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The issue of personal safety on dolomite: Update of a probability-based evaluation with respect to single-storey residential houses

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Sinkholes may be treated as a chance phenomenon of which the likelihood of occurrence is a key parameter in determining their effect on public safety in the development of dolomite land. However, determining the likelihood of the occurrence of sinkholes remains a complex if not evasive problem in view of the sparsely populated datasets available at present.

Three approaches to determining the likelihood of sinkhole occurrence that were developed over a period of time are reviewed and comparatively evaluated. The geological time-based return periods postulated for a residential infiltration regime in the third approach is applied to determine the maximum allowable population densities for a single-storey residential house development as an update of a previous paper on the problem.

The maximum population densities based on this updated approach were found to be one and a half to three times larger than those previously considered to be allowable for Inherent Hazard Classes 1, 2 and 3. This relaxation on the historically established empirical rules on population density is considered by the authors to better represent observations in recent years.

INTRODUCTION

Sinkholes are treated in the paper as a chance phenomenon of which the likelihood of occurrence is a key parameter in determining their effect on public safety in the development on dolomite land. To determine the likelihood of sinkhole occurrence, however, remains a complex if not evasive problem.

Two approaches were pursued by Kirsten *et al* (2009) to determine the likelihood of sinkhole occurrence. The first approach was based on the severity of the hazard in terms of anecdotal ground movement intensity as proposed by Buttrick *et al* (2001). Buttrick *et al* defined the inherent hazard of a site as the chance for a certain size of sinkhole to occur in a dolomite residuum as a result of a misuse of the site in terms of water management and proposed land usages. Sinkholes are considered to be mostly circular in shape and their size is accordingly determined by diameter. Buttrick *et al* expressed the hazard in three anecdotal categories, namely low, medium and high, and defined a matrix of eight Inherent Hazard Classes and four sinkhole sizes of varying likelihood of occurrence as shown in Table 1. Sinkhole size is referred to in the literature in terms of intervals as given in this table, which is retained in the paper for the sake of compatibility. The categories

low, medium and high are defined in Table 2 after Buttrick *et al* (2001) in terms of the number of sinkholes that can potentially form per hectare in a period of twenty years.

The second approach was based on estimates of the likelihood of sinkhole occurrence in the eight hazard classes and four sinkhole sizes by professionals familiar with dolomite ground, since the anecdotal categories (low, medium and high) were restricted in range. The results from the two approaches to determine the likelihood of sinkhole occurrence were not entirely satisfactory and could not be applied to the situation in a city centre where the infiltration regime is entirely different from those in the two approaches considered. Infiltration regime denotes the particular characteristics of the water-bearing services, measures to control stormwater, landscaping and irrigation provisions, occurrence of impermeable pavements and dewatering protocols that apply to a particular use of the land.

A third approach has since been developed which consistently applies to the entire spectrum of infiltration regimes that are usually encountered in the development of dolomite land. This third approach comprises sinkhole return periods assigned by the authors and Dr R J Kleywegt for natural, residential and city centre infiltration regimes in terms of

Table 1 Inherent hazard of sinkholes and dolines forming – after Buttrick *et al* (2001)

Inherent hazard class	Sinkhole size (m)				Doline	Recommended type of development in order to maintain acceptable development risk
	Small < 2	Medium 2 – 5	Large 5 – 15	Very large > 15		
Class 1	Low	Low	Low	Low	Low NDS or DS	Residential, light industrial and commercial development provided that appropriate water precautionary measures are applied. Other factors affecting economic viability such as excavatability, problem soils, etc, must be evaluated.
Class 2	Medium	Low	Low	Low	Medium NDS	Residential development with remedial water precautionary measures. No site and service schemes. May consider for commercial or light industrial development.
Class 3	Medium	Medium	Low	Low	Medium NDS	Selected residential development with exceptionally stringent precautionary measures and design criteria. No site and service schemes. May consider for commercial or light (dry) industrial development with appropriate precautionary measures.
Class 4	Medium	Medium	Medium	Low	Medium NDS	Selected residential development with exceptionally stringent precautionary measures and design criteria. No site and service schemes. May utilise for commercial or light (dry) industrial development with appropriate stringent precautionary measures.
Class 5	High	Low	Low	Low	High NDS	These areas are usually not recommended for residential development, but under certain circumstances selected residential development (including lower-density residential development, multi-storied complexes, etc), may be considered, commercial and light industrial development. The risk of sinkhole and doline formation is adjudged to be such that precautionary measures, in addition to those pertaining to the prevention of concentrated ingress of water into the ground, are required to permit the construction of housing units.
Class 6	High	High	Low	Low	High NDS	These areas are usually not recommended for residential development, but under certain circumstances high-rise structures or gentleman's estates (stands 4 000 m ² with 500 m ² proven suitable for placing a house) may be considered, commercial or light industrial development. Expensive foundation designs may be necessary. Sealing of surfaces, earth mattresses, water in sleeves or in ducts, etc.
Class 7	High	High	High	Low	High NDS	No residential development. Special types of commercial or light industrial (dry) development only (e.g. bus or trucking depots, coal yards, parking areas). All surfaces sealed. Suitable for parkland.
Class 8	High	High	High	High	Low-High	No development, nature reserves or parkland.

Abbreviations: DS denotes dewatering scenarios and NDS non-dewatering scenarios

the underlying dolomite ground profile, and a longstanding empirical understanding of dolomite land stability and the effectiveness of remedial interventions. The considerable differences in infiltration regime for natural land, residential development and city centres had previously not been considered.

The longstanding empirical understanding of dolomite land stability and the effectiveness of precautionary interventions refer to the experiential guidelines on residential land development densities that the authors and Dr Kleywegt had come to accept as reliable measures over many years of observation. Historically, population densities of 300, 200, 75, 100, 20, 0 and 0 people per hectare on residential land in the eight hazard classes respectively had become an established guideline. However, over the past ten to twenty years it has become necessary to consider much higher population densities, as presented further on in the paper.

The purpose of this paper is, firstly, to compare and evaluate the different determinations of sinkhole frequency of occurrence, and secondly, to demonstrate how the geological time-based return periods for a residential infiltration regime may be applied to determine population densities for single-storey residential house developments as an update to Kirsten *et al* (2009). A further objective is to present an improved method

of calculation that has since been developed for this purpose.

The sinkhole return periods for a natural infiltration regime were assigned in terms of the lengths of time over which geological processes may extend, which in this instance could be a million years or more. The sinkhole return periods for residential and city centre infiltration regimes were then assigned relative to those for natural infiltration regimes in terms of engineering geological judgement of the comparative differences in infiltration potential.

Treating sinkholes as a chance phenomenon is subject to qualification. Sinkholes occur invariably as a result of untoward ingress of water into the ground or untoward abstraction of water that is already in the ground. Owing to the presence of water and wastewater reticulation infrastructure that leak and act as the trigger, sinkholes are located at or not very distant from water-bearing services. Strictly speaking, sinkholes are therefore not a chance phenomenon, although it could be argued that the causative leakages or abstractions of water due to human action or inaction are chance phenomena. Because of this, only one sinkhole occurs at a time in a particular neighbourhood or stretch of land, as water-bearing services tend to leak at isolated locations. A related aspect is that the causes

Table 2 Inherent hazard characterisation in terms of frequency of ground movement events

Range	Inherent hazard characterisation	Ground movement events per hectare per 20 years
1	Low	≤ 0.1
2	Medium	> 0.1 and ≤ 1.0
3	High	> 1.0

of a sinkhole in a residential township and the sinkhole itself are generally repaired soon after it has occurred. More than one sinkhole therefore does not occur in principle in the same neighbourhood over a very long period of time. These qualifications have a bearing on the mathematical modelling of sinkhole occurrence as presented in this paper.

RETURN PERIOD BASED ON INTENSITY OF GROUND MOVEMENT EVENTS GIVEN BY BUTTRICK *et al*

The first approach presented by Kirsten *et al* (2009) to determine the likelihood of sinkhole occurrence was based on an interpretation of the class-size distribution for sinkhole occurrence given by Buttrick *et al* (2001) as shown in Tables 1 and 2. This involved adoption of a geometric series of values of

Table 3 Original Buttrick *et al* number of sinkholes/ha/20 years

Hazard class	Sinkhole size (m)						
	< 1	1 – 5	5 – 15	> 15	–	–	Total
1	0.100	0.100	0.100	0.100	–	–	0.400
2	0.316	0.100	0.100	0.100	–	–	0.616
3	0.316	0.316	0.100	0.100	–	–	0.832
4	0.316	0.316	0.316	0.100	–	–	1.049
5	1.000	0.100	0.100	0.100	–	–	1.300
6	1.000	1.000	0.100	0.100	–	–	2.200
7	1.000	1.000	1.000	0.100	–	–	3.100
8	1.000	1.000	1.000	1.000	–	–	4.000
Total	5.049	3.932	2.816	1.700	–	–	13.500

Table 4 Adjusted Buttrick *et al* number of sinkholes/ha/20 years

Hazard class	Sinkhole size (m)						
	< 1	1 – 5	5 – 15	15 – 25	25 – 40	> 40	Total
1	0.100	0.032	0.010	0.003	0.001	0.0003	0.146
2	0.316	0.100	0.032	0.010	0.003	0.001	0.462
3	1.000	0.316	0.100	0.032	0.010	0.003	1.461
4	3.162	1.000	0.316	0.100	0.032	0.010	4.620
5	1.000	0.316	0.100	0.032	0.010	0.003	1.461
6	3.162	1.000	0.316	0.100	0.032	0.010	4.620
7	10.000	3.162	1.000	0.316	0.100	0.032	14.610
8	31.623	10.000	3.162	1.000	0.316	0.100	46.201
Total	50.364	15.926	5.036	1.593	0.504	0.159	73.580

Table 5 Return period for adjusted Buttrick *et al* class size distribution (years for one sinkhole/ha)

Hazard class	Sinkhole size (m)					
	< 1	1 – 5	5 – 15	15 – 25	25 – 40	> 40
1	100	316	1 000	3 333	10 000	33 333
2	32	100	316	1 000	3 333	10 000
3	10	32	100	316	1 000	3 333
4	3	10	32	100	316	1 000
5	10	32	100	316	1 000	3 333
6	3.20	10	32	100	316	1 000
7	1	3.2	10	32	100	316
8	0.32	1	3.2	10	32	100

Table 6 Invited expert class size distribution (relative number of sinkholes out of 100)

Hazard class	Sinkhole size (m)						
	< 1	1 – 5	5 – 15	15 – 25	25–40	> 40	Total
1	0.603	0.603	0.121	0.024	0.005	0.001	1.357
2	1.225	0.442	0.241	0.048	0.010	0.002	1.968
3	1.673	2.463	1.408	0.603	0.121	0.024	6.291
4	0.684	1.974	2.342	0.845	0.169	0.034	6.047
5	8.379	9.858	3.017	0.764	0.153	0.031	22.202
6	2.638	15.361	6.002	1.609	0.321	0.065	25.996
7	1.207	4.425	9.567	3.805	0.761	0.153	19.916
8	0.362	2.952	4.934	6.431	1.286	0.257	16.222
Total	16.770	38.079	27.632	14.130	2.825	0.566	100.000

0.1, 0.3162 and 1.0 for the three ranges as given in Table 2. The common ratio of the geometric series is equal to $10(0.1)^{0.5} = 3.162$. Substituting the values 0.1, 0.3162 and 1.0 for the descriptors low, medium and high in Table 1, enables the number of sinkholes per hectare per 20-year period of time to be presented as shown in Table 3. The term inherent hazard in Tables 1 and 3 and throughout the paper indicates potential hazard. It so happens that low inherent hazard represents an acceptable level of hazard in terms of SANS 2012:1936. The purpose of precautionary measures according to this standard is to reduce the medium and high levels of inherent hazard to a low level of inherent hazard on an engineering judgemental basis.

The distribution in Table 3 was adjusted and expanded with regard to Inherent Hazard Class and sinkhole size in terms of engineering judgement as shown in Table 4. The values highlighted in light blue correspond to those published by Buttrick *et al* (2001). The two larger size ranges were added as systematic extensions of the class-size distributions.

The sinkhole return period was determined, as shown in Table 5, as the quotient of 10 ha-yr and the frequencies of ground movement events given in Table 4. If A denotes the area in hectares, Q the number of potential sinkholes of a particular size in a particular Inherent Hazard Class, and N the number of years in which the potential sinkholes occur, the return period for the sinkholes, T, is by definition given by AN/Q . Adopting a conversion factor of $AN = 10$ ha-yr instead of 20 ha-yr as originally proposed by Buttrick *et al* (2001), ensured longer return periods, which enabled allowable population densities to be determined that compared better with the historically accepted population densities for single-storey dwelling houses. The comparison with the historically accepted population densities of 300, 200, 75, 100, 20, 0, 0 and 0 people per hectare for the eight Inherent Hazard Classes was, however, not entirely satisfactory. This gave rise to further developments as presented below.

RETURN PERIOD BASED ON EXPERIENTIAL JUDGEMENT OF SINKHOLE FREQUENCY

The second approach pursued by Kirsten *et al* (2009) to determine the likelihood of sinkhole occurrence was based on estimates of the relative potential occurrence of sinkholes in terms of sinkhole size and Inherent Hazard Class by engineering geologists familiar with sinkhole occurrence in dolomite land, as shown in Table 6. The participating engineering geologists were requested to assign the relative numbers of potential sinkholes for the different size

intervals and inherent hazard classes out of a total of 100. The sinkhole return period was determined by substituting the relative number of sinkholes Q from Table 6 in the expression $AN/Q = 100/Q$ as shown in Table 7. A conversion factor of 100 ha-yr resulted in return periods that enabled allowable population densities to be determined which optimally compared with the historically acceptable population densities for single-storey residential houses of 300, 200, 75, 100, 20, 0, 0 and 0 people per hectare for the eight Inherent Hazard Classes respectively. The number of sinkholes per hectare per 20 years for the invited expert class-size distribution was obtained by substituting the return period T from Table 7 in the expression $Q = AN/T = 20/T$ as shown in Table 8, which is required for comparative purposes later in the paper.

RETURN PERIOD DIRECTLY ASSIGNED IN TERMS OF DOLOMITE GROUND PROFILE AND GEOLOGICAL TIME

Based on experience, the authors and Dr Kleywegt assigned potential sinkhole return periods per 100 hectares for three comparative regimes of infiltration, namely natural, residential and city centre, as shown in Table 9 as a further development. The assignments represented the perceived fundamental differences between the infiltration regimes and were based on a fundamental sense of geological time and a fundamental knowledge of the geological profile of dolomite ground. The return periods for particular infiltration regimes in general do not vary for the smaller sinkhole sizes, and significantly increase for the larger sinkhole sizes. Smoothly increasing return periods with sinkhole size would have been artificial. The return periods shown were rather selected to represent large-scale phenomena, which is all that can be done in terms of current knowledge. The return periods for the residential infiltration regime were adjusted to result in maximum population densities of 800, 400, 150, 100, 30, 0, 0 and 0 people per hectare for the eight Inherent Hazard Classes respectively, as determined later in this paper. As submitted further on, these population densities were considered by the authors as permissible instead of the corresponding historically acceptable densities of 300, 200, 75, 100, 20, 0, 0 and 0 people per hectare.

COMPARISON OF ALTERNATIVE SINKHOLE FREQUENCIES

The numbers of sinkholes per hectare per 20 years corresponding to the return periods for the three infiltration regimes were obtained by substituting the return

Table 7 Return period for invited expert class size distribution (years for one sinkhole/ha)

Hazard class	Sinkhole size (m)					
	< 1	1 – 5	5 – 15	15 – 25	25 – 40	> 40
1	166	166	826	4 167	20 000	100 000
2	82	226	415	2 083	10 000	50 000
3	60	41	71	166	826	4 167
4	146	51	43	118	592	2 941
5	12	10	33	131	654	3 226
6	38	7	17	62	312	1 538
7	83	23	10	26	131	654
8	276	34	20	16	78	389

Table 8 Invited expert number of sinkholes/ha/20 years

Hazard class	Sinkhole size (m)						Total
	< 1	1 – 5	5 – 15	15 – 25	25 – 40	> 40	
1	0.121	0.121	0.024	0.005	0.001	0.000	0.271
2	0.245	0.088	0.048	0.010	0.002	0.000	0.394
3	0.335	0.493	0.282	0.121	0.024	0.005	1.258
4	0.137	0.395	0.468	0.169	0.034	0.007	1.210
5	1.676	1.972	0.603	0.153	0.031	0.006	4.440
6	0.528	3.072	1.200	0.322	0.064	0.013	5.199
7	0.241	0.885	1.913	0.761	0.152	0.031	3.984
8	0.072	0.590	0.987	1.286	0.257	0.051	3.244
Total	3.354	7.616	5.526	2.826	0.565	0.113	20.000

Table 9 Geological time-based assessment of sinkhole return period per 100 ha (years)

Hazard class	Township	Sinkhole size (m)					
		Less than 2	2 – 5	5 – 15	15 – 25	25 – 40	Larger than 40
1	Natural	100 000	100 000	100 000	100 000	100 000	500 000
	Residential	100	100	100	4 500	27 000	135 000
	City Centre	50	50	50	2 000	12 000	60 000
2	Natural	100 000	100 000	100 000	100 000	100 000	500 000
	Residential	45	45	45	2 200	12 000	70 000
	City Centre	20	20	20	800	5 000	30 000
3	Natural	10 000	10 000	10 000	2 0000	100 000	500 000
	Residential	25	25	25	800	6 000	30 000
	City Centre	10	10	10	300	2 000	12 000
4	Natural	10 000	10 000	10 000	20 000	100 000	500 000
	Residential	15	15	15	550	3 000	15 000
	City Centre	5	5	5	250	1 250	6 000
5	Natural	1 000	1 000	1 000	10 000	500 000	5 000 000
	Residential	5	5	5	150	2 500	25 000
	City Centre	1	1	2	10	100	1 000
6	Natural	1 000	1 000	1 000	10 000	200 000	1 000 000
	Residential	3	3	3	3	500	5 000
	City Centre	0.5	0.5	0.5	2	50	500
7	Natural	1 000	1 000	1 000	1 000	20 000	1 000 000
	Residential	3	3	3	3	500	5 000
	City Centre	0.5	0.5	0.5	0.5	50	500
8	Natural	800	800	800	800	10 000	100 000
	Residential	3	3	3	3	500	5 000
	City Centre	0.5	0.5	0.5	0.5	0.5	100

periods, T, for 100 ha from Table 9 in the expression $Q = AN/T = (1/100)20/T = 0.2/T$ as given in Tables 10, 11 and 12 respectively. The overall numbers of sinkholes per hectare per 20 years given in Tables 3, 4, 8, 10, 11 and 12 are summarised in Table 13. The column and row totals in these tables are non-weighted arithmetic totals as a standard means of evaluating and communicating the frequency of sinkholes of all sizes in large tracts of dolomite land that may comprise any number of Inherent Hazard Classes.

The overall numbers of sinkholes in Tables 3, 4 and 8 are comparable in order of magnitude. The differences between these cases are mainly due to differences in Inherent Hazard Classes 5, 6, 7 and 8, because the contributions to the overall numbers of sinkholes principally derive from these classes. The main contributions to the overall numbers of sinkholes in Tables 10, 11 and 12 also derive from Inherent Hazard Classes 5, 6, 7 and 8.

The progression in the overall numbers of sinkholes of 0.00326, 1.007 and 5.839 in Tables 10, 11 and 12 respectively is expected for the three infiltration regimes. However, the last two numbers, 1.007 and 5.839, are low compared to the overall totals in Tables 3, 4 and 8. This is partly due to the fact that the associated return periods in Tables 3, 4 and 8 were based on worst-case postulations with regard to water ingress and infiltration compared to the return periods in Tables 10, 11 and 12 which are effectively based on water ingress and infiltration relative to geological time.

The overall number of 1.007 sinkholes per hectare per 20 years for the geological time-based evaluation of the return periods for sinkholes in a residential infiltration regime in Table 11 in principle agrees with the longstanding experience of the authors on a considerable number of residential townships on dolomite. The veracity of the overall number of 5.839 sinkholes per hectare per 20 years for the city centre infiltration regime in Table 12 cannot yet be adjudged. Centurion City, as the only city centre on dolomite in South Africa, provides a limited database, apart from the fact that, in terms of the observations of the authors over a long period of time, it is estimated that the water-bearing services in such an infiltration regime would take thirty to forty years to deteriorate sufficiently to give rise to the accelerated formation of sinkholes.

BASIC PROBABILITY CONCEPTS

The relevant basic probability concepts may be summarised as follows from Kirsten *et al* (2009a).

Table 10 Geological time-based number of sinkholes/ha/20 years for natural infiltration regime

Hazard class	Sinkhole size (m)						
	< 1	1 – 5	5 – 15	15 – 25	25 – 40	> 40	Total
1	0.00000	0.00000	0.00000	0.00001	0.00000	0.00000	0.00002
2	0.00000	0.00000	0.00000	0.00001	0.00000	0.00000	0.00002
3	0.00002	0.00002	0.00002	0.00001	0.00000	0.00000	0.00007
4	0.00002	0.00002	0.00002	0.00001	0.00000	0.00000	0.00007
5	0.00020	0.00020	0.00020	0.00002	0.00000	0.00000	0.00062
6	0.00020	0.00020	0.00020	0.00002	0.00000	0.00000	0.00062
7	0.00020	0.00020	0.00020	0.00020	0.00001	0.00000	0.00081
8	0.00025	0.00025	0.00025	0.00025	0.00002	0.00000	0.00102
Total	0.00089	0.00089	0.00089	0.00053	0.00004	0.00000	0.00326

Table 11 Geological time-based number of sinkholes/ha/20 years for residential infiltration regime

Hazard class	Sinkhole size (m)						
	< 1	1 – 5	5 – 15	15 – 25	25 – 40	> 40	Total
1	0.002	0.002	0.002	0.000	0.000	0.000	0.006
2	0.004	0.004	0.004	0.000	0.000	0.000	0.013
3	0.008	0.008	0.008	0.000	0.000	0.000	0.024
4	0.013	0.013	0.013	0.000	0.000	0.000	0.040
5	0.040	0.040	0.040	0.001	0.000	0.000	0.121
6	0.067	0.067	0.067	0.067	0.000	0.000	0.267
7	0.067	0.067	0.067	0.067	0.000	0.000	0.267
8	0.067	0.067	0.067	0.067	0.000	0.000	0.267
Total	0.268	0.268	0.268	0.202	0.001	0.000	1.007

Table 12 Geological time-based number of sinkholes/ha/20 years for city centre infiltration regime

Hazard class	Sinkhole size (m)						
	< 1	1 – 5	5 – 15	15 – 25	25 – 40	> 40	Total
1	0.004	0.004	0.004	0.004	0.004	0.004	0.024
2	0.020	0.020	0.020	0.020	0.010	0.010	0.100
3	0.020	0.020	0.020	0.020	0.010	0.010	0.100
4	0.040	0.040	0.040	0.040	0.020	0.002	0.182
5	0.200	0.200	0.100	0.020	0.002	0.000	0.522
6	0.400	0.400	0.400	0.100	0.004	0.000	1.304
7	0.400	0.400	0.400	0.400	0.004	0.000	1.604
8	0.400	0.400	0.400	0.400	0.400	0.002	2.002
Total	1.484	1.484	1.384	1.004	0.454	0.029	5.839

Table 13 Total numbers of sinkholes/ha/20 years alternatively determined

Alternative	Table number	Total number
Original Buttrick <i>et al</i> sinkhole class size distribution	3	13.50
Adjusted Buttrick <i>et al</i> sinkhole class size distribution	4	73.58
CGS expert sinkhole class size distribution	8	20.00
Geological time-based natural infiltration regime	10	0.00326
Geological time-based residential 1 infiltration regime	11	1.007
Geological time-based city centre infiltration regime	12	5.839

Probability of occurrence

An event of recurrence interval T years, i.e. a T -year event, is an event of such magnitude that the average time between events of larger magnitude is T years. This length of time is also referred to as the return period. The events considered in the paper refer to sinkholes in intervals of increasing diameter. Reference to a T -year event is therefore with regard to the occurrence of a size of sinkhole in a particular interval.

Let D denote the lifetime of a residential development. The probability, P_1 , that a T -year event will be exceeded at least once in the lifetime of the development is given by Expression 1. It is not necessary to consider the occurrence of more than one sinkhole in a neighbourhood, because of the way in which sinkholes occur and are repaired, as explained in the Introduction. A lifetime is defined as 70 years.

$$P_1 = 1 - (1 - \frac{1}{T})^D \quad (1)$$

The return periods per hectare, T^{14} , for the geological time-based residential infiltration regime given in Table 14 were obtained by substituting the return periods for 100 ha, T^9 , from Table 9 in the expression $T^{14} = N/[(1/100)N/T^9] = 100T^9$.

Probability of coincidence

Sinkholes in principle affect houses as shown in Figure 1. A sinkhole is considered to affect a house if the house is located anywhere within the perimeter of the sinkhole, as shown in Figure 2.

Let a denote the diameter of the sinkhole and c the diameter of the inscribed circle to the house as shown in Figure 2. The house would therefore be affected by the sinkhole if the house falls anywhere in a circle of diameter $(2a - c)$ as shown in the figure, and the probability of coincidence would be P_h given by Expression 2:

$$P_h = \frac{\pi(2a - c)^2}{40000} \quad (2)$$

For a residential development in which the floor area of a house is 120 m^2 , the diameter of the inscribed circle is of the order of 10 m. Expression [2] does not apply to sinkholes less than 10 m in diameter. The following power function in terms of sinkhole diameter a may be fitted to Expression [2] for values for $10 \text{ m} \leq a \leq 50 \text{ m}$ and a value for $c = 10 \text{ m}$. The correlation coefficient for the fitted function is very close to 100%.

$$P_f = \frac{4a^{2.4877}}{10000} \quad (3)$$

Table 14 Sinkhole return period per hectare for residential infiltration regime

Hazard class	Sinkhole size (m)					
	Less than 2	2 – 5	5 – 15	15 – 25	25 – 40	Larger than 40
1	10 000	10 000	10 000	450 000	2 700 000	13 500 000
2	4 500	4 500	4 500	220 000	1 200 000	7 000 000
3	2 500	2 500	2 500	80 000	600 000	3 000 000
4	1 500	1 500	1 500	55 000	300 000	1 500 000
5	500	500	500	15 000	250 000	2 500 000
6	300	300	300	300	50 000	500 000
7	300	300	300	300	50 000	500 000
8	300	300	300	300	50 000	500 000



Figure 1 Coincidence of sinkhole with residential house (Centurion, South Africa)

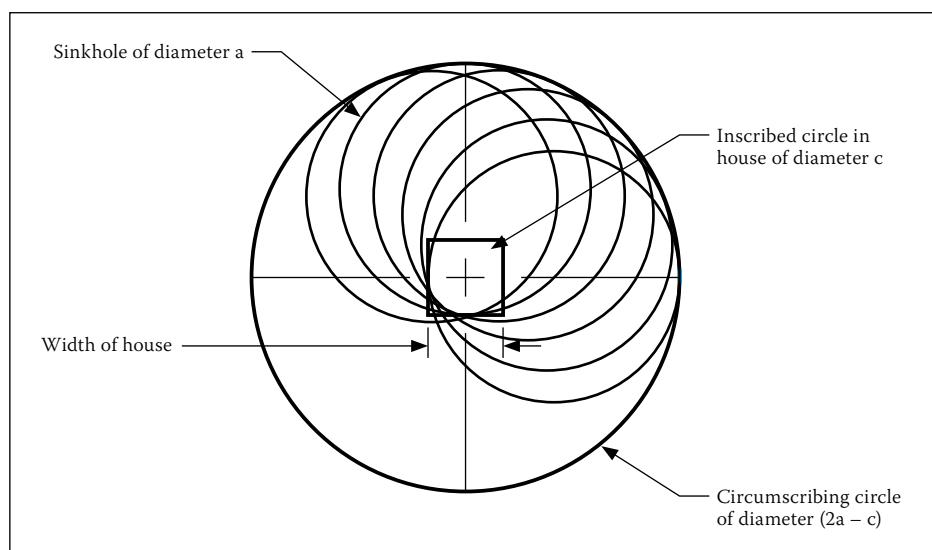


Figure 2 Area in which sinkhole coincides with house

For sinkholes less than 10 m in diameter, the area of the house affected by the sinkhole is equal to that of the sinkhole. As a result, Expression [3] applies to sinkholes less than 10 m in diameter, i.e. it applies to all sinkholes from zero to 50 m in diameter.

A representative layout of a township consisting of single-storey residential houses is shown in Figure 3. Consider a typical layout of single-storey houses as shown in Figure 4. Let U denote the number of houses per hectare. Every house in a particular

area has a probability of being affected by a sinkhole as given in Expression [3]. The probability, P_2 , that any of the houses would be affected by a sinkhole is therefore given by Expression [4] since only one sinkhole occurs in general in a neighbourhood and may occur anywhere in the neighbourhood:

$$P_2 = 1 - \left(1 - \frac{4a^{2.4877}}{10\,000}\right)^U \quad (4)$$

Expression [4] is not dependent on the distribution of the houses or their separation from one another. The potential number of fatal injuries is determined by the number of houses affected by a sinkhole, and that will depend on the size of the houses, their separation from one another and the size of the sinkhole.

Probabilities of additional dependent events

The following mutually dependent events have to occur in addition to sinkhole formation and coincidence with the house to cause fatal personal injury:

- P_3 Probability of building collapsing when affected by a sinkhole
- P_4 Probability of building occupied when affected by a sinkhole
- P_5 Probability of occupants at home when affected by a sinkhole
- P_6 Probability or relative number of fatal injuries of occupants of house when affected by a sinkhole to provide for the case in which not all the occupants are fatally injured

Values for these probabilities may be assumed for single-storey dwelling houses in terms of engineering judgement as given in Table 15 for a range of sinkhole sizes.

The values shown for P_3 and P_4 are considered to be realistic. P_4 enables long-term occupation of the building to be separately accounted from P_5 that enables shorter absences during the day or night to be considered. The values for P_5 represent the situation for the occupants at home for 12 out of 24 hours per day, which is realistic. The values for the probability of fatal injury, P_6 , may be somewhat conservative, especially for the smaller sizes of sinkhole, but no reliable statistics are available.

Threshold levels for fatal injury

Lifetime probability is defined as the probable unit number of times that a detrimental event could occur during the life of the person affected. A natural lifetime is on average 70 years. Parameter D in Expression [1] is therefore taken as 70.

Threshold levels for fatal injury may be presented after Whitman (1984) as shown



Figure 3 Illustrative layout of single-storey residential houses (Pretoria, South Africa)

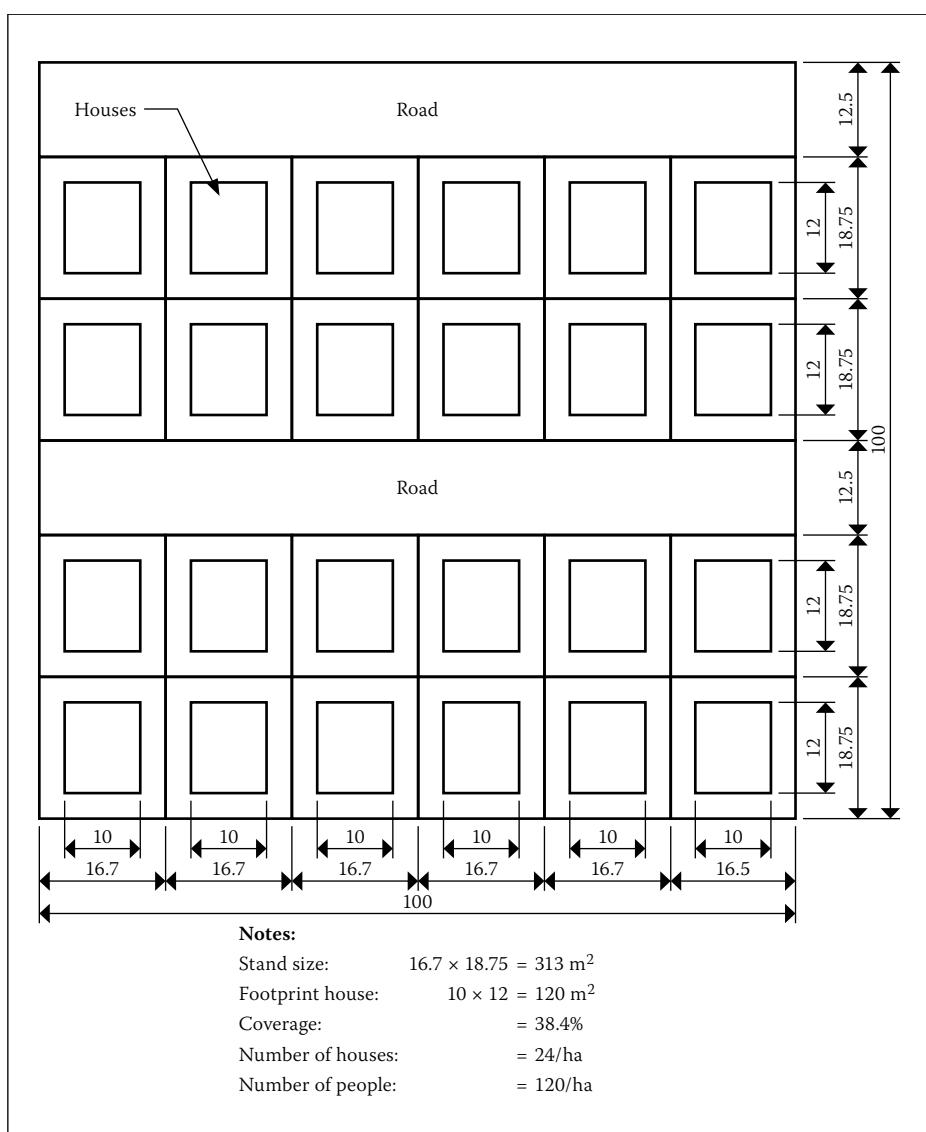


Figure 4 Typical layout of single-storey residential houses

Table 15 Assumed probabilities for relevant additional events

Probability	Sinkhole size (m)					
	< 1	1 – 5	5 – 15	15 – 25	25 – 40	> 40
P_3	0.0001	0.01	1	1	1	1
P_4	1	1	1	1	1	1
P_5	0.5	0.5	0.5	0.5	0.5	0.5
P_6	0.5	0.5	0.5	0.5	0.5	0.5

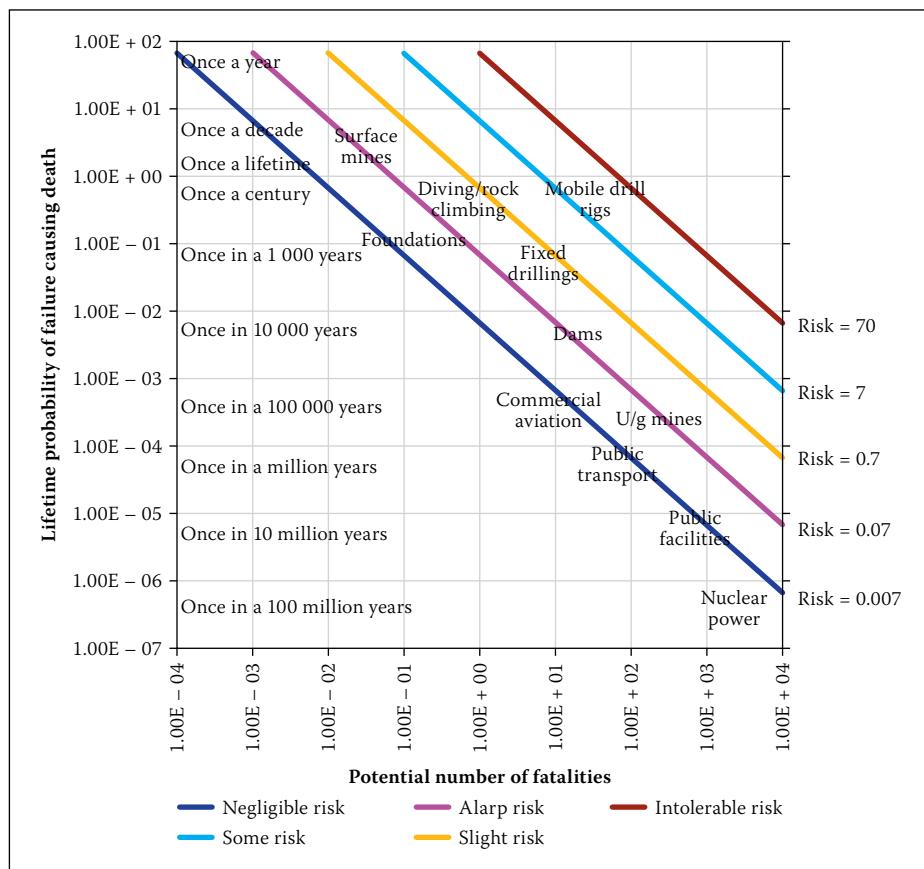


Figure 5 Risk criteria for fatal injury (after Whitman 1984)

in Figure 5 in terms of lifetime probability, P_t , and the potential number of lives lost per house, N . The ordinate axis may alternatively represent the relative lapse of time in anecdotal terms as shown. The product of the probability of fatal injury along the ordinate axis and the potential number of lives lost per house along the abscissa axis is defined as the risk. The product of the probability of fatal injury and the potential number of lives lost per house are constant along the inclined lines in Figure 5. The inclined lines represent constant levels of risk that increase from “negligible risk” at the bottom to “intolerable risk” at the top. The descriptions of the different levels of risk in Figure 5 correspond to international usage. The relationship between lifetime probability of injury, P_t , and the potential number of lives lost per house, N , for any particular risk level, R , is given by Expression [5]:

$$R = P_t N \quad (5)$$

Population density

Let j denote the average number of people per house. In this instance $j = 5$. By definition the proportion N/j represents the probability of people in a house fatally injured by a coincident sinkhole, that is, P_6 . Thus:

$$\frac{N}{j} = P_6 \quad (6)$$

Let h denote the number of people and U the number of houses per hectare. Thus:

$$h = Uj \quad (7)$$

Overall probability of fatal injury

As submitted, a number of events need to happen simultaneously for people in a house to be fatally injured, namely, the sinkhole has to occur, the sinkhole has to coincide with the house, the house has to collapse catastrophically, the house has to be occupied by people, the people have to be at home at

the time and people in the house need to be fatally injured. The overall probability, P_O , that people in the house are fatally injured can therefore be obtained by multiplying the probabilities for the six underlying events as in Expression [8]:

$$P_O = P_1 P_2 P_3 P_4 P_5 P_6 \quad (8)$$

As indicated in the definition of P_6 , inclusion of P_6 in Expression [8] provides for the situation in which not all the occupants of the house are fatally injured when affected by a sinkhole.

The overall probability, P_O , should be $\leq P_t$, the threshold probability of fatal injury for the particular risk level considered. The cumulative number of people per hectare, h , in respect of whom this condition is satisfied, i.e. the permissible population density, corresponds to all values for which $F = P_t/P_O \geq 1.0$. Thus, by substitution from Expressions [5], [6] and [8],

$$F = \frac{P_t}{P_O} = \frac{P_t}{P_1 P_2 P_3 P_4 P_5 P_6} = \frac{R}{j P_1 P_2 P_3 P_4 P_5 P_6} \geq 1.0 \quad (9)$$

The number of people per hectare, h , appears in the power $U = h/j$ in the expression for P_2 in Expression [9].

EVALUATION OF PERMISSIBLE POPULATION DENSITIES

Values for ratio F in Expression [9] are shown for ranges of population density and sinkhole size for Inherent Hazard Classes 2 and 7 in Tables 16 and 17 respectively. Similar tables can be developed for the remaining six Inherent Hazard Classes. The values for F correspond to a “negligible risk level” as defined in Figure 5, i.e. to $R = 0.007$. The values for F in Tables 16 and 17 have a minimum turning point character for particular population densities. The maximum population density for any Inherent Hazard Class corresponds to that value for which ratio $F \geq 1.0$ for all sinkhole sizes.

The maximum permissible population densities determined as described above for single-storey residential houses are summarised in Table 18 in terms of the sinkhole size for which factor F is approximately equal to unity. Owing to the minimum turning point character of factor F , the maximum population densities in Table 18 apply to all sinkhole sizes.

The maximum population densities correspond to 800, 400, 150, 100, 30, 0, 0 and 0 people per hectare for the eight Inherent Hazard Classes respectively. These densities are one-and-a-half to three times larger

Table 16 Factor F for single-storey residential houses – inherent hazard Class 2

Number of people/ha	Sinkhole size (m)					
	1.0	3	10	20	32.5	50
1	9.069E + 08	589 558	294	2 481	3 756	5 579
3	3.023E + 08	196 544	98	839	1 318	2 288
6	151 153 629	98 290	49	428	711	1 515
12	75 578 628	49 163	25	223	411	1 202
25	36 279 628	23 617	12	117	263	1 124
50	18 141 628	11 827	6	69	207	1 120
100	9 072 628	5 932	3	46.29	193	1 120
150	6 049 628	3 967	2	39.88	192	1 120
200	4 538 129	2 984	2	37.35	192	1 120
400	2 270 880	1 510	1.15	34.32	192	1 120
800	1 137 257	774	0.8	35.21	192	1 120

Table 17 Factor F for single-storey residential houses – inherent hazard Class 7

Number of people/ha	Sinkhole size (m)					
	1.0	3	10	20	32.5	50
1	67 171 299	43 667	21.74	3.79	157	398
3	22 390 612	14 557	7.27	1.28	55	163
6	11 195 440	7 280	3.65	0.65	29.64	108
12	5 597 855	3 641	1.84	0.34	17.13	86
25	2 687 110	1 749	0.90	0.18	10.96	80
50	1 343 689	876	0.46	0.11	8.63	80
100	671 979	439	0.25	0.07	8.05	80
150	448 076	294	0.17	0.06	8.01	80
200	336 124	221	0.14	0.06	8.01	80
400	168 196	112	0.09	0.05	8.01	80
800	84 223	57	0.06	0.05	8.01	80

Table 18 Summary of permissible population densities for single-storey residential houses

Inherent hazard class	Number of people/ha	Sinkhole size (m)					
		1.0	3	10	20	32.5	50
1	> 800	2 517 000	1 712	1.86	72.0	432	2 160
2	400	2 270 880	1 510	1.15	35.3	192	1 120
3	150	3 381 512	2 217	1.31	14.5	96.0	480
4	100	3 070 711	2 008	1.12	11.6	48.3	240
5	30	3 854 634	2 510	1.29	7.35	43.1	401
6	0	22 390 612	14 557	7.29	1.28	55.0	163
7	0	22 390 612	14 557	7.29	1.28	55.0	163
8	0	22 390 612	14 557	7.29	1.28	55.0	163

than the historically adopted maximum densities of 300, 200, 75, 100, 20, 0, 0 and 0 people per hectare for the eight Inherent Hazard Classes respectively. Based on observation over the past twenty years, the maximum population densities in Table 18 are considered by the authors to more accurately represent the perceived inherent hazard of sinkhole formation than the historically adopted densities.

As a result, the underlying sinkhole return periods for the residential infiltration regime in Table 9 are considered to be representative of the conditions in residential developments.

The number of people who can be accommodated per hectare at a risk R of 0.007 may not necessarily be able to be housed in single-storey dwelling houses at a maximum number of people per house of

five. For example, the permissible number of people of 800 per hectare for Inherent Hazard Class 1 would require 160 houses, which cannot physically be fitted into one hectare. The maximum number of people who can be accommodated in the size of house considered amounts to 120 per hectare. A township consisting of two- and three-storey clusters can in principle accommodate three times as many people per hectare for the same indoor floor area per person. This density approaches that permissible in Inherent Hazard Class 2 of 400 people per hectare, but is still far below that permissible in Class 1 of 800 people per hectare. In effect, population density is not limited in Inherent Hazard Class 1.

The most probable number of fatal injuries will be determined by the number of houses affected by a 10 m sinkhole, since this size of sinkhole corresponds to the minimum turning point with regard to population density as shown in Table 18. A 10 m sinkhole can only affect one house of 120 m² in floor area, following from which the most probable number of fatalities will in principle be $0.5 \times 5 = 2.5$ people.

EVALUATION OF SANS 1936:2012 LAND USAGE REQUIREMENTS

The permissible land usage requirements in terms of SANS 1936 – 1:2012 are compared with the population densities determined in this paper in Table 19, which is an extract of the land usage requirements in the standard on single-storey dwelling houses. The land usage requirements in SANS 1936 – 1:2012 are expressed in terms of maximum permissible population densities for four levels of precautionary measures represented by area designations D1, D2, D3 and D4 respectively, and briefly defined as follows:

- D1 No precautionary measures considered
- D2 Precautionary measures prevent concentrated ingress of water into the ground
- D3 Additional precautionary measures to D2 requirements as provided for in the standard
- D4 Precautionary measures determined rationally and specifically for the particular site

It follows by inference that the maximum permissible population densities for land usage requirements D1, D2 and D3 by definition correspond to minimum population densities for land usage requirements D2, D3 and D4 respectively, i.e. for an area designation one level higher in each instance, hence the minimum inferred population densities denoted by superscript "1" in Table 19.

The permissible population densities determined in the paper are not subject to the implementation of any specified precautionary measures. SANS 1936:2012 does not allow any people to be accommodated on dolomite land unless precautionary measures as specified in Table 19 are provided. The population densities corresponding to area designation D1 in the table are accordingly zero.

SANS 1936:2012 land usage requirement D2 for Inherent Hazard Class 1 is sensible, because it is always good practice to prevent the concentrated ingress of water into dolomite ground.

SANS 1936:2012 land usage requirement D3 for Inherent Hazard Class 2 is considerably more onerous than determined in the paper. Inferred land usage requirement D4 for Inherent Hazard Class 2, if appropriate, may be unduly onerous.

SANS 1936:2012 land usage requirement D3 for Inherent Hazard Class 3 is more onerous than determined in the paper, especially with regard to the limitation on sinkhole size. Inferred land usage requirement D4 for Inherent Hazard Class 3 is sensible.

SANS 1936:2012 land usage requirement D3 for Inherent Hazard Class 4 is sensible compared to that determined in the paper, except for the limitation on sinkhole size. Inferred land usage requirement D4 for Inherent Hazard Class 4 is sensible.

SANS 1936:2012 land usage requirement D3 + FP1 for Inherent Hazard Class 5 is sensible compared to that determined in the paper, except perhaps for the limitation on sinkhole size. Inferred land usage requirement D4 for Inherent Hazard Class 5 is sensible.

SANS 1936:2012 land usage requirements D1, D2 and D3 for Inherent Hazard Classes 6, 7 and 8 are sensible compared to that determined in the paper. Inferred land usage requirement D4 for Inherent Hazard Classes 6, 7 and 8 is sensible.

CONCLUSION

The objective of this paper is to present a methodology in terms of which the various factors that determine the risk of fatal injury when a sinkhole gives rise to the collapse of a single-storey dwelling house, can be rigorously accounted for on the basis of probability theory and in terms of which it can in principle be shown how mathematical modelling can be applied to address some of the issues on personal safety that are involved.

Table 19 Permissible land usages for single-storey residential houses

Inherent hazard class	Number of people/ha (stand size ha)			Permissible land usage requirement									
				Sinkhole size (m)									
Table 18	SANS 1936:2012 (various alternatives)			2.0	5	10	20	32.5	50				
1	> 800	0	0	0	D1								
		≤ 300 (> 150)	≤ 200 (> 300)	≤ 60 (2 500)	D2								
		≤ 300 (> 150) ¹	≤ 200 (> 300) ¹	≤ 60 (2 500) ¹	D3 or D4								
2	400	0	0	0	D1 or D2								
		≤ 300 (> 150)	≤ 200 (> 300)	≤ 60 (2 500)	D3	–							
		0	0	0	–	D3							
3	150	0	0	0	D1 or D2								
		–	≤ 200 (> 300)	≤ 60 (2 500)	D3	–							
		0	0	0	–	D3							
4	100	0	0	0	D1 or D2								
		–	≤ 200 (> 300)	≤ 60 (2 500)	D3	–							
		0	0	0	–	D3							
5	30	0	0	0	D1 or D2								
		–	–	≤ 60 (2 500)	D3 + FP1	–							
		0	0	0	–	D3 + FP1							
6, 7 & 8	0	0	0	0	D1, D2 & D3								
		≤ 300 (> 150)	≤ 200 (> 300)	≤ 60 (2 500)	D4								

Note 1: Minimum population densities inferred from SANS 1936:2012

The values for the underlying parameters are estimates based on subjective engineering judgement and can be adjusted considerably. However, the overall result of the proposed methodology, based on the parameter values considered, is in principle compatible with long-standing observation by the authors.

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