Field and laboratory investigations were conducted in order to determine rock mass behaviour. A genetic algorithm code that uses the Simplified Bishop method as an objective function was developed for finding the safety factor of circular slip surfaces. Sensitivity analysis was applied to determine the optimum values of the genetic algorithm variables, such as population size, selection method, crossover and mutation rates. After finding the critical circular slip surface, slope modification is carried out by removing unstable sections from marked critical slip surfaces, and this process is repeated until the last unsafe section is removed. Based on this code, modification occurred during 7 steps, by reaching a safety factor of 1.3 in the last step. Finally, the modified slope angle of the southern wall of the 6th Golbini Jajarm bauxite mine was determined to be 48.44 degrees.

Keywords: Genetic algorithm, slope modification, circular slips surface, 6th Golbini Jajarm bauxite mine .

Introduction

Slope stability is one of the most important issues in stability analysis in geomechanics. Of the various available methods (numerical and analytical), the limit equilibrium is widely used due to its simplicity and the results are found to be close to the rigorous methods[1]. The conventional limit equilibrium method for slope stability analysis consists of two steps: (a) calculation of the safety factor for a slip surface and (b) determination of the critical slip surface with the lowest factor of safety. Many methods (Bishop, Jumbo, Morgenson and Price, and Spencer) have been presented to compute the factor of safety using limit equilibrium with a critical slip surface[2]. A simple circular slip surface is sufficient for a slope in homogenous rock masses. But the analysis of slope stability requires many analyses of different potential slip surfaces in order to reach the surface with the lowest factor of safety. To avoid the difficulty in determining the global minima, evolutionary methods such as the genetic algorithm (GA) are being used, and are more robust to achieve the optimal solution in many complex problems[3]. The genetic algorithm differs from other methods since it searches among a population of points and works with a coding of parameters set rather than the parameters themselves. Goh (1999) has used the GA to work out the critical surface and the factor of safety using the method of wedges[4]. McCombie and Wilkinson (2002) used Bishop's method to determine the critical surface using GA[5]. In the above studies, GA proved a better solution compared to other traditional approaches[4,5].

This paper presents a method for determining the critical circular slip surface using GA with an objective function based on Bishop's simplified method. The modification of the slope is carried out by removing unstable sections from marked critical slip surfaces. This process is repeated until the last unsafe section is removed. Consequently, the analysis can be performed through the following stages: (a) development of an objective function and (b) the application of GA to solve the objective function and reaching an optimum slope angle.

Development of the objective function

Several slope stability analysis methods (e.g. Spencer) are more rigorous and should be favoured for the detailed evaluation of the final designs. Other methods (e.g., Spencer, Modified Swedish, and Wedge) can be used to analyse non-circular slip surfaces. Certain methods (e.g., Ordinary Method of Slices,

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Table I

Comparison of features of limit equilibrium methods[6]

Feature	Ordinary Method of Slices	Simplified Bishop	Spencer	Modified Swedish	Wedge	Infinite Slope
Accuracy		●	●			●
Plane slip surfaces						●
Circular slip surfaces	●	●	●	●		
Wedge failure mechanism			●	●	●	
Non-circular slip surface			●	●		
Suitable for hand calculation	●	●		●	●	●

Simplified Bishop, Modified Swedish and Wedge) can be used without computer aid and are therefore convenient for independently checking the results obtained using computer programs. Also, when these latter methods are implemented in the software, they execute extremely fast and are useful where very large numbers of trial slip surfaces are to be analysed. The limit equilibrium methods are summarized in Table I[6].

The geological study of the southern wall of the 6th Golbini Jajarm bauxite mine shows that the rock mass is homogenous and under dry conditions[7]. Circular slip surfaces are widely used in slope stability analysis of homogenous rock masses as they are a fair reflection of actual failure mechanisms, and can be analysed reliably without the need to justify the assumption made, or complex testing. Among the various methods that can be used for analysing circular slip surfaces (e.g., ordinary method of slices, Simplified Bishop, modified Swedish and Spencer), the Simplified Bishop and the Spencer lead to more accurate results. Also the Simplified Bishop is extremely fast and suitable to use in GA, as the slope stability comes with large numbers of trial slip surfaces to be analysed. Therefore, the Simplified Bishop Method can be the best objective function to calculate the factor of safety in GA.

Simplified Bishop method

The Simplified Bishop Method was developed by Bishop[6]. This method is based on the assumption that the inter slice forces are horizontal ($E_i = E_{i+1}$), as shown in Figure 1.

A circular slip surface is assumed in the Simplified Bishop method. Forces are summed in the vertical direction. The resulting equilibrium equation is combined with the Mohr-Coulomb equation and the definition of the factor of safety to determine the forces on the base of the slice. Finally, moments are summed about the centre of the circular slip surface to obtain the following expression for the factor of safety, F,[6]:

$$F = \frac{\sum \left[\frac{c'\Delta x + (W + P\cos\beta - u\Delta x \sec\alpha)\tan\phi'}{m_\alpha} \right]}{\sum W \sin\alpha - \frac{\sum M_P}{R}} \qquad [1]$$

Where Δx is the width of the slice, W is the weight of the slice, c' and φ' are shear strength parameters for the centre of the base of the slice, α and β are the slice inclinations, u is pore water pressure at the centre of the base of the slice, P is the resultant water force acting perpendicular to the top of the slice, M_p is the moment acting on the centre of the circle induced by the water force acting on the top of the slice, R is the radius of the slip circle and m_α is defined by the following Equation[6] :

$$m_\alpha = \cos\alpha + \frac{\sin\alpha \tan\phi'}{F} \qquad [2]$$

The factor of safety calculated from Equation [1] satisfies the equilibrium of forces in the vertical direction and overall equilibrium of moments to the centre of the circle. Since the value of the term m_α depends on the factor of safety, it appears on both sides of Equation [1]. Consequently, Equation [1] cannot be manipulated in a such way that an explicit expression is obtained for the factor of safety and an iterative, trial and error procedure is applied to solve it[6].

Slice number is a critical parameter in the Simplified Bishop Method. Accuracy in the factor of safety values is increased with higher slice numbers, but an increase in slice numbers needs more calculations and more time. The optimum slice number is required for faster and accurate values. Analysis on slice numbers was implemented using the Simplified Bishop Method for four slip surfaces and slice numbers from 10 to 100 with steps of 5. As seen from Figure 2, after slice number 40, there is no significant improvement in the factor of safety, so, slice number 40 was selected as the objective function.

Genetic algorithm

The genetic algorithm, developed by Holland, relies on the principle of Darwin's theory of the survival of the fittest[8].

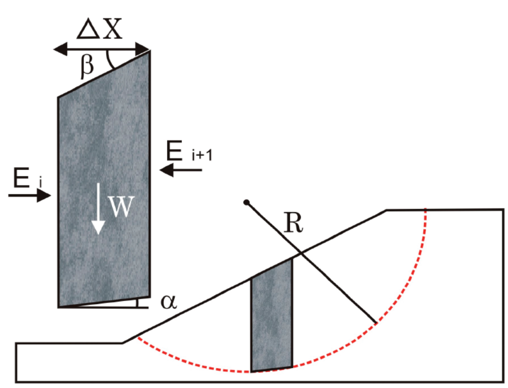

Figure 1—Typical slice and inter slice forces for Simplified Bishop method[6]

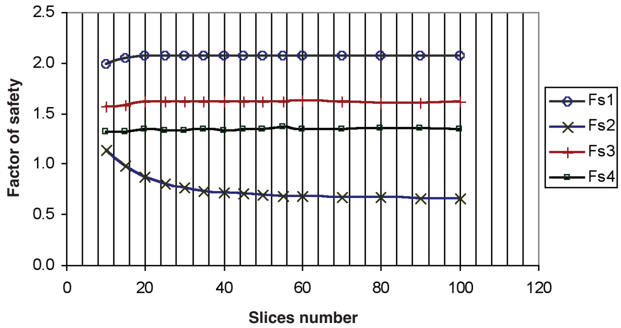

Figure 2—Factor of safety vs. slices number

Solution of the problem can be obtained through evolution. The algorithm is started with a set of possible solutions called 'population'. Each possible solution within the population is called a 'chromosome'. Each chromosome is assigned a fitness value based on the fitness function that reaches from the objective function. Solutions from one population are taken and used to construct a new population called 'offspring'. So, the offspring will be fitter than the old population. This process is repeated until termination criteria are met. For example, the reproduction will stop when the total number of generations reaches a specified maximum number or best fitness is constant for a while. The basics of GA are described in the following sections.

Coding

The first step in the GA is translating the real problem into biological terms. For this, all variables are represented by chromosomes and this process is called coding. There are various coding methods in the GA. The method applied in this GA is decimal coding that consists of strings of bits, 0 to 9.

Selection

In order to reproduce offspring, parents need to be selected. The two most commonly used methods are the roulette wheel and tournament selections. In the roulette wheel selection, the chromosomes are ranked in an ascending order based on their fitness values, F_j (j=1, 2, n). Next, a probability value, P_j, is assigned to each chromosome (j=1, 2, n) that gives a higher probability to the chromosome with a higher fitness value. After ranking the chromosomes according to their fitness values, the best chromosome is placed first with the greatest probability value (P_1) and the worst chromosome is placed last with the least probability value (P_n). The probability values for other chromosomes are linearly interpolated as:

$$P_j = \frac{F_j}{\sum_{j=1}^{n} F_j} \qquad [3]$$

The other method is tournament selection. In this method, a small subset of the chromosomes (two or three chromosomes) is selected randomly and the one with the best fitness will be considered as a parent. In some genetic algorithms, elitism is applied to keep the best chromosome in the next generation.

Crossover

The crossover strategy determines how the parent chromosomes are combined to generate offspring. Crossover is applied randomly to selected pairs of parents with a probability equal to a specified crossover rate. A single-point crossover is the most popular operator. One crossover point is randomly selected along the parent chromosomes. The coded bits from the beginning of the first parent to its crossover point are copied to the first offspring in the same position. The rest of the bits from the same crossover point of the second parent to its tail are copied to the first offspring in the same position. This action is then conducted for the second offspring (Figure 3).

Mutation

After a crossover is performed, mutation takes place. Mutation is the genetic operator that randomly changes one or more of the chromosome's genes. The purpose of the mutation is to prevent the genetic population from converging to a local minimum and to introduce new possible solutions to the population. The mutation is carried out according to the mutation rate (Figure 4).

Initializing GA parameters

A flow chart depicting the major operations of the GA is shown in Figure 5. To enhance the performance, the analysis is based on different combinations of various parameters such as population size, selection methods, crossover rate and mutation rate.

9	0	7	4	5	1	4	2	2	9	1	4	3	4	2	5	2	3	9	1	3	1	3	6	8	5	1
9	0	2	3	4	5	8	0	7	9	2	1	6	0	4	2	8	8	9	1	6	2	4	1	9	3	6

▼

9	0	7	4	5	1	4	2	2	9	1	4	3	0	4	2	8	8	9	1	6	2	4	1	9	3	6
9	0	2	3	4	5	8	0	7	9	2	1	6	4	2	5	2	3	9	1	3	1	3	6	8	5	1

Figure 3—Single-point crossover for 27 genes length chromosomes

9	0	7	4	5	1	4	2	2	9	1	4	3	0	4	2	8	8	9	1	6	2	4	1	9	3	6
				▼																			▼			
9	0	7	4	7	1	4	2	2	9	1	4	3	0	4	2	8	8	9	1	6	2	4	5	9	3	6

Figure 4—Mutation for 27 genes length chromosome

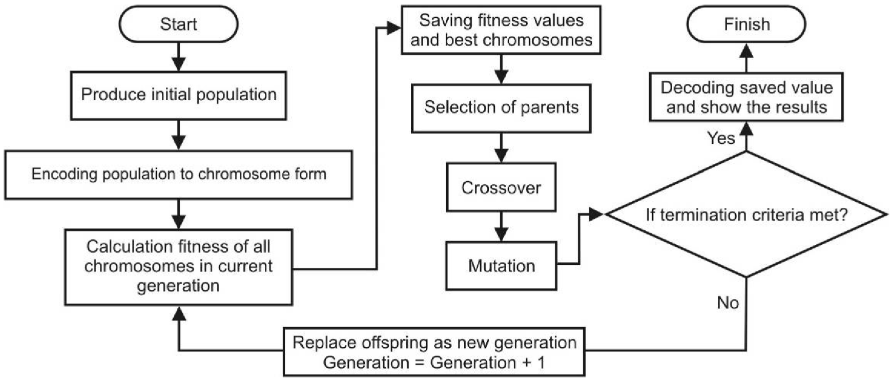

Figure 5—Main flow chart of a genetic algorithm

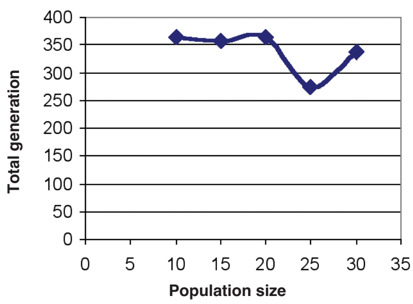

Figure 6—Total generation vs. population size

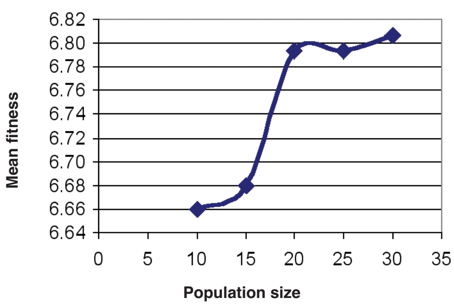

Figure 7—Mean fitness vs. population size

Population size

The GA is implemented for population sizes between 10 to 30 chromosomes, with step of 5, and 15 repeats for each step. As seen in Figures 6 and 7, the total generation and mean fitness values change with population size. A population of 20 chromosomes can satisfy a reasonable fitness value.

Selection method

The GA is executed for the roulette wheel and the tournament selection methods with 15 repeats for each one. As can be seen in Figures 8 and 9, generation number and mean fitness values change in each repeat. In general, the tournament selection is more stable.

Mutation and crossover rates

The GA was applied for mutation and crossover rates with 15 repeats for each rate. As can be seen from Figures 10–13, best results achieved for a mutation rate of 0.050 and a crossover rate of 0.700. Note that the mutation rate of 0.001 causes the GA to converge to local optimum points.

Rock mass strength parameters

The RMR rock mass classification system[9] was utilized for the assessment of dolomite wall strength parameters in the 6th Golbini Jajarm bauxite mine. As a first step, a joint study and site investigation were carried out for RQD and RMR determinations. The results from these studies show that there are four major discontinuities. The second step was involved in laboratory tests and determining groundwater conditions that can be used to assess the RMR. The unconfined compressive strength of intact dolomite specimens was determined in the laboratory. The findings are shown in Table II. With respect to Table II, the RMR_B is calculated as follows:

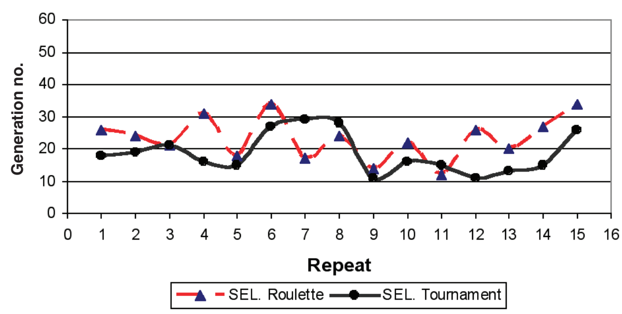

Figure 8—Generation number vs. selection methods

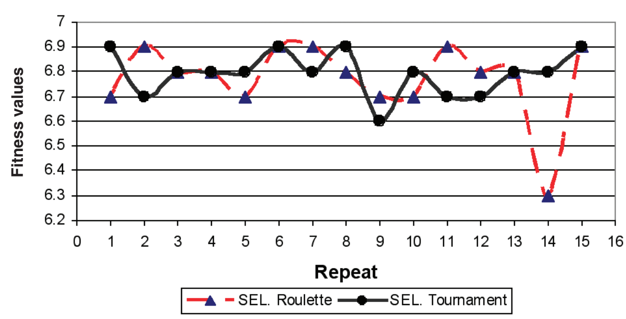

Figure 9—Fitness values vs. selection methods

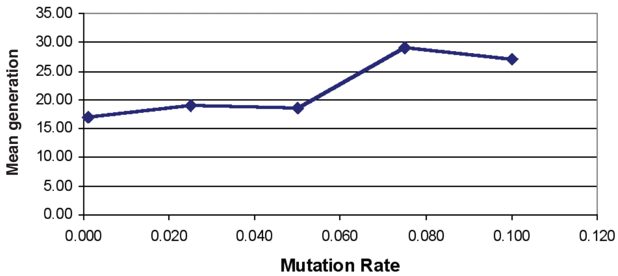

Figure 10—Mean generation vs. mutation rate

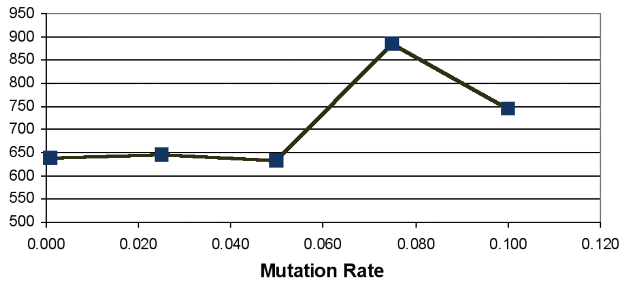

Figure 11—Mean time vs. mutation rate

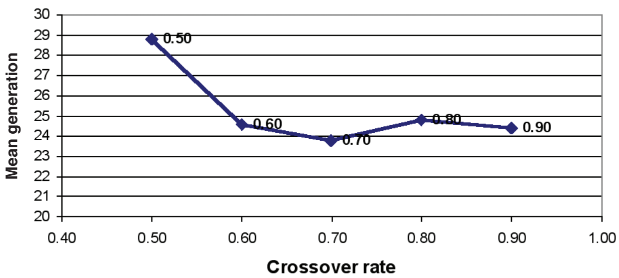

Figure 12—Mean generation vs. crossover rate

$$RMR_B = 7 + 13 + 5 + 16 + 15 = 56$$

Finally, the required rock mass strength parameters were calculated using GSI (Geological Strength Index) and RocLab® software[10]. With this regard, the Mohr-Coulomb strength parameters, cohesive strength (c') and friction angle (φ') of the rock mass were calculated to be 0.243 MPa and 44.60 degrees, respectively.

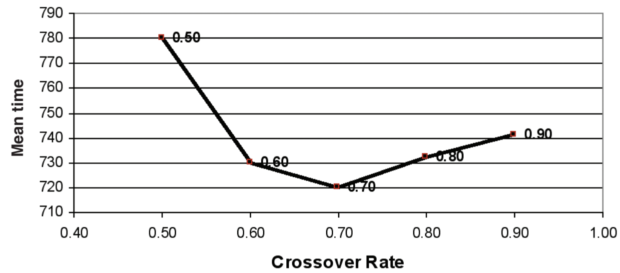

Figure 13—Mean time vs. crossover rate

Application of GA to find the critical circular slip surface

To determine the critical slip surface, the objective function must find the slip surface with the minimum factor of safety. Each population chromosome has tree sections: the first and second sections are the X and Y coordinates of the slip circle, and the third section shows the radius of the slip surface. The GA is implemented with an initial random population of 20 chromosomes, using the tournament selection method, a crossover rate of 0.70, and a mutation rate of 0.050. The criterion for termination of the algorithm is to reach a difference of 0.01 between the best fitness of current and previous population chromosomes for the last 10 generations or for a maximum of 50 generations. The results show that the algorithm converged after 21 generations and reach the critical slip surface with a factor of safety equal to 0.58. The results are shown in Figures 14–16.

Slope modification

There is sufficient control of optimization between safety and economic parameters in slope modification using the GA. This is because most economic savings will be achieved with minimum removal of the unstable sections that are marked. Slope modification uses several iterations and the best result is gained for 7 slope modification steps within 27 generations of GA (Table III). Finally, a factor of safety of 1.3 is reached for a slope angle of 48.44 degrees (Figure 17).

Table II

RMR parameters and ratings. RMR_B= Basic RMR = Σ ratings[10]

Parameter	Intervals				
UCS (MPa)	>250	250–100	100–50	50–25	25-5 5-1 <1
Rating	15	12	7 ←*	4	2 1 0
RQD (%)	100–90	90–75	75–50	50–25	<25
Rating	20	17	13 ←*	8	3
SPACING (mm)	>2 000	2 000–600	600–200	200–60	<60
Rating	20	15	10	8	5 ← *
Condition of Discontinuities	Very rough surfaces No separation Unweatheredwall rock Not continuous	Slightly rough Separation < 1 mm Slightly weathered walls Not continuous	Slightly rough Separation < 1 mm Highly weathered Walls	Slicken sided walls Or gouge < 5 mm or separation 1–5 mm	Soft gouge >5 mm or separation continuous
Rating	30	25	20 (16) ←*	10	0
Groundwater	Completely dry	Damp	Wet	Dripping	Flowing
In joints	0	(0–0.1)	(0.1–0.2)	(0.2–0.5)	-0.5
Rating	15 ←*	10	7	4	0

Conclusion

The present paper suggests a slope modification method based on the GA for locating the critical circular slip surface and modification of unstable marked sections. The proposed technique has a simple structure and is easily programmable. In addition, a case study has been presented to demonstrate the capabilities of the proposed approach. The results show that the GA can be successfully employed to locate the critical failure surface in a homogenous rock mass slope, and the Simplified Bishop Method can be easily solved with the GA in order to obtain the factor of safety. The study on GA parameters concluded that a population size of 20, a tournament selection method, a single point crossover with the rate of 0.7 and a mutation rate of 0.05 can locate the critical circular slip surface. Finally, in a case study, slope modification was implemented using the GA. The slope modification process of the southern wall of the 6th Golbini Jajarm bauxite mine terminated by 7 slope modification steps within 27 generations. A factor of safety of 1.3 is reached for a slope angle of 48.44 degrees.

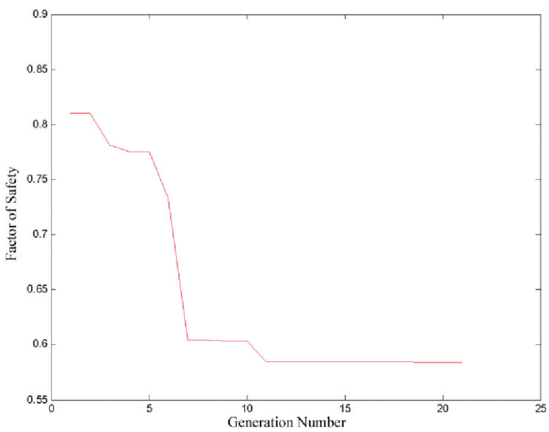

Figure 14—Lowest factor of safety vs. generation number

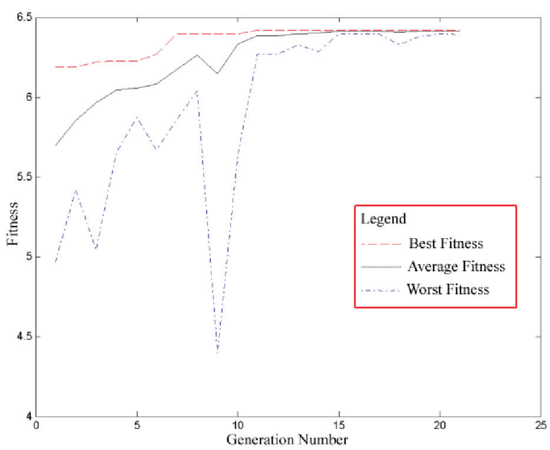

Figure 15—Best, average and worst fitness vs. generation number

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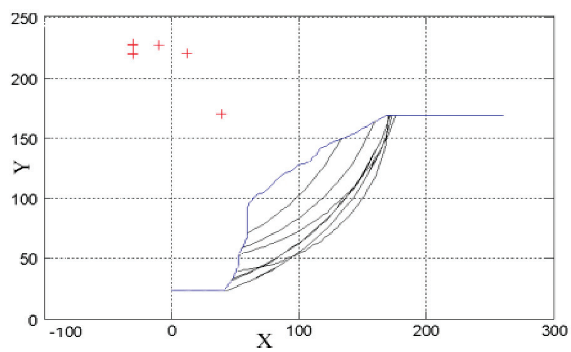

Figure 16—Critical slip surfaces in consecutive generations

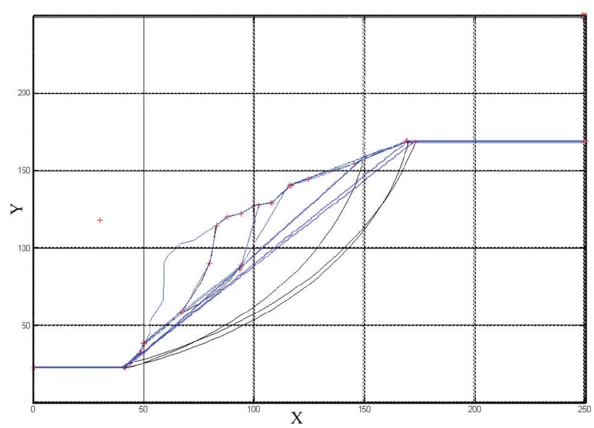

Figure 17—Steps of slope modification using genetic algorithm

Table III

Slope modification details

Modification step	1	2	3	4	5	6	7
Generation number	25	26	13	12	19	25	27
Safety factor of the critical surface	0.5800	0.6910	0.9408	0.8414	0.9631	1.1529	1.3115
Total time (hh:mm:ss)	00:20:32	00:47:10	00:58:54	01:07:22	01:45:12	02:32:18	02:45:15