The relationship between irrational beliefs, socio-affective variables and secondary school learners' achievement in mathematics[1]

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ABSTRACT

Theoretically, the study was based on ten irrational beliefs that Ellis identified as part of his ABC-model. Activating events (A) in a mathematics context can trigger certain beliefs (B) that may have affective consequences (C), which, in turn, can have an influence on achievement in mathematics. The main objective of the research was to investigate how these irrational beliefs and affective consequences relate to one another and to achievement in mathematics. A sample of 306 secondary school learners was selected. The learners completed questionnaires on irrational beliefs and socio-affective variables. Their achievement scores in Mathematics were also obtained. The analysis of the data showed that irrational beliefs correlate negatively with mathematics achievement. Self-concept and teacher-learner relationship had a partially mediating effect on the relationship between irrational beliefs and achievement in mathematics. The direct and indirect effects of irrational beliefs explained as much as 40% of the variance in mathematics achievement. Most teachers of mathematics will at one stage or another be confronted with learners' irrational beliefs, and it is, therefore, recommended that teachers know how such beliefs should be disputed and replaced with rational beliefs as a way of enhancing academic performance in mathematics.

Keywords: irrational beliefs, mathematics achievement, stress, anxiety, motivation, self-concept, teacher-learner relationships, parental involvement

INTRODUCTION

One of the most well-known psychologists that explored the phenomenon of human beliefs was Albert Ellis (Austad, 2009). Ellis was strongly inspired by the ancient Greek philosopher Epictetus, who reasoned that the view people take of events in their lives has far greater implications for their well-being than the events themselves. Ellis labelled two major categories of beliefs, namely (i) irrational and (ii) rational beliefs (Mpofu, 2006). Rational beliefs are characterised by objective information which people apply in order to achieve personally chosen goals (Feldman, 2009). In contrast, irrational beliefs relate to people's own, subjective approach to life as well as to their wishes and demands (Ellis, 2004). Irrational beliefs focus on ideal experiences and, since those cannot always be accomplished, these beliefs cause emotional setbacks and behavioural difficulties (Austad, 2009). The most prominent irrational beliefs identified by Ellis are listed below, with an abridged form in brackets:

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- It is a dire necessity to be accepted and admired by others (*demand acceptance*)
- One should always be competent to be recognised (*demand for competency*)
- Inappropriate behaviour should consequently be punished (*rigid punishment*)
- It is catastrophic when things do not happen as planned (*non-adaptability*)
- Human misery is caused by external factors you cannot control (*external control*)
- One has to be constantly worried about possible dangers in life (*constant worries*)
- Avoid problems instead of facing difficulties (*problem avoidance*)
- Without the help of others, you will never succeed (*dependence mentality*)
- It is impossible to overcome difficulties of the past (*captured by the past*)
- It is catastrophic if problems in your life cannot be solved perfectly (*perfectionism*).

Irrational beliefs has been an active research field, but only a few studies have investigated the possible relationship between irrational beliefs and achievement in mathematics. Tsui and Mazzocco (2007) focused on perfectionism which can be associated with the irrational belief that everything should be done in a correct and flawless way. Perfectionism and anxiety correlated negatively with achievement in mathematics, and both explained a significant proportion of the variance in mathematics achievement. However, the study by Tsui and Mazzocco did take other irrational beliefs into consideration. Boehnke (2008) found that the irrational belief that one should always be accepted and respected by others can influence learners' performance in mathematics. Again, no other irrational beliefs were considered in the present study.

In his ABC-model, Ellis explains the implications of beliefs for a person's psychological functioning (Khaledian et al., 2013). The 'A' represents an activating stimulus within the person or within the person's environment (Zionts & Zionts, 1997). Writing a test in mathematics would be a typical example of an activating event. 'B' symbolises a person's beliefs which are applied to interpret the activating event. These beliefs can be rational or irrational, and, consequently, people differ in the ways they interpret a particular event. If a learner in the mentioned example fails the test, but displays rational beliefs, failure might be viewed as a temporary setback. That will be in contrast with a learner who irrationally believes that failure in mathematics will indefinitely occur. 'C' represents the consequence of a person's beliefs which manifests in certain emotions and behaviour. The learner who considers failure in mathematics as inevitable will most probably be unmotivated to study for a test, whereas the learner who demonstrates rational beliefs might be motivated to overcome the situation. If a person demonstrates rational beliefs, the discomfort or disappointment that originates from the activating event is treated in a balanced and sensible way. If the beliefs are irrational, the emotions are usually incongruous and self-defeating, accompanied by maladaptive behaviour (Rosner, 2011).

Theoretically, the ABC-model of Ellis suggests that beliefs, both rational and irrational, may relate to socio-affective variables. Socio-affective variables happen to be an integral part of the teaching and learning process in mathematics. Earlier research, for example, Mellet (1986), has shown that socio-affective variables, such as self-concept, motivation, teacher-learner relationship and anxiety, are generally associated with mathematics achievement. It is, therefore, possible that learners' beliefs in a mathematics context may relate to the above-mentioned socio-affective variables which will, in turn, relate to mathematics achievement.

SOCIO-AFFECTIVE VARIABLES, IRRATIONAL BELIEFS AND MATHEMATICS ACHIEVEMENT

In general, studies that analysed the relationship between the self-concept of learners and their achievement in mathematics obtained a positive correlation between the two variables (see e.g., Wang, Osterlind & Bergin, 2012). Research results also suggest a relationship between irrational beliefs and the self-concept of learners. Arai (2001) found that the impact of irrational beliefs intensifies if a discrepancy exists between learners' actual and ideal self-concepts, while a study by Ferla, Valckle and Cai (2009) revealed that learners' beliefs regarding their academic self-efficacy relate to their academic self-concept. The results of Tella (2007) showed that learners with different levels of motivation differed with regard to their academic performance, and a study by Md. Yunus and Ali (2009) identified significant positive correlations between motivation, effort, self-efficiency and academic achievement. It is possible that some of the irrational beliefs previously mentioned may also relate to motivation, for example, the irrational belief regarding *problem avoidance*. Learners who avoid doing challenging academic tasks and are reluctant to act responsibly in an academic context are usually characterised by low achievement motivation (Eccles & Wigfield, 2002). With regard to stress, Byrne, Davenport and Mazanov (2007) presented a list of possible stressors to school-going adolescents and conducted a factor analysis with the data obtained. Four of the 10 factors that were identified could be associated with academic work. According to Sulaiman, Hassan, Sapian and Abdullah (2009), the climate in a school, the education level of parents and their parenting styles are possible variables that might have an effect on the stress that adolescents experience. A number of studies investigated the relationship between stress and academic achievement, though not necessarily in mathematics. Kauts and Sharma (2009) showed that learners who experience stress at a low level outperformed those who experience stress at a high level. Irrational beliefs, for example, that 'one has to be constantly worried', might create stress in a learning context, which will then have a negative influence on academic achievement. This is confirmed in a study by Beilock (2008), who indicated that self-sustaining worries relate to high-stress situations which increase the failure rate in mathematics. Thompson and Henderson (2007) drew attention to Ellis's statement that anxiety cannot be considered irrational but rather that anxiety emanates from irrational thinking. Anxiety should, therefore, be seen as an emotion that is unnecessary and one that can be prevented. Mathematics learners who, for example, irrationally believe that 'something that once affected them in the past will indefinitely affect them in future' may experience anxiety. In general, negative correlations have been obtained between mathematics anxiety and achievement in mathematics (Sherman & Wither, 2003). Zakaria and Nordin (2008) obtained a negative correlation as high as −0.72.

Teacher-learner relationships and parental involvement are two social relationships that may relate to achievement in mathematics and irrational beliefs regarding mathematics. According to Orton (2004), teachers are an important factor in the entire teaching and learning process. In most instances, adolescents want to receive approval from their teachers. Some adolescents might irrationally demand such approval, and, if teachers criticise their schoolwork or behaviour in class, they may experience it as catastrophic (Thompson & Henderson, 2007). Another irrational belief of learners, which involves teachers, is the belief that external factors cause misery and unhappiness since one cannot control these. Learners with such beliefs blame curriculum implementers (who happen to be teachers) for the difficulty level of mathematics content, which is subsequently interpreted as a form of suffering enforced upon them (Osler, 2010). Besides teacher-learner relationships, Gonzalez and Wolters (2006) found that parental involvement also has an effect on academic achievement. Adolescents whose parents participate in school functions and are involved in their learning activities at home adopt achievement goals, not only to outperform their peers, but also the standards set by their parents. If learners are aware that their parents maintain regular contact with the school, they often surpass their parents' expectations in order to ensure a positive report from the school's side. However, not all parental involvement is supportive in nature. Levpuscek and Zupancic (2009) showed that parents can have a negative influence if they put extensive pressure on their children to perform well in mathematics or when they distrust the efficiency of the study methods of their children.

RESEARCH OBJECTIVES

Since multiple types of variables relate to achievement in mathematics, a noticeable gap exists in the sense that previous research did not consider irrational beliefs as possible variables in this regard. To obtain information on learners' perceptions of mathematics as a subject and how they perceive themselves as students of mathematics, the relationship between irrational beliefs and other socio-affective variables which relate to achievement in mathematics should be analysed jointly. This is what the current study attempted to establish, guided by the following three objectives. The first objective was to determine if any of the well-known irrational beliefs (Ellis, 2004) relate to achievement in mathematics. The second objective was to establish if socio-affective variables, such as self-concept, motivation, teacher-learner relationships, parental involvement, anxiety and stress, relate to achievement in mathematics. The third objective was to determine whether socio-affective variables have a mediating effect on the relationship between irrational beliefs and achievement in mathematics.

RESEARCH METHODOLOGY

Sampling procedure and ethical considerations

The population comprised form 3 and form 4 learners (Ordinary-level learners) in the Masvingo Province of Zimbabwe who took mathematics as a school subject. Before the study commenced, ethical clearance was obtained from the University, under whose supervision the study was conducted. Permission to conduct the research was then granted by the Education Department in Zimbabwe. A list of schools within the Masvingo Province was provided by the Education Department. From this list, using a table of random numbers, five schools were randomly selected. Letters were sent to the School Governing Bodies of the selected schools to obtain permission to conduct the study at the respective schools. Three secondary schools agreed to participate in the investigation. Letters were then sent to the parents of learners in forms 3 and 4 of the three schools that volunteered to participate. In the letter that parents received, the aim of the research project as well as the voluntary nature of participation in the study were explained. Ethical principles, such as privacy, anonymity and confidentiality, were assured. In total, the parents of 306 learners granted their children permission to participate in this study. The number of respondents from school 1 was 72, 177 from school 2, and 57 from school 3. The sample consisted of 124 male and 182 female learners. The respondents had a mean age of 15.79 years with a standard deviation of 0.814.

Measuring instruments

Irrational beliefs

Jones (1968) used the 10 irrational beliefs, originally identified by Ellis, to develop the Irrational Belief Test (IBT). Each irrational belief is measured by 10 items. There are 100 items in total. This test was used in the current investigation, but since the study focused on irrational beliefs in mathematics, the wording of the items of the original test was adapted. The items were responded to by using a six-point Likert scale ranging from 6 (definitely the case) to 1 (not the case at all). Due to item changes, an item analysis had to be done. For this purpose, an item-total correlation was calculated. An item was omitted if the item showed a low or a negative correlation with the total and if the omission of the item increased the Cronbach's alpha reliability coefficient. A reliability coefficient of 0.91 was obtained for the final test with 88 items.

Socio-affective variables

Socio-affective factors were measured using a questionnaire initially developed by Bester (2003) to measure socio-affective variables in a drama context. Since the current investigation focused on learning activities in a mathematics context, the wording of the items was adapted. Learners had to respond to 122 items using a six-point Likert scale similar to that of the IBT. The items measured socio-affective variables in a mathematics context, such as self-concept, motivation, teacher-learner relationships, parental involvement, anxiety and stress. The changing of the original items required an item analysis as well as a recalculation of the reliability coefficient of each construct. As in the case of the irrational belief test, an item was omitted if it correlated low or negatively with the total and if the omission of an item resulted in a significant increase in the Cronbach's alpha reliability coefficient. The reliability coefficient for each variable was as follows: self-concept 0.83 (18 items); motivation 0.79 (18 items); anxiety 0.85 (20 items); stress 0.79 (18 items); teacher-learner relationship 0.84 (20 items); and parental involvement 0.76 (10 items).

Mathematics achievement

The mathematics achievement scores of the respondents during the written examination in June were obtained. The mathematics syllabus is structured in such a way that learners write two examination papers. The duration of each of the two examination papers is 2 hours 30 minutes, and both papers total 100 marks. The content of one of the examination papers focuses on algebra (e.g., factorisation and equations). In the other examination paper, the focus is on geometry and elementary trigonometry. The examinations were conducted under formal examination conditions. The final score for each candidate is obtained by averaging the scores of the two examination papers. The average mathematics achievement for the sample was 50.25 per cent with a standard deviation of 16.61.

Procedure of the investigation

The questionnaires were distributed to the respondents, who were guided through the instructions that accompanied the questionnaire thereafter. The Likert scale and how it should be interpreted were explained to the respondents. The respondents were required to respond to each item by expressing their personal experience and how they feel about the circumstances related to a particular item. The completion of the questionnaires took place during the morning, and most of the learners completed it in 55 to 65 minutes.

RESULTS

The first objective was to determine if any of the well-known irrational beliefs relate to achievement in mathematics. Correlation coefficients between the measured irrational beliefs and mathematics achievement were calculated. These coefficients appear in Table 1.

Table 1:
Correlation coefficients between irrational beliefs and mathematics achievement

Irrational Belief	Mathematics Achievement
Demand for acceptance	-0.15
Demand for competency	-0.01*
Rigid punishment	-0.09
Non-adaptability	-0.29
External control	-0.29
Constant worries	-0.29
Problem avoidance	-0.27
Dependence mentality	-0.22
Captured by the past	-0.26
Perfectionism	-0.04*

N= 306.
*$p>0.05$. For all the other correlation coefficients, $p<0.01$

Significant negative correlation coefficients were obtained between eight of the irrational beliefs and mathematics achievement. Three irrational beliefs, namely (i) non-adaptability, (ii) external control and (iii) constant worries, showed the highest negative correlations. In all three instances, the correlation coefficients were -0.29. To determine the importance of a particular irrational belief, the underlying relationships among the irrational beliefs had to be taken into account. For this purpose, a forward, stepwise regression analysis was performed. In a regression analysis, the independent variable that shows the highest correlation with the dependent variable enters the model first. After the first variable is taken up in the model, the order of the remaining variables depends on their correlation with the dependent variable as well as their correlation with the variable(s) already in the model. The independent variable that explains the largest significant proportion of the remaining unexplained variance of the dependent variable enters next. In the current analysis, all the irrational beliefs were used as independent variables and mathematics achievement as the dependent variable. The results appear in Table 2.

Table 2:
Proportion of the variance in mathematics achievement explained by irrational beliefs

Dependent variable	Independent variables	R^2	*F*	df	*P*
Mathematics achievement	1. External control	0.08	28.07	(1,304)	p<0.01
	2. Problem avoidance	0.11	7.42	(2,303)	p<0.01
	3. Perfectionism	0.12	4.72	(3,302)	p<0.05
	4. Constant worries	0.13	4.81	(4,301)	p<0.05

The first irrational belief to enter the model was external control, explaining 8% (R^2=0.08) of variance in mathematics achievement. This proportion was significant: F(1,304) = 28.07; p<0,01. The second irrational belief to enter was problem avoidance which explained an additional 3% of the variance in mathematics achievement. This additional proportion was significant: F(2,303) = 7.42; p<0.01. The next irrational belief to enter was perfectionism, which explained a further 1% of the variance in mathematics achievement with F(3,302) = 4.72; p<0.05. Constant worries entered the model last explaining 1% more of the variance already explained by the previous variables; F(4,301) = 4.81; p<0.05. In total, external control, problem avoidance, perfectionism, and constant worries explained 13% of the variance in mathematics achievement.

The second objective was to determine how socio-affective variables relate to achievement in mathematics. For this purpose, correlation coefficients between self-concept, motivation, anxiety, stress, teacher-learner relationships, parental involvement and mathematics achievement were calculated. These coefficients appear in Table 3.

Table 3:
Correlation coefficients between socio-affective variables and mathematics achievement

Socio-affective Variables	Self-concept	Motivation	Anxiety	Stress	Teacher-learner Relationship	Parental Involvement
Mathematics Achievement	0.40	0.34	-0.29	-0.26	0.34	0.13

N= 306. For all the correlation coefficients *p*<0.01

The strongest correlation was between learners' self-concept and achievement in mathematics (r = 0.4). Motivation and teacher-learner relationships also correlated positively with mathematics achievement. In both instances, the correlation coefficient was 0.34. Anxiety and stress correlated negatively with mathematics achievement (r = -0.29 and r = -0.26 respectively), while parental involvement showed a low positive correlation (r = 0.13). To identify the variables that explain the largest proportion of the variance in mathematics achievement (controlling for the relationships among the socio-affective variables), a forward, stepwise regression analysis was performed using all the socio-affective variables as independent variables and mathematics achievement as the dependent variable. The results appear in Table 4.

Table 4:
Proportion of the variance in mathematics achievement explained by self-concept and teacher-learner relationship

Dependent variable	Independent variables	R^2	F	df	P
Mathematics achievement	1. Self-concept	0.16	58.04	(1,304)	p<0.01
	2. Teacher-learner relationship	0.18	5.83	(2,303)	p<0.05

The first variable to enter the model was self-concept, explaining 16% of variance in mathematics achievement. This proportion was significant: F(1,304) = 58.04; p<0,01. The second variable was teacher-learner relationship, explaining an additional 2% of the variance in mathematics achievement; F(2,303) = 5.83; p<0.05. No other variable made a significant contribution to the variance in mathematics achievement. In total, self-concept and teacher-learner relationship explained 18% of the variance in mathematics achievement.

According to Ellis's ABC-model, irrational beliefs in a mathematics context (B) will have affective consequences (C), and these affective consequences may relate to achievement in mathematics. Irrational beliefs can thus directly or indirectly relate to achievement in mathematics. If indirectly related to achievement in mathematics, the two prominent socio-affective variables (self-concept and teacher-learner relationship) will mediate the relationship between irrational beliefs and achievement in mathematics. The third objective of the research project was to investigate such a possibility.

Two regression analyses were performed. In the first instance, mathematics achievement was used as the dependent variable and all the irrational beliefs as well as the two socio-affective variables (self-concept and teacher-learner relationship) as independent variables. A R_a^2-value = 0.1962; p<0.01 was obtained. In the second analysis, an R_b^2-value = 0.1340; p<001 was obtained with only the irrational beliefs as independent variables. R_a^2 - R_b^2 = 0.0311 was significant with F(2,293) = 11.35; p<0.01. Since the inclusion of self-concept and teacher-learner relationship in the first analysis contributed to a significantly larger proportion of the variance in Mathematics achievement, the partial mediating effect of self-concept and teacher-learner relationship is supported.

To analyse the mediating role of learners' self-concept and their relationship with teachers, two regression analyses were performed. In the first analysis, self-concept was used as the dependent variable and the irrational beliefs as independent variables. The results appear in section A of Table 5. Problem avoidance was the first irrational belief to enter the model, explaining 31% of variance in the self-concept of learners [F(1,304) = 134.69; p<0,01], followed by constant worries, which explained 7% more of the variance [F(2,303) = 35.18; p<0.01]. Rigid punishment entered next, explaining an additional 1% of the variance with F(3,302) = 6.42; p<0.05. The last irrational belief to enter was non-adaptability, which also explained an additional 1% of the variance; F(4,301) = 4.79; p<0.05. In total, four irrational beliefs, that is (i) problem avoidance, (ii) constant worries, (iii) rigid punishment and (iv) non-adaptability, explained 40% of the variance in the self-concept of learners. In the second regression analysis, teacher-learner relationship was used as the dependent variable and the irrational beliefs as independent variables. The results appear in section B of Table 5.

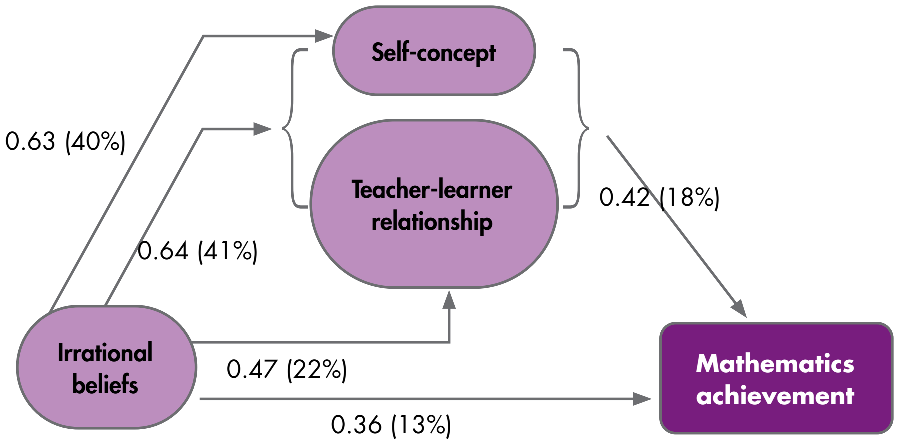
Table 5:
Proportion of the variance that irrational beliefs explained for self-concept and teacher-learner relationship

Dependent variable	Independent variables	R^2	F	df	P
A. Self-concept	1. Problem avoidance	0.31	134.69	(1,304)	p<0.01
	2. Constant worries	0.38	35.18	(2,303)	p<0.01
	3. Rigid punishment	0.39	6.42	(3,302)	p<0.05
	4. Non-adaptability	0.40	4.79	(4,301)	p<0.05
B. Teacher-learner relationship	1. Non-adaptability	0.16	57.09	(1,304)	p<0.01
	2. Problem avoidance	0.19	10.67	(2,303)	p<0.01
	3. Demand for acceptance	0.21	8.03	(3,302)	p<0.01
	4. Dependence mentality	0.22	4.15	(4,301)	p<0.05

The first irrational belief to enter the model was non-adaptability, explaining 16% of variance in teacher-learner relationship with F(1,304) = 57.09; p<0,01. The second irrational belief to enter was problem avoidance, explaining an additional 3% of the variance [F(2,303) = 10.67; p<0.01]. The next irrational belief to enter was demand for acceptance, which explained an additional 2% of the variance with F(3,302) = 8.03; p<0.01. Dependence mentality entered the model last, explaining an additional 1% of the variance already explained by the previous variables; F(4,301) = 4.15; p<0.05. In total, (i) non-adaptability, (ii) problem avoidance, (iii) demand for acceptance and (iv) dependence mentality explained 22% of the variance in teacher-learner relationship.

Due to the intervening role of self-concept and teacher-learner relationship, the effect of irrational beliefs on mathematics achievement can be either direct or indirect as portrayed in Figure 1. The different correlation coefficients are also presented in Figure 1, and, in each instance, the proportion of the variance explained appears in brackets. The direct effect is represented by the Multiple correlation coefficient between the irrational beliefs and mathematics achievement which is R=0.36 (R^2=0.13; Table 2). To determine the size of indirect effect, two correlation coefficients were calculated. In the first instance, a Canonical correlation was calculated [R_c=0.64, F(14,594)=16.53; p<0.01] between the irrational beliefs and the two prominent socio-affective variables – that is, self-concept and teacher-learner relationship. In the second instance, the Multiple correlation coefficient between the two socio-affective variables and mathematics achievement was used; R=0.42 (R^2=0.18; Table 4).

Figure 1:
The direct and indirect effects of irrational beliefs on mathematics achievement (the proportion of the variance is provided in brackets)



The size of the indirect effects can be obtained by multiplying the Canonical correlation coefficient between the irrational beliefs and the two socio-affective variables with the Multiple correlation coefficient between the socio-affective variables and mathematics achievement (Hair et al., 2006). That will be 0.64 x 0.42 = 0.27. The size of the direct effect is 0.36 (the Multiple correlation coefficient between the irrational beliefs and mathematics achievement). The total effect of irrational beliefs on mathematics achievement is the sum of the direct effect and the indirect effects, which is 0.36 + 0.27 = 0.63. It seems, therefore, that 40% (0.63^2) of the variance in mathematics achievement can be explained by the direct and indirect effects of irrational beliefs.

DISCUSSION OF THE RESULTS

To reduce the complexity regarding the number of variables used as independent variables in the regression analyses, the discussion will mainly focus on those variables that contributed more than one percent to the variance of a particular dependent variable.

Eight of the 10 irrational beliefs measured in this investigation showed significant negative correlation coefficients with learners' achievement in mathematics. This suggests that the more learners apply irrational thinking in a mathematics context, the lower their achievement in mathematics tends to be. The results support the opinion that irrational beliefs prevent people from obtaining their goals in life (Bermejo-Toro & Prieto-Ursua, 2006) and foster a sense of hopelessness (Ullusoy & Duy, 2013). Based on the results of the regression analysis, two notable irrational beliefs that relate to mathematics achievement emerged, namely (i) external control and (ii) problem avoidance. The two irrational beliefs explained 11% of the variance in mathematics achievement. If learners believe that misery in mathematics is caused by external factors they cannot control, there will be little effort on their part to improve their performance. The opposite is true of learners with an internal locus of control. Bishara and Kaplan (2018) indicated that a high level of internal locus of control can be associated with increased use of metacognitive knowledge in mathematics. They also found that mathematics achievement was enhanced by both variables – that is, the internal locus of control and the use of metacognitive knowledge. A second notable irrational belief that relates to achievement in mathematics was problem avoidance. Some learners believe it is better to avoid problems related to mathematics than to face difficulties and responsibilities associated with the subject. Such an approach may result in frequent postponement of home assignments, which can have a detrimental effect on mathematics achievement (Asikhia, 2010). Due to the hierarchical structure of mathematical content, certain content must be mastered before new content can be introduced. Learners who avoid their responsibilities or postpone them will not be able to develop a proper basis of existing knowledge and will, therefore, find it difficult, if not impossible, to understand new content. Their learning experiences in a mathematics context will also be affected in a negative way. Skaalvik (2018) found that an avoidance perspective predicted lower grades in mathematics and higher levels of anxiety.

The finding of Skaalvik (2018) corresponds with Ellis's ABC-model. Activating events (A) in a mathematics context will result in certain beliefs (B) which may have affective consequences (C). These affective consequences, in turn, can relate to achievement in mathematics. In this investigation, the most important variables in this regard were the self-concept of learners and their relationship with teachers. The correlation coefficient between the self-concept of learners and mathematics achievement was 0.4, which corresponds with the 0.42 that Perry, Catapano and Ramon (2016) found between the self-concept of learners and their achievement in mathematics. Self-concept development has a strong social component since the feedback that learners receive from significant others is often used as a criterion to judge their own behaviour. For learners, 'significant others' may include teachers, parents and friends; therefore, researchers have found that teachers can have an effect on a learner's academic self-concept (Perry, Catapano & Ramon, 2016). This might explain the results of the regression analysis in the current investigation. The self-concept of learners on its own explained 16% of the variance in mathematics achievement and, when the relationship with teachers entered the model, an additional 2% could be explained. The correlation coefficient between teacher-learner relationship and mathematics achievement in the current investigation was 0.34. It corresponds with the results of Mikk, Krips, Säälik and Kalk (2016), who investigated the relationship between teacher-learner relationship and mathematics achievement in different countries. They found significant positive correlation coefficients that varied between 0.35 (Estonia) and 0.11 (Brazil) with a median correlation coefficient of 0.20. Briede (2016) found that learners relate positively to teachers who follow an advisor role and encourage cooperation, while Mikk et al. (2016) concluded that teachers who are warm and supportive create a secure environment where learners feel free to actively participate in learning events.

Irrational beliefs relate directly to mathematics achievement; they also relate to mathematics achievement indirectly due to the relationship between irrational beliefs and socio-affective variables (the self-concept of learners and their relationship with teachers). With regard to the self-concept of learners, the results of the regression analysis indicated that the irrational belief problem avoidance explained 31% of its variance. According to Skaalvink (2002), learners' ego orientations activate them to compare their abilities and achievement to other learners, and they are preoccupied with how other learners perceive them. Such an orientation can manifest in two ways. It can either be self-enhancing or self-defeating. Self-enhancing-orientated learners take part in certain activities to demonstrate their abilities, whereas self-defeating-orientated learners will rather avoid participation because of a fear for being negatively evaluated. Skaalvink (2002) found that a self-defeating ego orientation in a mathematics context correlated negatively with the self-concept of learners. It can, therefore, be assumed that an irrational belief which reinforces avoidance, instead of facing difficulties and responsibilities (problem avoidance), will most probably strengthen a self-defeating ego orientation, resulting in a lower self-concept. In the current investigation, constant worries explained 7% more of the variance in the self-concept of learners not already explained by problem avoidance. 'Worries' is characterised by concerns over evaluation and expectations and, consequently, relates to negative thoughts and self-doubts (Eysenck et al., 2007). In a mathematics context, 'constant worries' refers to the belief that it is expected of one to brood, on a continuous basis, over the demands and requirements that accompany the study of mathematics. Lauermann et al. (2017) found that learners' worries were the strongest when they perceived mathematics as a valuable subject but rated their own abilities and expected success relatively low. As in the case with the self-concept of learners, irrational beliefs also relate indirectly to mathematics achievement due to the relationship between learners' irrational beliefs and their relationship with teachers. From the results of the regression analysis, 16% of the variance in learners' relationship with teachers can be explained by the irrational belief that it is catastrophic when things do not happen as planned (non-adaptability). For some learners, it is catastrophic if mathematics content cannot easily be understood or if mistakes are made when homework is done, since it jeopardises their relationship with teachers. Some teachers of mathematics reinforce such an irrational belief because of their teaching approach. Maulana, Opdenakker, den Brok and Bosker (2012) found that most mathematics teachers follow a directive teaching approach. The classrooms of these teachers are task-oriented, well-structured and efficiently organised. They complete lessons on time, dominate class discussion and are very demanding. These teachers are not very close to students, occasionally friendly and not very cooperative. Learners who find themselves in such a milieu may believe that it is disastrous if mathematics cannot be done faultlessly. To avoid mistakes, some learners may avoid studying some content or doing homework that appears to be difficult. Therefore, it is not surprising that the irrational belief problem avoidance was the next variable to enter the regression model accounting for an additional 3% of the variance in learners' relationship with teachers. Over and above non-adaptability and problem avoidance, the irrational belief that it is a dire necessity to be accepted and admired by others (demand acceptance) explained 2% more of the variance in teacher-learner relationship. Learners who experience challenges in their mathematics studies will occasionally also experience tension in their relationship with teachers. Martin and Collie (2019) found that lower engagement of learners related to a negative teacher-learner relationship in a mathematics context. Experiencing a negative relationship with the mathematics teacher might intensify the necessity of learners to be accepted. Such a desire for acceptance will only be fulfilled if learners act more responsibly with regard to their studies and if teachers become more socially and emotionally supportive in the classroom. According to Prewett, Bergin and Huang (2019), concrete prosocial behaviour of mathematics teachers, such as scaffolding instruction and a sensitivity to learners' needs, are indicators to learners that teachers 'like' them.

Most learners sustain irrational beliefs over time through self-talk and by indoctrinating themselves (Prout & Brown, 2007). In a mathematics context, the teacher is primarily the person that can overturn the situation. Teachers should dispute learners' irrational beliefs on a continuous basis to prevent them from indoctrinating themselves. The results of this investigation provide guidelines with regard to specific irrational beliefs that can be expected in a mathematics context. Teachers should prepare themselves to challenge these beliefs and to create a classroom environment which prevents the onset of such beliefs. For example, to address the irrational belief that problems should be avoided instead of facing difficulties, the teacher should convince learners to do homework assignments despite the difficulty level of the content and the possibility of mistakes being made. However, if homework is done, learners should be supported and not criticised for the mistakes they might have made. The same would apply to the irrational belief that mathematics achievement is a result of external factors one cannot control. Teachers should convince learners that hard work and not luck is the key to success in mathematics. At the same time, reasonable tests and examination papers should be set. If a well-prepared learner struggles because of an extremely difficult assessment, the disputing arguments of the teacher will be doubted. As in the two examples above, appropriate counterarguments for each of the irrational beliefs identified in this investigation should be formulated. The more prepared a teacher is to deal with these irrational beliefs, the more successfully they can be disputed.

There are limitations to this study that open possibilities for future research. Firstly, the current study did not focus primarily on age; the chosen sample only involved form 3 and 4 learners. However, learners from all levels of the secondary school phase can in future be approached to establish whether age differences contribute to irrational thoughts about mathematics. For the same reason, primary school learners can also be considered to verify the onset of irrational beliefs. Secondly, the gender phenomenon was not dealt with in this investigation. However, there are indications that such differences may exist, which necessitates future research. For example, external control, which was identified as an irrational belief directly related to mathematics achievement, may differ between boys and girls. Many people wrongly believe that mathematics is a subject that falls in a male domain (Haylock & Thangata, 2007). When girls are compelled by their parents and teachers to study mathematics, they may, in comparison with boys, be more inclined to believe that their misery is externally caused by factors beyond their control. Thirdly, in this investigation, ten irrational beliefs were measured and some of them were identified as beliefs that directly or indirectly relate to mathematics achievement. The 10 beliefs used in the investigation were the well-known irrational beliefs identified by Ellis, but they may not be the only ones that operate in a mathematics context. The possibility to identify and investigate other irrational beliefs that exclusively manifest in a mathematics context should be considered.

CONCLUSION

Irrational beliefs affected mathematics achievement directly and indirectly. With regard to the direct effect, negative correlations were obtained between irrational beliefs and achievement in mathematics. With regard to the indirect effect, two intervening variables were identified, namely (i) the self-concept of learners and (ii) their relationships with teachers. Irrational beliefs relate to the self-concept of learners and to teacher-learner relationship which, in turn, relate to mathematics achievement. In this regard, the results offer support for Ellis's ABC-model. The direct and indirect effects of irrational beliefs accounted for 40% of the variance in mathematics achievement, which is a substantial proportion. It is reasonable to assume that every mathematics teacher will at some stage have to deal with irrational beliefs of learners. Therefore, teachers should take note of typical irrational beliefs that learners hold in a mathematics context and proactively think of suitable means to dispute such beliefs. Disputing irrational beliefs and replacing them with empowering rational beliefs can be very supportive in a learning context (Austad, 2009). The irrational belief that one should rather avoid problems instead of facing difficulties and responsibilities not only showed a direct effect on mathematics achievement, but the threat of such a belief also had implications for the self-concept of mathematics learners and for teacher-learner relationship. It should be considered a prominent irrational belief that manifests in a mathematics context, and teachers should be able to deal with learner beliefs in this regard.

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