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Risk-based member reliability in structural design

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The balance between safety and economy in structural design was explored in the context of the member cost, liability and location in a structure. A model was developed giving the optimal reliability of a member, taking account of the tradeoff between cost and risk in maximising the long-term expected benefit derived from the structure.

The model was first applied to a single independent member to derive a relationship which expresses the reliability required for optimal benefit as a function of the liability-cost ratio. Next the model was applied to two test structures: a determinate steel truss and a multi-storey reinforced concrete frame. Reliability analysis for both structures revealed that members can be treated as independent, and that marginal benefit is greatest for members with the highest liability-cost ratio values.

It was shown that the relationship of liability-cost ratio versus optimal reliability provides a guideline for the improvement of existing structural design. Structures with reliabilities less than the optimal value can most effectively be improved by strengthening members with the highest liability-cost ratio values, while structures with reliabilities greater than optimal are improved by economising on members with the lowest liability-cost ratio values.

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INTRODUCTION

The primary goal of engineering design is to balance safety and economy. The simplest approach to avoid failure is to overcompensate for the expected loading conditions by means of safety factors (load and material factors), usually at the expense of economy.

Safety factors can be intuitively understood to decrease the probability of failure of a design, as it accounts for reasonable variation in applied loads and member resistance values. The statistical interpretation of safety factors was formalised with the development of mathematical statistics, which enabled the development of the theory for structural reliability (Freudenthal 1947; Freudenthal & Gumbel 1953; Pugsley 1955).

It is customary to quantify structural reliability via the reliability index β, which expresses the separation between expected failure and the mean loading and resistance conditions in units of standard deviations. β is associated with the failure probability within a given reference period via the standard normal distribution as (Rackwitz & Fiessler 1978):

$$\beta = \sqrt{2}\ \mathrm{erf}^{-1}(1 - 2p_f) \qquad (1)$$

where p_f is the probability of failure of the member in its design life, and erf(·) is the error function (e.g. McQuarrie 2003). For small p_f a member's expected lifespan will far exceed the design lifetime, and the probability of failure can be taken as constant during the design life.

Calibration studies that aim to determine partial factors of safety from a statistical basis (Milford 1988; Holický *et al* 2010) indicate that the long-standing empirical range of factors used in permissible stress design correspond to β values in the range of 3 to 5.

As part of an effort to establish a robust reliability basis for structural design, the Joint Committee on Structural Safety (JCSS 2008) considered the tradeoff between the cost of a safety measure and the risk associated with fatalities due to structural failure. Based on this analysis, ISO 2394:2014 provides 50-year reference period target structural reliability values ranging between β = 2.0 and 3.8, for structural classes depending on the cost of safety and the consequences of failure. Many modern limit-states design standards for loads acting on structures use target β that comply with this range. In particular, SANS 10160:2011 uses β = 3.0 (Retief & Dunaiski 2010, also assuming a 50-year reference period). For consistency with these values, 50-year reference periods will be assumed throughout this study.

Material-focused design codes (e.g. SANS 10162:2011 and SANS 10100:2000)

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all focus on failure of individual members, specifying partial factors calibrated to target reliability values that depend on the dominant mode of failure. Milford (1988) recommends that, for South African materials codes, β = 3.0 be used for ductile failure modes, β = 4.0 for brittle modes and β = 4.5 for connections.

However, these factors do not take the location of a member in the structure into consideration. Almost all structures have members of greater and lesser importance; determinate structures being an exception. Yet, when designed according to current building standards, all members will tend to have a similar target reliability level, so that some members may be under- or over-designed from the perspective of risk. If the relative importance of members in a structure in the context of reducing overall risk exposure is taken into account, the appropriate adjustment of the design value for member resistance capacity needs to be investigated.

This work explores the tradeoff between safety cost and failure risk in determining the member reliability that is most favourable in terms of the total expected benefit over the lifetime of the structure. It develops the theory that considers optimal adjusted reliability for each member in a structure taking risk into account. The theory is then applied to two types of structures: determinate structures in which failure of one member implies failure of all, and hierarchical structures where the consequences of member failure vary with position.

THEORETICAL DEVELOPMENT

Expected benefit

Consider a structure with a design life of τ years. The structure consists of N members, grouped into n member types. Members of a given type have identical design specification, reliability, cost, and liability payable upon failure. Member dependence is specified via the $N \times N$ matrix $\mathbf{\Gamma}$, in which entry Γ_{ij} is 1 if member i supports member j, and 0 otherwise. For a determinate structure, all entries of $\mathbf{\Gamma}$ will be 1; a structure with a high degree of redundancy will have a sparse $\mathbf{\Gamma}$.

The total expected benefit Ψ derived from the use of the structure over the course of its design life is given by:

$$\Psi = I - C - R \qquad (2)$$

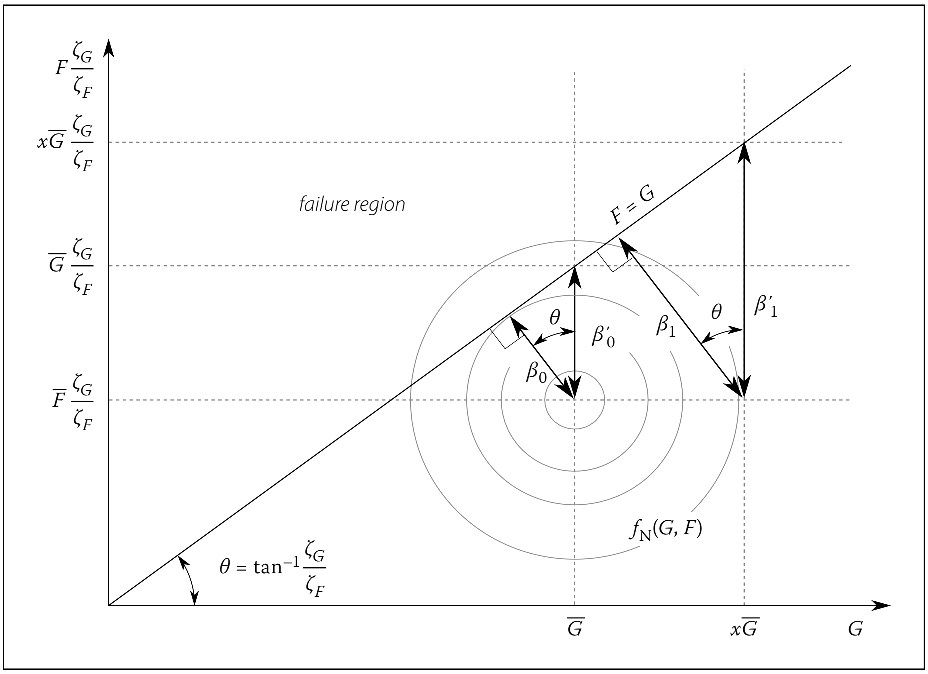

Figure 1 **Geometrical representation of the derived scaling relationship between β_0 and β_1 (Equation 10)**

where I is the total revenue generated from the structure over its lifetime, C is the cost of construction and commissioning, and R is the risk due to failure of any part of the structure. Risk, defined as the probability of an event times the potential loss resulting from it, accounts for both the magnitude and the likelihood of payable damages.

Member i costs c_i to construct, so that the total cost of the structure C can be taken as:

$$C = \sum_i^N c_i \qquad (3)$$

Secondary factors, such as maintenance cost, construction time and depreciation can be taken into account. However, they are ignored here to keep the theory tractable.

In the event of failure of member i, a set of dependent members will be affected, described by matrix $\mathbf{\Gamma}$. Liability d_i will be payable by the owner of the structure, and the failed member together with all dependent members will have to be replaced. The total cost of the failure would then be:

$$C_{fi} = d_i + \sum_i^N \Gamma_{ij} c_j \qquad (4)$$

Members are assumed to have been designed to meet a minimum target reliability β_0, for example by adhering to the specifications set out in SANS 10100:2000, SANS 10162:2011 and SANS 10160:2011. This implies a failure probability for member i of p_{fi} within the design life of the structure. The total risk over the lifetime of the structure is then:

$$R = \sum_i^N p_{fi} c_{fi} = \sum_i^N p_{fi} \left[d_i + \sum_j^N \Gamma_{ij} c_j \right] \qquad (5)$$

Member reliability adjustment

The characteristic internal force F_k and resistance G_k values used in the design of a member reflect conservative upper and lower bounds on these design parameters, respectively. For simplicity, F and G are assumed to be normally distributed with means $\overline{F}$ and $\overline{G}$, and coefficients of variation ζ_F and ζ_G.

Now suppose that the resistance capacity of a member with reliability β_0 is adjusted by a factor x, that is $G_1 = G_0 x$. A relation is required for the new reliability β_1 of the adjusted member.

Figure 1 illustrates geometrically the derivation of x in terms of β_0 and β_1 that follows. The vertical axis is scaled by the ratio of coefficients of variation to indicate values of $F\zeta_G/\zeta_F$. This ensures that the contours of the bi-variate normal density distribution $f_N(G, F)$ are circular, so that lines β_0 and β_1 are normal to the failure boundary ($F = G$), which is inclined at:

$$\tan\theta = \frac{F\zeta_G/\zeta_F}{G} = \frac{\zeta_G}{\zeta_F} \qquad (6)$$

According to its multidimensional geometrical interpretation (Rackwitz & Fiessler 1978), β_0 is the distance (in units of

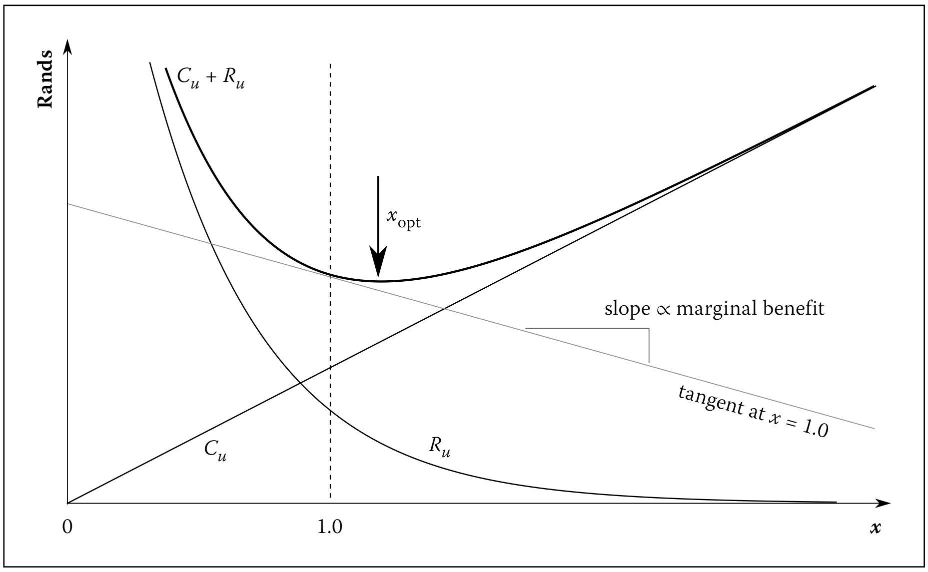

Figure 2 An illustration of the tradeoff between increasing cost C_u and decreasing risk R_u as members of type u are strengthened by a factor x (not to scale; $m = 1$)

standard deviation) from the mean to the point on the failure boundary where the multivariate probability density function is a maximum.

For $G = \overline{G}$ fixed, the marginal reliability index is then:

$$\beta'_0 = \frac{\overline{G} - \overline{F}}{\zeta_F \overline{F}} = \frac{1}{\zeta_F}\left(\frac{\overline{G}}{\overline{F}} - 1\right) \tag{7}$$

If the member resistance capacity is now adjusted, the mean shifts, and

$$\beta'_1 = \frac{\overline{G}x - \overline{F}}{\zeta_F \overline{F}} = x\beta'_0 + \frac{x-1}{\zeta_F} \tag{8}$$

Defining the total coefficient of variation as $\zeta_T = \sqrt{\zeta_G^2 + \zeta_F^2}$, it is clear from the geometry that:

$$\cos\theta = \frac{\beta_0}{\beta'_0} = \frac{\beta_1}{\beta'_1} = \frac{\zeta_F}{\zeta_T} \tag{9}$$

so that the new relationship is:

$$\beta_1 = x\beta_0 + \frac{x-1}{\zeta_T} \tag{10}$$

which can be rearranged to give:

$$x = \frac{\beta_1 + 1/\zeta_T}{\beta_0 + 1/\zeta_T} \tag{11}$$

Member cost adjustment

Let the resistance G of member i be scaled by factor x. Given this linear scaling of resistance, it will be assumed that the effect on member cost c_i can be represented as:

$$c_{ix} = c_i x^m \tag{12}$$

where m is a constant.

For example, in simple tensile failure, where the member resistance is related to the yield stress σ_y via the section area A as:

$$G_{\text{tension}} = \sigma_y A \tag{13}$$

increasing G by a factor x would imply increasing the cross-sectional area and thus the volume by the same factor. If cost is taken to be proportional to member mass, this type of failure would imply $m = 1$.

Failure in bending of a square or circular sectioned member implies moment resistance:

$$G_{\text{mom}} \propto \sigma_y A^{3/2} \tag{14}$$

Scaling G by a factor x now implies increasing the volume by a factor of $x^{2/3}$. For cost proportional to member mass, this therefore gives $m = 2/3$. Sections of more complex geometry can only be approximately represented via Equation 12.

The empirical relations used in design against buckling failure (e.g. SANS 10162:2011) cannot be directly adapted to conform with Equation 12. However, if buckling failure is described by the Euler equation, resistance of a square or circular sectioned member implies:

$$G_{\text{buckle}} \propto I/L^2 \propto A^2/L^2 \tag{15}$$

so that increasing G by a factor x implies an increase in the volume by a factor $x^{1/2}$, that is $m = 1/2$.

Although only exact for a few special cases, the preceding discussion suggests that m values can be expected to range between 0.5 and 1.0 for basic member failure modes.

Marginal benefit of increasing member reliability

Let the G resistance of all members of type u in the structure be scaled by a factor x, resulting in new member reliabilities of β_{1x} and failure probabilities p_{fx}.

Changes in the expected benefit will result only from changes in the unit cost and failure probability of members of type u. The contribution to the total cost and risk from members of type u is:

$$C_u = \sum_{i \in u} c_i x^m = C_{u0} x^m \tag{16}$$

$$R_u = \sum_{i \in u} p_{fx}\left(d_i + \sum_j^N \Gamma_{ij} c_j x_j^m\right) \tag{17}$$

where $x_j = x$ for $j \in u$ and $x_j = 1$ otherwise.

With increasing x, C_u increases linearly, while R_u decreases asymptotically to zero as the probability of failure p_{fx} decreases.

These trends are schematically illustrated in Figure 2. As a result of these opposing trends, an x value x_{opt} exists where $C_u + R_u$ is a minimum, i.e. where Ψ is a maximum.

At x_{opt} the member design represents an optimal balance between safety and economy (see Equation 2). If the member group is under-designed (with respect to Ψ), the benefit can be increased by making members more reliable, so that $x_{opt} > 1$; if the member group is over-designed, members can be more affordable and $x_{opt} < 1$.

The extent to which a member group is over- or under-designed is quantified via the effect of spending (or saving) on Ψ. This 'marginal benefit' ψ is given by:

$$\psi = \left.\frac{d\Psi}{dC_u}\right|_{x=1} = \frac{1}{mC_{u0}} \left.\frac{d\Psi}{dx}\right|_{x=1} \tag{18}$$

Under-designed members require additional spending, and so $\psi > 0$; over-designed members imply too much has been spent, so $\psi < 0$.

Comparison of ψ among member types in a structure indicates where in a design the greatest change in Ψ can be affected for a unit amount of expenditure/savings. In addition $\psi = 0$, implies that the member type design is optimal. This special scenario is described by β_{opt}, the value of β_0 for which $x_{opt} = 1$.

Optimal reliability of an independent member

Consider now a single, independent member designed to reliability β_0 with cost c and liability d. Given coefficients of variation ζ_G and ζ_F for the resistance and internal forces, what is the optimal reliability of the member β_{opt}?

If member resistance is increased by a factor x, the reliability becomes β_1 with failure probability p_1, and the expected benefit is:

$$\Psi = I - x^m c - p_1 c\,[d/c + x^m] \tag{19}$$

At maximum expected benefit, $x = x_{opt}$ and $d\Psi/dx = 0$. At the point of optimal marginal benefit where $\psi = 0$ and $x_{opt} = 1$, the design reliability is also optimal, so that $\beta_0 = \beta_1 = \beta_{opt}$, and $p_1 = p_{opt}$. Taking $d\Psi/dx = 0$ and setting $x_{opt} = 1$ yields:

$$1 + p_{opt} + \frac{d/c + 1}{m}\,p'_{opt} = 0 \tag{20}$$

where, from the density function of the standard normal distribution (e.g. McQuarrie, 2003):

$$p_{opt} = \frac{1}{2}\left[1 + \mathrm{erf}\left(-\frac{\beta_{opt}}{\sqrt{2}}\right)\right] \tag{21}$$

$$p'_{opt} = \left.\frac{dp}{dx}\right|_{x=x_{opt}} = \frac{\beta_{opt} + 1/\zeta_T}{-\sqrt{2\pi}}\exp\left(-\frac{\beta^2_{opt}}{\sqrt{2}}\right) \tag{22}$$

If $d/c \gg 1$, the relation becomes:

$$1 + p_{opt} + \frac{d/c}{m}\,p'_{opt} = 0 \tag{23}$$

With β_{opt} known, the required adjustment to the design resistance is then:

$$x_{adjust} = \frac{\beta_{opt} + 1/\zeta_T}{\beta_0 + 1/\zeta_T} \tag{24}$$

Equation 20 provides an implicit relationship for β_{opt} of an independent member in terms of d/c, ζ_T, and m. As can be seen in Figure 3, the dominant factor determining the value of β_{opt} is the liability-cost ratio d/c.

This dependence implies two important concepts. Firstly, for the same member cost, a greater failure liability requires a greater member reliability. That is, a greater risk warrants higher safety levels. Secondly, for a given failure liability, greater member cost results in lower optimal member reliability. That is, increased safety is more affordable for less expensive members.

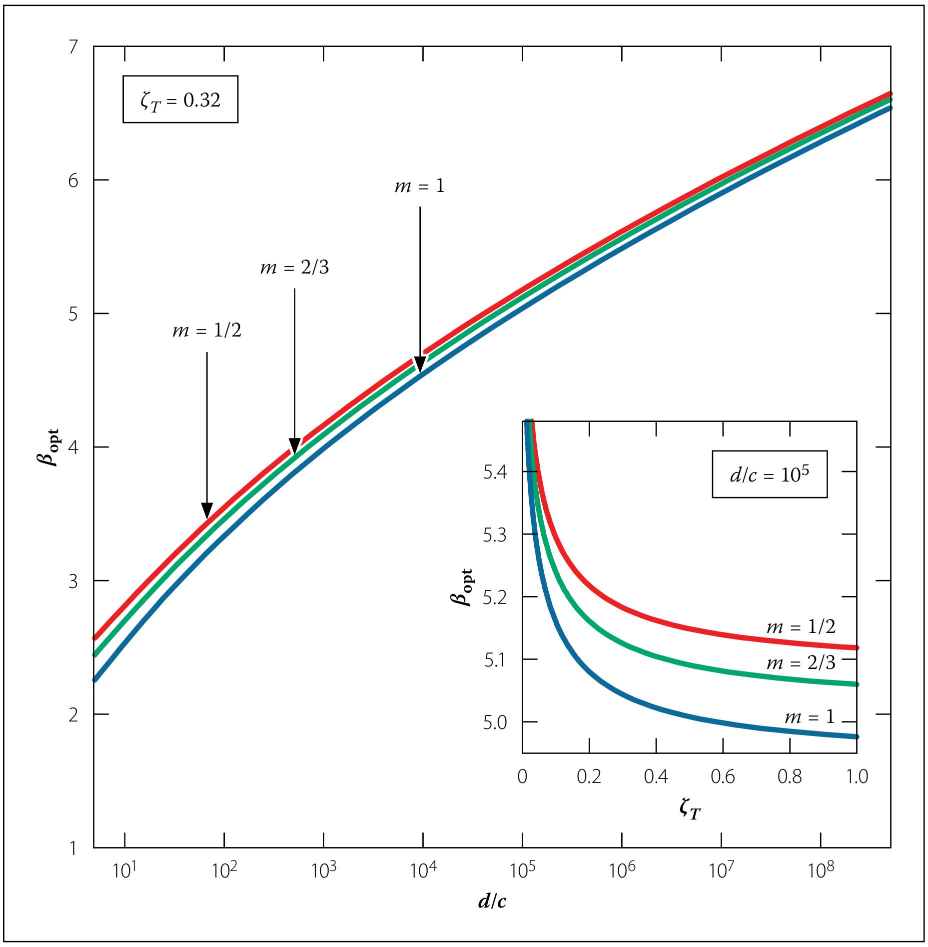

Figure 3 Optimal reliability β_{opt} of an independent member; main plot determined for $\zeta_T = 0.32$; inset for $d/c = 10^5$

The relationship is not exact for members forming part of a structure, as the risk includes the cost of dependent members (see Equation 17). Nonetheless, if $d_i \gg \sum_j \Gamma_{ij} c_j$ the coefficient of p'_{opt} would again tend to $(d/c)/m$. Therefore, Equation 20 can be used for members that are part of structures as well, provided that the liability due to member failure exceeds the cost of repair by a sufficient margin for the latter to be negligible.

APPLICATION TO TEST STRUCTURES

The theory developed in the preceding section is now applied to two different example test structures. First, a determinate steel truss, where failure of any member compromises the entire structure, so that the failure liability is the same for all members. Second, a three-storey reinforced concrete frame, in which the failure liability of a member depends on its position within the structure.

The primary aim of this reliability analysis is to explore the potential spread of optimal member reliabilities β_{opt} within a structure. In addition, the extent of strengthening required to upgrade members in a structure from design reliability β_0 to optimal β_{opt} is of interest, together with the most effective modification by which the expected benefit of an existing design can be increased.

To perform the analysis, coefficients of variation ζ for applied loads and member resistance capacities must be assigned. Table 1 summarises the range of values suggested in the literature. To conform with values used in the calibration of limit-states design codes (Holický *et al* 2010; Holický & Retief 2005; Kemp *et al* 1987; Milford 1988), $\zeta_G^{steel} = 0.10$ and $\zeta_G^{concrete} = 0.20$, together with imposed loads $\zeta_F = 0.25$, were used in the test examples.

As failure modes are not specified, it will be assumed that the cost adjustment relation (Equation 12) is linear, that is $m = 1$ and $c_{ix} = xc_i$. As noted in Figure 3, the effect of this assumption on β_{opt} values is expected to be minor.

Finally, the analysis requires $d\Psi/dx$ to be evaluated, but direct values of Ψ are not

Table 1 Values for the coefficient of variation of loads ζ_F and material strength ζ_G reported in the literature

Loads	ζ_G
Dead	0.1[a,b]
Imposed	0.20–0.25[a,b,c]
Wind	0.25–0.52[a,b,c]
Materials	ζ_F
Steel	0.10–0.13[d,e]
Reinforced concrete	0.20–0.25[f,g]

a – Holický & Retief 2005; *b* – Ellingwood 1982; *c* – Retief & Dunaiski 2010; *d* – Kemp *et al* 1987; *e* – Galambos 1990; *f* – Holický *et al* 2010; *g* – MacGregor 1983

determined. The revenue I earned from use of the structure is therefore not needed, and will not be set.

Determinate steel truss bridge

The first test structure is a Pratt-type determinate truss (Figure 4), in which failure of any member results in failure of the entire structure.

In the current analysis only the primary support trusses are considered, with the lateral bracing and the bridge deck excluded. These trusses support a deck of width sufficient to accommodate a single vehicle lane, so that only one vehicle would use the bridge at a given time. It is further assumed that connections are significantly stronger than the members, so that only member reliability needs to be considered.

The bridge is designed assuming a travelling design load of two 150 kN point loads 5 m apart (TMH7:1981). Sections are designed according to the SANS 10162:2011 specification, with sections of similar

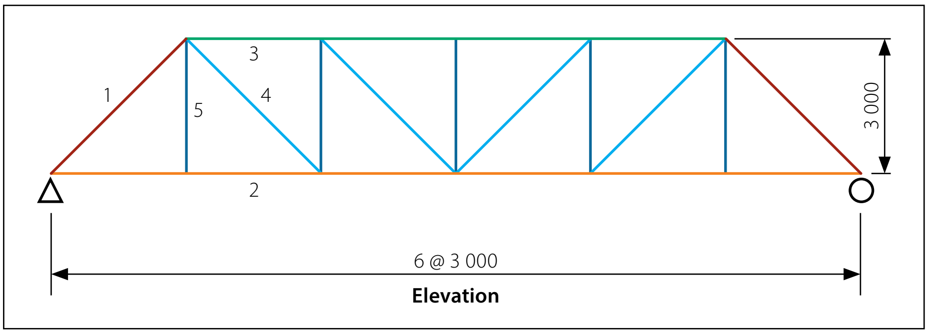

Figure 4 Test structure made up of a Pratt-type truss; the five member types (numbered) are identified by the line colours

Table 2 Section design and reliability analysis parameters for the truss test structure

Member design					
Member type	F_{compr}(max) (kN)	$F_{tension}$(max) (kN)	A_{req} (mm^2)	$A_{section}$ (mm^2)	**Section (circular hollow)**
1	430	–	2 260	2 270	165.1 × 4.5
2	–	470	1 470	2 270	165.1 × 4.5
3	510	–	2 050	2 270	165.1 × 4.5
4	135	330	1 360	1 370	101.6 × 4.5
5	165	200	1 030	1 370	101.6 × 4.5

Structure-level reliability parameters				
$\zeta_F = 0.25$	$\zeta_G = 0.10$	$\zeta_T = 0.26$	$m = 1$	$\tau = 50$ years
Member-level reliability parameters				
Member type	**Number of members**	c	d	d/c
1	2	R 1.5k	R 2.0m	1.33×10^3
2	6	R 1.2k†	R 2.0m	1.67×10^3
3	4	R 1.0k	R 2.0m	2.00×10^3
4	4	R 0.75k	R 2.0m	2.67×10^3
5	5	R 0.5k	R 2.0m	4.00×10^3
	Total:	**R 19.7k**		

† 20% additional cost for fabrication necessary to allow erection of the bridge deck, based on the number of joints along the bottom chord and the manufacturing cost fraction

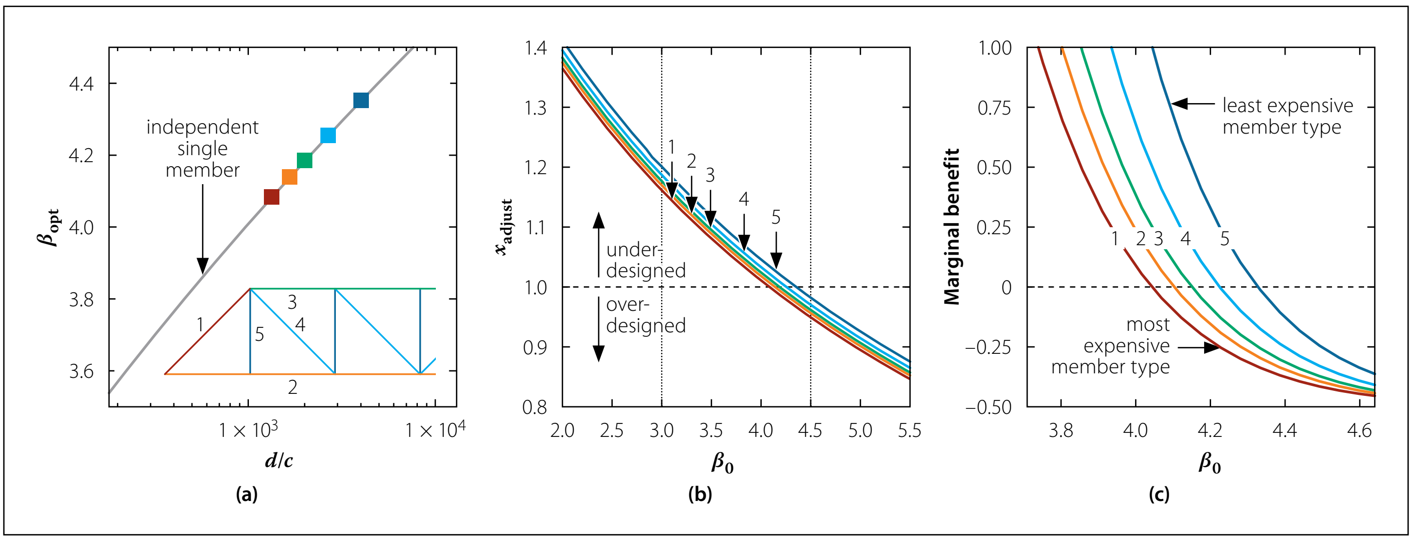

Figure 5 Analysis results for the truss test structure; independent single member trend determined using $m = 1$ and $\zeta_T = 0.26$

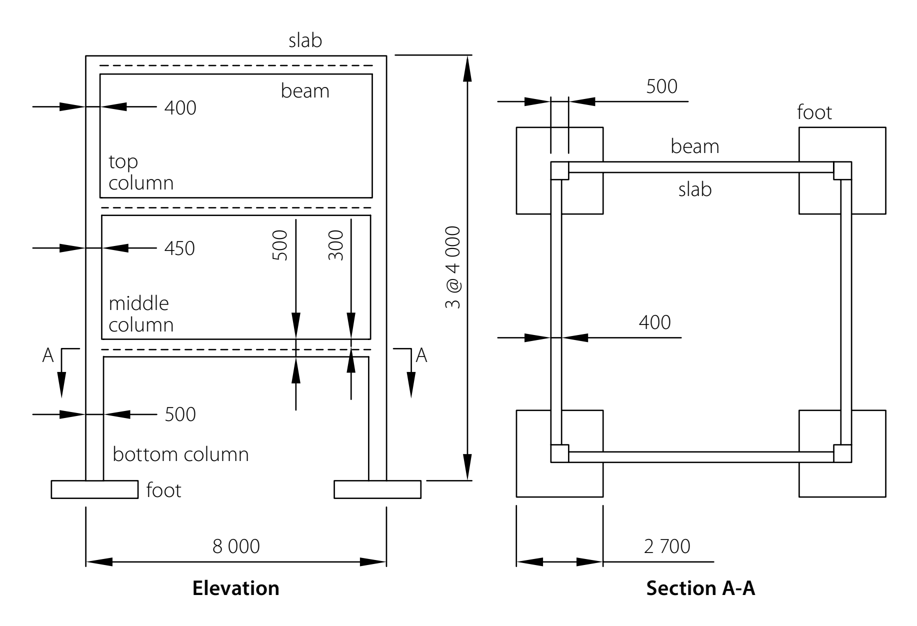

Figure 6 Test structure of a three-storey frame; the six member types are labelled

loading grouped into five member types for ease of construction. Analysis results and section design are summarised in Table 2.

Estimated costs and liabilities for each member group are given in Table 2. Costs are assumed to be R16k/tonne, based on a cost breakdown of 40% material, 40% fabrication and transport, and 20% construction and labour (McNamara 2017). Taking both primary support trusses into account, the total cost of the structure is calculated to be R39.4k.

To determine the associated liability, the legal damages due to injury/death of individuals using the structure must be accounted for. Given the size of the bridge, it is unlikely that more than one vehicle would be on the bridge if it were to fail. Damages should then be expected to be payable for two persons, at a sum of R2.0m. This value is based on an assessment of reasonable damages for injury/death in the context of South African law, performed by Koch (2011); occupants are assumed to be one breadwinner and one non-breadwinner.

Starting from the assumption that the structure is of sound design, i.e. every member satisfies a minimum design reliability β_0, the optimal reliability value β_{opt} for each member type is determined numerically by finding the β_0 value for which $d\Psi/dx = 0$ at $x = 1$. From this, the required resistance adjustment factor x_{adjust} is determined using Equation 24, and marginal benefit ψ as defined in Equation 18.

Results of the analysis are shown in Figure 5. As seen in Figure 5(a), member reliabilities (β_{opt}) that maximise the expected benefit correspond closely to the independent member values predicted from their liability-cost ratios (d/c values), as determined using Equation 23.

The range of x_{adjust} values needed to adjust members to β_{opt} falls between 1.2 and 0.9 (Figure 5(b)) for reasonable design reliability values β_0 associated with structural design (3.0 to 4.5, see Introduction).

Marginal benefit derived from improving any member with reliability β_0 towards its β_{opt} value is shown in Figure 5(c). Marginal benefit decreases with increasing β_0, becoming negative for $\beta_0 > \beta_{opt}$. At a given β_0, the marginal benefit is higher for more affordable members, corresponding to an increase in ψ with d/c values.

Multi-storey reinforced concrete building

The second test structure is a three-storey frame building constructed from reinforced concrete, consisting of a series of slab-beam-columns, and supported by square footing shallow foundations (Figure 6).

The structure is analysed assuming an imposed load of 4 kN/m^2 on each floor and a peak wind speed pressure of 1.3 kPa, as specified by SANS 10160:2011. The six reinforced concrete member types are designed according to SANS 10100:2000. Analysis results and section design are summarised in Table 3.

Estimated costs and liabilities for the member groups are also given in Table 3. Costs are determined using a unit cost of

Table 3 Section design and reliability analysis parameters for the three-storey frame test structure

Member design (30 MPa concrete)

Member type	F_{M1}(max) (kNm)	F_{M2}(max) (kNm)	F_{compr}(max) (kN)	Dimensions (mm)	Reinforcing
slab	21	–38	–	300 × 8 000^2	Y10-150
beam	160	–180	–	400 × 500	3Y25 B&T
foot	520	–	1 240	500 × 2 700^2	Y16-150
top column	50	50	360	400 × 400	8Y12
middle column	110	140	760	450 × 450	12Y20
bottom column	390	0	1 240	500 × 500	12Y32

Structure-level reliability parameters

$\zeta_F = 0.25$	$\zeta_G = 0.20$	$\zeta_T = 0.32$	$m = 1$	$\tau = 50$ years

Member-level reliability parameters

Member type	Number of members	c	d	d/c
slab	3	R100k	R10m	1.00 × 10^2
beam	12	R7k	R10m	1.43 × 10^3
foot	4	R15k	R30m	2.00 × 10^3
top column	4	R4k	R10m	2.50 × 10^3
middle column	4	R5k	R20m	4.00 × 10^3
bottom column	4	R6k	R30m	5.00 × 10^3
	Total:	R504k		

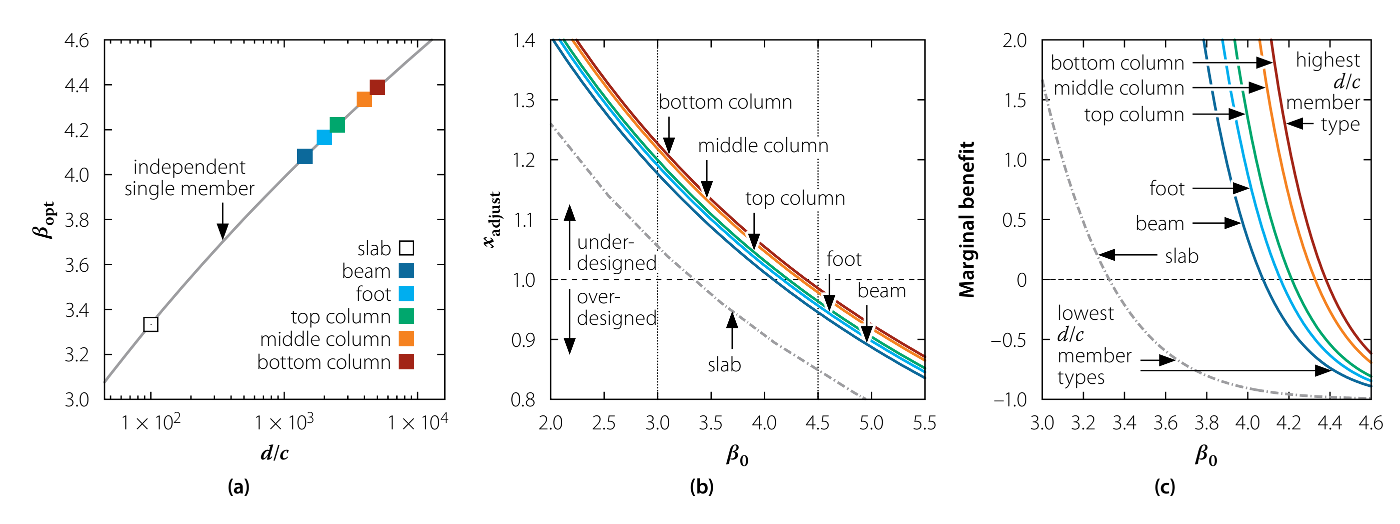

Figure 7 Analysis results for the three-storey frame test structure; independent single member trend determined using $m = 1$ and $\zeta_T = 0.32$

R6k/m^3 (Robberts & Marshall 2010, adjusted for inflation), based on a breakdown of 35% concrete, 35% reinforcing steel, 15% formwork and 15% labour. The total cost of the structure is calculated to be R504k.

In contrast to the truss example, this structure has a unique set of supported members associated with each individual member. Failure of a given member is assumed to compromise only its supported members (as described via the matrix $\mathbf{\Gamma}$). For example, failure of a beam would compromise its supported slab, but leave its supporting column unaffected. Failure of a column would compromise the members it supports: two beams, their supported slab, and recursively the column on the next level up with all the members it supports. The number of compromised members due to failure of a column will therefore increase towards the base of the structure: if a ground floor column fails, all columns and their associated beams and slabs above it will no longer have sufficient support; if a column in the topmost storey fails, only two top beams together with their supported slab are compromised.

Assuming the building to be residential, with a normal-use occupation of 10 persons per storey, legal damages due to serious injury or death is estimated at R10m per floor (Koch 2011).

Starting from the assumption that every member in the structure satisfies a minimum target reliability β_0, the optimal reliability β_{opt} is determined numerically for each member type, together with the required resistance scaling x_{adjust} and the marginal benefit ψ.

Results of the analysis are presented in Figure 7. Similar to the results obtained for the truss test structure, optimal member reliabilities β_{opt} correspond closely to the predicted independent member values (Figure 7(a)). x_{adjust} values needed to adjust members to β_{opt} vary from about 1.25 for β_0 = 3.0 to 0.95 for β_0 = 4.5. Marginal benefit again decreases with increasing β_{opt} values, being negative for $\beta_0 > \beta_{opt}$. At a constant β_0, members with higher d/c values have higher marginal benefit; however, there is no longer a simple correlation with member cost.

DISCUSSION

Basic trends

The close correspondence of member values to the independent member β_{opt} values predicted from their d/c ratios (Figures 5(a) and 7(a)) indicates that the relationship for optimal reliability of independent members (Equation 20) can be applied for members in structures, provided that the liability d of the member in question is much greater than the cost of its supported members.

For example, in the truss test structure d = R2.0m for all members, which is much larger than the cost of repair, as the cost of the entire structure is R39.4k. In the frame, the liability of the slab is R10m, while the cost of repair is only the cost of the slab, R100k; the liability of the bottom column is R30m, while the cost of replacement is R357k. This comparison ignores demolition costs, and loss of revenue is also not accounted for in the model. Both these factors will increase the liability expenses by amounts comparable to or smaller than estimates used here.

This result suggests that knowledge of the cost of a member, the liability implications of failure, and estimates of the coefficients of variation can provide an indication of the optimal design reliability of a member.

For South African design standards, Milford (1988) recommends β_0 = 3.0 for ductile failure modes and β_0 = 4.0 for brittle failure modes. Ductility allows for load redistribution and provides time for remedial action to be taken prior to collapse. The reliability analysis of the two test structures considered here assumes that failure leads to collapse, implying that either brittle failure occurs, or no remedial action had been taken when ductile failure started. In this sense, the calculated β_{opt} values are broadly consistent with the recommended β_0 value for brittle failure of Milford (1988).

The direct relationship between marginal benefit ψ and the d/c ratio of a member can be understood as follows. The marginal benefit decreases with increasing β_0 (Figures 5(c) and 7(c)) and is zero for $\beta_0 = \beta_{opt}$. Members with higher β_{opt} values, and thus higher d/c ratios, will therefore have higher marginal benefit values at a given β_0 value.

This suggests a simple guideline for improving the expected benefit of an existing structural design: for members that are under-designed relative to β_{opt}, the greatest impact on expected benefit is obtained by strengthening the member with the highest d/c value; for over-designed members, the greatest impact on expected benefit is obtained by reducing the size of the members with the lowest d/c value.

This principle is illustrated in the two examples. In the truss, all members have the same liability, so that the highest d/c value corresponds to the least expensive member type. Strengthening this member type would be the most affordable way to increase the expected benefit. In the frame, the columns have similar costs, but very different liabilities. Strengthening the columns with the greatest liability, i.e. the bottom columns, thus reduces the risk by the greatest amount, bringing about the largest increase in expected benefit.

Effect of assumptions

As shown by Equation 23, m affects β_{opt} by acting as an adjustment to d/c. As seen in Figure 3, the effect is relatively small: assuming $m = 1$ results in β_{opt} about 2% lower than for $m = 1/2$. For the two test structures it was assumed that $m = 1$, as the nature of failure is not specified in either example. As shown in Equations 13–15, m values can be expected to vary between 1/2 and 1. The effect of $m \neq 1$ would therefore be at most a 2% increase in the values of β_{opt}.

The relationship for the reliability of the adjusted member (Equation 10) was derived assuming that both the internal force F and the member resistance G are normally distributed. This assumption allows the simple form of Equation 7 upon which the derivation is based. Depending on the nature of loading, this assumption does not always hold. The Gumbel distribution is generally used for wind loading, while imposed loading is often represented via a log-normal distribution (Retief & Dunaiski 2010). These distributions are all positively skewed (asymmetrical with positive tails), so that transformation to normal space (via the Rosenblatt transformation equations, Ang & Tang 1984) would distort the F–G line to be concave down. The result would be that, for a given β_0 value, the corresponding β_1 would be somewhat smaller than predicted by Equation 10. β_{opt} values determined in this work can therefore be viewed to represent upper bounds on the values for non-normally distributed parameters.

The effect of time on the value of money via interest rates and inflation is ignored in the model. This effect would enter the model in the revenue derived from the structure, and would also be needed if the probability of failure is not taken as constant during the lifetime of the structure. The latter would be the case if the lifetime was similar to the mean time to failure. However, for the failure probabilities associated with structural members, mean time to failure is in the order of 10^3 to 10^5 years, which is far greater than the common 50 to 100-year design lifetimes.

The analyses performed in this work focus on the member level only; system level reliability is not accounted for, and is assumed to be a cumulative, linear result of the individual members. Similarly, it is assumed that failure liabilities are the cumulative result of individual member failure liabilities. Member interaction effects at system level are therefore not accounted for. For example, redundancy in the structure due to ductility and load redistribution can limit the cumulative increase in liability, and thus reduce β_{opt} for some members in the structure.

CONCLUSION

A model was developed for the adjusted reliability of a member strengthened by a multiplicative factor. The model allows the degree of strengthening required to maximise overall expected benefit derived from the structure to be determined together with the marginal benefit. This provides an indication of the greatest change in benefit brought about by strengthening a given member type.

The model was first applied to a single independent member to derive a relationship expressing the reliability β_{opt} required for optimal benefit as a function of the liability-cost ratio d/c for the member. Other parameters in the equation, i.e. the coefficients of variation and the cost scaling power, have only a minor effect on β_{opt}.

Next the model was applied to analyse the reliability of members in two example test structures: a determinate steel truss and a three-storey reinforced concrete frame. Reliability analysis of members in both structures reveals that the independent member $d/c - \beta_{opt}$ relationship is applicable to members that form part of a structure, while marginal benefit was found to be greatest for members with the highest β_{opt} and d/c values.

The relationship therefore provides a guideline for the improvement of existing structural designs. Structures that are under-designed with respect to β_{opt} can be most effectively improved by strengthening members with the highest d/c values; over-designed structures are most effectively improved by reducing member sizes with the lowest d/c values.

While the model provides useful trends, its quantitative value is limited by the various assumptions made. Most notable of these are: (a) the normal distribution of applied loads and material resistances, (b) the power-law scaling of member cost with resistance adjustment, (c) the focus on reliability solely at the member level, and (d) neglecting connections. Exploring each of these assumptions will be the focus of future research work.

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