An Optimal Rural Community PV Microgrid Design Using Mixed Integer Linear Programming and DBSCAN Approach

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***Abstract*—The deployment of microgrids has been identified as one of the fastest ways of bringing electricity to the large number of people that are currently without electricity access. In developing countries in Africa, most of these people live in rural locations that are not served by the main grid making it necessary to establish community microgrids. These microgrids should be optimally sized so as to meet the electrical needs of the communities cost effectively. This work presents an efficient and robust sizing approach for off-grid PV microgrid systems that has been named as the ComµGrid Sizing Approach in this research. This approach utilizes "Mixed Integer Linear Programming (MILP)" to optimally size the PV microgrid. The ComµGrid optimization algorithm uses hourly load variation, hourly solar irradiance values and hourly ambient temperature to optimally size the system. This approach also uses the "Density Based Spatial Clustering of Applications with Noise (DBSCAN)" algorithm to aggregate load and meteorological data. MATLAB software is used to execute the optimization algorithm. The results show that it is possible to achieve accuracy and a faster convergence to the solution with the proposed approach than that of the iterative method.**

***Index Terms*—Microgrid, Mixed Integer Liner Programming, Reliability, Rural Electrification, Solar Photovoltaics**

I. INTRODUCTION

ACCESS to electricity is one of the major factors that contribute to economic growth of communities and countries. With increased level of electrification, people's standards of living are improved and numerous opportunities open up due to the availability of electric power. Developing countries in Africa are faced with the challenge of having to electrify remote rural areas where most of the people live. Extending the main grid to these areas cannot be easily achieved because of the technical and financial difficulties associated with such projects. Microgrids offer a cost effective option for electrifying these remote areas due to their ability to function independently from the main grid and can also be designed to interconnect with the main grid should there be an opportunity for extension of the grid to the remote community. This has led to what are known as community microgrids. According to [1], a microgrid can be defined as a "localized grouping of electricity generation, energy storage, energy control and conversion, energy monitoring and management, and load management tools, capable of operating while connected to the traditional main grid or function independently". Microgrids offer the opportunity to utilize renewable energy resources such as solar photovoltaics (PV), wind, hydro and bio gas reducing dependency on fossil fuels that are harmful to the environment and human health.

Microgrid design consists of several aspects such as generation modelling, load modelling, storage, local network, sizing of the components and determination of the control strategy. One of the critical steps in the design process of PV microgrids is the determination of the sizes of system components. In this research, an efficient and improved approach to PV system sizing is presented. The limitations of the current sizing approaches are discussed in section II of this paper. It should be noted that when sizing PV systems, the main aim of the process should be to obtain a realistic possible optimum combination of the system components. The optimum solution is one that satisfies the load at a given level of reliability while minimizing capital and operational costs [2].

II. THE CURRENT TYPICAL PV SYSTEM SIZING APPROACH

The current typical approach to sizing of PV systems provided by PV manuals such as [3] involves the use of sizing sheets to determine the number and specifications of the components required [4][5][6][7][8][9][10]. The sizing procedure is based on average values of daily load and peak sun hours for the month with least solar irradiance. The assumption is that if the PV system is designed to meet the load demand at this radiation, then it will be sufficient to meet load demand throughout the year.

The battery capacity is determined using (1). The acronyms used in the equations are defined in the Appendix.

$$\mathrm{Bat_{cap}} = \frac{\mathrm{Z} * \mathrm{D_{aut}}}{\mathrm{DOD}} \tag{1}$$

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The number of PV modules is determined by (2), (3) and (4).

$$\mathrm{PV_{par}} = \frac{\mathrm{Z}}{\eta_{\mathrm{bat}} * \mathrm{PSH} * \mathrm{I_{mp}}} \tag{2}$$

$$\mathrm{PV_{ser}} = \frac{\mathrm{V_{dc}}}{\mathrm{V_{mp}}} \tag{3}$$

$$\mathrm{PV} = \mathrm{PV_{pal}} * \mathrm{PV_{ser}} \tag{4}$$

The size of the inverter, controller and the system wiring are determined as defined in [3] [11][12].

A. Merits of the Current Typical PV System Sizing Approach

The above procedure is explained in [3] which presents simple equations that can be used to size solar PV systems. The equations can be scripted in available tools such as Microsoft Excel to further ease computation and replication of the procedure to various locations [13] [6].

B. Shortcomings of the Current Approach to Sizing PV Systems

The current typical PV system sizing procedure does not consider the variation in the load demand and irradiance with time throughout the year. If the load demand consists of consecutive days where the daily load is higher than the average daily load, the battery capacity designed using this simple procedure may be insufficient to meet the desired days of autonomy and therefore the system is undersized. However, if the days of high load demand are relatively spread over time, the simple sizing procedure may oversize the system since the same system reliability could be achieved by designing with a lower daily load [14] [15] [11] [16].

Numerical methods are among the methods proposed in literature to improve the accuracy of PV system sizing . These methods evaluate the load, the PV output energy and battery state of charge for each hour throughout the year. The battery charges when the PV output energy is greater than the load demand current and discharges when the PV output is less than the load demand. If the output from the PV and battery cannot meet the load demand, there is a deficit [11]. A Loss of Load probability (LLP) is used as the measure of the system's reliability. The LLP is defined as the ratio of the total deficit energy to the total load demand over the period of consideration, typically one year. With a predefined LLP based on the user's satisfaction, an optimal configuration of the PV and battery is determined.

Numerical methods yield more reliable and optimum systems. The shortcoming with the numerical method discussed above is that the process of iterating through hourly data leading to long execution times and may sometimes lead to inability to converge to an optimal solution [16]. One way of overcoming this challenge is to reduce the time steps of the iteration without affecting the outliers in the hourly load demand and PV output energy.

This research presents the ComμGrid Sizing approach as a new method for PV system sizing based on the Density Based Spatial Clustering of Applications with Noise (DBSCAN) algorithm and Mixed Integer Linear Programming (MILP) algorithm to determine the sizing of the solar PV system components that will give a minimum Annualized Total Life Cycle Cost (ATLCC) for a predefined desired LLP. The ComμGrid Sizing approach provides a more reliable sizing procedure than the basic sizing method since it considers the variation in the load demand and irradiance throughout the year instead of average values of load and irradiance. It also provides improvement to the numerical method discussed above by reducing the time steps of the iteration. The next sections of the paper will describe DBSCAN clustering algorithm, the MILP algorithm and the optimization process of the proposed sizing approach.

III. The Proposed ComμGrid Sizing Approach

In this research a solar PV community microgrid (ComμGrid) whose components are indicated in the block diagram shown in Fig. 1 is presented and optimally sized.

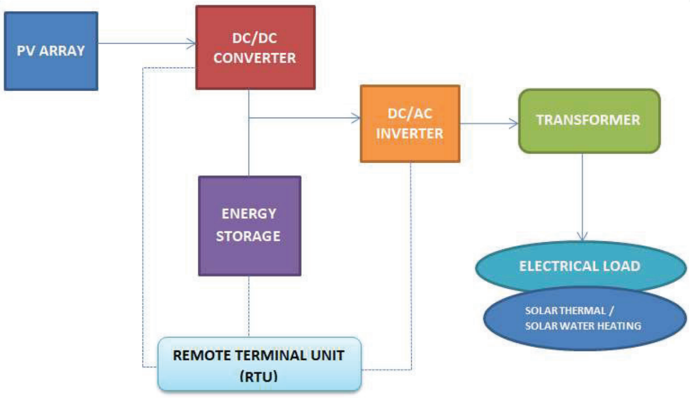


Fig. 1. Block Diagram for the Proposed Community Microgrid (ComμGrid)

The steps involved in the sizing of the microgrid components using the proposed approach are described below.

Step1: Input the load and irradiance data

In this step, the hourly load current $\mathrm{I_L(t)}$ is determined based on the community's appliance usage and ratings. The specifications of the PV and battery are defined. The hourly irradiance, $\mathrm{S(t)}$, and temperature,$\mathrm{T(t)}$, of the community's location are also defined.

Step2: Calculate the hourly PV output current

The hourly PV output current $\mathrm{I_{pv}(t)}$ is calculated using (5).

$$\mathrm{I_{pv}(t)} = \frac{\mathrm{S(t)}}{1000} + \mathrm{Alpha}(\mathrm{T(t)} - 25) \tag{5}$$

Step 3: Consolidate the Load and Irradiance for consecutive time steps

This step involves consolidation of the load and PV outputs into fewer time steps. Subsection A presents the detailed description of this consolidation process.

Step 4: Determine the range of number of PV modules ($\mathrm{PV_{min}}, \mathrm{PV_{max}}$) and batteries ($\mathrm{Bat_{min}}$, $\mathrm{Bat_{max}}$) for the optimization process.

Use the method based on sizing sheets to determine the initial number of PV modules and Batteries. Assume an initially large range where the number of PV modules and batteries range from one to ten times the values obtained using the sizing sheets.

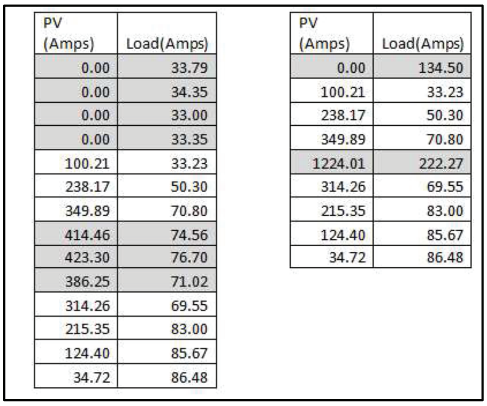
In order to determine the minimum values ($\mathrm{PV_{min}}, \mathrm{Bat_{min}}$), consider the month with the least average load and assume that the LLP is larger than the desired LLP for the community. Using the range derived from the basic method above and the optimization procedure described in subsection B, determine the values ($\mathrm{PV_{min}}, \mathrm{Bat_{min}}$).

In order to determine the maximum values ($\mathrm{PV_{max}}, \mathrm{Bat_{max}}$), consider the month with the highest average load and assume that the LLP is smaller than the desired LLP for the community. Using the range derived from the basic method above and the optimization procedure described in subsection B, determine the values ($\mathrm{PV_{max}}, \mathrm{Bat_{max}}$).

Step 5: Use the MILP procedure as described in subsection B to determine the size of system components that give minimum ATLCC for the desired LLP.

A. DBSCAN Clustering Algorithm

The main concept of the DBSCAN algorithm is that all the points within a cluster are such that the distance between them is less than a predefined radius referred to as the eps [17]. This concept is applied to the hourly irradiation and load data as follows: A three dimensional point$(t, I_L(t), I_{PV(t)})$, is defined for each hour t where , $I_L(t)$ is the load demand and $I_{PV(t)}$ is the Output current of a single PV module at each hour. First, all consecutive points where the PV output is zero are clustered. These correspond to the night hours of each day. The points are consolidated into one time step t where the PV output for the time step is zero and the load is the sum of the respective loads. Thereafter, all consecutive points for which the PV output $I_{PV(t)}$ are in a range of m% of each other and the loads $I_L(t)$ are also in a range of m% of each other are clustered and consolidated into one time step. The PV output for the time step is the sum of the respective PV outputs and the load is the sum of the respective loads. This paper has considered a value of m equal to 10%. The consolidation is illustrated in Fig. 2. All the shaded values in the first table on the left are consolidated into one time step resulting into the second table on the right with fewer time steps.



PV (Amps)	Load(Amps)
0.00	33.79
0.00	34.35
0.00	33.00
0.00	33.35
100.21	33.23
238.17	50.30
349.89	70.80
414.46	74.56
423.30	76.70
386.25	71.02
314.26	69.55
215.35	83.00
124.40	85.67
34.72	86.48

PV (Amps)	Load(Amps)
0.00	134.50
100.21	33.23
238.17	50.30
349.89	70.80
1224.01	222.27
314.26	69.55
215.35	83.00
124.40	85.67
34.72	86.48

Fig. 2. Illustration of the Consolidation of Hourly PV and Load Data into Fewer Time Steps.

B. Mixed Integer Linear Programming Approach

The objective of this optimization method is to obtain the optimal design of a system that minimizes the ATLCC while meeting the load demand with a desired LLP that is dependent on the customer's satisfaction.

Optimization Problem

The objective function for the mixed-integer linear programming (MILP) problem is;

$$minimise\ ATLCC$$

subject to the constraints defined by (6) to (32). The ATLCC is defined by (55). The acronyms used in the equations are defined in the Appendix.

The PV System Model

The number of PV modules is within a maximum and minimum range determined above.

$$PV_{min} \leq PV \leq PV_{max} \tag{6}$$

$$IPV(t) = I_{PV(t)} * PV_{par} \tag{7}$$

$$\mathrm{PV} = PV_{par} * PV_{ser} \tag{8}$$

The current balance at each time period is given as

$$IPV(t) + I_{dis}(t) + I_{def}(t) - I_L(t) - I_{ch}(t) - I_{sur}(t) = 0 \tag{9}$$

$$I_{dis}(t) \geq 0,\ I_{def}(t) \geq 0, I_{L(t)} \geq 0, I_{ch}(t) \geq 0, I_{sur}(t) \geq 0 \tag{10}$$

Battery Model

The number of batteries is within a maximum and minimum range determined above. It is assumed that initially, the battery is full. Thereafter, in each time period, the battery cannot charge to more than the battery capacity and cannot discharge to less than the minimum allowable battery capacity.

$$Bat_{min} \leq Bat \leq Bat_{max} \tag{11}$$

$$Bat = Bat_{par} * Bat_{ser} \tag{12}$$

$$Bat_{cap} \leq Bat_{AH} * Bat_{\max\_par} \tag{13}$$

$$Bat_{par} = Bat_{cap}/Bat_{AH} \tag{14}$$

$$SOC(0) = Bat_{cap} \tag{15}$$

$$SOC(t) \leq Bat_{cap} \tag{16}$$

$$SOC(t) \geq (1 - DOD)Bat_{cap} \tag{17}$$

In each time period, the battery shall be either in charging or discharging state as represented by (18) [18]

$$\varphi_{ch}(t) + \varphi_{dis}(t) = 1 \tag{18}$$

Where $\varphi_{ch}(t)$ and $\varphi_{dis}(t)$ are binary variables representing charging and discharging modes.

During charging mode ($\varphi_{ch}(t) = 0$), the PV system charges the battery. There is no discharge current and deficit current.

$$I_{def}(t) \leq M * \varphi_{ch}(t) \tag{19}$$

$$I_{dis}(t) \leq M * \varphi_{ch}(t) \tag{20}$$

When the battery is full, there is a surplus due to excess energy from the PV.

$$I_{sur}(t) \leq M * \varphi_{ch\_sur}(t) + M * \varphi_{ch}(t) \tag{21}$$

$$Bat_{cap} - Soc(t-1) - I_{ch}(t) * Bat_{c_{eff}} * Bat_{d_{eff}} \leq \left(1 - \varphi_{ch\_sur}(t)\right)M + M * \varphi_{ch}(t) \tag{22}$$

The charge and surplus currents cannot exceed the maximum PV output for each hour

$$I_{sur}(t) \leq I_{PV(t)} * PV_{\max\_par} \tag{23}$$

$$I_{ch}(t) \leq I_{PV(t)} * PV_{\max\_par} \tag{24}$$

During the discharge mode $\varphi_{dis}(t) = 0$, the PV system current is insufficient to meet the load demand. The battery supplies the extra current to meet load demand. There is no charge current and surplus current.

$$I_{ch}(t) \leq M * \varphi_{dis}(t) \tag{25}$$

$$I_{sur}(t) \leq M * \varphi_{dis}(t) \tag{26}$$

If State of Charge (SOC) of the battery falls below DOD, there is a deficit.

$$I_{def}(t) \leq M * \varphi_{dis\_def}(t) + M * \varphi_{dis}(t) \tag{27}$$

$$Soc(t-1) - I_{def}(t)/\left(Bat_{c_{eff}} * Bat_{d_{eff}}\right) - (1 - DOD)Bat_{cap} \leq M(1 - \varphi_{dis\_def}(t)) + M * \varphi_{dis}(t) \tag{28}$$

The Deficit and Discharge currents cannot exceed the Load current

$$I_{def}(t) \leq I_L(t) \tag{29}$$

$$I_{dis}(t) \leq I_L(t) \tag{30}$$

The energy balance equation for the battery is given by (31) [19]

$$SOC(t) - SOC(t-1) - I_{ch}(t) * Bat_{c_{eff}} * Bat_{d_{eff}} + \frac{I_{dis}(t)}{Bat_{c_{eff}} * Bat_{d_{eff}}} = 0 \tag{31}$$

The LLP is less than or equal to a desired LLP [2]

$$\frac{\sum I_{def}(t)}{\sum I_L(t)} <= Desired\_LLP \tag{32}$$

Controller Specification

The controller is sized using (33) to (35). [2]

$$\mathrm{Ctrler_{SCC}} = \mathrm{I_{sc}} * \mathrm{PV_{par}} * \mathrm{SF} \tag{33}$$

$$\mathrm{Ctrler_V} = \mathrm{V_{dc}} \tag{34}$$

$$\mathrm{Ctrler_{Total}} = \frac{\mathrm{Ctrler_{SCC}}}{\mathrm{Ctrler_{Sel}}} \tag{35}$$

Where; SF is a safety factor to ensure that the array can withstand high currents. $\mathrm{I_{sc}}$ is the module short circuit current, $\mathrm{Ctrler_V}$ is the voltage rating of the controller and is equal the DC system voltage, $\mathrm{Ctrler_{Sel}}$ is the current rating of the controller selected basing on the voltage rating and current ratings ($\mathrm{Ctrler_{SCC}}$) and $\mathrm{Ctrler_{LdAmps}}$. The total number of controllers $\mathrm{Ctrler_{Total}}$ is then obtained.

Inverter Sizing and Specification

The inverter is sized using (36) and (37). [2]

$$\mathrm{Max_{DCAC}} = \frac{\mathrm{P_c}}{\mathrm{V_{dc}}} \tag{36}$$

$$\mathrm{I_{inv}} > \mathrm{Max_{DCAC}} \tag{37}$$

Cost Analysis

The capital cost for the PV array and the battery storage is given by (38) and (39) respectively [2].

$$CC_{PV} = PV * C_{PV} \tag{38}$$

$$CC_{Bat} = Bat * C_{Bat} \tag{39}$$

Maintenance cost of PV modules and Batteries per year is assumed to be 2% of the capital costs [4]. This is presented in (40) and (41) respectively.

$$MCC_{PV} = CC_{PV} * 0.02 \tag{40}$$

$$MCC_{Bat} = CC_{Bat} * 0.02 \tag{41}$$

The annualized capital and maintenance cost of the PV modules is estimated using (42) [16].

$$C_{PVa} = \frac{CC_{PV} + (L_s * MCC_{PV})}{L_{PV}} \tag{42}$$

The annualized capital maintenance cost and replacement cost of the batteries is calculated using (43) [16].

$$C_{Bata} = \frac{CC_{Bat} * (1 + Y_{Bat}) + MCC_{Bat} * (L_s - Y_{Bat})}{L_{Bat}} \tag{43}$$

Where;

$$Y_{Bat} = \frac{L_s}{L_{Bat}} - 1 \tag{44}$$

The capital cost for the transformers is calculated using (45).

$$CC_{TR} = N_{TR} * C_{Tr} \tag{45}$$

Maintenance cost of the Transformers per year is given by (46).

$$MCC_{TR} = CC_{TR} * 0.02 \tag{46}$$

The annualized capital and maintenance cost of the transformers is estimated using (47).

$$C_{TRa} = \frac{CC_{TR} + (L_s * MCC_{TR})}{L_{TR}} \tag{47}$$

The total capital costs for other components are lamped together as 20% of PV cost as given in (48).

$$CC_{CW} = CC_{PV} * 0.02 \tag{48}$$

The total capital costs for other components aside PV and Batteries are given by (49)

$$CC_{OC} = CC_{CW} + CC_{TR} \tag{49}$$

The annualized Capital costs of other components is estimated using (50). [2]

$$CC_{OCa} = \frac{CC_{OC}}{((1 + ndr)^{L_s} - 1)\,/(ndr * (\,1 + ndr)^{L_s}\,)} \tag{50}$$

Where;

ndr is the net of discount inflation rate and is given by (51). ir is the real interest rate and fr is the inflation rate [2].

$$ndr = \frac{1 + ir}{1 + fr} - 1 \tag{51}$$

The total life cycle cost is the sum of the respective component costs and is given by (52). [2]

$$LCC = CC_{OCa} + C_{PVa} \tag{52}$$

The annualized salvage value of the system is given by (53). [2]

$$CS_a = \frac{0.13 * CC_D}{((1 + ndr)^{L_s} - 1)\,/(ndr * (\,1 + ndr)^{L_s}\,)} \tag{53}$$

Where;

CC_D is the capital cost of the disposable component and is given by (54). [2]

$$CC_D = CC_{PV} + CC_{Bat} \tag{54}$$

The annualized total life cycle cost is given by (55). [2]

$$ATLCC = LCC - CS_a \tag{55}$$

The levelized cost of energy (LCE) is obtained using (56) below [16].

$$LCE = \frac{ATLCC}{E_{TOT}} \tag{56}$$

IV. Case Study Using the ComμGrid Sizing Approach

The ComμGrid Sizing approach proposed in this research was implemented using MATLAB software and it was used to design a community microgrid for a village in the district of Tororo in eastern Uganda. The location chosen is not connected to the main utility grid. The village considered has 100 households, a maize mill, ladies and men's salons, a primary school and a clinic. The household categories and appliance usage were estimated based on the energy survey conducted by the Uganda Bureau of Statistics and Ministry of Energy and Mineral Development for 2012 [19] as well as the Uganda National household survey for 2016 [20]. The average load demand per day is 485kWh.

The make and model of the PV module chosen for the system is the TT, Auversun, AV275M96NB-5P while the battery chosen is the Concorde Sun Xtender PVX-2580L. These types of battery and PV module were chosen as they have been successfully used in other studies on PV systems such as those in [4]. A depth of discharge of 0.8 was used.

The renewables.ninja tool [20] [21] [22] was used to generate the hourly solar data and ambient temperature for the target location. The community microgrid was also sized using the basic sizing method defined in [3] and the results from the two methods were compared. The basic sizing was done using 4 days of autonomy.

The comparison between the basic sizing approach and the ComμGrid approach is shown in Table I.

TABLE I
COMPARISON BETWEEN RESULTS FROM THE PROPOSED SIZING APPROACH AND THE BASIC SIZING METHOD

Parameter	Basic Method	Proposed ComμGrid MILP based Method
Total Number of Batteries	944	281
Battery Capacity (AH)	114,500	72,240
Total Number of PVs (Modules)	1176	2032
LLP	0.2	0.01
ATLCC(£)	108,226	75,136
LCE(£)	0.62	0.43

The ATLCC and the LCE obtained using the ComμGrid Sizing approach are lower than those obtained using the basic sizing approach. The LLP for the ComμGrid Sizing approach is lower than that for the basic sizing approach.

The performance of the two methods was compared considering the month with highest load. Fig. 3 and Fig. 4 show the sample hourly performance of such a system when sized using the two methods. During this period, The LLP for the system sized using the basic method was 0.44 and the average state of charge was 0.21. This means that the system sized using the basic method was unable to meet the load demand for a period equivalent to 13 days and the battery was operating near minimum DOD for most of the time as shown in Fig. 4. The period of deficit is longer than the 4 days of autonomy used during the sizing. The LLP for the system sized using the proposed ComμGrid MILP based method was 0.05 and the average state of charge was 0.5. This means that the system sized using the proposed ComμGrid MILP based method was unable to meet the load demand for a period equivalent to just 1.5 days.

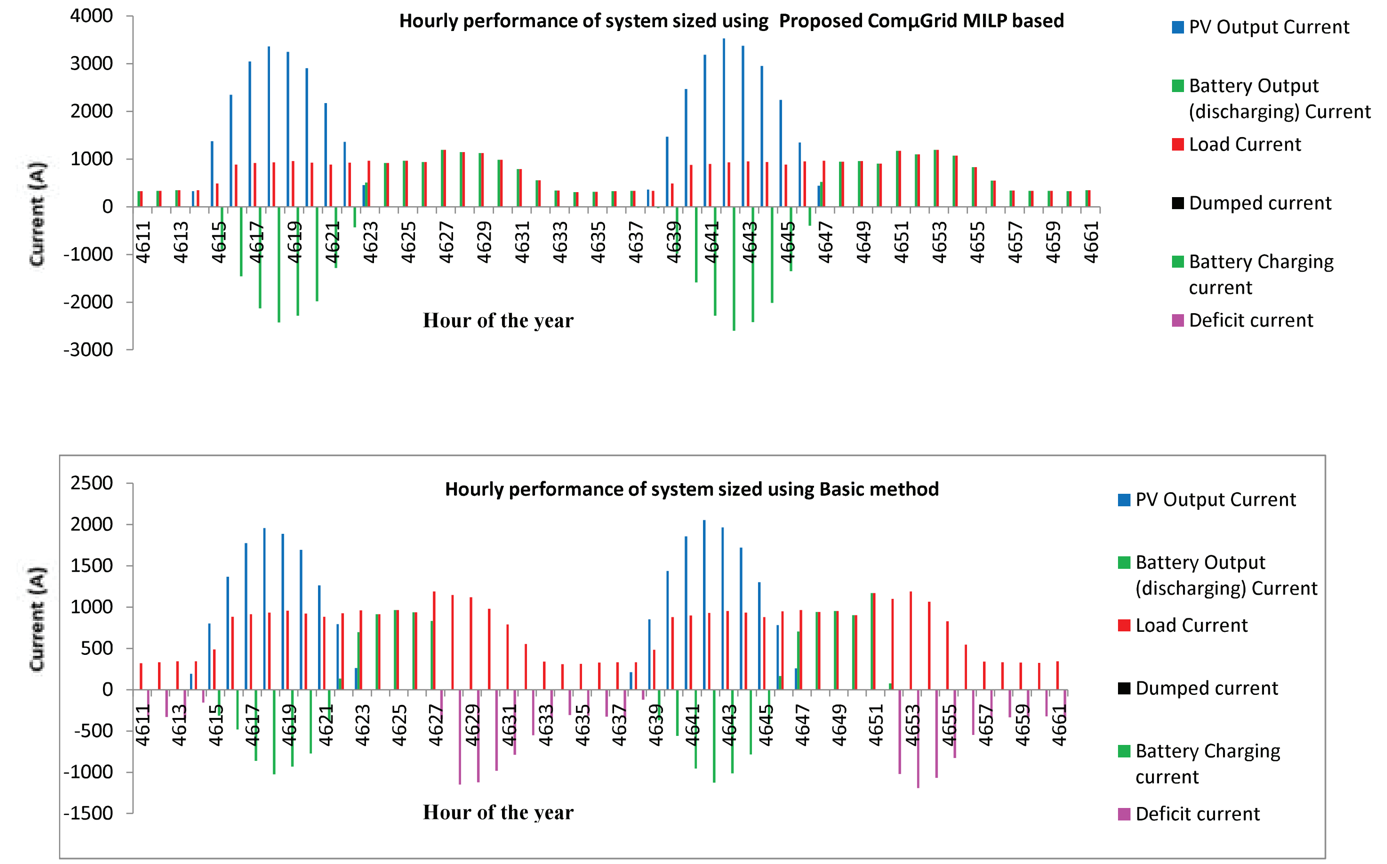


Fig. 3. Sample Hourly Performance of the Proposed ComμGrid MILP based Sizing and Basic Sizing Methods

The results discussed above show that the ComµGrid Sizing approach yields a better system that is both technically reliable and cost effective as compared to the one achieved using the basic sizing method.

It is however noted that the Levelized Cost of Energy obtained using the ComµGrid approach (£0.42/kWh approximately 2,000/= UGX) is higher than that for the main utility grid, which is 752.5/= UGX [23]. This is due to the cost of storage. When storage is not considered, the price for the electricity is £0.07/kWh (approximately 350/= UGX).

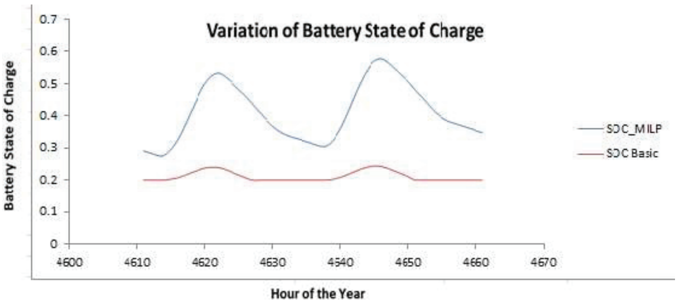


Fig. 4. Comparison of Battery State of Charge for the Basic and Proposed ComµGrid MILP based Sizing Methods

The microgrid was sized using different consolidation values for $m = 5\%, 10\%, 20\%$ and 30%. The results are shown in Table II.

TABLE II
COMPARISON BETWEEN RESULTS FOR DIFFERENT VALUES OF CONSOLIDATION PARAMETER M

	m = 5%	**m=10%**	**m=20%**	**m=30%**
No. of Batteries	281	281	280	281
No. of PV modules	2034	2032	2033	2031
LLP	0.009	0.01	0.01	0.01
ATLCC(£)	72,240	75,136	75,602	75,055
LCE(£)	0.43	0.43	0.43	0.43
Total Time steps	3,876	3456	3084	2724

The results show that the total time steps processed reduce with an increase in m. However the results for the ATLCC, LCE, Number of PV modules and batteries are similar for the different values of m. This is because the consolidation maintains the outlier periods for the irradiance and load demand.

V. CONCLUSION

Many developing countries are aiming to improve the electrification rate of their countries especially in the remote areas that are very costly to connect to the main grid. In this regard there is urgent need for optimum, reliable and cost-effective off-grid electrification projects. These projects should be sustainable and serve the growing needs of the communities for which they are designed. In this paper a community microgrid system has been proposed that uses an improved PV sizing approach taking into consideration hourly load variation together with the hourly variation of solar irradiance and ambient temperature of the area. This is an improvement from the basic sizing approach presented in solar PV design and installation manuals. The results have shown that a system sized using the basic method is unreliable and more expansive than one sized using the proposed ComµGrid sizing approach that utilizes MILP. The main contribution of this paper is that it has provided a method for reducing the hourly load and irradiance data into fewer time steps to aid in faster execution and convergence to the optimal solution. In addition, it has provided a MILP based method and a process on how to derive the search space of the optimization starting from the basic sizing method. Lastly from the results it can be seen that the cost of electricity from off-grid systems incorporating storage is still higher than that supplied by the main grid. This cost can be expected to go down with reduction in prices for battery storage and provision of attractive subsidies for investment in rural electrification systems.

APPENDIX

SYMBOLS AND ACRONYMS

ATLCC	*Annualized Total Life Cycle Cost*
Alpha	*PV panel temperature coefficient*
Bat	*Total number of Batteries*
$\mathrm{Bat}_{\mathrm{c_{eff}}}$	*Battery Charging Efficiency*
$\mathrm{Bat}_{\mathrm{d_{eff}}}$	*Battery Discharging Efficiency*
$\mathrm{Bat}_{\mathrm{AH}}$	*AH rating of one Battery*
$\mathrm{Bat}_{\mathrm{cap}}$	*Battery Storage Capacity (AH)*
$\mathrm{Bat}_{\mathrm{max}}$	*Maximum Number of Batteries*
$\mathrm{Bat}_{\mathrm{max\_par}}$	*Maximum number of Batteries in Parallel*
$\mathrm{Bat}_{\mathrm{min}}$	*Minimum Number of Batteries*
$\mathrm{Bat}_{\mathrm{par}}$	*Number of Batteries in Parallel*
$\mathrm{Bat}_{\mathrm{ser}}$	*Number of Batteries in Series*
$\mathrm{C}_{\mathrm{Bat}}$	*Cost of Battery*
$\mathrm{C}_{\mathrm{Bata}}$	*Annualized capital maintenance cost and replacement cost of the batteries*
$\mathrm{CC}_{\mathrm{Bat}}$	*Capital Cost of Batteries*
$\mathrm{CC}_{\mathrm{CW}}$	*Total capital costs for other components*
CC_{D}	*capital cost of the disposable components*
$\mathrm{CC}_{\mathrm{OC}}$	*Total capital costs for other components aside PV and Batteries*
$\mathrm{CC}_{\mathrm{OCa}}$	*Annualized Capital costs of other components*
$\mathrm{CC}_{\mathrm{PV}}$	*Capital Cost of PV modules*
$\mathrm{CC}_{\mathrm{TR}}$	*Capital Cost of transformers*
C_{PV}	*Cost of one PV module*
$\mathrm{C}_{\mathrm{PVa}}$	*Annualized capital and maintenance cost of the PV modules*
CS_{a}	*Annualized salvage value of the system*
$\mathrm{C}_{\mathrm{TRa}}$	*Annualized capital and maintenance cost of the transformers*
C_{Tr}	*Cost of one transformer*
$Ctrler_{LdAmps}$	*Maximum DC Load Amps that controller must handle*

$\text{Ctrler}_{\text{SCC}}$	*Controller short circuit current*
$\text{Ctrler}_{\text{Sel}}$	*Controller current rating*
$\text{Ctrler}_{\text{Total}}$	*Number of Charge Controllers in operation during the system lifetime*
Ctrler_{V}	*Voltage rating of the controller*
DBSCAN	*“Density Based Spatial Clustering of Applications with Noise”*
Desired_LLP	*Desired Loss of Load Probability*
DOD	*Depth of Discharge for the Battery*
D_{aut}	*Days of Autonomy*
E_{TOT}	*Total energy consumed by the load from the system per year*
fr	*Inflation Rate*
$\text{I}_{\text{ch}}(\text{t})$	*Battery Charge Current at time (t)*
$\text{I}_{\text{def}}(\text{t})$	*Deficit Current at time (t)*
$\text{I}_{\text{dis}}(\text{t})$	*Battery Discharge Current at time (t)*
I_{inv}	*Inverter current rating*
I_{mp}	*Peak Amps per module at STC*
$\text{I}_{\text{L}}(\text{t})$	*Load demand current at time (t)*
$\text{I}_{\text{PV(t)}}$	*Output current of a single PV module at time (t)*
IPV(t)	*Total output current of the PV array at time (t)*
I_{sc}	*PV Module Short Circuit Current*
$\text{I}_{\text{sur}}(\text{t})$	*Surplus Current at time (t)*
ir	*Real Interest Rate*
L_{Bat}	*Lifetime of the Battery*
L_{PV}	*PV system Lifetime*
L_{s}	*Microgrid System Lifetime*
L_{TR}	*Lifetime of the transformer*
LCC	*Sum of annualized capital, annualized maintenance and annualized replacement costs*
LCE	*Levelized cost of energy*
LLP	*Loss of Load Probability*
M	*A large number*
ndr	*Net of discount inflation rate*
Max_{DCAC}	*Maximum Continuous Direct Current of the controller*
MCC_{Bat}	*Maintenance cost of batteries*
MCC_{PV}	*Maintenance cost of PV modules*
MCC_{TR}	*Maintenance cost of transformers*
MILP	*“Mixed Integer Linear Programming”*
N_{TR}	*Number of transformers*
P_{c}	*Total Connected AC Power*
PSH	*Peak sun hours per day*
PV	*Number of PV Modules*
PV_{max}	*Maximum Number of PV Modules*
$\text{PV}_{\text{max\_par}}$	*Maximum Number of PV Modules in Parallel*
PV_{min}	*Minimum Number of PV Modules*
PV_{par}	*Number of PV Modules in Parallel*
PV_{ser}	*Number of PV Modules in Series*
$S(t)$	*Solar Radiation(W/m^2) at time (t)*
SF	*safety factor*
SOC(t)	*Battery State of Charge*
STC	*Standard Test Conditions*
T(t)	*Ambient Temp (°C)at time (t)*
V_{dc}	*DC System voltage*
V_{mp}	*Nominal Module Voltage (Voltage at MPP under STC)*
Y_{Bat}	*Number of times of Battery Replacement*
Z	*Average Daily Load (Amp-Hour per Day)*
$\varphi_{\text{ch}}(\text{t})$	*Binary variable for charging mode*
$\varphi_{\text{ch\_sur}}$	*Binary variable for the discharge/deficit mode*
$\varphi_{\text{dis}}(\text{t})$	*Binary variable for discharging mode*
$\varphi_{\text{dis\_def}}$	*Binary variable for the charge/surplus mode*
η_{bat}	*Battery Efficiency*

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