

# Experimental study on heat and mass transfer for heating milk

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### Abstract

In this paper, an attempt has been made to estimate the convective heat transfer coefficient for sensible heating of milk in a stainless steel pot during khoa, made by traditional method. Various indoor experiments were performed for simulation of a developed thermal model for maximum evaporation by varying heat inputs from 240 watts to 420 watts. The experimental data was used to determine values of constants in the well known Nusselt expression by simple linear regression analysis and, consequently, convective heat transfer coefficients were determined. It is found that the convective heat transfer coefficients decrease with an increase in rate of heating. The experimental error in terms of percent uncertainty was also evaluated.

Keywords: khoa making, milk heating, sensible heating, convective heat transfer coefficient

### Nomenclature

$A_p$	Area of pan, $m^2$
$C$	Experimental constant
$C_v$	Specific heat of humid air, $J/kg\ ^\circ C$
$g$	Acceleration due to gravity, $m/s^2$
$Gr$	Grashof number = $b\ g\ X^3\ \nu^2\ DT/mv^2$
$h_c$	Convective heat transfer coefficient, $W/m^2\ ^\circ C$
$h_{c,av}$	Average convective heat transfer coefficient, $W/m^2\ ^\circ C$

$K_v$	Thermal conductivity of humid air, $W/m\ ^\circ C$
$m_{ev}$	Mass evaporated, kg
$n$	Experimental constant
$N$	Number of observations in each set of heat input
$Nu$	Nusselt number = $hc\ X / K_v$
$Pr$	Prandtl number = $mv\ C_v / K_v$
$P(T)$	Partial vapour pressure at temperature $T$ , $N/m^2$
$\dot{Q}_e$	Rate of heat utilized to evaporate moisture, $J/m^2\ s$
$t$	Time, s
$\Delta T$	Effective temperature difference, $^\circ C$
$w_1$	Weight of milk/water, g
$w_2$	Weight of empty pot, g
$W$	Heat input, watts
$X$	Characteristic dimension, m

### Greek symbols

$\beta$	Coefficient of volumetric expansion ( $K^{-1}$ )
$\gamma$	Relative humidity (%)
$\lambda$	Latent heat of vaporization, $J/kg$
$\mu v$	Dynamic viscosity of humid air, $N\ s/m^2$
$\rho v$	Density of humid air, $kg/m^3$

## 1. Introduction

India is the largest milk producing country with an annual production of more than 91 million tones, which is about 15 percent of total milk production in the world. It has been estimated that nearly 7% of the total milk production is being utilized for making khoa due to its large scale consumption. Khoa is an important indigenous heat coagulated, partially dehydrated milk product which is popular in large sections of the Indian population throughout the country. It is obtained by heat desiccation of whole milk to 65 to 70 percent milk solids without the addition of any foreign ingredients. Khoa has considerable economic and dietary importance to the Indian population. It forms an important base for preparation of milk sweets which are an integral part of Indian food heritage. The total Indian sweet market is around 520 billion in terms of annual sales (Pal, 2008; Kumar *et al.*, 2010).

Dunkle (1961) developed a semi-empirical relation to determine the rate of evaporation for distillation under indoor conditions with few limitations. Clark (1990) also developed a thermal model for a higher operating temperature range under simulated conditions for a small inclination of the condensing surface. Later on, Tiwari and Lawrence (1991) attempted to incorporate the effect of inclination of the condensing surface by choosing the values of constants (C & n) as proposed by Dunkle. Adhikari *et al.*, (1990; 1991; 1995) who attempted to modify the values of these coefficients under simulated conditions. Kumar and Tiwari (1996) and Tiwari *et al.*, (1997) have developed a thermal model for heat and mass transfer for indoor as well as outdoor conditions by using simple regression analysis. Tiwari *et al.*, (2003) studied the heat and mass transfer behaviour of sugarcane juice during natural convection heating for preparation of jaggery under the open and closed conditions. They observed that the convective heat transfer coefficients increase significantly with an increase in heat input and it was reported to vary from 1.87 W/m<sup>2</sup> °C to 8.72 W/m<sup>2</sup> °C.

The heating of milk during khoa making involves sensible heating (natural convection) and boiling convection heat transfer mechanisms. The present study has been set forth to experimentally investigate the sensible heating behaviour of milk under open conditions in a circular stainless steel pot for different heat inputs varying from 240 W to 420 W. For the present study the following temperature ranges have been classified: sensible heating of milk is up to 90 °C and pool/nucleate boiling starts at 90-95 °C (Kumar *et al.*, (2011). Experimental data was analyzed by using the Nusselt expression for the natural convection of fluids to determine the convective heat transfer coefficient. The present research work would be useful in designing an evaporator for khoa production.

## 2. Thermal modelling and theoretical considerations

The convective heat transfer coefficient for evaporation was determined by using the following relation (Anwar and Tiwari, 2001):

$$Nu = \frac{h_c X}{K_v} = C(GrPr)^n$$

$$\text{Or } h_c = \frac{K_v}{X} C(GrPr)^n \quad (1)$$

The rate of heat utilized to evaporate moisture is given as (Tiwari *et al.*, 2003; Anwar and Tiwari, 2001):

$$\dot{Q}_e = 0.016 h_c [P(T_c) - \gamma P(T_e)] \quad (2)$$

( $T_c = T_1$  and  $T_e = T_6$  Used from Appendix A, Tables A1-A7)

On substituting  $h_c$  from Eq. (1), Eq. (2) becomes

$$\dot{Q}_e = 0.016 \frac{K_v}{X} C(GrPr)^n [P(T_c) - \gamma P(T_e)] \quad (3)$$

The moisture evaporated is determined by dividing Eq. (3) by latent heat of vaporization ( $\lambda$ ) and multiplying the area of pan ( $A_p$ ) and time interval ( $t$ ).

$$m_{ev} = \frac{\dot{Q}_e}{\lambda} A_p t = 0.016 \frac{K_v}{X \lambda} C(GrPr)^n [P(T_c) - \gamma P(T_e)] A_p t \quad (4)$$

Let

$$0.016 \frac{K_v}{X \lambda} [P(T_c) - \gamma P(T_e)] A_p t = K$$

$$\frac{m_{ev}}{K} = C(GrPr)^n \quad (5)$$

Taking the logarithm of both sides of Eq. (5),

$$\ln \left[ \frac{m_{ev}}{K} \right] = \ln C + n \ln(GrPr) \quad (6)$$

This is the form of a linear equation,

$$y = mx + c \quad (7)$$

Where

$$y = \ln \left[ \frac{m_{ev}}{K} \right], m = n, x = \ln(GrPr) \text{ and } c = \ln C$$

Values of  $m$  and  $c$  in Eq. (7) are obtained by using the simple linear regression method by using the following formulae

$$m = \frac{N \sum xy - \sum x \sum y}{N \sum x^2 - (\sum x)^2}$$

$$\text{and } c = \frac{\sum x^2 \sum y - \sum x \sum xy}{N \sum x^2 - (\sum x)^2} \quad (8)$$

Then the constant 'C' and exponent 'n' can be obtained from the above equations.

The different thermal physical properties of humid air, such as specific heat ( $C_v$ ), thermal conductivity ( $K_v$ ), density ( $\rho_v$ ), viscosity ( $\mu_v$ ), and partial vapour pressure,  $P(T)$  were determined by using following expressions (Tiwari *et al.*, 2003):

$$C_v = 999.2 + 0.1434T_i + 1.101 \times 10^{-4} T_i^2 - 6.7581 \times 10^{-8} T_i^3 \quad (9)$$

$$K_v = 0.0244 + 0.7673 \times 10^{-4} T_i \quad (10)$$

$$\rho_v = \frac{353.44}{(T_i + 273.15)} \quad (11)$$

$$\mu_v = 1.718 \times 10^{-5} + 4.620 \times 10^{-8} T_i \quad (12)$$

$$P(T) = \exp \left[ 25.317 - \frac{5144}{(T_i + 273.15)} \right] \quad (13)$$

Where  $T_i = (T_c + T_e)/2$

The experimental errors were evaluated in terms of percent uncertainty (internal + external) for the mass of water vapour evaporated. The following two equations were used for internal uncertainty (Kumar *et al.*, 2011; Nakra and Chaudhary, 1985):

$$U_i = \frac{\sqrt{\sigma_1^2 + \sigma_2^2 + \dots + \sigma_N^2}}{N_o} \quad (14)$$

Where  $\sigma$  is the standard deviation and  $N_o$  are the number of sets.

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The percent internal uncertainty therefore was determined using the following expression:

$$\% \text{ internal uncertainty} = (U_i / \text{mean of the total observations}) \times 100 \quad (15)$$

For external uncertainty, the least counts of all the instruments used in measuring the observation data were considered.

### 3. Experimental set-up and procedure

The schematic view of the experimental set-up is shown in Figure 1. It consists of a hot plate (1000W capacity and 178 mm in diameter) which is connected through a variac to control the rate of heating of the milk in a stainless steel pot of 3.2 litres capacity. Temperatures were measured at various locations namely milk ( $T_1$ ), pot bottom ( $T_2$ ), pot outer side ( $T_3$ ) and room ( $T_4$ ) by a ten channel digital temperature indicator (least count of 0.1 °C; accuracy  $\pm 0.1\%$ ) with calibrated copper-constantan thermocouples. Milk surface temperature ( $T_5$ ) was measured by infra-red thermometer (Raytek-MT4), having a least count of 0.2 °C with an accuracy of  $\pm 0.2\%$ . The relative humidity ( $\square$ ) and temperature above the milk surface ( $T_6$ ) were measured by a digital humidity/temperature meter (model Lutron-HT3006 HA). It had a least count of 0.1% RH (accuracy of  $\pm 3\%$ ) and 0.1 °C temperature with an accuracy of  $\pm 0.8$  °C. The heat input was measured by a calibrated digital wattmeter having a least count of 1 watt. The mass of moisture evaporated during heating of milk was measured by an electronic weighing balance (capacity 6 kg; Scaletech, model TJ-6000) having a least count of 0.1g with an accuracy of  $\pm 2\%$ .

In order to determine the convective heat transfer coefficient of milk for a sensible heating mode during khoa making under open conditions, the following procedure was employed:

1. A fresh milk sample obtained from a herd of 15 cows was heated in a stainless steel cylindrical

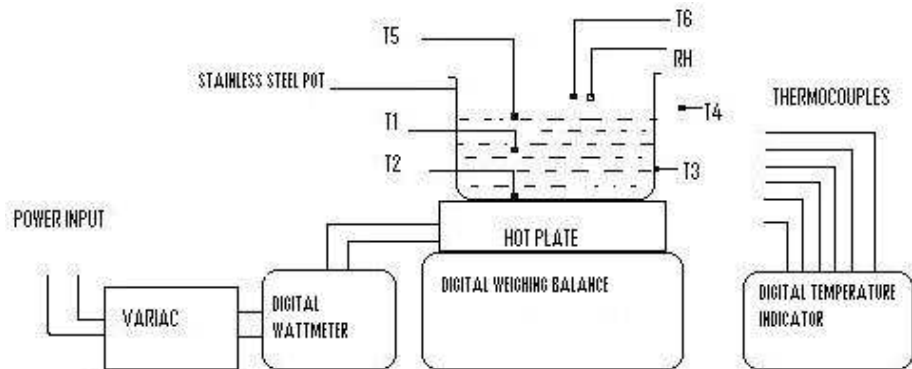


Figure 1: Schematic view of experimental unit

pot (200 mm in diameter, 102 mm deep and 1.6 mm thick) for different heat inputs ranging from 240 W to 420 W. For every run of the milk heating, constant mass of the milk sample was taken i.e. 935g.

2. In traditional method of khoa making, milk is heated in an open pan and simultaneously it is stirred and scraped with a palta (iron scraper) to prevent the scorching of milk solids sticking to the pan. In the present research work, light manual stirring and scraping of milk was carried out with the help of a Teflon scraper (width=20mm, length=200mm, having a wooden handle) to avoid the scaling and burning of the product. Skin formed at the milk surface had been broken 4 to 5 times per minute by stirring the milk gently at frequent intervals.
3. The necessary data of temperatures, mass evaporated and relative humidity (RH) was recorded up to 90 °C (i.e. sensible heating range). The experimental data was recorded after every 10 minute time intervals.
4. Different sets of heating of milk were obtained by varying the input power supply from 240 W to 420 W. The experimental data for different sets of heating are reported in Appendix-A (Tables A1-A6). In order to make a comparison the same process was also repeated for water under the same working conditions at 240 W. The experimental data for water heating is given in Table A7 (Appendix-A). The mass evaporated during heating of milk and water for each set of observations were obtained by subtracting two consecutive readings in a given time interval.

#### 4. Results and discussion

The convective heat transfer coefficients for sensible heating of milk were calculated by using the experimental data from Tables A1- A6 (Appendix A). This data was used to determine the values of constant 'C' and exponent 'n' in the well known Nusselt expression. The values of 'C' and 'n' obtained for sensible heating of milk at different rate of heat

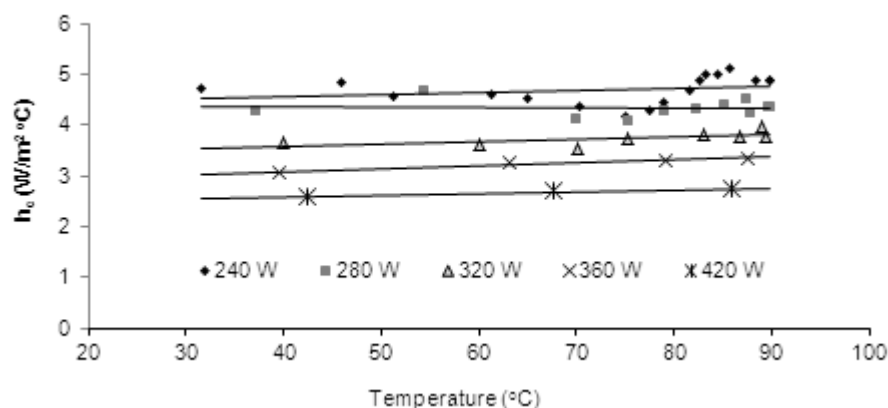
inputs are reported in Table 1. After evaluating the values of constants (C & n) the values of convective heat transfer coefficients were determined from Eq. (1). The results for the convective heat transfer coefficients are also reported in Table 1. It can be seen from Table 1 that the values of convective heat transfer coefficients decrease with the increase in the rate of heat inputs.

**Table 1: Values of the constants (C & n) and 'h<sub>c</sub>' of milk at different heat inputs**

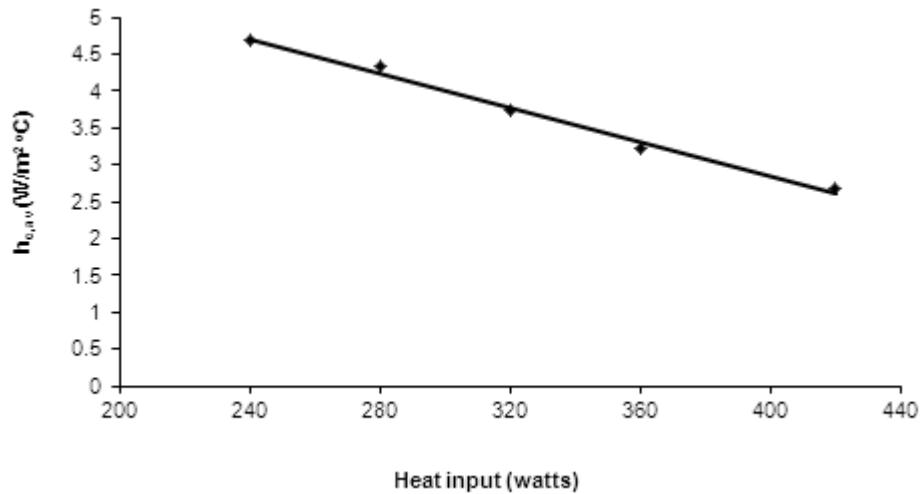
Heat input (W)	Weight (g)	C	n	h <sub>c</sub> (W/m <sup>2</sup> °C)
240	935	1.07	0.23	4.15-5.13
280	935	1.01	0.23	4.09-4.67
320	935	1.02	0.22	3.53-3.96
360	935	1.00	0.21	3.06-3.34
420	935	1.00	0.19	2.59-2.74

The variation in convective heat transfer coefficients with respect to change in operating temperatures for the given range of heat inputs are shown in Figure 2. From this Figure, it can be observed that the convective heat transfer coefficients increase with an increase in the operating temperature for each rate of heat inputs. From this Figure, it can also be seen that convective heat transfer coefficients decreases significantly with the increase in heat inputs.

The average values of convective heat transfer coefficients for the given heat inputs were also calculated and are illustrated in Figure 3. From Figure 3 it can be clearly seen that convective heat transfer coefficients decrease with the increase in heat input. This may be due to the facts that increase in the rate of heating causes faster deposition of the milk solids on heating surface of the pot. The heating of milk below 90 °C results in partial unfolding of proteins which makes it to adhere to heating surface (Prakash *et al.*, 2005; Jun and Puri, 2005). These milk



**Figure 2: Variation of h<sub>c</sub> vs. Temperature for milk at different heat inputs**



**Figure 3: Variation of  $h_{c,av}$  vs heat inputs**

solids deposits are poor thermal conductor, and as they accumulate there is a decline in the thermal performance of the heating surface.

In order to obtain the effect of milk solids deposits on the pot surface, an experiment was also conducted without any scraping of milk at 280 W. The results so obtained are reported in Table 2 from which it is observed that convective heat transfer coefficients decrease drastically, as expected. The average value of convective heat transfer coefficient was found 2.10 times lower than that when milk was scrapped. These results are also illustrated in Figure 4, which clearly shows the marked decrease in heat transfer coefficients.

**Table 2: Values of the constants (C & n) and ' $h_c$ ' of milk without scraping at 280 W**

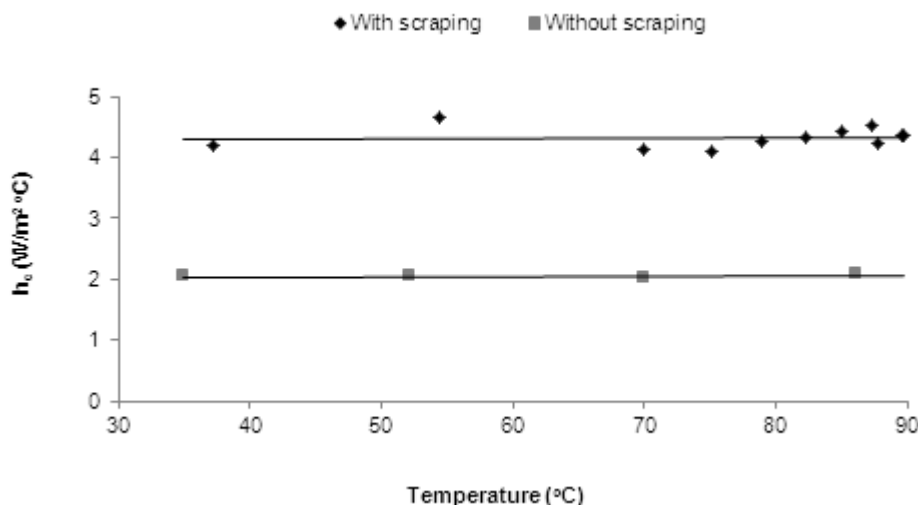
Heat input (W)	Weight (g)	C	n	$h_c$ ( $W/m^2 \cdot ^\circ C$ )
240	935	0.99	0.18	2.02-2.10

For comparison purpose, the values of convective heat transfer coefficients for water were also determined at 240W which are given in Table 3. These results are illustrated in Figure 5 from which it can be seen that the convective heat transfer coefficients for milk are lower in comparison to water. The average value of convective heat transfer coefficient for milk was observed 1.31 times lower than that of water. This may be due to the facts that water is pure in comparison to milk.

**Table 3: Values of the constants (C & n) and ' $h_c$ ' of milk without scraping at 240 W**

Heat input (W)	Weight (g)	C	n	$h_c$ ( $W/m^2 \cdot ^\circ C$ )
240	935	1.01	0.26	5.64-6.64

The percent uncertainty (internal + external) was found to be in the range of 28.06 % to 57.84% and the different values of heat transfer coefficients



**Figure 4: Comparison of  $h_c$  for scraping and without scraping of milk at heat input = 280 W**

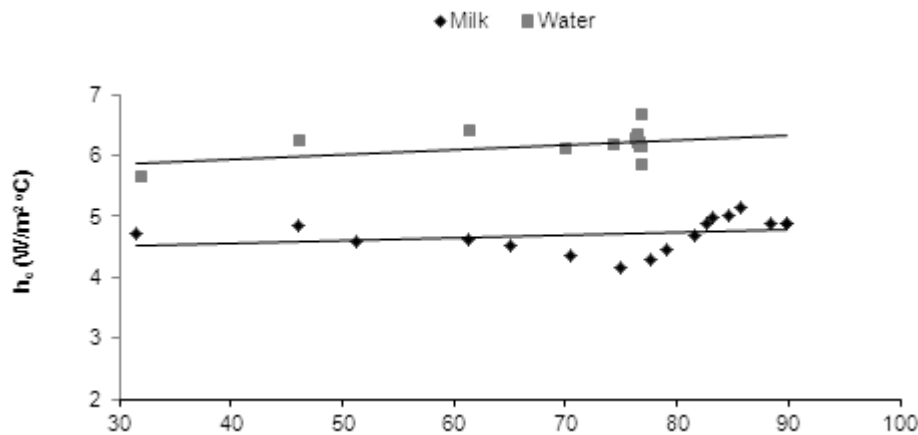


Figure 5: Comparison of  $h_c$  for milk and water at heat input = 240W

were found to be within the range of the percent experimental error. Error bars for convective heat transfer coefficients are depicted in Figure 6.

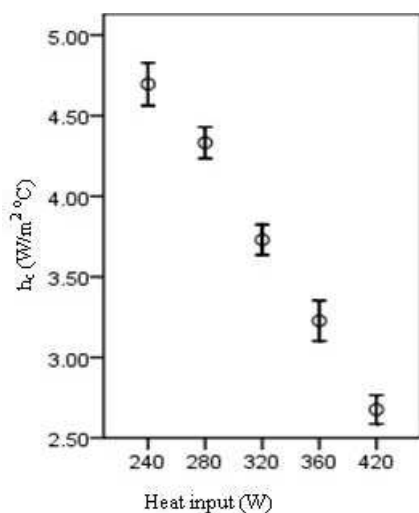


Figure 6: Error bars for convective heat transfer coefficients

## 5. Conclusions

The following results have been drawn from the present research work in which the convective heat transfer coefficients for sensible heating of milk during khoa making in a stainless steel pot under open conditions were investigated.

1. The values of convective heat transfer coefficients decrease with an increase in rate of heat inputs from 240 watts to 420 watts. It was observed to decrease about 75.37% for the given range of heat inputs. This is because of the faster deposition of the scale of milk solids on heating surface which acts as a thermal barrier.
2. The convective heat transfer coefficient increases with an increase in operating temperature.
3. The value of convective heat transfer coefficient of milk was observed 31.28% lower in comparison to water which may be due to the presence of fats, sugar, proteins and other minerals particulates.

4. It is concluded, that for khoa making very high rate of heating will not be beneficial until fast rate of stirring and scraping is employed.
5. The high rate of heating also changes the colours and taste of the khoa.
6. The convective heat transfer coefficients were observed to vary between  $2.59 \text{ W/m}^2 \text{ } ^\circ C$  to  $5.13 \text{ W/m}^2 \text{ } ^\circ C$  for the given range of heat inputs and the experimental errors in terms of percent uncertainty were found in the range of 28.06 % to 57.84%.

## References

- Pal, D. (2008). New innovations in the processing of traditional Indian dairy products. *Indian Dairyman*, 60(3), 127-131.
- Kumar, M., Prakash, O., Kasana, K.S. and Dabur, R.S. (2010). Technological advancement in khoa making. *Indian Dairyman*, 62(1), 64-70.
- Dunkle, R.V. (1961). Solar water distillation: the roof type still and a multiple effect diffusion still, International Development in Heat Transfer, ASME, Proc. International Heat Transfer, Part V, University of Colorado, 895-902.
- Clark, J.A. (1990). The steady state performance of a solar still. *Solar Energy* 44, 43-49.
- Tiwari, G.N. and Lawrence, S.A. (1991). New heat and mass transfer relations for a solar still. *Energy Conversion Management* 31, 201-203.
- Adhikari, R.S., Kumar, A. and Kumar, A. (1990). Estimation of mass transfer rates in solar stills. *Energy Res.* 14, 737-744.
- Adhikari, R.S., Kumar, A. and Sodha, M.S. (1991). Thermal performance of a multi-effect diffusion solar still. *Energy Res.* 15, 769-779.
- Adhikari, R.S., Kumar, A. and Sotha, G.D. (1995). Simulation studies on a multi stage tray solar still. *Solar Energy* 54(5), 317-325.
- Kumar, S. and Tiwari, G.N. (1996). Estimation of convective mass transfer in solar distillation system. *Solar Energy* 57(6), 459-464.
- Tiwari, G.N., Minocha, A., Sharma, P.B. and Khan, M.E. (1997). Simulation of convective mass transfer in

solar distillation process. *Energy Conversion Management* 38(8), 761-770.

Tiwari, G.N., Kumar, S. and Prakash, O. (2003). Study of heat and mass transfer from sugarcane juice for evaporation. *Desalination* 159, 81-96.

Kumar, M., Prakash, O. and Kasana, K. S. (2011). An experimental study on pool boiling of milk. Heat transfer – *Asian research*, 40(2), 159-170.

Anwar, S.I. and Tiwari, G.N. (2001). Evaluation of convective heat transfer coefficient in crop drying under open sun drying conditions. *Energy Conversion*

*Management* 42(5), 627-637.

Nakra, B.C. and Chaudhary, K.K., 'Instrumentation, measurement and analysis', Tata McGraw Hill, New Delhi, (1985), 33.

Prakash, S., Datta, N. and Deeth, H.C. (2005). Methods of detecting fouling caused by heating of milk. *Food Reviews International* 21, 267-293.

Jun, S. and Puri, V.M. (2005). Fouling models for heat exchangers in Dairy Processing: a review. *Journal of Food process Engineering*. 28, 1-34.

## Appendix A

**Table A1: Observations for heating the milk at atmospheric pressure (heat input=240 W,  $w_1=935g$ ,  $w_2=1191g$ )**

Time interval (min)	$T_1$ (°C)	$T_2$ (°C)	$T_3$ (°C)	$T_4$ (°C)	$T_5$ (°C)	$T_6$ (°C)	$\gamma$ (%)	$w_1$ (g)	$m_{evp}$ (g)
-	21.0	21.5	18.7	17.4	20.4	17.2	56.9	935.0	-
10	31.5	37.1	24.5	17.5	30.0	18.0	69.9	934.3	0.7
10	46.0	52.3	32.0	17.6	42.6	19.4	82.2	930.8	3.5
10	51.3	56.3	34.2	17.6	43.7	23.2	90.5	921.2	9.6
10	61.4	66.6	38.0	17.6	56.4	24.1	91.4	905.9	15.3
10	65.1	69.9	43.7	17.8	58.6	24.2	91.3	883.9	22.0
10	70.4	74.6	44.2	17.8	64.8	25.9	92.4	859.9	24.0
10	75.0	78.4	44.4	18.0	68.2	27.9	92.7	839.0	20.9
10	77.6	81.6	45.4	18.0	71.6	29.3	92.6	812.1	26.9
10	79.0	83.7	46.5	18.0	72.8	30.8	93.4	786.0	26.1
10	81.6	87.4	46.4	18.1	76.8	29.3	93.2	759.5	26.5
10	82.6	89.6	46.3	18.3	76.8	30.0	93.1	733.5	26.0
10	83.3	91.0	46.1	18.6	78.2	29.4	93.4	705.9	27.6
10	84.6	92.5	46.1	18.9	78.4	31.5	94.1	678.0	27.9
10	85.7	94.4	45.9	19.0	79.8	30.8	94.6	649.2	28.8
10	88.4	95.4	46.4	19.3	81.8	29.3	93.9	622.3	26.9
10	89.9	97.0	46.3	19.5	82.4	31.1	93.4	596.5	25.8
10	89.8	96.8	46.5	19.5	82.2	29.9	93.7	567.9	28.6

**Table A2: Observations for heating the milk at atmospheric pressure (heat input=280 W,  $w_1=935g$ ,  $w_2=1191g$ )**

Time interval (min)	$T_1$ (°C)	$T_2$ (°C)	$T_3$ (°C)	$T_4$ (°C)	$T_5$ (°C)	$T_6$ (°C)	$\gamma$ (%)	$w_1$ (g)	$m_{evp}$ (g)
-	26.4	25.9	24.5	20.3	25.6	20.9	55.4	935.0	-
10	37.2	42.0	29.1	20.9	36.6	21.2	69.6	933.2	1.8
10	54.4	61.5	41.4	21.0	52.8	23.5	88.3	927.1	6.1
10	70.0	74.3	44.7	21.0	64.8	26.6	91.3	911.5	15.6
10	75.2	79.3	46.5	21.0	67.6	27.3	91.0	890.5	21.0
10	78.9	83.9	50.8	21.0	71.4	28.9	91.5	866.5	24.0
10	82.3	87.7	50.3	21.0	75.0	30.2	92.6	833.1	33.4
10	85.1	90.9	50.6	21.0	76.6	30.4	91.7	798.0	35.1
10	87.8	92.7	51.6	21.0	80.0	32.2	93.4	761.9	36.1
10	87.3	93.7	50.4	21.0	78.6	28.9	92.5	722.9	39.0
10	89.6	95.1	51.0	21.0	84.6	29.9	92.4	685.8	37.1
10	89.8	95.3	51.0	21.2	86.2	29.2	93.0	646.8	39.0

**Table A3: Observations for heating the milk at atmospheric pressure  
(heat input=320 W, w<sub>1</sub>=935g, w<sub>2</sub>=1191g)**

Time interval (min)	T <sub>1</sub> (°C)	T <sub>2</sub> (°C)	T <sub>3</sub> (°C)	T <sub>4</sub> (°C)	T <sub>5</sub> (°C)	T <sub>6</sub> (°C)	γ (%)	w <sub>1</sub> (g)	m <sub>evp</sub> (g)
-	28.8	28.4	23.8	19.8	27.8	20.8	75.3	935.0	-
10	39.9	44.3	29.7	20.5	38.8	21.6	81.7	934.5	0.5
10	60.0	64.2	39.5	20.6	56.8	23.1	84.3	925.3	9.2
10	70.2	74.1	42.5	20.7	60.2	26.3	89.9	907.9	17.4
10	75.2	80.3	46.1	20.9	68.2	27.6	92.0	887.9	20.0
10	83.0	88.4	50.2	20.8	72.6	25.1	90.1	857.2	30.7
10	86.8	92.2	50.0	20.9	77.6	29.5	92.7	818.1	39.1
10	89.5	94.9	50.4	21.0	83.0	30.6	93.4	773.5	44.6
10	89.0	95.7	49.6	20.9	84.6	30.0	92.8	729.1	44.4

**Table A4: Observations for heating the milk at atmospheric pressure  
(heat input=360 W, w<sub>1</sub>=935g, w<sub>2</sub>=1191g)**

Time interval (min)	T <sub>1</sub> (°C)	T <sub>2</sub> (°C)	T <sub>3</sub> (°C)	T <sub>4</sub> (°C)	T <sub>5</sub> (°C)	T <sub>6</sub> (°C)	γ (%)	w <sub>1</sub> (g)	m <sub>evp</sub> (g)
-	26.3	26.1	20.5	18.0	25.6	18.9	73.9	935.0	-
10	39.5	43.7	28.9	18.7	37.8	19.6	84.9	934.0	1.0
10	63.2	68.8	40.0	18.8	58.6	20.4	92.0	924.4	9.6
10	79.1	85.1	48.9	18.9	68.8	21.7	94.6	899.5	24.9
10	87.5	94.1	50.6	18.9	79.0	25.8	94.8	858.1	41.4

**Table A5: Observations for heating the milk at atmospheric pressure  
(heat input=420 W, w<sub>1</sub>=935g, w<sub>2</sub>=1191g)**

Time interval (min)	T <sub>1</sub> (°C)	T <sub>2</sub> (°C)	T <sub>3</sub> (°C)	T <sub>4</sub> (°C)	T <sub>5</sub> (°C)	T <sub>6</sub> (°C)	γ (%)	w <sub>1</sub> (g)	m <sub>evp</sub> (g)
-	26.8	26.1	17.9	17.0	26.0	18.1	64.4	935.0	-
10	42.4	50.9	31.2	17.8	40.6	20.0	87.1	933.6	1.4
10	67.6	78.1	43.0	18.0	60.2	21.1	92.5	923.6	10.0
10	85.9	97.2	51.5	18.2	75.4	24.2	93.3	891.9	31.7

**Table A6: Observations for heating the milk without scraping at atmospheric pressure  
(heat input=280 W, w<sub>1</sub>=935g, w<sub>2</sub>=1191g)**

Time interval (min)	T <sub>1</sub> (°C)	T <sub>2</sub> (°C)	T <sub>3</sub> (°C)	T <sub>4</sub> (°C)	T <sub>5</sub> (°C)	T <sub>6</sub> (°C)	γ (%)	w <sub>1</sub> (g)	m <sub>evp</sub> (g)
-	27.2	27.0	19.0	17.0	26.8	17.9	77.5	935.0	-
10	35.0	40.7	23.6	17.6	33.6	18.7	81.5	933.9	1.1
10	52.1	58.1	30.5	18.1	50.4	20.4	88.6	928.8	5.1
10	70.0	75.3	35.6	18.4	67.4	22.0	86.0	920.9	7.9
10	86.1	92.5	39.8	18.6	81.8	23.5	84.7	913.6	7.3

**Table A7: Observations for heating the water at atmospheric pressure  
(heat input=240 W, w<sub>1</sub>=935g, w<sub>2</sub>=1191g)**

Time interval (min)	T <sub>1</sub> (°C)	T <sub>2</sub> (°C)	T <sub>3</sub> (°C)	T <sub>4</sub> (°C)	T <sub>5</sub> (°C)	T <sub>6</sub> (°C)	γ (%)	w <sub>1</sub> (g)	m <sub>evp</sub> (g)
-	22.0	22.4	22.6	22.6	21.6	21.9	55.1	935.0	-
10	32.1	34.5	28.6	23.2	31.2	22.4	59.0	933.4	1.2
10	46.3	49.9	35.3	23.5	44.6	23.5	76.8	929.5	3.9
10	61.5	65.6	40.9	23.9	58.4	26.6	89.9	917.5	12.0
10	70.2	73.7	44.4	24.0	66.2	31.5	92.6	896.4	21.1



10	74.4	78.1	46.8	24.0	69.6	32.3	92.6	868.5	27.9
10	76.6	80.4	47.3	24.2	70.0	33.1	93.0	831.0	37.5
10	76.7	80.4	47.5	24.3	71.4	33.4	93.4	796.0	35.0
10	77.0	80.0	46.9	24.3	71.4	32.6	93.0	757.5	38.5
10	76.8	80.6	47.8	24.4	71.0	33.0	93.3	720.9	36.6
10	76.9	80.6	47.3	24.6	71.0	32.5	92.9	682.5	38.4
10	76.5	80.6	47.4	24.6	71.2	32.0	92.8	644.3	38.2
10	76.9	80.5	46.9	24.6	71.6	33.4	94.3	605.5	38.8
10	76.4	81.9	48.0	24.8	71.6	36.5	93.8	568.2	37.3

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