Estimating the modulus of elasticity of the rock material from compressive strength and unit weight

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Synopsis

The modulus of elasticity of rock material (E_i) is an important rock property that is used as an input parameter in the design stage of engineering projects such as dam and tunnel constructions, mining excavations, and so forth. However, determination of the modulus of elasticity is sometimes difficult to obtain by laboratory tests because high-quality cores are required. For this reason, empirical methods for predicting the modulus of elasticity of rock material have been popular research topics in recently published literature. In this study, the relationships between the uniaxial compressive strength (UCS), unit weight (γ) and modulus of elasticity for different types of rocks were analysed by using 177 data obtained from laboratory tests carried out on cores obtained from drill holes within the area of the Kadıköy-Kartal Metro line (Istanbul, Turkey).

Keywords: Kadikoy–Kartal Metro, uniaxial compressive strength, unit weight, estimating the modulus of elasticity.

Introduction

The modulus of elasticity of rock material is one of the most important rock properties used in designing civil and mining projects such as dam and tunnel constructions or mine layout design. High-quality core samples are needed to obtain this parameter using laboratory tests. Sometimes it is very difficult to obtain high-quality cores from particularly thinly bedded and heavy rock masses. Because of this, the planning engineer may estimate the modulus of elasticity from the other rock properties by using predictor equations published in the literature.

Aufmuth (1973), Sachpazis (1990), and Xu *et al.* (1990) used a Schmidt hammer to estimate the modulus of elasticity. Some of the other researchers, including Sachpazis (1990), Rohde and Feng (1990), Tuğrul and Zarif (1999), Palchik (1999), and Lashkaripour and Nakhaei (2001) preferred using uniaxial compressive strength to estimate the modulus of elasticity. Another team of researchers (Sonmez *et al.* 2004a and Sonmez e*t al.* 2004b) constructed recent empirical approaches by using multiple input parameters such as unit weight and UCS for the estimation. Sonmez *et al.* (2006) proposed an artificial neural-network-based prediction chart that considered unit weight and UCS as input parameters.

In this study, 177 data sets including UCS, E_i and γ have been used. These data have been obtained from 177 drilling holes along the 21.6 km route between the Kadikoy and Kartal Metro tunnels. The ongoing construction work is being conducted by the Istanbul Metropolitan Municipality. The tests of UCS (Figure 1), unit weight and modulus of elasticity were performed on the core samples in accordance with the procedure suggested by ISRM (1981). Average core length is 13.8 cm, average diameter is 6.13. Length/diameter ratio is about 2.2. The average UCS value is 43.8 MPa. The average modulus of elasticity value is 10.0 GPa. The average unit weight value is 26.6 kN/m^3. The mean values of some geotechnical parameters of the rock samples collected from the tunnel route are given in Table I. The geological section of the region is given in Figure 2.

General information about the project of the Kadikoy-Kartal rail transport system and metro tunnels

The Kadikoy-Kartal rail transport system project starts at Kadikoy and passes through the districts of Uskudar, Maltepe and Kartal. The length of the rail transport system is 21.6 km and construction of 16 stations is planned. The elevation of the railway at the Kadikoy station is 36.0 metres below the sea level. The Kadikoy-Kartal rail transport system is integrated with the Marmaray undersea rail

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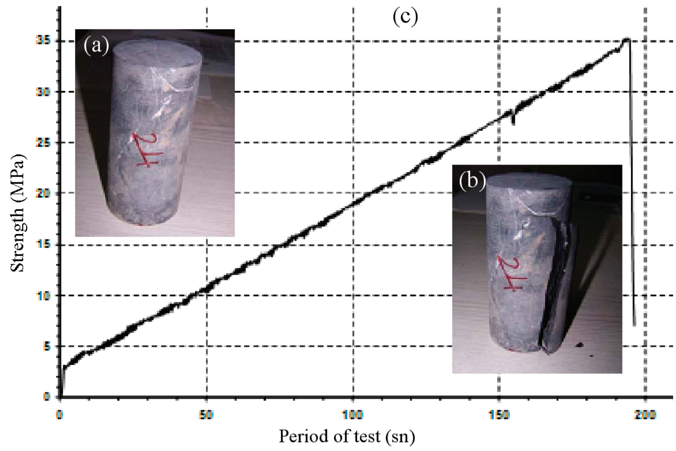

Figure 1—(a) Test specimen before UCS test (b) test specimen after UCS test (c) a typical strength–period of test curve obtained from the UCS tests

Table I

The mean values of geotechnical properties of the rock samples (IBB, 2005a)

Formation	Lithology	Unit weight (kN/m³)	Poisson ratio	Internal friction angle (º)	Cohesion (MPa)	Uniaxial comp. strength (MPa)	Modulus of elasticity (GPa)
Trakya	Sandst.-siltst.-clayst.	26.5	0.27	37.9	13.9	50.9	10.9
Tuzla	Shale	26.8	0.27	43.1	6.4	32.6	8.4
Kartal	Shale-limestone	26.2	0.28	40.0	18.6	34.0	7.9
Kurtkoy	Sandstone-conglom.	26.9	0.29	43.1	19.5	74.5	11.5
Dolayoba	Limestone	27.0	0.30	49.4	10.6	43.6	18.9

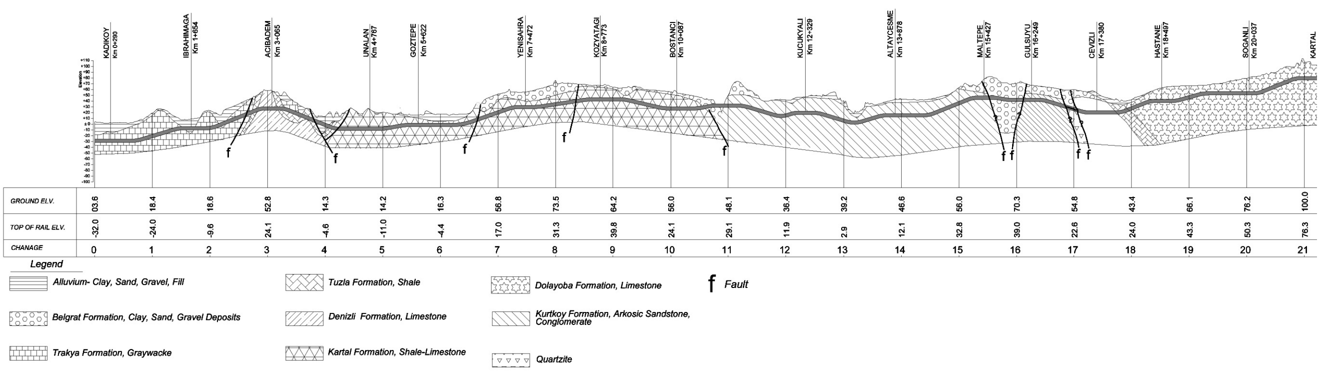

Figure 2—Kadikoy-Kartal metro system general geology (IBB, 2005b)

tunnel project, which will join the European and Asian halves of Istanbul at the Ibrahimaga Station with a tunnel under the sea. Both passenger transportation and the connection to the European side of Istanbul with the Marmaray Project will be provided with the integration with the Marmaray Project (Figure 3).

The metro system is composed of double tunnels having a diameter of 6.10 metres. The depth of the tunnels from the surface is approximately 30 metres and the distance between two tunnels is 32 metres. The passenger capacity for one way is planned at about 60 000 persons/hour. The tunnels will be excavated by two tunnel boring machines (TBM) and two earth pressure balance machines (EPBM). The station platform tunnels, access tunnels, shafts, switch tunnels and connection tunnels are excavated considering the new Austria tunneling method (NATM), using impact hammers and road headers.

Database used for statistical analysis

The data sets, composed of uniaxial compressive strength, unit weight and modulus of elasticity, were obtained from laboratory experiments carried out on the drilling cores obtained by the Directorate General of IETT, Istanbul Metropolitan Municipality for Kadıköy Kartal Rail Transport System. While the values of uniaxial compressive strength

Figure 3—Main route of Kadikoy-Kartal metro system

vary between 1.9 MPa and 152.2 MPa, the modulus of elasticity changes from 0.7 GPa to 38.4 GPa. The unit weights vary between 22.5 kN/m^3 and 28.9 kN/m^3. The variation histograms of the data are given in Figure 4. The distribution of the data on the modulus ratio graph suggested by Deere and Miller (1966) is shown in Figure 5.

The relationships between the modulus of elasticity, compressive strength and unit weight

The relationships between the uniaxial compressive strength and the modulus of elasticity of the rock material collected from each formation along the Kadikoy–Kartal Metro System route have been investigated separately using data sets obtained from each geologic formation (Table II). The relations are also analysed by using the whole database without separating the data into geological formations (Table II and Figure 6). While the correlation coefficient of the relation between E_i and UCS was obtained as 0.809, the relationship between E_i and γ was also sought (Figure 7) and a correlation coefficient of 0.50 was obtained. To increase the prediction capacity, the relation between E_i and multiple input parameters such as UCS and γ was also investigated by using a combined parameter (CP) as used in the literature (Sonmez *et al.* 2004a). When the whole database was considered, the relation between E_i, UCS and γ having a correlation coefficient of 0.834 was obtained (Figure 8 and Table II).

Prediction performance of suggested equation

The measured and predicted values of the modulus of elasticity using equations from 1 to 5 are given in Figure 9. The predicted and measured values for all the formations give very significant correlation coefficients along a 1:1 line except for the Dolayoba formation, with a correlation coefficient of 0.48. The other values are as follows: for the Trakya, Tuzla, Kartal and Kurtköy formations, the correlation coefficients are 0.92, 0.90, 0.75, and 0.91, respectively; for all the formations, the correlation coefficients are 0.81 (from Equation [6]) and 0.83 (from Equation [7]). In short, the equations obtained from the statistical analysis show that the elasticity modulus of rock samples representing different geological formations can be predicted sufficiently from compressive strength and unit weight values except for those of the Dolayoba formations.

The prediction performances of suggested relations are given in Table III. Prediction performances of these equations were evaluated by using both a correlation coefficient (r) and the root mean square error (RMSE), given in Equations [8a] and [8b] respectively.

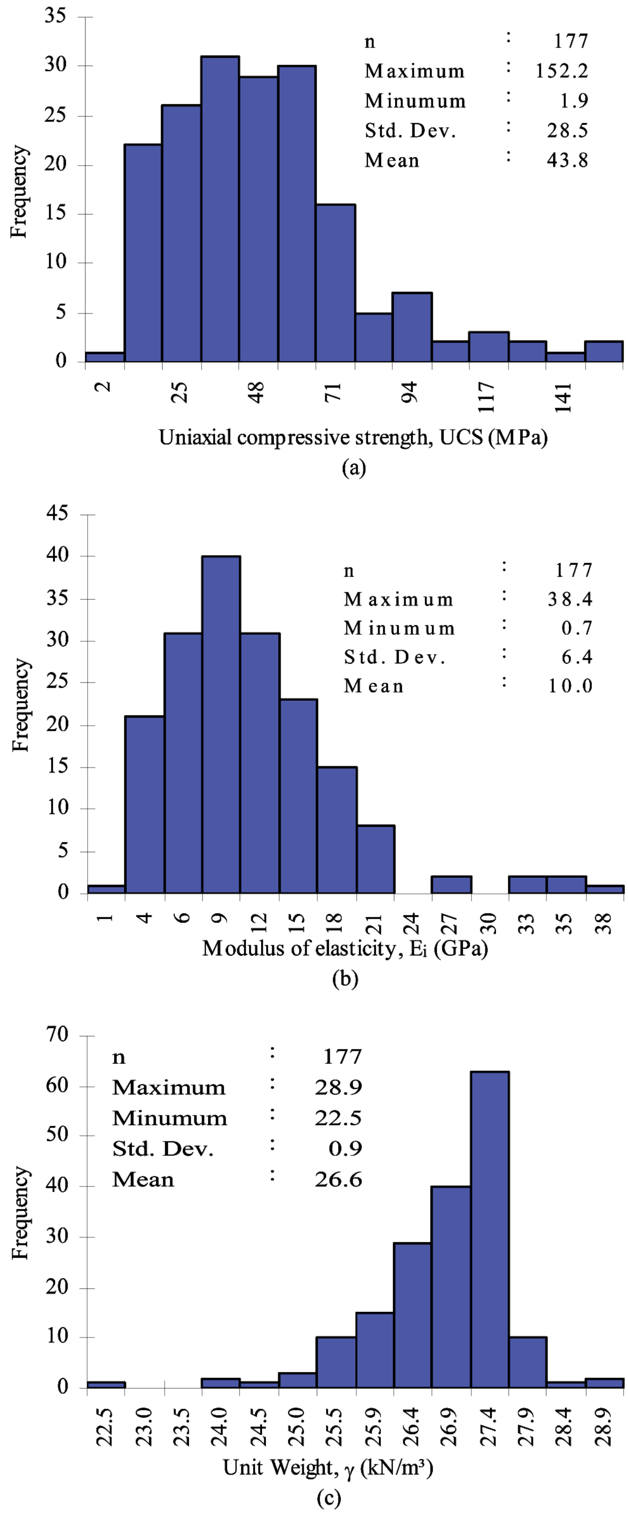

Figure 4—The histograms and statistical evaluations of the data used in to predict E_i

Estimating the modulus of elasticity of the rock material

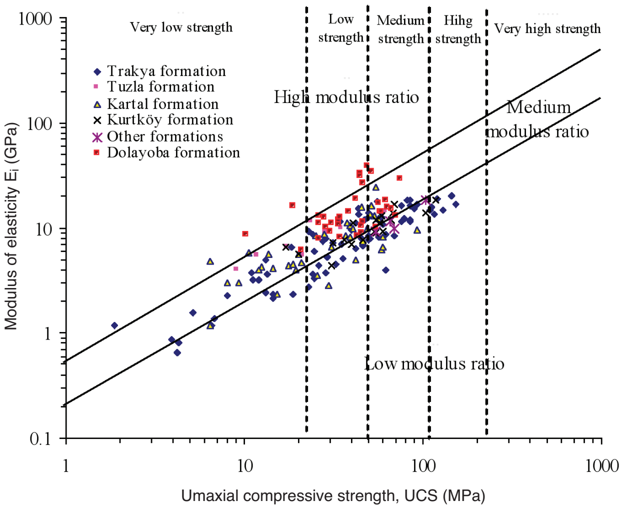

Figure 5—The distribution of the data base on the modulus rate graphic by Deere and Miller (1966)

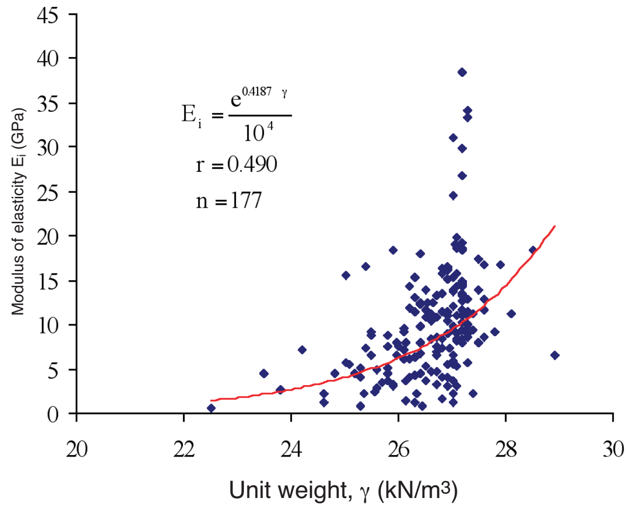

Figure 7—Correlation between unit weight and modulus of elasticity for all formations

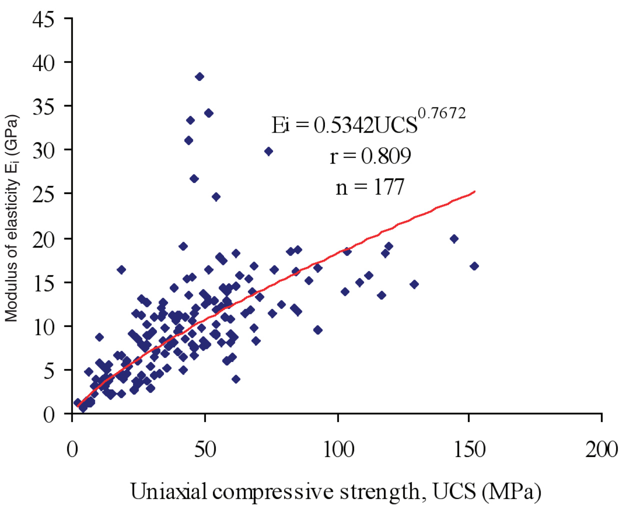

Figure 6—Correlation between UCS and modulus of elasticity for all formations

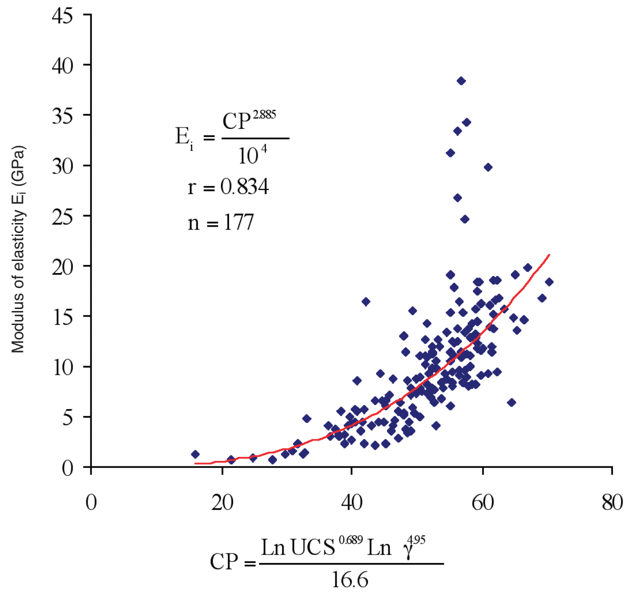

Figure 8—Correlation between UCS, γ and modulus of elasticity for all formations

Table II

Statistical relationships for estimation of modulus of elasticity for different geologic formations and all formations together

Equations		Number of data	Formation	Lithology	r
$E_i = 0.3663UCS^{0.8213}$	(1)	73	Trakya	Sandst.-siltst.-clayst.	0.915
$E_i = 1.0331UCS^{0.6443}$	(2)	8	Tuzla	Shale	0.903
$E_i = 0.7498UCS^{0.6495}$	(3)	38	Kartal	Shale-limestone	0.747
$E_i = -24.7 + 0.102UCS + 1.1\gamma$	(4)	20	Kurtköy	Sandstone-conglemerat.	0.908
$E_i = 2.0562UCS^{0.5238}$	(5)	34	Dolayoba	limestone	0.478
$E_i = 0.5342UCS^{0.7672}$	(6)	177	All form.	All lithology above	0.809
$E_i = \frac{CP^{2.885}}{10^4}$ $CP = \frac{Ln\ UCS^{0.689} Ln\gamma^{4.95}}{16.6}$	(7)	177	All form.	All lithology above	0.834

Table III

List of suggested equations for estimating the modulus of elasticity, required parameters and prediction capacity of these

Equation(s)		Lithology	Required parameter(s)	r	RMSE
$E_i = 0.3663UCS^{0.8213}$	(1)	Sandst.-siltst.-clayst.	UCS or UCS and γ	0.882	4.4
$E_i = 1.0331UCS^{0.6443}$	(2)	Shale			
$E_i = 0.7498UCS^{0.6495}$	(3)	Shale-limestone			
$E_i = -24.7 + 0.102UCS + 1.1\gamma$	(4)	Sandstone-conglemerat.			
$E_i = 2.0562UCS^{0.5238}$	(5)	Limestone			
$E_i = 0.5342UCS^{0.7672}$	(6)	All lithology above	UCS	0.809	5.2
$E_i = \frac{CP^{2.885}}{10^4}$ $CP = \frac{Ln\, UCS^{0.689} Ln\gamma^{4.95}}{16.6}$	(7)	All lithology above	UCS and γ	0.834	4.9

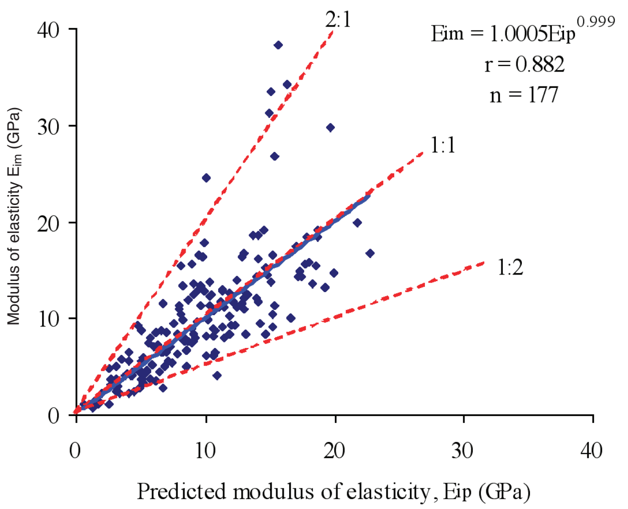

Figure 9—Cross-correlation between predicted and measured values of E_i from Equations [1–5]

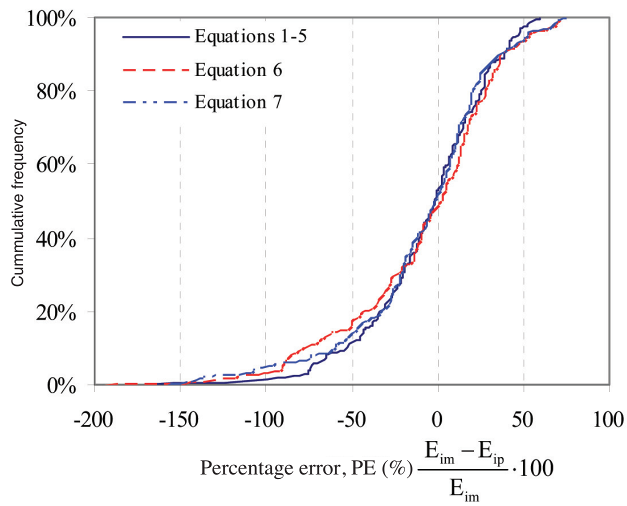

Figure 10—The relation between percentage error and cumulative frequency

$$r(x,y) = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2 \sum (y-\bar{y})^2}} \quad [8a]$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (x - x')^2} \quad [8b]$$

where x and y are standard deviations, x and x' are the measured and predicted values, respectively, and n is the number of data. If the model has excellent prediction capacity, the r and RMSE will be 1 (or -1) and zero, respectively.

In addition, by using 177 datasets collected from various lithologic types of rock, percentage errors are drawn (Figure 10). Percentage errors for approximately 75% of the data are less than 50%. Therefore, the proposed empirical equations have a strong prediction capacity and can be used to estimate the modulus of elasticity of intact rock for practical purposes.

Conclusions

Determination of the modulus of elasticity of rocks requires high quality core samples; therefore, it is sometimes difficult to determine the modulus of elasticity using direct methods applied to core samples obtained from difficult ground conditions such as stratified (thinly bedded), highly fractured and block-in-matrix rock. In this study, to overcome these problems, several basic equations were proposed by using an extensive database including modulus of elasticity, unit weight and uniaxial compressive strength of intact rock.

The significant relationships in the statistical analysis were evaluated by using a wide range of data from different rocks of different geologic formations. The database used is of great importance for current engineering applications in Istanbul since it is obtained from data of 177 drillings carried out along a 21.6 km tunnel route on the Asian side of Istanbul. The empirical equation presented has a strong prediction capacity and can be used to estimate the modulus of elasticity of intact rock for practical purposes.

It is concluded that the modulus of elasticity can be estimated significantly from the uniaxial compressive strength and unit weight of the rock using presently proposed prediction equations. It is suggested that the given prediction equations may also be used in the nine ongoing metro projects under construction in Istanbul that pass through the geologic formations from which the data used in this study was gathered.

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