The influence of different teaching and learning strategies in mathematics – a case study[6]

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ABSTRACT

This article is a case study that examines how the different teaching and learning strategies influence mathematical knowledge acquisition at university. The research hinged on how mathematics lecturers at universities teach and how students acquire the disseminated knowledge. Research stresses that most lecturers have teacher-centred teaching approaches and mathematics teaching is with minimal student participation. Research further asserts that teaching approaches that emphasise student participation is critical in enhancing effective classroom interaction. This means that, as students' learning strategy is dominantly a participatory and collaborative one, lecturers are challenged to create a learning-as-participation environment for effective mathematics classroom interaction. The article reports a study that was conducted among university students and lecturers in a mathematics course at a South African university. The findings were that most lecturers at the university use traditional non-interactive teaching approaches that create passive environments in contrast with the predominant participative learning strategies of most students.

KEYWORDS: teaching strategy, learning strategy, mathematics class interaction, student-centred strategy, university lecturers

INTRODUCTION

How can effective classroom interaction take place and enhance learning among a diverse group of students in a mathematics course? This is a relevant question that most mathematics lecturers ask themselves. Recently there have been calls for reforming mathematics teaching that is rooted in the social constructivist perspective related to Vygotskian principles (Abdulwahed, Jaworski & Crawford, 2012). This perspective emphasises classroom interaction, offering a way forward in that silent and solitary learning is transformed into a sharing activity of rich mathematical ideas between and among students and the lecturer.

In undergraduate mathematics, traditional non-interactive or lecture-centred approaches have been criticized for leaving little time for interaction and conversations among students regarding the constructs and the mathematics content (Skemp, 1986; Schoenfeld & Floden, 2014; Luneta, 2012). To enhance classroom interaction at university, many alternative ways have been recommended. These include two-way communication (Chang, 2011); the capacity of lecturers to enact whole-class discussions (Speer & Wagner, 2009); or finding ways to cater for students' individual differences (Mokhtar, Yusof & Misiran, 2012). These suggestions indicate 'student-centred' approaches, which have been emphasised in undergraduate mathematics as opposed to 'teacher-centred' or 'content-centred' approaches (Johnson et al., 2009; Mesa, Celis & Lande, 2014).

There have been several approaches to classroom interaction and several studies have been conducted on how to promote a student-centred teaching strategy and these have included an analysis of inventories of students' learning strategies and interviews about perceptions of lecturers yet, studies on the interaction between lecturers and students are rarely conducted (Ashwin,2012. As Kolb (1984) and Grasha (1996) suggested, the relationship between students' preferred learning strategies and a lecturer's teaching strategy is one of many influential variables that affect classroom interaction. This indicates that for students to respond to the classroom environment and to enhance their capability to learn and take part in learning, the teaching strategy should correlate with students' learning strategies to a greater degree.

In this article, we attempt to show what is happening in a mathematics classroom at tertiary level with respect to learning and teaching strategies and provide reasons why it is important for lecturers to use the appropriate approaches suitable for the majority of their students.

The purpose of the research was (1) to identify the dominant learning strategies exhibited by students, as well as the dominant teaching strategies employed by mathematics lecturers at one urban university in South Africa, and (2) to provide research-based evidence of the appropriate teaching strategies that could correlate with most of the students' learning strategies and improve achievement of learning outcomes. For this purpose, an explanatory sequential mixed-methods approach was used.
The article is organised around two research questions:

1) What are the prominent learning strategies and teaching strategies exhibited in mathematics classes at one urban university in South Africa?
2) What are the most appropriate teaching strategies that address the most prominent learning strategies among mathematics students?

The findings generated through this study provide a foundation for the improvement of mathematics classroom interaction in South Africa and other countries with students with diverse abilities and backgrounds.

Classroom interaction in mathematics education

There has been a shift in how learning mathematics is viewed: from 'learning-as-acquisition' (something that happens within the minds of individuals) towards 'learning-as-participation' (Sfard, 2001). In other words, participation in the learning process becomes one of the significant ways to understand mathematics concepts (Goos, 2004).

In a traditional mathematics classroom, effective participation involves activities that require students to listen to and watch mathematical procedures demonstrated by the lecturer and then to complete textbook exercises afterwards (Luneta, 2014. However, research (Chambers et al., 2008) shows that memorization with understanding and reproduction of procedures, discussion, collaboration, and participation in reform-oriented mathematics classrooms effectively build intellectual challenges. Instead of relying on the lecturer, students are expected to actively engage with mathematics by learning how to develop explanations, make predictions and debate alternative approaches to problems. These classroom interactions between and among students and a lecturer can be regarded as important for teaching of and learning mathematics.

Mathematical conversations among students and the teacher play a key role in shaping mathematical proficiency, competence, and disposition (Gresalfi et al., 2009). Engaging with mathematical constructs helps students to discover important mathematical concepts by making conjectures, talking, questioning, and debating problems in the interactive learning process (Stein, 2007). Participation as a collective activity would promote mathematical critique and argumentation, leading toward the proof, justification, and development of important mathematical concepts as a necessary step for conceptual understanding (Truxaw & DeFranco, 2007).

In tertiary education, undergraduate students are regarded as practitioners along with lecturers because of the significant role of their participation (Abdulwahed et al., 2012). However, studies assert that students in higher education do not actively interact or participate in the teaching process (Rudduck & McIntyre, 2007). Particularly mathematics classes have been criticized for failing to encourage and motivate students to participate (Turner et al., 1998).

According to Gresalfi et al. (2009), the relationship among individual students and how they conceive their classroom environment are critical for learning mathematics. Frenzel, Pekrun and Goetz (2007) provided evidence that the environmental characteristics perceived by students closely relate to their motivation, academic engagement, and mathematics achievements. Their findings were compatible with a study conducted by Gilbert et al. (2014) which pointed out that classroom contexts affect mathematics classroom interactions.

Other key factors in classroom interaction are the knowledge of students' learning strategies and an understanding of the nature of their relationships (Cheng & Zhang, 2017; Iurea et al., 2011). Since students learn better and engage more in a classroom environment congruent to their preferences (Wong, Ding & Zhang, 2016), catering for the differences between the lecturers and students have been discussed and studied for decades. It was a focus of teaching in the UK in the late 20th century termed as the differentiation approach (Chambers, 2008). This approach entailed providing teaching to students based on their differentiated abilities. Chowdhury (2015) also suggested that it is both effective for students and comfortable for the lecturers to maintain and understand each other's teaching and learning strategies.

As mentioned above, not much research has been conducted on classroom interaction strategies between lecturers and students in undergraduate mathematics classes. Furthermore, the extent to which preferred teaching and learning strategies relate to classroom interaction has also received little attention. In short, there was a need to identify and examine teaching and learning strategies' relationships and how these enhance mathematics classroom interaction at university.

Teaching and learning strategy in classroom interaction

The use of social constructivist teaching – which is designed to promote students' participation in the learning process – compared to more traditional teaching methods, such as lecturing, has been associated with higher grade achievement, enhanced intellectual curiosity, and the development of superior creativity and leadership skills (Henson, 2003). To achieve all these, learning should focus on interaction, negotiation, discussion, and collaboration instead of instructing only on isolated facts. It is important to enhance mutual construction of conceptual understanding through classroom interaction, taking into consideration the teaching style of the lecturer and students' preferred learning strategies (Grasha, 1996; Kolb, 1984). The traditional lecture method was popularized by the Greeks in the 5th century BC. It was widely used in universities when books were scarce to motivate and create interest among large numbers of students (Friesen, 2014).

Grasha (1996) defines teaching strategies and learning strategies respectively as interpersonal relationships with students and responses to the needs of the classroom environment, and as an individual characteristic that influences a learner's capability to understand new knowledge and to participate in learning environments. In this definition, communicative and interactive aspects of strategies in the classroom are emphasised and teaching strategies and learning strategies are considered to be constructs related to classroom interactions (McCaskey, 2010).

Firstly, teaching strategy refers to all of the teaching techniques, activities, and approaches that a teacher employs in the classroom. It can be associated with a teacher's behaviour in the class (Cotton, 2000), the context of the teaching (Rahimi & Nabilou, 2011) and how they manage their classes (Yilmaz & Çavaş, 2008). It is evident that teaching strategy is a very influential factor in students' learning experiences since teachers provide the 'vital human connection between the content and the environment and the learners' (Heimlich & Norland, 1994: 109).

Though there are various teaching strategies, Grasha identifies five specific teaching strategies. These strategies be a continuum of teaching (see Table 1) and were observed amongst teachers at university. Out of these five teaching strategies, the facilitation approach is claimed to enhance classroom interaction (e.g., Wachira, Pourdavood & Skitzki, 2013; White, 2003; Gresalfi, 2009). These strategies require (university) teachers to be flexible and able to interweave knowledge with skills (Sherin; 2002) or construct their thoughts and decision-making processes during teaching (Thomson, 1984). A lecturer with a facilitator teaching strategy can listen to classroom conversations and observe classroom interactions between and among students and themselves. Mathematical knowledge acquisition is enhanced when there is synergy between teaching and learning.

Table 1:
Grasha's Five Teaching Strategies

Expert	Formal authority	Personal model	Facilitator	Delegator
▪ Experienced, ▪ Eager to transfer information, ▪ Focusing on preparing students completely.	▪ Considering the positive and negative feedbacks of students' behaviour, ▪ Meeting the objectives. ▪ Having clear expectations from their students.	▪ Believing in the personal example on how the students can think, ▪ Encourag-ing the students to observe & compete.	▪ Supervising ▪ Emphasising the interaction (asking questions & expressing opinions), ▪ The students' capacity in gaining practical independency is imperative.	▪ Developing the students' ability to function indepen-dently & autono-mously.

Teacher-centred Student-centred

Learning strategies consist of different dimensions of interaction between and among students and their lecturer. Grasha and Riechmann identified three bipolar dimensions, namely Dependent-Independent, Avoidant-Participatory, and Competitive-Collaborative. These dimensions represent six learning strategies which measure the preferences for classroom interactions that focus on student attitudes toward classroom activities, learning content, lecturers, and peers (see Table 2).

Table 2:
Riechmann and Grasha's Six Learning Strategies

Dependent	▪ Little intellectual curiosity. ▪ Learning only what is required.
Independent	▪ Confident in their learning abilities. ▪ Prefer to work alone.
Avoidant	▪ Not enthusiastic about learning content. ▪ No participation.
Participatory	▪ Good citizens in class. ▪ Enjoy attending classes & participating in class activities.
Competitive	▪ Study to perform better than others. ▪ Like to receive attention & recognition.
Collaborative	▪ Learn by sharing ideas. ▪ Like to work with others.

According to literature, the Participatory and Collaborative strategies can be regarded as the more interactive strategies, which involve sharing ideas and active participation in small-group problem-solving or group works Participatory and Collaborative learning strategies, could possibly improve classroom interaction, and create a learning-conducive environment for both the lecturer and students (Könings, Seidel & Van Merriënboer, 2014).

As earlier alluded to, there are controversial opinions on the effectiveness of matching teaching and learning strategies. Thus, the meshing hypothesis (teaching that matches a student's learning strategies leads to greater learning than mismatched teaching) is controversial. It is because of this tension that the study engaged with the empirical data to investigate the importance or invalidity of such a claim.

RESEARCH DESIGN AND METHODS

The research was a mixed method design, which made use of quantitative and qualitative methods. It employed a sequential explanatory mixed method design to obtain rich data for pragmatic reasons. The first procedure gathered qualitative data followed by quantitative data. These two different data collection approaches allow the examination of the overlapping phases of data gathering (Creswell & Clark, 2007).

Participants

The participants were chosen from two groups: students who were registered for mathematics modules and lecturers who were teaching mathematics at an urban university in South Africa. 276 university students participated (147 male & 129 female) of which 141 were natural science students, 71 education students, and 64 engineering students. 29 lecturers or professors participated from the Department of Pure Mathematics and Applied Mathematics and the Faculty of Education (16 males & 13 females). With the intent of gaining more in-depth information, another group of 24 participants were purposely selected for interviews (16 students & seven lecturers). The criteria for selecting the participants for the qualitative phase included: (i) being participants in the quantitative phase; and (ii) students had to have high scores in Collaborative and Participatory learning strategies (being the most popular learning strategies) (eight students with Collaborative & eight students with Participatory learning strategies); and (iii) seven lecturers with more teacher-centred (T-D) teaching strategies (high scores in Formal Authority & Personal Model) and two lecturers with more student-centred (S-C) teaching strategies (high scores in Facilitator & Delegator). The Expert score was not considered because 97% of lecturers had a high score in this teaching strategy.

Instruments

The 'Grasha-Riechmann Student Learning Strategy Scales (GRSLSS)' and the 'Grasha (1996) Teaching Strategy Inventory' (GTSI) were used for the quantitative data. Each of the six learning strategies in the GRSLSS is measured with 10 items according to modes of student behaviour in a tertiary learning environment: Independent, Dependent, Collaborative, Competitive, Participatory, and Avoidant. The learning strategies were evaluated on a 5-point Likert type scale (Grasha, 1996). The GTSI instrument was used to assess the five teaching strategies (Expert / Formal Authority / Personal Model / Facilitator / Delegator) and involved all 29 lecturers. The instrument has been widely used and yield valid and reliable results (Behnam & Bayazidi, 2013). We took cognisance of the fact that this instrument was used in an Iranian context and among predominantly adult scholars. We were therefore careful in our analysis of how we interpreted the students' views on learning.

An unstructured interview protocol was used to collect qualitative data. The content of the informal interview questions focused on students' participation and the interaction between a lecturer and students in real mathematics classrooms, personal attitudes toward learning and teaching mathematics.

Procedure and analysis

The data collection took place at the university during the first and second semesters of 2018. In the first phase, frequency counts and cross-tabulation were utilized to analyse the demographic information and the participants' answers to separate items in each domain. To compare means of learning strategies based on gender and studying year, an 'Independent-samples T-test' (for parametric data) and a 'Mann–Whitney U Test' (for nonparametric data) were used. The 'Kruskal-Wallis test' (for nonparametric data) was used for the comparison of learning strategies among students from different faculties.

For the richness and the depth of description in the second phase, three data sources were used: (i) in-depth unstructured interviews; (ii) researcher's reflection notes on each participant's responses immediately after the interviews; and (iii) electronic follow-up interviews with each participant to secure additional information on the emerging themes.

In the analysis of qualitative data, a grounded theory approach to gathering and coding data was adopted. When researchers attempt to make sense of their data by organizing and interpreting them, a classificatory scheme needs to be chosen. Corbin and Strauss (1990) state that the process of analysis in the grounded theory is dependent on the purpose: very useful description, conceptual ordering (classifying and elaborating) or developing a theory. A conceptual ordering (high-level description) is important for the generation of knowledge, and it can make a valuable contribution to a discipline. Since the analysis of this research in the qualitative phase was to classify and elaborate the qualities of learning mathematics (conceptual ordering) perceived by lecturers and students. The open code was used to identify concepts and to discover their properties and dimensions, and the axial coding was used to connect and group the identified codes.

To consolidate the results of the research further, the findings from the quantitative and qualitative phases were integrated with existing literature. The integration of the quantitative and qualitative data was essential to provide an informed thick description of findings and further to develop implications for the development of mathematics classroom discourses.

The dominant teaching strategies

Table 3 shows that the Expert teaching strategy was the most frequent modality of high scores; the second most popular strategy was Formal Authority followed by Personal Model teaching strategy, which all fell into the cluster of T-D teaching strategies.

Table 3:
Comparison of the Mean and Standard Deviation of Teaching Strategies

	Minimum	Maximum	Mean (SD)
Expert	3.38	5.00	4.10 (.39)
Formal Authority	3.13	4.88	4.03 (.42)
Personal Model	3.00	5.00	3.90 (.54)
Facilitator	1.50	4.88	3.23 (.85)
Delegator	2.13	4.13	3.03 (.55)

The fact that 97% (n = 29) of lecturers had high scores in Expert (see Table 4) indicates clearly that mathematics lecturers believe their role is mainly to teach and to set high standards.

Table 4:
The Score and Frequency of Grasha Teaching Strategies

		Low	Moderate	High
Expert	Score	1.0-2.29	2.3-3.43	3.44-5.0
	Freq. (%)	▪	**1** (3.4)	**28** (96.6)
Formal Authority	Score	1.0-2.96	2.97-3.86	3.87-5.0
	Freq. (%)	▪	**10** (34.5)	**19** (65.5)
Personal Model	Score	1.0-3.07	3.08-4.07	4.08-5.0
	Freq. (%)	**1** (3.4)	**19** (65.5)	**9** (31.0)
Facilitator	Score	1.0-2.64	2.65-3.79	3.80-5.0
	Freq. (%)	**7** (24.1)	**14** (48.3)	**8** (27.6)
Delegator	Score	1.0-1.86	1.87-3.0	3.01-5.0
	Freq. (%)	▪	**17** (58.6)	**12** (41.4)

With an in-depth analysis of the quantitative data from 'Teaching Strategy Inventory', the first five most frequent items that mathematics lecturers regarded as 'Very important' related closely to T-D teaching strategies (See Table 5).

Table 5:
The Five Frequent Statements that Lecturers valued highly

Question No.	Statements	Teaching strategy
5	Sharing my knowledge and expertise with students is very important to me.	Expert
9	I typically show students how and what to do in order to master the mathematics lessons content.	Personal Model
10	I want to students to leave this course well prepared for further work in this area.	Expert
20	This course has very specific goals and objectives that I want to accomplish.	Formal Authority
2	I set high standards for students in my mathematics class.	Formal Authority

It means most lecturers still employ the traditional teaching T-D strategies in mathematics classrooms at university. This shows that mathematics, engineering, and natural science faculties are generally less inclined to adopt student-centred pedagogies than other faculties (Lindholm & Astin, 2008).

The dominant learning strategies

The Statistical Package of the Social Sciences (SPSS) 24.0 was used to analyse the quantitative data using descriptive and parametric statistics (or non-parametric alternatives if necessary) (Pallant, 2020). Table 6 shows the frequency distribution of samples.

Table 6:
Frequency Distribution of Samples based on Gender, Discipline, Studying Year

	Gender		Studying Year			
Discipline	Male	Female	1st	2nd	3rd	**Total**
Engineering	44	20	26	24	14	
Science	84	57	82	37	22	
Education	19	52	12	30	29	
Total	147	129	120	91	65	276

The range of the mean scores in each domain was marked as low, moderate, or high according to the ranges suggested by Grasha (1996). Collaborative (M = 3.69 / 3.5-5.0 scores were regarded as 'High'), Competitive (M = 3.06 / 2.9-5.0 scores were regarded as 'High') and Participatory (M = 4.03 / 3.9-5.0 scores were regarded as 'High') were found to be more prevalent among the university students sampled.

There were differences in learning strategies based on gender, field of study, and year of study. With regard to gender difference (see Table 7), there was a significant difference in the scores of the independent learning strategy (t (276) = 2.528 p = .012, two-tailed Independent-sample T test) and the Dependent learning style (U = 7206.5, Z = -3.368, *p* = .001 two=tailed Mann–Whitney U Test). Male students tended to be more independent and prefer to study on their own. On the other hand, female students prefer strong guidance and concrete hands-on experience and benefited more from well-organised and structured teaching. This could be interpreted to mean that female students are more agreeable and conscientious than male students (Rubinstein, 2005).

Table 7:
Mean and Standard Deviation Distribution of Learning Strategies by Gender

	Male		Female	
	Mean	SD	Mean	SD
Independent	**3.47**	**.58**	**3.29**	**.59**
Dependent	3.86	.51	4.04	.48
Collaborative	3.64	.68	3.67	.78
Competitive	3.14	.76	2.99	.79
Participatory	3.98	.60	4.05	.58
Avoidant	2.44	.55	2.43	.58

There was a significant difference of learning strategy between junior and senior students (see Table 8). A 'Mann–Whitney U Test' revealed the differences in the bipolar relationship of the Participatory–Avoidant. (Participatory learning strategy (U = 6479.0, Z = -2.649, p = .008 two-tailed) and Avoidant learning style (U = 6010.5, Z = -2.666, p = .008 two-tailed)). 1st-year students were more Participatory than 2nd and 3rd year students.

Table 8:
Mean and Standard Deviation Distribution of Learning Strategies by Studying Year

	Junior (1st year)		Senior (2nd & 3rd year)	
	Mean	SD	Mean	SD
Independent	3.35	.54	3.42	.63
Dependent	4.00	.54	3.94	.48
Collaborative	3.74	.73	3.61	.73
Competitive	3.10	.81	3.07	.75
Participatory	**4.11**	**.59**	**3.94**	**.58**
Avoidant	**2.32**	**.55**	**2.52**	**.57**

In relation to the field (Science, Education, Engineering) (see Table 9), the 'Kruskal-Wallis' test was conducted, and a significant difference was found in the score of Collaborative learning strategies across three different disciplines (Engineering (n = 64); Science (n = 141); and Education (n = 71)), χ^2 (2, n = 276) = 10.79, p = .05). The students in the Education Faculty tended to acquire more information by sharing and cooperating with lecturers and peers than in other faculties.

Table 9:
Mean and Standard Deviation Distribution of Learning Strategies by Field

	Engineering		Natural Science		Education	
	Mean	SD	Mean	SD	Mean	SD
Independent	3.40	.54	3.41	.58	3.32	2.57
Dependent	3.93	.46	3.91	.53	4.04	.48
Collaborative	**3.77**	**.66**	**3.54**	**.68**	**3.84**	**.85**
Competitive	3.10	.75	3.01	.80	3.15	.75
Participatory	4.10	.58	4.00	.59	4.07	.60
Avoidant	2.37	.42	2.40	.55	2.57	.68

The perceived qualities of the mathematics classroom discourse

The transcribed interviews of the 16 students (eight students with Participatory and Collaborative learning strategy respectively) and seven lecturers (five lecturers with T-D teaching strategy and two lecturers with S-C teaching strategy) were analysed. Out of the three dominant learning strategies, the Participatory learning strategies and the Collaborative learning strategy were chosen. The Competitive learning strategy was not chosen because there was a positive correlation between the Participatory learning strategy and the Competitive learning strategy. Students who tend to compete are 'good classroom citizens' and more willing to do what the teacher wants them to do (Diaz & Cartnal, 1999). Also, on-campus students prefer the Collaborative while off-campus students prefer the Competitive learning strategy (McCaskey, 2010).

Table 10 shows the characteristics of the mathematics classroom interaction that were perceived by different teaching strategies and learning strategies respectively: the two dominant learning strategies (Participatory and Collaborative) and the two opposed teaching strategies (T-D / S-C). Each number in table 10 indicates how many times lecturers or students mentioned those characteristics during the interviews.

Table 10:
The Characteristics of the Mathematics Classroom Interaction (n=23; 7 lecturers and 16 students)

	The characteristics of the mathematics classroom interaction	A (%)	B (%)	C (%)	D (%)	Total (%)
1	Positive attitude to participate	4 (80)	2(100)	6 (75)	5 (62.5)	79.38
2	Little confidence & low self-efficacy	5 (100)	2 (100)	4 (50)	5 (62.5)	78.13
3	Realistic examples	3 (60)	2 (100)	5 (62.5)	5 (62.5)	71.25
4	Careful attention to students' conceptual understanding	4 (80)	2 (100)	4 (50)	4 (50)	70.00
5	The importance of what lecturers question	3 (60)	2 (100)	6 (75)	3 (37.5)	68.13
6	Tutorial period	3 (60)	1 (50)	5 (62.5)	5 (62.5)	58.75
7	Break time during the class	1 (20)	2 (100)	5 (62.5)	4 (50)	58.13
8	Mutual communication	2 (40)	2 (100)	3 (37.5)	4 (50)	56.88
9	Small group discussion	2 (40)	1 (50)	4 (50)	5 (62.5)	55.63
10	Lack of interest in mathematical knowledge	4 (80)	2 (100)	1 (12.5)	1 (12.5)	51.25
11	Linkage to students' previous knowledge	3 (60)	1 (50)	1 (12.5)	5 (62.5)	46.25
12	Unprepared students	4 (80)	2 (100)	1 (12.5)	1 (12.5)	43.75

Note: A – 5 lectures with T-C teaching strategy / B – 2 lecturers with S-C teaching strategy / C – 8 Students with participatory learning strategy / D – 8 Students with collaborative learning strategy

The 12 characteristics were chosen because these were expressed by either over 50% of lecturers or 50% of students. Out of the 12 characteristics, only nine (over 55% of total percentage) were categorised under the four themes (see Table 11): (i) articulating conceptual understanding; (ii) shaping mathematical argumentation; (iii) collaborative setting; and (iv) strategic environment.

Table 11:
Categories and Subcategories

Categories	Sub-categories	Characterised actions
Articulating Conceptual Understanding	Authentic (real life) examples	An instrument for students to reflect on what they have learned & for lecturers to gain ideas of classroom interaction.
	Better listener	Lecturers' careful attention to students' level of conceptual understanding.

Shaping Mathematical Argumentation	Low self-efficacy	Little confidence to discuss, debate, & reason.
	Thought-provoking questions	The fundamental cause of students' conjectures and justification.
Collaborative Setting	Positive attitude	Students' positive attitude to participate in classroom interaction.
	Effective tutorial	Tuition period with interactive tasks.
	Small group discussion	Interactive discussion among peers during classroom.
Strategic Environment	Purposive interval	Break time to think over and to reflect.
	Reciprocal environment	Dialogue format & approachable atmosphere.

DISCUSSION AND IMPLICATIONS

Articulating conceptual understanding: The first disparity between lecturers and students in terms of classroom interaction was how much they comprehended mathematical concepts. Abstract mathematical information easily grasped by lecturers was not always easy for students to understand. As a result, mathematical concepts explained and discussed in the classroom were frequently beyond students' understanding and mathematical knowledge acquisition was hampered. A lack of interaction in mathematics classrooms was one of the obstacles students faced. This theme reveals the students' need for concreteness. To minimize this disparity and to make a classroom more active, 'Authentic or real-life examples' were frequently mentioned by lecturers and students (the first sub-category). Although undergraduate mathematics is primarily symbolic mathematics that requires abstract thinking, 'real maths' (everyday experiences or objects) would possibly increase classroom interaction.

Yet, how they viewed authentic examples varied: lecturers with the S-C teaching strategy prefer instruments to trigger students' mathematical insight; students with a Participatory learning strategy preferred opportunities to reflect on what they learned; or students with a Collaborative learning strategy used real examples to explain concepts to peers. Regardless of their different views, lecturers and students all agreed that authentic examples could keep classrooms alive and assist lecturers to build up solid interaction in their class.

Classroom interaction in mathematics has been playing a central role in enhancing students' conceptual understanding in comparison to instrumental or procedural understanding, which is obtained in a traditional lecture-based classroom (Steinberg, Empson & Carpenter, 2004). Traditional lecture-based methods hamper conceptual knowledge acquisition. However, 90% of the lecturers mentioned (no.11 in Table 10) that most students were not well prepared to grasp mathematical concepts at the level of university mathematics. Studies (Luneta, 2014) in South Africa show that there is a big gap between what the universities require from first-year students and the mathematical knowledge they come with from high schools. As a result, lecturers are expected to be 'better listeners' (the second sub-category) who orchestrate classroom interaction that helps students to construct mathematical concepts and facilitates a conceptual understanding of key information. Lecturers with an S-C teaching strategy suggested constant but indirect evaluation of students' understanding in order to assist better classroom interaction.

Shaping mathematical argumentation

The development of mathematical proficiency has been emphasised not only for those who study mathematics as their major, but also as part of non-major subject choices. It is critical for students to develop deep conceptual understanding, and also to be able to make concrete their understanding through explanation and argumentation (Carpenter, Franke & Levi, 2003). In this respect, shaping mathematical arguments in class is one of the essential aspects of classroom interaction.

All lecturers were aware of the importance of mathematical arguments. However, from both the students' and the lecturers' input there was very little effective and efficient mathematical argumentation that took place. Not many students actively participated, and classroom discussion did not always include significant mathematical content. There might be two reasons for this: students' 'low self-efficacy' and their lack of interest in mathematics.

Regardless of the different learning strategies, students did not believe they had enough mathematical knowledge to discuss, debate and reason ('Low self-efficacy' – self-efficacy is defined as the judgments students make about their potential to learn mathematics successfully and their belief in their own capabilities). Self-efficacy has the potential to facilitate or hinder a mathematics learner's motivation, use of knowledge, and disposition to learn (Tait-McCutcheon, 2008). It is closely related to students' learning strategies and affects classroom participation (Yates, 2002). In this regard, to make classroom interaction mathematically rich and sufficient, students' self-efficacy should be considered, and it is critical to create an environment that encourages students to do the necessary activities that result in enhancing classroom interaction.

'Thought-provoking questions' emerged as the second sub-category. This was because the questions lecturers provided seemed appropriate and could be the platform for shaping mathematical argumentation. Lecturers with a T-D teaching strategies employed the questions to develop students' mathematical thinking, whereas with an S-C teaching strategies lecturers analysed students' answers to their questions to understand students' way of reasoning. On the other hand, students viewed lecturers' questions as the source of preparing for exams. Participatory students used the questions as the resources to think further and Collaborative students had discussion time among peers based on those questions. This implies that lecturers should carefully consider how and what kinds of questions they offer in the classroom. By asking thought-provoking questions, lecturers create a problematic interaction and informal conjectures turn out to be a formal fact in the process of debating.

Collaborative setting

Many lecturers mentioned that the ineffective collaborative setting was mainly due to the passive attitude of students and partly because of time constraints. Students also acknowledged their unwillingness to participate. Students' positive attitude towards participation could be critical to create successful classroom interaction ('Positive attitude'– the first sub-category). Tutorials are good opportunities for students to establish their confidence to discuss and debate mathematical concepts and it helps them to take part in small group discussions in class (the second and third sub-categories). The tutorial, including many hands-on activities and interactive tasks, has been used for collaborative learning by lecturers and it has played a significant role for students to practice classroom interaction regardless of their learning strategies (no. 8 in Table 10).

Classroom interaction goes hand in hand with collaborative learning, which leads to co-construction of knowledge and the acquisition of communication skills (Bouta & Retailis, 2013). Effective classroom interaction depends upon the extent to which collaborative settings are established in the classroom. To make a productive collaborative setting, students' 'Active attitude' is required.

Many students preferred to join small group discussions instead of whole classroom discussions. What students experienced in small group discussions could develop not only mathematical knowledge, but also their skills to listen to each group member and to give constructive feedback. Many studies stated that learning in small co-operative groups fosters students' cognitive learning processes and motivation (e.g., Schmidt & Moust, 2000; Langer-Osuna, 2016).

Strategic environment

Participatory students mentioned that an 'approachable atmosphere' would be helpful, where a lecturer accepted students' answers and occasionally followed up their responses. In line with 'approachable atmosphere', S-C teaching approach' lecturers suggested that the dialogue format created for students should be ideally informal.

Students wanted to have more time in class to practice and to reflect on what they have been learning. Providing intervals (the first sub-category) was constantly mentioned by students instead of filling class time with lecturing and writing on the board. During the intervals students might have opportunities to think over what the lecturer explained or to reflect on what has been discussed (Parsons & Brown, 2002). Given that learning mathematics needs introspective and reflective thinking, the intervals provided by lecturers could improve the quality of classroom interaction.

To enhance classroom interaction, the environment should be more reciprocal (the second sub-category). According to students, the long presentation with high technology caused less communication between a lecturer and students. The 'Chalk and Talk' approach, that is writing out a mathematical narrative on the board while talking aloud, (Artemeva & Fox, 2011) was recommended. The verbal explanations, gestures in interacting with writing on the board and the use of questions might be used both as rhetorical devices and in interaction with students.

Effective mathematics classroom interaction and differences in teaching and learning strategy

Orchestrating productive mathematics classroom interactions require different teaching skills from conventional university lectures. This traditional approach focuses mostly on high level mathematics content and differs from teaching that includes mathematics classroom interaction where students' participation forms a significant part of mathematical content.

The qualitative analysis (Table 11) showed that effective mathematics classroom interaction requires a learning-conducive environment. If a lecturer with a T-D teaching strategy wants to be attentive to students' conceptual understanding (Better listener) or to create dialogue format teaching (Reciprocal environment), he or she needs to make an extra effort (See Table 11). In short, learning-as-participation becomes auspicious not only for students but also for a teacher, because teachers should become learners themselves (Schifter, 1998).

In this learning-conducive environment the teaching and learning strategies and classroom interaction can be seen as interdependent. For instance, if a lecturer with a T-D teaching style is likely to focus on mathematical content rather than students' participation, then the teaching becomes highly directive with less interaction. The teaching focused mainly on mathematical content might be preferable to dependent students but easily fails to support students with an independent learning strategy, who are willing to develop their own reasoning and justification skills. On the other hand, excessive focus on participation by a lecturer with an S-C teaching strategy can result in classroom interaction that appears to promote classroom reform. It might reinforce Collaborative and Participatory learning styles but fails to produce substantive mathematical outcomes.

In the analysis of quantitative data, most lecturers had T-D teaching strategies (see Table 3 & 4) and the dominant learning strategies were Competitive, Participatory and Collaborative learning strategies, which befit S-C teaching strategies. The qualitative data showed that the differences between T-D and S-C teaching strategies were clearer than the differences among students according to their learning strategies (see Table 10). There were important differences of teaching strategies in respect of what effective classroom interaction entailed.

Firstly, there was a difference between T-D and S-C teaching strategies with regard to mathematics content. Lecturers with S-C teaching strategies were more aware of the needs of realistic and authentic tasks (100% over 60%, no. 3 in Table 10) and also gave more consideration to the questions they asked students (100% over 60%, no. 5 in Table 10).

Mathematics relating to the real-world application has been generally encouraged (Matthews, Adams & Goos, 2009) with realistic and authentic mathematical tasks encouraging student participation in classroom interaction. Providing authentic tasks help students to make concrete their uncertain and partial ideas in real time and it would be one of the ways to make classroom interaction productive (Felder & Silverman, 1988).

As evident in Table 11, thought-provoking questions, is another difference between T-D and S-C teaching strategies, which encourages students to question their own assumptions and relocate their errors, thus formulating mathematical argumentation. As a result, these questions should be carefully formed by considering students' level of conceptual understanding instead of structuring highly mathematically directive discussion.

This requires lecturers to be better listeners, who facilitate conceptual understanding of crucial concepts in general and orchestrate the whole classroom mathematical discussion. This new role of lecturers espoused by Davis (1997, p.366) as 'a participant in the exploration' (hermeneutic listening) and is further embraced by Rasmussen as 'generative listening' for designing mathematical teaching. Paying more attention to students' reasoning and conjectures transforms the mode of the classroom into 'more a matter of flexible response to ever-changing circumstances than of unyielding progress toward imposed goals' (Johnson & Larsen, 2012, p.123). This two-way communication between lecturers and students keeps students engaged and motivates learning (Chang, 2011). To make sense of students' responses and their struggles, lecturers may also learn some new mathematics and discover an opportunity to make mathematics classroom interaction productive and mathematically rich (Johnson & Larsen, 2012).

Secondly, Table 11 shows that there were several differences between T-D and S-C teaching strategies in terms of maintaining the process of participation. To create the 'Strategic environment', as one of the perceived qualities of mathematics classroom interaction, both T-D and S-C teaching strategies lecturers agreed that interactions between a lecturer and students and among students were important. However, they had preferred different interactive formats. These are a structured format where students could participate in the traditional chalk-and-talk environment and a dialogue format where there were open discussions and various contributions.

The structured format in a chalk-and-talk environment, which lecturers with T-D teaching strategy preferred could be helpful for students with a Dependent and Participatory learning strategy, but it might cause classroom interaction that is inflexible and inactive. On the other hand, the dialogue format which lecturers with an S-C teaching strategy preferred requires the skills of idea sharing, intellectual explanation or social interactive collaboration, which might be difficult for many students. Moreover, without a lecturer playing a central role, the classroom interaction easily becomes unbalanced without substantive mathematical outcomes.

The process of mathematical interaction is closely related to the creation of a classroom climate that changes different roles and responsibilities for the lecturer and the students. This change should envision the engagement of all students in interaction by monitoring their participation in discussions and by examining the nature and type of diverse students (White, 2003).

Thirdly, to build effective mathematics classroom interaction, affective factors should be considered. Rovai (2001) explains that a sense of trust and interaction and a sense of classroom interaction are mutually interdependent. This means that the members of a classroom interaction, including the lecturer, should have shared goals and values. Among the characteristics of the mathematics classroom interaction, the most frequently mentioned by lecturers and students related to affective factors (attitude, confidence & self-efficacy, see Table 10). Affective factors, especially self-efficacy, is critical for students to utilize cognitive and metacognitive learning strategies more vigorously in mathematics (Mousoulides & Philippou, 2005) and is closely connected to a self-motivation and active attitude (Wang et al., 2017).

This affective influence on students could be one of the reasons why there is little difference among learning strategies when it comes to classroom interaction. There was a slight difference among learning strategies – the importance of what lecturers' question (no. 5 in Table 10). Participatory students were more concerned about what lecturers questioned than Collaborative students (75% over 37.5%). This slight difference did not indicate that the difference between these two learning strategies were directly linked to mathematics classroom interaction. There might have been other influential variables: students' focusing mainly on passing exams, as many lecturers mentioned (no.10 in Table 10) or the correlation between the Participatory and Dependent learning strategy.

Students who took up the mathematical modules might not have been well prepared to practise learning-as-participation in classroom interaction or they might have not been much interested in mathematical knowledge. Regardless of teaching strategies, lecturers agreed that students were interested not in gaining knowledge but in passing exams thus, they were not prepared to study mathematics at university (nos. 9 & 11 in Table 10).

In conclusion, for effective classroom interaction, as the quantitative and qualitative data analysis shows, lecturers are recommended to have more S-C teaching strategies. To lay the platform for learning-as-participation, lecturers' focus should be not only on mathematical content, but also on what students need in class, which would be realistic and authentic mathematical tasks, thought-provoking questions based on students' conceptual understanding and a more dialogue-format classroom environment.

CONCLUSION

Based on the analysis of the quantitative and qualitative data presented in this article, two issues are proposed as being more at the core of lecturers' roles to meet the complex demands for the effective mathematics classroom interaction. These are, firstly, the need for a new perspective for teaching mathematics through classroom interaction, and secondly, as this article shows, according to their teaching strategies lecturers may choose to emphasise mathematical content over students' participation or vice versa in classroom interaction. Clearly, a variety of variables along with teaching strategies influence teachers' decisions about how to facilitate classroom discourse and to create two-way communication. There has been substantive research on how a lecturer's knowledge constrains and enables teaching. This study shows that the relationship between their knowledge and the ways in which lecturers facilitate classroom interaction is also of importance.

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