

# Effect of the minimum void ratio on the vertical intercept of the steady state line of non-plastic soils

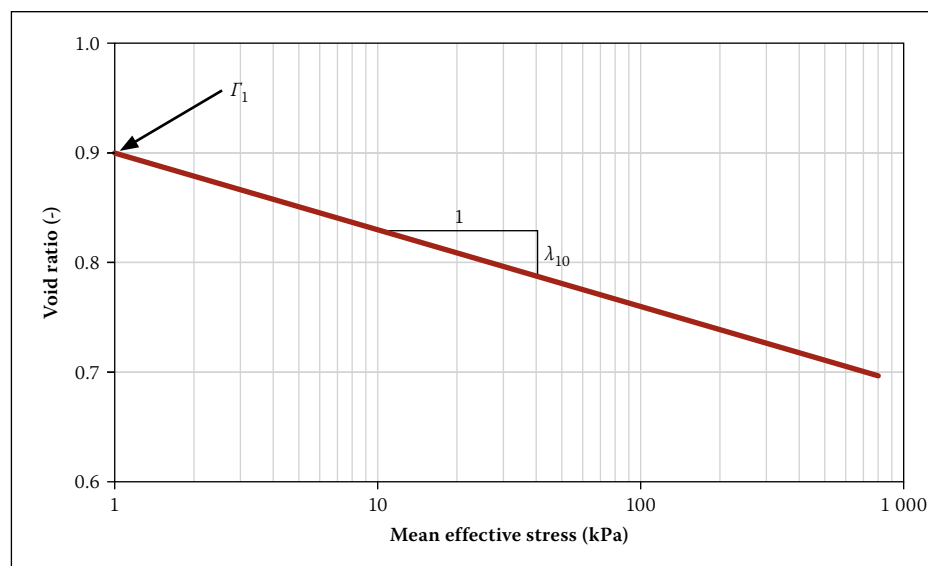
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The steady state line (SSL) plays a key role in understanding and modelling the mechanical response of soils. Accordingly, understanding how the SSL correlates to soil index properties is of primary importance. A previous study reported that the vertical location of the SSL ( $\Gamma_1$ ) in void ratio ( $e$ ) versus mean effective stress ( $p'$ ) space is correlated to the minimum void ratio ( $e_{min}$ ). However, the correlation only included soils with narrow particle size distributions (PSD) and low fines content (FC). In the current study, published data corresponding to 30 non-plastic soils were re-processed to further explore the applicability of the  $\Gamma_1$ - $e_{min}$  correlation. The results indicate that the  $\Gamma_1$ - $e_{min}$  correlation is linear ( $R^2 = 0.85$ ) and valid regardless of the coefficient of uniformity ( $C_u$ ), FC, and particle shape. The  $\Gamma_1$ - $e_{min}$  dataset presented herein was also compared to a previously published dataset, and good agreement was observed. It is proposed that the  $\Gamma_1$ - $e_{min}$  correlation can be very useful to understand how the  $\Gamma_1$  of different non-plastic soils compare to one another, and to minimise the extent of triaxial testing required when characterising a soil deposit from an SSL standpoint. Limitations of the  $\Gamma_1$ - $e_{min}$  correlation are also discussed.

## INTRODUCTION

Soils reach constant values of void ratio ( $e$ ), mean effective stress ( $p' = (\sigma'_1 + \sigma'_2 + \sigma'_3)/3$ ), and deviator stress ( $q = \sigma'_1 - \sigma'_3$ ) when sheared to large strains (Castro 1969). The locus of steady state ( $e, p', q$ ) coordinates attained when shearing from different initial states, defines the steady state line (SSL). Because the SSL represents the stress and void ratio conditions towards which a soil evolves when sheared, it plays a key role in defining mechanical response. Two projections are typically used to define the SSL:  $q$ - $p'$  (stress plane) and  $e$ - $p'$  (compression plane). The

$q$ - $p'$  projection, which reflects steady state frictional properties, is strongly dependent on particle shape (Cho *et al* 2006) and largely independent of particle size distribution (PSD) (Carrera *et al* 2011; Rahman *et al* 2014). By contrast, the  $e$ - $p'$  projection, representative of stiffness, is affected by particle shape (Cho *et al* 2006), PSD (Thevanayagam *et al* 2002; Rahman & Lo 2008; Muir-Wood & Maeda 2008; Li *et al* 2013), and void ratio limits (Cho *et al* 2006; Cubrinovski & Ishihara 2000; Hemer *et al* 2016). Given the greater number of factors that affect the  $e$ - $p'$  projection, it has received attention from a significant number



**Figure 1** Idealisation of the SSL using Equation 1

## TECHNICAL NOTE

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of researchers and is the focus of this note. The  $e$ - $p'$  projection is commonly modelled with Equation 1:

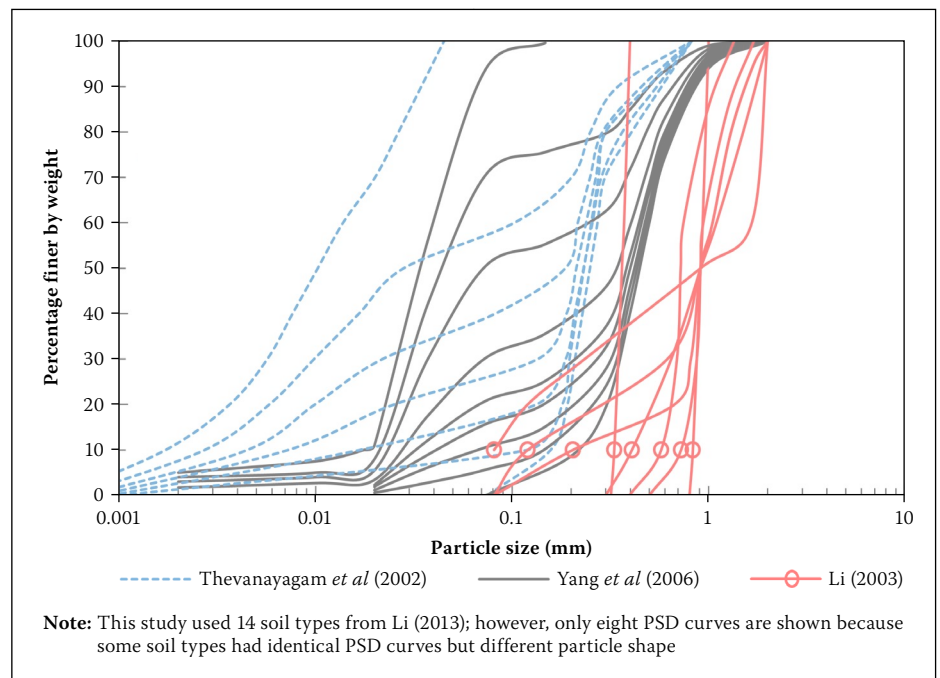
$$e = \Gamma_1 - \lambda_{10} \log_{10}(p') \quad (1)$$

Where  $\lambda_{10}$  is the slope of the SSL in semi-logarithmic space and  $\Gamma_1$  is the void ratio at  $p' = 1$  kPa (Figure 1). The current note will explore the correlation between the minimum void ratio ( $e_{min}$ ), which is associated with a defined maximum density state, and  $\Gamma_1$ .

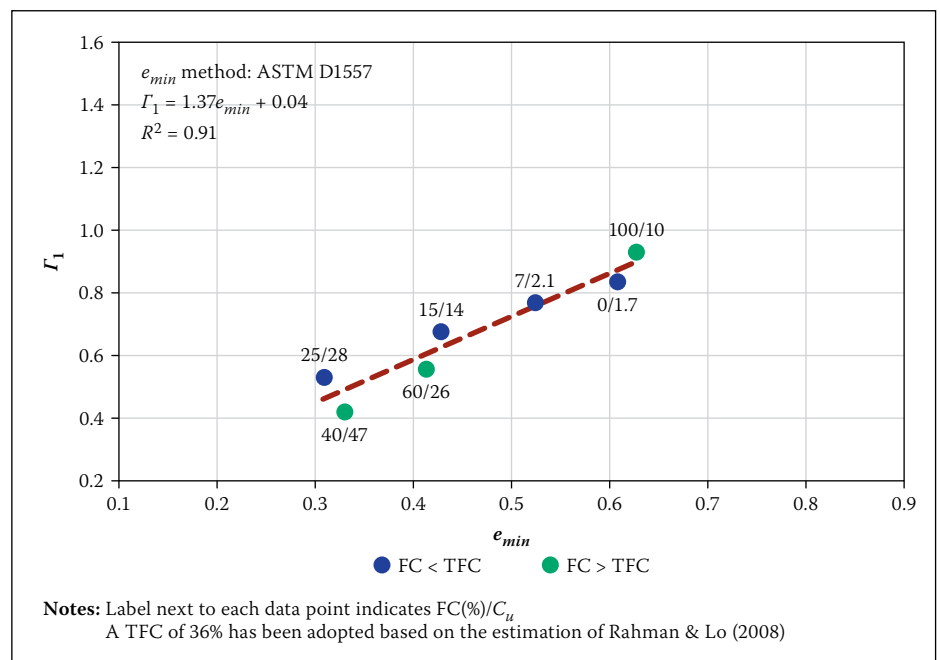
Several studies have investigated how the SSL is affected by soil index properties. Thevanayagam *et al* (2002) tested gap-graded mixtures composed of sand and non-plastic fines and concluded that the vertical location of the SSLs could be explained by the fines content (FC). They noted that as FC increases from zero, the SSL shifts downwards in  $e$ - $p'$  space ( $\Gamma_1$  decreases), and that beyond a certain FC value it shifts upwards ( $\Gamma_1$  increases). The FC value at which the SSL shift reverses direction was termed by Thevanayagam *et al* (2002) as the threshold FC (TFC). The calculation of the parameters proposed by Thevanayagam *et al* (2002) to explain the effect of FC required knowledge of the SSLs of the different sand-fines mixtures. Consequently, the framework lacked predictive power. To overcome this, Rahman & Lo (2008) developed semi-empirical equations to calculate, as a function of FC and other PSD descriptors, the parameters in the framework proposed by Thevanayagam *et al* (2002). This allowed the prediction of the SSL of sands with  $FC < TFC$ , provided that the SSL of another sand-fines mixture with  $FC < TFC$  was known. More recent works show that the effect of non-plastic fines on the SSL continues to be investigated (e.g. Mohammadi & Qadimi 2015; Rahman *et al* 2014; Yang *et al* 2015).

Despite the success of Rahman & Lo (2008) in predicting several SSLs, their framework has limitations that hinder its wider applicability. For example, the framework is limited to soils with FC smaller than the TFC, which tends to be close to 40%. Additionally, the framework cannot explain the differences between the SSLs of soils with no fines but different PSDs or grain shape.

Cho *et al* (2006) explored the correlation between  $\Gamma_1$  and  $e_{min}$  of 49 natural and crushed sands with mostly little to no fines (only six sands had  $FC > 12\%$ ) and a maximum coefficient of uniformity ( $C_u$ ) of 6.2 (only three sands had  $C_u > 4$ ). They found that the correlation was linear, independent of particle shape, and of modest strength ( $R^2 = 0.54$ ). However, the validity of the  $\Gamma_1$ - $e_{min}$  correlation for high FC values or widely graded soils remains untested. The objective of this



**Figure 2** PSD curves of the 30 soils whose data was processed by the current authors



**Figure 3**  $\Gamma_1$  vs  $e_{min}$  plot corresponding to soil types reported in Thevanayagam *et al* (2002)

note is to present evidence that expands the applicability of the  $\Gamma_1$ - $e_{min}$  correlation to a wide range of non-plastic soils, regardless of PSD descriptors such as FC and  $C_u$ . The wide applicability of this correlation is considered a step towards overcoming the limitations encountered when using FC and TFC to explain the location of the SSL.

## METHODOLOGY

Datasets from three references were processed to explore the validity of the  $\Gamma_1$ - $e_{min}$  correlation: Thevanayagam *et al* (2002), Yang *et al* (2006) and Li (2013). The dataset from Thevanayagam *et al* (2002) includes seven soils composed of foundry sand mixed with

non-plastic crushed silica fines. The resulting mixtures have FCs varying from 0% to 100% and  $C_u$  varying from 1.7 to 47. The dataset from Yang *et al* (2006) includes nine soils composed of Høksund sand mixed with non-plastic Chengbei silt. The resulting mixtures have FCs varying from 0% to 94% and  $C_u$  varying from 2 to 14. The Thevanayagam *et al* (2002) and Yang *et al* (2006) datasets were used herein to assess the validity of the  $\Gamma_1$ - $e_{min}$  correlation over a wide range of FC and  $C_u$  values. The dataset from Li (2013) includes 14 soils of which six were made of glass balls (spherical particles) and eight of Hostun sand (angular particles). Two soils, one each of glass balls and Hostun sand, had  $FC = 10\%$ , whereas the remaining 12 had

**Table 1** Values of  $p'$  and  $e$  used to calculate  $\Gamma_1$ 

Soil type	$p'$ (kPa)	$e$	Soil type	$p'$ (kPa)	$e$	Soil type	$p'$ (kPa)	$e$	Soil type	$p'$ (kPa)	$e$
SIM <sup>a</sup> (0/1.7) <sup>b</sup>	5	0.801	SIM (100/10)	13	0.854	GB (0/2.5)	277.9	0.577	HCM (20/13)	83	0.534
	10	0.796		157	0.821		543.2	0.566		115	0.560
	23	0.774		315	0.767		840.3	0.559		221	0.543
	41	0.774	HS <sup>d</sup> (0/1.1/0.9) <sup>e</sup>	25.9	0.779	GB (0/5)	131.6	0.470		268	0.556
	48	0.765		162.5	0.732		268.6	0.460		515	0.482
	157	0.746		328.3	0.719		360.4	0.462	HCM (30/14)	0.1	0.626
	473	0.718	HS (0/1.1/0.35) <sup>e</sup>	639.0	0.700	GB (0/10)	546.0	0.448		0.1	0.612
	<b>937<sup>c</sup></b>	<b>0.668</b>		158.9	0.730		136.6	0.421		0.5	0.603
	<b>937</b>	<b>0.645</b>		315.3	0.706		144.8	0.413		0.7	0.588
	<b>1 099</b>	<b>0.681</b>		632.6	0.697		271.1	0.404		1.0	0.532
	<b>1 217</b>	<b>0.595</b>	HS (0/1.4/0.9) <sup>e</sup>	11.5	0.780	GB (10/20)	572.6	0.392		22	0.477
	<b>1 407</b>	<b>0.606</b>		158.4	0.726		75.1	0.342		108	0.544
SIM (7/2.1)	5	0.731		159.2	0.727		131.0	0.332		209	0.536
	14	0.717		160.6	0.723		274.6	0.321		272	0.530
	216	0.670		224.3	0.720	HCM <sup>g</sup> (0/2.4)	579.1	0.304		528	0.448
	236	0.665		249.5	0.720		14	0.854	HCM (50/8.9)	0.1	0.758
	434	0.628		316.4	0.713		29	0.850		0.4	0.727
	<b>547</b>	<b>0.595</b>		497.5	0.700		77	0.842		1.2	0.746
	<b>1 234</b>	<b>0.553</b>		544.1	0.690		111	0.828		1.8	0.667
SIM (15/14)	<b>1</b>	<b>0.617</b>	HS (0/1.4/0.75) <sup>e</sup>	629.9	0.691		215	0.825		3	0.635
	17	0.589		165.3	0.723		268	0.807		109	0.683
	42	0.575		318.1	0.710		335	0.809		139	0.567
	94	0.600	HS (0/2.5)	627.1	0.698	HCM (5/3.4)	9	0.789		165	0.523
	124	0.575		9.6	0.774		54	0.771		208	0.673
	306	0.522		12.8	0.770		98	0.781		262	0.673
	306	0.513		61.3	0.743		121	0.779	HCM (70/2.2)	1.0	0.981
	<b>1 289</b>	<b>0.421</b>		159.7	0.714		236	0.760		1.3	1.006
SIM (25/28)	<b>1</b>	<b>0.477</b>		321.3	0.695		275	0.756		2.2	0.958
	11	0.461		640.5	0.675		306	0.752		3	0.815
	69	0.430	HS (0/5)	162.1	0.694	HCM (10/6.6)	51	0.715		4	0.790
	73	0.416		323.4	0.671		64	0.693		12	0.748
	83	0.430		326.3	0.681		111	0.675		105	0.900
	232	0.404	HS (0/10)	644.1	0.662		115	0.709		126	0.723
	690	0.357		164.7	0.618		221	0.650		194	0.877
	787	0.357		324.7	0.608		286	0.665		253	0.885
SIM (40/47)	4	0.410	HS (10/20)	652.0	0.606	HCM (15/11)	495	0.679	HCM (94/2)	0.7	1.242
	7	0.401		164.5	0.602		7	0.678		2.3	1.267
	13	0.390		325.1	0.595		7	0.670		2.3	1.219
	64	0.390	GB <sup>f</sup> (0/1.1)	645.2	0.586		61	0.658		4	0.983
	265	0.363		138.1	0.671		70	0.636		5	1.094
SIM (60/26)	5	0.547		275.6	0.663		109	0.633		6	1.046
	5	0.530		556.5	0.655		226	0.607		97	1.150
	11	0.539	GB (0/1.4)	1 363.7	0.645	HCM (20/13)	283	0.615		152	0.929
	112	0.514		119.9	0.643		314	0.584		184	1.125
	138	0.514		252.6	0.633		1.4	0.650		240	1.113
	157	0.511		498.3	0.623		2.5	0.638			
	184	0.497		1 254.8	0.621		7	0.624			
SIM (100/10)	10	0.877	GB (0/2.5)	138.2	0.586		32	0.579			

a. SIM = Silica sand-silt mixtures tested by Thevanayagam *et al* (2002)b. Values in parentheses indicate FC(%)/ $C_u$ c. Values of  $p'$  and  $e$  that appear in bold-italic font were not used to calculate  $\Gamma_1$  as they could not be adequately fitted with Equation 1

d. HS = Hostun sand tested by Li (2013)

e. Third value inside the parentheses indicates the mean grain size in mm

f. GB = Glass balls tested by Li (2013)

g. HCM = Mixtures of Hokksund sand and Chengbei silt tested by Yang *et al* (2006)

FC = 0%.  $C_u$  varied from 1.1 to 20. This dataset was used herein because: (i) some of the soils made up of glass balls had considerably low  $e_{min}$  values which allowed a significant extension of the lower bound of the domain of the  $\Gamma_1$ - $e_{min}$  correlation; (ii) given that the particle shape of the glass balls is distinctly different from that of Hostun sand, this dataset enables a straightforward assessment of whether particle shape affects the  $\Gamma_1$ - $e_{min}$  correlation; and (iii) this dataset also allows assessment of the validity of the  $\Gamma_1$ - $e_{min}$  correlation at different  $C_u$  values. Figure 2 presents the PSDs of the 30 soils considered.

Values of  $\Gamma_1$  were calculated by fitting Equation 1 to the  $(p', e)$  points that defined the SSL of each soil (Table 1). Some of the SSLs reported by Thevanayagam *et al* (2002) cannot be modelled with Equation 1, due to the curvature of the SSL in  $e$ - $\log_{10} p'$  which some soils exhibit at high stress levels (e.g. Been *et al* 1991; Li & Wang 1998). Consequently, some  $(p', e)$  points with high  $p'$  values were excluded from the fitting process. Similarly, given the experimental difficulties and uncertainties involved in performing triaxial tests at very low values of effective stress, two  $(p', e)$  points with  $p' = 1$  kPa were also disregarded when calculating  $\Gamma_1$  (see footnote c in Table 1).

As annotated in Figures 3 to 5, different methods were used to determine the  $e_{min}$  values of each dataset. ASTM D1557 refers to the modified Proctor compaction test, and ASTM D4253 refers to the method of soil densification using a vibratory table. Although the current authors acknowledge that  $e_{min}$  values from different methods are not strictly comparable, it is hypothesised that, regardless of the method, the resulting  $e_{min}$  provides a reasonable indicator of packing efficiency.

## RESULTS AND DISCUSSION

Figures 3 to 5 suggest strong ( $R^2 \geq 0.90$ ) linear  $\Gamma_1$ - $e_{min}$  correlations. The data point labels further suggest that the correlations are valid regardless of FC or  $C_u$ . The independence of the correlation from FC observed in Figures 3 and 4 is at odds with previous works (e.g. Thevanayagam *et al* 2002; Rahman & Lo 2008; Rahman *et al* 2014) which have suggested that  $\Gamma_1$  is fundamentally correlated to FC. Furthermore, Figure 3 explicitly shows that essentially the same  $\Gamma_1$ - $e_{min}$  correlation is followed regardless of whether FC is smaller or greater than TFC. Additionally, the angular Hostun sand and the glass balls follow the same  $\Gamma_1$ - $e_{min}$  correlation despite their significantly different particle shapes (Figure 5). This result agrees with Cho *et al* (2006) who reported

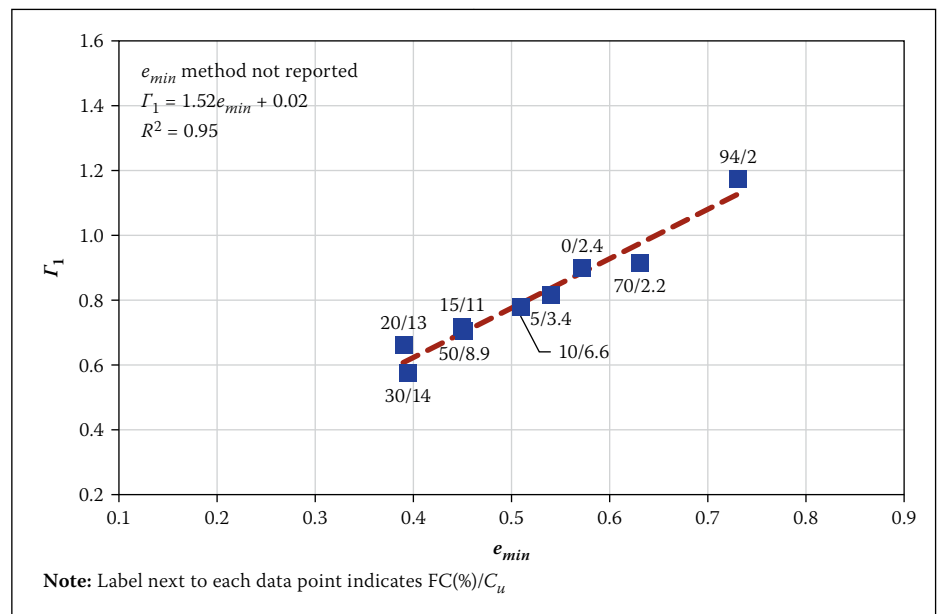


Figure 4  $\Gamma_1$  vs  $e_{min}$  plot corresponding to soil types reported in Yang *et al* (2006)

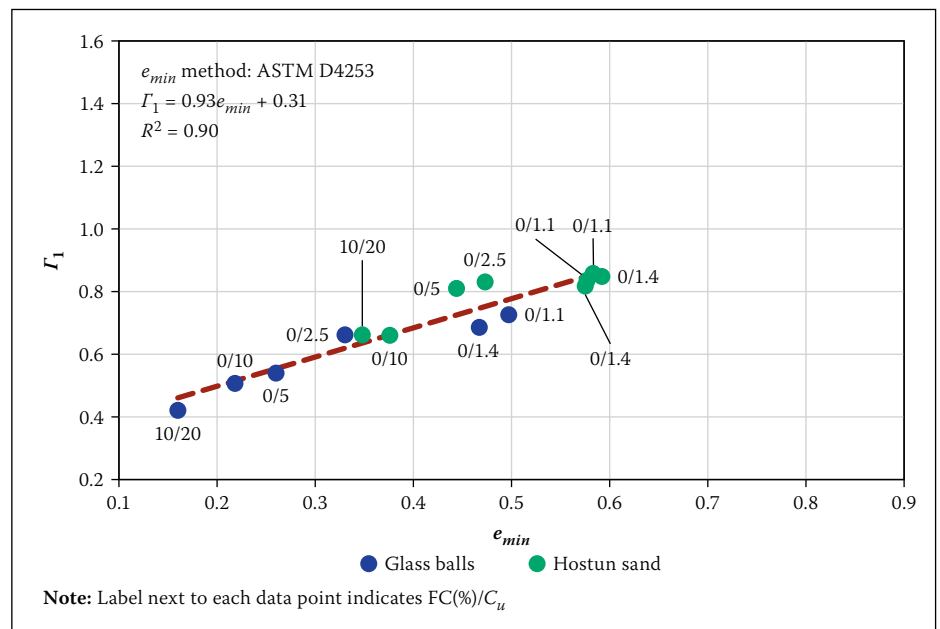


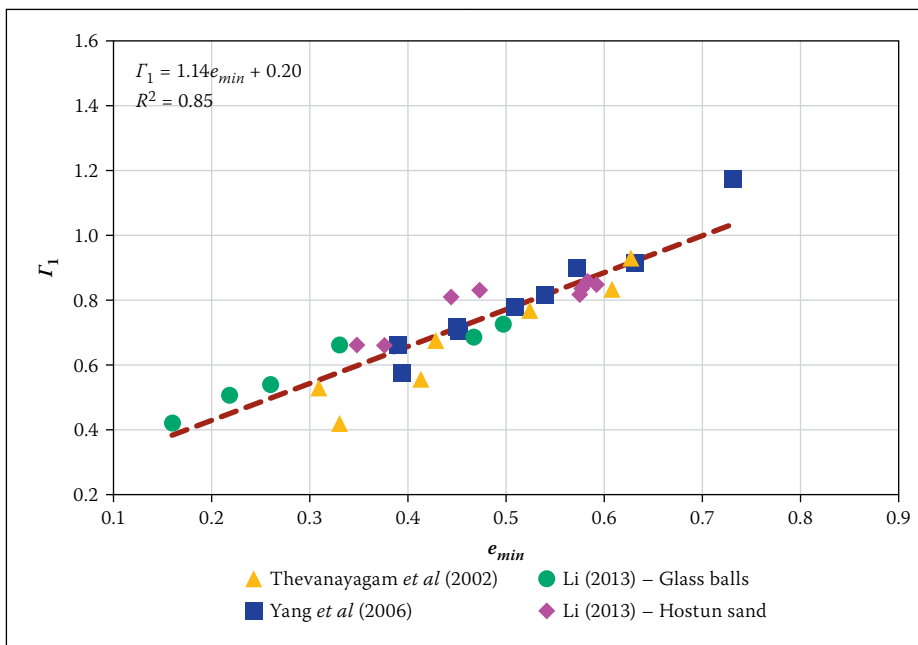
Figure 5  $\Gamma_1$  vs  $e_{min}$  plot corresponding to soil types reported in Li (2013)

the independence from particle shape of the  $\Gamma_1$ - $e_{min}$  correlation for narrowly graded sands with low values of FC.

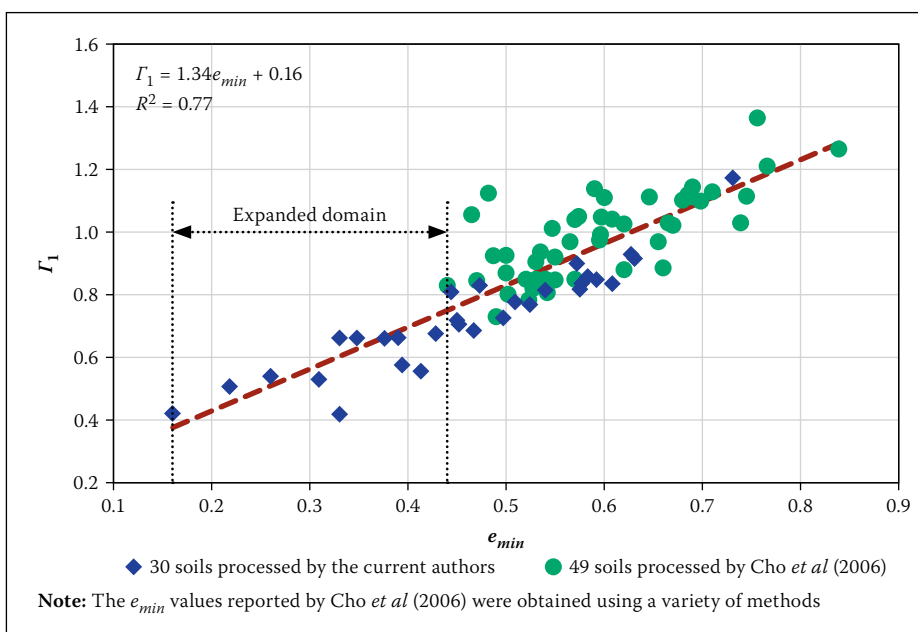
When all 30 soils are collectively plotted (Figure 6), a single linear correlation emerges ( $R^2 = 0.85$ ). The slight decrease in  $R^2$  (compare to Figures 3 to 5) is likely a consequence of combining SSLs calculated from triaxial tests conducted in different laboratories and following slightly different protocols, and  $e_{min}$  values obtained through different procedures. For example, Prochaska & Drnevich (2005) showed that the maximum dry unit weight, which is associated to  $e_{min}$ , can show variances of the order of  $\pm 3\%$  when estimated from different compaction techniques. A unique linear correlation ( $R^2 = 0.77$ ) continues to be apparent when the data corresponding to Cho *et al* (2006) is included (Figure 7). This indicates that

the  $\Gamma_1$ - $e_{min}$  correlation observed by Cho *et al* (2006) in narrowly graded sands with low FC values is approximately the same for soils with significant amounts of non-plastic fines and large  $C_u$  values, such as those represented in Figures 3 to 5. The data analysed in this study has also been useful to expand the lower bound of the domain of the  $\Gamma_1$ - $e_{min}$  correlation reported by Cho *et al* (2006) (Figure 7).

The authors suggest that the validity of the  $\Gamma_1$ - $e_{min}$  correlation over such a wide range of non-plastic soil types is explained by the similarity in which both  $\Gamma_1$  and  $e_{min}$  are affected by a soil's fundamental properties. For example, they are both directly correlated to particle angularity (Li 2013; Biarez & Hicher 1994; Cho *et al* 2006), inversely correlated to  $C_u$  (Li 2013; Biarez & Hicher 1994; Poulos *et al* 1985), and respond in a similar



**Figure 6**  $\Gamma_1$  vs  $e_{min}$  plot including all the data points from Figures 3 to 5



**Figure 7**  $\Gamma_1$  vs  $e_{min}$  plot including all the data points from Figures 3 to 5 and the data points from Cho *et al* (2006)

manner to changes in FC (Lade *et al* 1998; Rahman & Lo 2008). It is also important to acknowledge that the use of  $e_{min}$  as a predictor of  $\Gamma_1$  may be limited by the differences between the remoulding and particle crushing mechanisms that a soil undergoes when sheared to steady state and when compacted to  $e_{min}$ .

The authors are not recommending the use of the correlations in Figures 3 to 7 to replace triaxial testing to determine the SSL, as doing so can result in significant errors. For example, in Figure 7 a deviation from the best fit line of  $\pm 0.1$  is observed in  $\Gamma_1$ , implying a potential error of 0.2. This value is significantly higher than the error of  $\pm 0.01$  in void ratio, which was suggested by Jefferies & Been (2006) as a reasonable

target error when calculating the SSL experimentally. Notwithstanding these potential errors, the correlation is very useful to qualitatively understand how the  $\Gamma_1$  values of different non-plastic soil types will compare to one another. Additionally, when studying a heterogeneous soil deposit that includes a variety of soil types, the correlation can help reduce the number of soil types whose SSLs have to be experimentally determined to fully characterise the deposit from an SSL standpoint (e.g. Hemer *et al* 2016).

## CONCLUSIONS

The  $\Gamma_1$ - $e_{min}$  correlation of 30 non-plastic soils has been investigated. The results indicate that the correlation is linear ( $R^2 = 0.85$ )

and valid regardless of FC,  $C_w$  and particle shape. Comparison of the results presented herein (Figure 6) with the  $\Gamma_1$  versus  $e_{min}$  dataset obtained by Cho *et al* (2006), indicates that the  $\Gamma_1$ - $e_{min}$  correlation originally observed by Cho *et al* (2006) for narrowly graded sands with small amounts of fines may be applicable to all non-plastic soils (Figure 7). The similarity in the way in which both  $\Gamma_1$  and  $e_{min}$  respond to changes in the fundamental properties of a soil is believed to be the reason why the  $\Gamma_1$ - $e_{min}$  correlation is valid over a wide variety of soil types.

Given that the effect of non-plastic fines on the SSL continues to be intensely researched, it is an important finding of this work that the  $\Gamma_1$ - $e_{min}$  correlation is not affected by FC (Figures 3 and 4). This is not entirely surprising though, as it seems unlikely that a fundamental soil property ( $\Gamma_1$ ) can be explained by a property such as FC which is arbitrarily defined as the percentage of particles smaller than 75  $\mu\text{m}$  (or 63  $\mu\text{m}$ , depending on the standard). Accordingly, it must be expected that the predictive power of FC and associated concepts like the TFC will have important limitations. The results presented herein suggest that a better understanding of the SSL can be achieved by correlating  $\Gamma_1$  to  $e_{min}$  rather than to FC. The authors are currently analysing an extended database and conducting triaxial experiments to continue exploring the strengths and limitations of the  $\Gamma_1$ - $e_{min}$  correlation.

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## NOTATION

- $e$  = void ratio  
 $p'$  = mean effective stress  $(\sigma'_1 + \sigma'_2 + \sigma'_3)/3$   
 $q$  = deviator stress  $(\sigma'_1 - \sigma'_3)$   
 SSL = steady state line  
 $\Gamma_1$  = steady state void ratio that corresponds to a mean effective stress of 1 kPa  
 $\lambda_{10}$  = slope of the SSL in  $e$ - $\log_{10}(p')$  space  
 FC = fines content ( $< 75 \mu\text{m}$ )  
 PSD = particle size distribution  
 $e_{min}$  = minimum void ratio  
 TFC = threshold fines content  
 $C_u$  = coefficient of uniformity ( $D_{60}/D_{10}$ )