ENERGY EFFICIENT STATISTICAL COOPERATIVE SPECTRUM SENSING IN COGNITIVE RADIO NETWORKS

E. Kataka∗ and T. Walingo†

∗ *Discipline of Electrical Electronic and Computer Engineering, University of KwaZulu-Natal, Durban 4041, South Africa, E-mail: kataka.edwin@gmail.com*

† *Discipline of Electrical Electronic and Computer Engineering, University of KwaZulu-Natal, Durban 4041, South Africa, E-mail:walingo@ukzn.ac.za*

Abstract: Cooperative spectrum sensing (CSS) alleviates the problem of imperfect detection of primary users (PU)s in cognitive radio (CR) networks by exploiting spatial diversity of the different secondary users (SUs). The efficiency of CSS depends on the accuracy of the SUs in detecting the PU and accurate decision making at the fusion center (FC). This work exploits the higher order statistical (HOS) tests of the PU signal for blind detection by the SUs and combination of their decision statistics to make a global decision at the FC. To minimize energy, a two stage optimization paradigm is carried out, firstly by optimal iterative selection of SUs in the network using Lagrange criterion and secondly optimized fusion techniques achieved by Neyman Pearson. The probability of detecting the PU based on HOS and hard fusion schemes is investigated. The results indicate that the Omnibus HOS test based detection and optimized majority fusion rule greatly increases the probability of detecting the PU and reduces the overall system energy consumption.

Key words: Cognitive radio, cooperative spectrum sensing, fusion techniques, higher order statistics, primary user, secondary user.

1. INTRODUCTION

Cooperative spectrum sensing (CSS) utilizes multiple secondary users (SUs) to sense the vacant spectrum and send their decision to the fusion center (FC) for a final global decision to be made regarding the presence of the primary user (PU) on the channel. CSS overcomes the challenges of wireless channel characteristics such as multipath fading, shadowing or hidden terminal problem experienced when only one SU is employed to detect the PU. This is due to the spatial diversity of the different SUs cooperating to make the final decision on the status of the PU on the channel [1, 2]. A number of spectrum detection schemes have been proposed to detect the presence or absence of PU, among them include energy, matched filter and cyclostationary methods [3]. In most practical systems the transmission channels are usually noisy hence causing tremendous reduction in signal to noise ratio (SNR) of the PU received signals. This has prompted the need for the higher order statistical (HOS) detection techniques which have very high sensitivity at low SNR signal condition while maintaining reasonable circuit complexity [4]. CSS can generally be divided into two detection stages; local update stage and global fusion stage. At the local update stage, the individual SUs detect the received PU's signals based on HOS. The SU then computes a local decision and sends it to the FC for fusion. The commonly used metrics that utilize the HOS properties to detect the PU's received signals include Jarque-Bera, kurtosis, skewness and omnibus tests. These statistical tests are utilized to determine the probability distribution function (PDF) of a group of data samples. This is crucial for benchmarking the distribution in order to make an informed inference on a physical phenomena (existence of PU on the channel) [5].

In this paper, the performance analysis of the HOS tests on the PU signal is investigated with aim of selecting the best statistical technique in determining the status of the PU on the channel. This has not been adequately addressed in literature.

The global fusion stage is performed at the fusion centre where either soft or hard combination schemes are employed to fuse the received signals from individual SUs [6]. Furthermore to reduce energy consumption in the cooperative network not all the SU need to report their individual decisions. To optimize on the number of SUs selected to participate in the fusion process, this paper proposes a two stage optimization strategy. The first stage is to select the SUs which qualify to transmit their individual decision data to the fusion center. To achieve this an iterative optimization threshold algorithm is employed and determined based on the SUs' SNR. However, this is at the cost of minimizing on the error probability formulated by the Lagrange optimization criterion. The rest of SUs that do not meet this threshold are rejected at this sensing point in time (they are not allowed to transmit). Those SUs selected during the first optimization stage are subjected to the second stage optimization process, realized by a prudent and optimal choice of hard fusion criteria taken to fuse the SUs binary decisions. A strategic *k out of n* counting rule is adopted to determine the optimal combinatorial order of the SUs to be considered for final global fusion. To realize this, Neyman-Pearson optimization criterion is employed through an iterative Bisection numerical search algorithm formulated on *k out of n* rule. The cost function is to maximize the probability of detection subject to minimizing of the probability of false alarm. In summary, a hybrid detection strategy of HOS local detection test and optimal global fusion technique was implemented. The simulated results show that an optimal *k out of n* fusion rule based on omnibus test perform better than other HOS tests in terms of detection probability. In this model, not all SUs participate in detection at any one sensing time frame hence great energy cost saving in the whole cooperative spectrum sensing network.

The rest of the paper is organized as follows. Section II presents the related work, section III describes the system model, section IV is devoted on local spectrum sensing, section V focuses on the fusion techniques, section VI presents the energy efficiency. Simulation results illustrating the effectiveness of the scheme are given in section VII and finally, section VIII, draws the conclusions.

2. RELATED WORK

Cooperative spectrum sensing schemes have not exhaustively been studied in the current literature. In [7], authors investigated the performance of energy based CSS scheme where a group of SUs cooperated to detect the presence or absence of primary user (PU) in fading channel environment. They also made comparative study on the three main hard fusion techniques i.e. OR-logic, AND-logic and Majority-logic to make global decisions at the fusion center. In [8], authors proposed selection technique based on iteratively setting different thresholds for different signal to noise ratio (SNR) of SUs in cooperative spectrum sensing with OR logic fusion technique done at the fusion centre. This scheme highly outperformed the traditional energy spectrum sensing with the same threshold in terms of reduced probability of false alarm. Higher order test (HOS) have been utilized in literature to analyze data distribution and its degree of departure from the normal distribution. The concept of separation is based on the maximization of the non-Gaussian property of separated signals to improve the robustness against noise uncertainty. The authors in [9], proposed kurtosis and skewness (goodness-of-fit) test to check the non-Gaussianity of an averaged periodogram of received SUs signal. This is computed from the Fast Fourier transform (FFT) of the PU signal to justify its existence and hence the availability or not of the spectrum for a cognitive radio transmission. Their findings showed improved detection of the PU signals especially under very low SNR conditions i.e. the SUs are able to detect the primary channel with certainty even under very noisy environment. In [10], authors proposed Jarque-Bera tests based spectrum sensing algorithm and compared it to a kurtosis & skewness combination test statistics. From their simulated results they concluded that Jarque-Bera showed better detection performance than the kurtosis & skewness in terms of the reliability i.e. improved probability of detection for different values of SUs' SNR. In the emerging research on spectrum sensing schemes, researchers considered a number of modulation schemes on multipath fading channel based on Jarque-Bera test in detection of the primary user. These schemes were considered to transcend the absence of a priori information of the spectrum occupancy under additive white Gaussian noise channel [4]. In [11], authors showed Jarque-Bera as having rather poor small data sample properties, slow convergence of the test statistic to its limiting distribution. In their findings the power of the statistical tests showed the same eccentric form, the reason being skewness and kurtosis are not independently distributed, and the sample kurtosis especially attains normality very gradually. However, the JB test is simple to calculate and its power has proved to match other powerful statistical tests. A genuine omnibus tests should be consistent to any departure from the null hypothesis. In [12], authors formulated omnibus test which is based on the standardized third and fourth moments. This was done to assess the normality of random variables by calculating the transformed samples of kurtosis & skewness. In the computational economics these authors showed omnibus's simplicity provided by the chi-squared framework. In this work the omnibus test is applied in CSS and compared to other well known Jarque-Bera, kurtosis and skewness tests. Fusion of the decisions received at the fusion center with a view to make the final global decision on the status of the primary user is also another important challenge that has not been exhaustively studied. Fusion techniques are classified into soft and hard combination schemes. In hard decision strategy the FC combines binary decisions using standard hard decision rules to achieve the global decision. Three hard combining decision rules used to arrive at the final decision are classified as AND, OR and majority also called *k out of n* counting rule [13]. In [14], authors made a comparative study of the performance of the three hard fusion techniques. In their findings they concluded that AND rule was the most reliable fusion scheme followed by majority and the lastly the OR rule. Another comparative study on the performance of hard fusion schemes and soft decision schemes was done by authors in [15]. In their study they confirmed earlier research done to justify that soft fusion decision reported better PU signal detection, albeit having significant data communication overheads. Hard combination schemes however have attracted most attention from researchers since these fusion schemes are easy to implement by simple logics gates. The authors in [16], proposed strategies on how the AND, majority and OR fusion rules are optimized based on the Neyman-Pearson criterion. Under this strategy the sensing objective was to maximize the probability of detection with the constraint on the probability of false alarm of less than 10 percent. Their findings showed AND rule had higher detection performance than the other two. Spectrum sensing in the IEEE 802.22 standard, for example requires stringent sensing of a false alarm probability of less than 0.1 for a signal as low as -20 dB (SNR) [17]. In [18], authors proposed an the iterative threshold cooperative spectrum technique. Their objective was to optimize the thresholds of the cooperative spectrum sensing with different fusion rules including AND logic & OR logic. This was done in order to obtain the optimal SUs in cooperative spectrum sensing and their optimal thresholds. Their algorithm achieved better detection performance for SUs' with different SNR. The optimal scheme also employed fewer SUs in collaborative sensing at the fusion center. In [19], the authors proposed an optimized detection threshold in order to minimize both the error detection probabilities of single-channel and multichannel cooperative spectrum sensing. In single-channel cooperative spectrum sensing, they performed an iterative optimal thresholds with AND logic, OR logic and *k out of n* rule respectively. Their findings showed a great decrease in the error on detecting PU status on the channel. Energy efficiency in the cognitive radio network is defined as the ratio of throughput (average amount of successfully delivered bits transmitted from SUs to the fusion centre) to the total average energy consumption in the system [20]. In order to reduce the energy consumed in spectrum sensing network, not all SUs in each cluster send their sensed results to the fusion center of local cluster. In [21], authors optimized *k out of n* by allowing those SUs with reliable sensing results to transmit to the FC. This showed some reduction in energy consumption of the cognitive radio network. In this paper an optimal *k out of n* is applied to improve on the probability of detection and reduce on the energy system consumption by employing fewer SUs in the final detection on the presence or absence of the PU.

Notations : $\mathbf{E}[\cdot]$ is the expectant operator, **var** is the variance, $\mathbf{Im}[\cdot]$ and $\mathbf{Re}[\cdot]$ are the imaginary and real parts of the signal $X(\cdot)$, $\mathbf{erfc}(\cdot)$ is complementary error function and $\mathbf{h}$ is the circular Gaussian channel.

3. SYSTEM MODEL

3.1 Practical cooperative sensing model

The system model in figure 1 shows a practical CSS network. In this scheme, a group of SUs sense the spectral band to determine the presence or absence of PU. They receive this information through the control channel and independently analyze it by utilizing the statistical properties of the received PU's signal, and subsequently communicate their individual decisions through the reporting channel to the FC. At the fusion center, the decisions from individual SUs are integrated together to finally make the global decision on whether the PU is transmitting on the channel or not. The SUs can then opportunistically access and transmit on the channel if found idle.

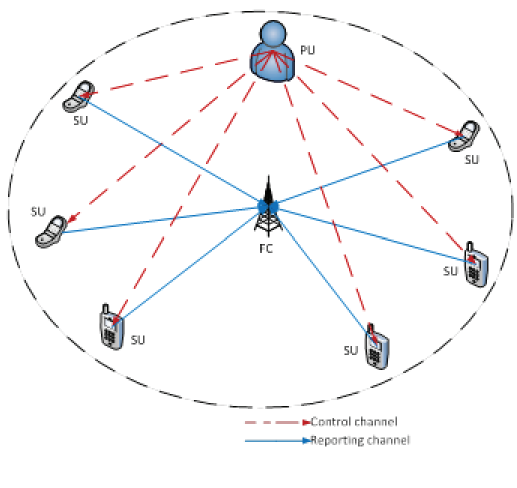

Figure 1: A practical cognitive radio network

3.2 Proposed Cooperative Spectrum Model

In the proposed lower level system model of figure 2, the secondary users $(SU_1, SU_2, ..., SU_n)$ collectively sense the PU channel based on HOS tests namely, kurtosis & skewness $(kurt\,\&\,skew)$, omnibus $(omnb)$ and Jarque-Bera (JB) statistics tests. The hard binary local decisions made by SUs are transmitted over wireless Gaussian channel represented as $(CH_1, CH_2, ..., CH_n)$ to the data FC. The binary data $(b_1, b_2, ..., b_n)$ is fused to achieve the final global decision on the presence or absence of the primary user.

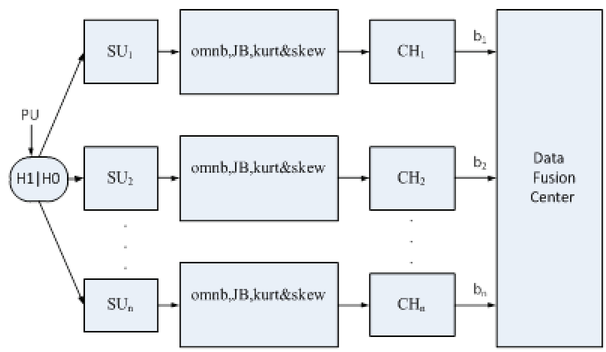

Figure 2: Proposed cooperative spectrum sensing model

4. LOCAL SPECTRUM SENSING

4.1 Spectrum sensing hypothesis

Generally the spectrum sensing problem can be formulated by the following two hypothesis [4, 9]

$$H_0 : x(t) = w(t) \qquad t = 0, ..., T-1 \tag{1}$$

$$H_1 : x(t) = s(t) + w(t) \qquad t = 0, ..., T-1, \tag{2}$$

where H_0 and H_1 are null and alternative hypothesis respectively, t is the digital samples numbering T, $w(t)$ is the additive white Gaussian noise, $s(t)$ is the PU's signal and $x(t)$ is the signal received at the fusion center. The received signal plus additive white Gaussian noise $x(t)$ as function of SNR (γ) is given as

$$x(t) = f\left([s(t) + w(t)], \gamma\right), \tag{3}$$

where γ is the PU signal to noise ratio (SNR). The probability of detection is formulated as hypothesis test $P_d = \text{Prob(Signal Detected} \,|\, H_1)$, whereas the probability of false detection is determined as $P_f = \text{Prob(Signal not Detected} \,|\, H_1)$. Another form of formulation is thresholding on the statistical test parameter. To detect the PU's spectrum effectively there is need to first estimate and analyze the power spectral density (PSD) of the SUs received signal. A strategic periodogram PSD estimation technique can be used to accurately present the frequency-domain statistical properties of a signal [9]. Based on the periodogram method and as formulated in algorithm 1, the received signal $x(t)$ of T samples is firstly subdivided into L smaller segments. Then the *i-th* segment signal can be formulated as [9]

$$x_i(t) = x[t+iM], \tag{4}$$

where $i = 0,...,T-1$ is the number of data samples, $M = T/L$ is the length of each segment and $t = 0,...,M-1$ are the Fast Fourier transforms (FFT) points in one segment. Performing FFT on signal sample $x_i(t)$, periodogram of the *i-th* SU, $y_i(t)$ is given by

$$y_i(t) = \frac{1}{M}\left|\sum_{t=0}^{M-1} x_i[t]\, e^{-\frac{j\omega t}{M}}\right|^2, \tag{5}$$

where $i \in [t,T]$ is the number of samples, M is the length of each segment representing the elements of discrete Fourier transform (DFT) and $\omega = 2\pi f$. The function $y_i(t)$ is modeled as the PU signal and is utilized in the next section to determine the skewness and kurtosis.

4.2 Spectrum sensing HOS techniques

Skewness and kurtosis: The estimated skewness ($skew$) is defined as third standard moment of a random variable $x_i(t)$ of a Gaussian distribution. Estimated kurtosis ($kurt$) on the other hand is given by fourth standard moment of a random distribution. The value tends to 3 as the sample size considered for the test increases [20]. For given sample set of $y_i(t)$ the estimated sample of $skew$ is given as

$$skew(y(t)) = \frac{\frac{1}{M}\sum_{i=0}^{M-1}(y_i(t)-\bar{y})^3}{\left(\frac{1}{M}\sum_{i=0}^{M-1}(y_i(t)-\bar{y})^2\right)^{\frac{3}{2}}}, \tag{6}$$

where $\bar{y}$ is the mean of a given signal data. Similarly, the estimated $kurt$ of a random sample is formulated as

$$kurt(y(t)) = \frac{\frac{1}{M}\sum_{i=0}^{M-1}(y_i(t)-\bar{y})^4}{\left(\frac{1}{M}\sum_{i=0}^{M-1}(y_i(t)-\bar{y})^2\right)^2}. \tag{7}$$

The test statistics $ST(s_t)$ of the periodogram (power spectral density) is represented as the square root of the sum of squares of $skew(y(t)$ and $kurt(y(t))$ as calculated in algorithm 1. When the value of test statistics is larger than a set threshold T_λ, the distribution of the received signals averaged periodogram deviates from the AWGN's power spectral density, which is an indicator of the presence of PUs signal. The test statistics of the periodogram estimate can be formulated as

$$ST(S_t) = \sqrt{skew(y(t))^2 + kurt(y(t))^2}, \tag{8}$$

where $skew(y(t))$ and $kurt(y(t))$ are the test statistics for $skew$ and $kurt$ respectively of the signal $x(t)$. For a given probability of false alarm (P_f), the threshold (T_λ) for $skew$ and $kurt$ tests the null hypothesis (H_0). This is a chi-squared distribution defined as $P_f = 1 - f(T_\lambda : H_0)$ and hence is formulated as [9]

$$T_\lambda = \sqrt{-\log(P_f)}. \tag{9}$$

In order to derive the probability of detection (P_d) and (P_f), the PDF for the test statistic is developed for both H_0 and H_1 as

$$\begin{cases} ST(S_t) \geq T_\lambda & H_1 \\ ST(S_t) < T_\lambda & H_0 \end{cases}. \tag{10}$$

Jarque-Bera (JB): The Jarque-Bera statistic has asymptotic chi-squared distribution with two degrees of freedom [10], formulated by considering the estimated $skew$ and $kurt$ on the transmitted PU signal, defined as [11]

$$JB = \frac{M}{6}\left[skew^2 + \frac{(kurt^2-3)^2}{4}\right], \tag{11}$$

where M=M_{FFT} is the number FFT points. In order to derive the P_d and P_f the hypothesis tests H_1 and H_0 are

Algorithm 1 Algorithm for kurtosis and skewness test

Input: $M = M_{FFT}$, $T = 3000$, $\gamma_j = -30:5$, $P_f = 0.1:1$
Output: $P_{d,kurt\,\&\,skew}, P_{d,JB}, P_{d,omnb}$
  $x(t) \leftarrow$ generate T random data, eqn. (2)
  $x_i(t) \leftarrow$ modulate $x(t)$ (16 QAM) plus noise, eqn. (4)
  fast Fourier transform on modulated signal
  $y_i(t) \leftarrow$ FFT on $x_i(t)(mod)$, eqn. (5)
  $y_{FFT} \leftarrow$ **break**$(y_i(t), M_{FFT}, T)$
  $y_i(t) \leftarrow$ **concatenation of** y_{FFT}
  $y_i(t) =$ real parts (y_{FFT}) + imaginary parts (y_{FFT})
  for $j =$ length (γ), $i =$ length (M_{FFT})
  Calculate kurtosis & skewness
  $skew(y(t)) \leftarrow$ **skewness test, eqn. (6)**
  $kurt(y(t)) \leftarrow$ **kurtosis test, eqn. (7)**
  while $\gamma_j \leq 0$, $n \leftarrow 0$ **do**
    $S_t \leftarrow$ **the test statistics, eqn.** (8)
    $T_\lambda \leftarrow$ **the threshold, eqn. (9)**
    if $ST(S_t) \geq T_\lambda$ **then**
      decision $= H_1$
      increment counter $\leftarrow H_1 = H_1 + 1$
    else $\{ST(S_t) \leq T_\lambda\}$
      decision $= H_0$
      increment counter $\leftarrow i = i+1, j = j+1$
      $P_{d,kurt/skew} = \text{sum}(\frac{H_1}{MFFT})$
    end if
  end while

formulated as

$$\begin{cases} JB \geq JB_\lambda & H_1 \\ JB < JB_\lambda & H_0 \end{cases}. \quad (12)$$

For a given probability of false alarm (P_f), the threshold for JB test based on null hypothesis (H_0), for an M_{FFT} points is expressed as [12]

$$JB_\lambda = 0.0688\, M_{FFT}. \quad (13)$$

For the null hypothesis to be accepted the test statistics must be smaller than a critical value that is positive and near zero. Higher values of JB indicate the sample do not follow the Gaussian distribution. The probability of detection is iteratively determined as shown in the pseudo code for algorithm 2.

Algorithm 2 Algorithm for Omnibus and Jarque-Bera

Input: $M = M_{FFT}, T = 3000, \gamma_j = -30:5$
Output: $P_{d,JB}, P_{d,omnb}$
  Calculate Jarque Bera test
  while $\gamma_j \leq 0,\ n \leftarrow 0$ **do**
    $JB \leftarrow$ **the test statistics, eqn.** (11)
    $JB_\lambda \leftarrow$ **the threshold, eqn. (13)**
    if $JB \geq JB_\lambda$ **then**
      decision $= H_1$
      increment counter $\leftarrow H_1 = H_1 + 1$
    else $\{JB \leq JB_\lambda\}$
      decision $= H_0$
      increment counter $\leftarrow i = i+1, j = j+1$
      probability of detection$(P_{d,JB}) = \text{sum}(\frac{H_1}{MFFT})$
    end if
  end while
  Calculate omnibus K^2 **test**
  while $\gamma_j \leq 0,\ m \leftarrow 0$ **do**
    $K^2 \leftarrow$ **the test statistics, eqn.** (14)
    $K^2_\lambda \leftarrow$ **the threshold, eqn. (16)**
    if $K^2 \geq K^2_\lambda$ **then**
      decision $= H_1$
      increment counter $\leftarrow H_1 = H_1 + 1$
    else $\{K^2 \leq K^2_\lambda\}$
      decision $= H_0$
      increment counter $\leftarrow i = i+1, j = j+1$
      probability of detection$(P_{d,omnb}) = \text{sum}(\frac{H_1}{MFFT})$
    end if
  end while

Omnibus (K^2) *test:* Omnibus is defined as the square root of a transformed skewness *(skewT)* and kurtosis *(kurtT)* test statistics. The asymptotic normal values for *(skew)* and *(kurt)* are used to construct a chi-squared test involving the first two moments of the asymptotic distributions [12], mathematically expressed as

$$K^2 = \sqrt{skewT^2 + kurtT^2}, \quad (14)$$

The hypothetical omnibus test is derived by comparing to defined threshold (K^2_λ) formulated as

$$\begin{cases} K^2 \geq K^2_\lambda & H_1 \\ K^2 < K^2_\lambda & H_0 \end{cases}. \quad (15)$$

For a predetermined P_f the threshold for omnibus test is a fixed value determined by

$$K^2_\lambda = 0.0688\, M_{FFT}, \quad (16)$$

where M_{FFT} is the number of FFT points. The $skewT$ on the estimated data sample is given as [11, 12]

$$skewT = \delta \log\left[\frac{Y}{\Phi} + \sqrt{\left(\frac{Y}{\Phi}\right)^2 + 1}\right], \quad (17)$$

where $\Phi = \sqrt{\frac{2}{W^2-1}}$ is a small deviation from the critical value on the skewness of the estimated distributed random data, $W^2 = (\sqrt{4B_2 - 4} - 1)$ is a constant of normalization on skewness, $\delta = \frac{1}{\sqrt{logW}}$ is the skewness parameter and (Y) is the estimated skewness value of the random distributed data given as

$$Y = skew\left[\frac{(M+1)(M+3)}{6(M-2)}\right], \quad (18)$$

where $skew = skew(y(t))$ is estimated skewness of the sampled signal data as given in equation (7), M is the number FFT data sample points. The skewness as a function of the variance $\mu_2(skew)$ is formulated as

$$\mu_2(skew) = B_2 = \frac{3(M^2 + 27M - 70)(M+1)(M+3)}{(M-2)(M+5)(M+7)(M+9)}. \quad (19)$$

The transformed kurtosis $(kurtT)$ on the random distributed received PU's signal is also formulated as [11, 12]

$$kurtT = \frac{(1 - \frac{2}{9D})\left[\frac{1-\frac{2}{D}}{1+x\sqrt{\frac{2}{D-4}}}\right]^{\frac{1}{3}}}{\sqrt{\frac{2}{9D}}}, \quad (20)$$

where D is a constant that denotes the degrees of freedom for the chi-squared distribution. Solving for D to equate the third moment of theoretical and sampling distributions, it is possible then to compute D as follows

$$D = 6 + \frac{8}{B_1}\left[\frac{2}{B_1} + \sqrt{1 + \frac{4}{B_1}}\right], \quad (21)$$

where $B_1 = \mu_1(kurt)$ is the kurtosis as a function of the mean (μ_1), given as

$$\mu_1(kurt) = B_1 = \frac{6(M^2 - 5M + 2)}{(M+7)(M+9)}\sqrt{\frac{6(M+3)(M+5)}{M(M-2)(M-3)}}, \quad (22)$$

where $kurt = kurt(y(t))$ is the estimated kurtosis given in equation (7) and M is the number of samples. It is possible to standardize kurtosis by formulating the expression as

$$x = \frac{kurt - \mathbf{E}[kurt]}{\sqrt{\mathbf{var}[kurt]}}, \tag{23}$$

where the mean as a function of kurtosis is given as $\mathbf{E}[kurt] = \frac{24M(M-2)(M-3)}{(M+1)^2(M+3)(M+5)}$ and variance as a function of kurtosis is expressed as $\mathbf{var}[kurt] = \frac{3(M-1)}{M+1}$, are all computed to determine transformed estimated kurtosis s shown in algorithm 2.

Algorithm 3 First Stage Optimal Selection of SUs

Input: $N = 15, SNR = -30:2:-5.0$
Output: λ_n^{opt}, n, $r = \frac{N}{2}$
  intialize: $n = 1 \leftarrow$ sort all SUs in descending order SNR
  calculate the following
  step1: $\lambda_i^* \leftarrow$ the threshold of *ith* SU, eqn. (33)
  step2: $P_e^{r,n} \leftarrow$ the error detection, eqn. (27)
  step3: $P_{f,1|\gamma,\theta}$ and $P_{d,1|\gamma,\theta} \leftarrow 1^{st}$ iterate, eqn. (26) & (30)
  step4: $Q_d^{r,n} \leftarrow$ the detection prob, eqn. (34)
  step5: $Q_f^{r,n} \leftarrow$ the false alarm , eqn. (35)
  step6: $Q_e^{r,n} \leftarrow$ the decremental error , eqn. (40)
  for $i =$ length (n) and $r =$ length($\frac{N}{2}$)
  while $n \le m$, $n \leftarrow 0$ **do**
    if $Q_e^{r,n} \ge 0$ **then**
      $i = n+1$
      increment counter $\leftarrow n = n+1$
      $\lambda_n^{opt} \leftarrow$ the optimal threshold, eqn. (39)
      go to step 4
    else $\{Q_e^{r,n} \le 0\}$
      $n = n-1 \leftarrow$ **delete the SU**
      go to step 4 otherwise have attained the solution
    end if
  end while

5. FUSION SCHEMES

5.1 Fusion strategy hypothesis tests

The null hypothesis (H_0) for decision statistics of the omnibus test can be derived as

$$\begin{cases} K^2 \ \ge \ H_1 & \lambda \\ K^2 \ < \ H_0 & \lambda \end{cases}, \tag{24}$$

where λ is the decision threshold which has to be optimized. The cost functions are formulated in terms of probability of misdetection and false alarm as conditioned on the channel, the probability of misdetection is formulated as [22]

$$P_{m,i|\gamma,\theta} = 1 - \frac{1}{2}\mathbf{erfc}\left(\frac{\lambda_i - K^2}{\sqrt{2}\sigma_1(\gamma,\theta)}\right) + \frac{1}{2}\mathbf{erfc}\left(\frac{\lambda_i + K^2}{\sqrt{2}\sigma_1(\gamma,\theta)}\right), \tag{25}$$

where $\gamma = |h|^2\left(\frac{\mathbf{E}[|\mathbf{x(t)}|^2]}{\mathbf{E}[|\mathbf{w(t)}|^2]}\right)$ is given as the instantaneous SNR. The instantaneous channel phase angle θ is defined as $\theta = \tan^{-1}\left(\frac{\mathbf{Im}[\mathbf{x(t)}^2]}{\mathbf{Re}[\mathbf{w(t)}^2]}\right)$, $w(t)$ is the AWGN. The probability of misdetection $(P_{m,i}|\gamma,\theta)$ is the sum of the lower bound probability $P_{m,1|\gamma,\theta} = \frac{1}{2}\mathbf{erfc}\left(\frac{\lambda_i - K^2}{\sqrt{2}\sigma_1(\gamma,\theta)}\right)$ and upper bound probability $P_{m,2|\gamma,\theta} = \frac{1}{2}\mathbf{erfc}\left(\frac{\lambda_i + K^2}{\sqrt{2}\sigma_1(\gamma,\theta)}\right)$. Unlike in [22], this paper uses omnibus test (K^2) instead of kurtosis. λ_i is the decision threshold, $\sigma_1(\lambda,\theta) = a_{00} + a_{10}\gamma + \left[a_{20} + a_{21}\sin^2(2\theta)\right]\gamma^2 + \left[a_{30} + a_{31}\sin^2(2\theta)\right]\gamma^3 + \left[a_{40} + a_{41}\sin^2(2\theta + a_{42}\sin^4(2\theta)\right]\gamma^4$ is expressed in terms of instantaneous SNR and phase angle of a circular Gaussian channel. The following constants; $a_{00}, a_{10}, a_{20}, a_{21}, a_{30}, a_{31}, a_{40}, a_{41}$ & a_{42} are given in table 1. The conditional (on the channel) probability of false alarm is given as

$$P_{f,i|\gamma,\theta} = \frac{1}{2}\mathbf{erfc}\left(\frac{\lambda_i - \mu_0}{\sqrt{2}\sigma_0}\right) + \frac{1}{2}\mathbf{erfc}\left(\frac{\lambda_i + \mu_0}{\sqrt{2}\sigma_0}\right), \tag{26}$$

where θ is the phase angle, γ is the SNR of the signal, σ_0 is the modulation constant and μ_0 is the mean of the data distribution as given in table 2.

5.2 First stage optimization on SU selection criteria

The aim of the first stage optimization is to iteratively select n SUs in $\forall\ n \in [1,N]$ SUs, in an *r out of n* counting rule where r is the number of SUs that form the combinatorial n fusion order and N is the total number of SUs in CSS network. The criteria on selection is based on SUs decrementing SNR as formulated in algorithm 3. The error probability is further expressed as

$$P_{e,i} = P(H_0)Q_f + P(H_1)Q_m, \tag{27}$$

where $P(H_0)$ is the null hypothesis, $P(H_1)$ is the alternative hypothesis, Q_f is the global probability of false alarm and Q_m is probability of misdetection. The sum of probability of false alarm and misdetection is derived as a cost function to determine the global decremental error probability (Q_e) in the detection of the primary user in CSS network. The minimization problem is formulated as [15, 16, 18, 19]

$$\begin{aligned} &\min_{\lambda}\left(Q_m(\lambda^{opt}) \text{ and } Q_f(\lambda^{opt})\right) \\ &\text{Subject to } Q_e > 0 \end{aligned}, \tag{28}$$

where λ^{opt} is the optimal decision threshold. Considering equation (25) and equation (26), the optimal threshold is formulated as

$$\lambda_i^* = \arg\min_{\lambda}\left(P_{e,i} = (\beta P_{f,i|\gamma,\theta} + P_{m,i|\gamma,\theta})P(H_1)\right), \tag{29}$$

where $\beta = \frac{P(H_0)}{P(H_1)}$ is the detection factor, $P_{f,i|\gamma,\theta}$ is the false alarm and $P_{m,i|\gamma,\theta}$ is the misdetection of the *ith* SU. From equation (25), the probability of detection is similarly given as

$$P_{d,i|\gamma,\theta} = 1 - P_{m,i|\gamma,\theta}. \tag{30}$$

Consequently from equation (29), the threshold is maximized as follows

$$\begin{aligned}\lambda_i^* =&\arg\max_{\lambda}\left((P_{d,i|\gamma,\theta}-\beta P_{f,i|\gamma,\theta}-1)P(H_1)\right)\\ =&\arg\max_{\lambda}\left(P_{d,i|\gamma,\theta}-\beta P_{f,i|\gamma,\theta}\right).\end{aligned} \tag{31}$$

By the Lagrange theorem, the maximum threshold is obtained by differentiating by parts as follows

$$\frac{\partial P_{d,i|\gamma,\theta}}{\partial\lambda_i}\bigg|_{\lambda_i^*}=\beta\frac{\partial P_{f,i|\gamma,\theta}}{\partial\lambda_i}\bigg|_{\lambda^*}, \tag{32}$$

where $i=1,...,n$ is the number of SUs selected to participate in fusion and λ_i^* is the initial optimal threshold derived as

$$\lambda_i^*=\frac{\sigma_s^2}{2}+\sigma_s^2\sqrt{\frac{1}{4}+\frac{\gamma_i}{2}+\frac{4\gamma_i+2}{M\gamma_i}\log\left(\beta\sqrt{2\gamma_i+1}\right)}, \tag{33}$$

where σ_s^2 is the noise variance, γ_i is the SNR of the *i-th* SU and M is the number of signal data samples. The global probability of detection in *r out of n* rule is derived as

$$Q_d^{(r,n)}=\sum_{j=r}^{n}\binom{n}{j}\prod_{i=1}^{j}P_{d,i|\gamma,\theta}\prod_{i=j+1}^{n}\left(1-P_{d,i|\gamma,\theta}\right), \tag{34}$$

where $n\in\{j=1,..,N\}$, N is the total number of SUs, $P_{d,i|\gamma,\theta}=1-P_{m,i|\gamma,\theta}$ is probability of detection as given in equation (25), r is the actual number of SUs that form *r out of n* counting rule and n is the total number of SUs selected to participate in decision making. Similarly, the global probability of false alarm is formulated as

$$Q_f^{(r,n)}=\sum_{j=r}^{n}\binom{n}{j}\prod_{i=1}^{j}P_{f,i|\gamma,\theta}\prod_{i=j+1}^{n}\left(1-P_{f,i|\gamma,\theta}\right), \tag{35}$$

where $n\in\{j=1,..,N\}$, $P_{f,i|\gamma,\theta}$ is probability of false alarm as given in equation (26). The selection criteria is done by the iterative calculation of global probability detection and false alarm simultaneously, as performed in algorithm 3. The minimization problem stated in equation (28) is formulated mathematically as

$$Q_d^{r,n}=Q_d^{(r-1,n-1)}(P_{d,n|\gamma,\theta})+Q_d^{(r,n-1)}(1-P_{d,n|\gamma,\theta}), \tag{36}$$

where $Q_d=1-Q_m$ is the global probability of detection, the probability of false alarm is similarly derived as

$$Q_f^{r,n}=Q_f^{(r-1,n-1)}(P_{f,n|\gamma,\theta})+Q_f^{(r,n-1)}(1-P_{f,n|\gamma,\theta}). \tag{37}$$

The final iteration gives the optimal threshold λ_n^{opt} given for n number of SUs, formulated as

$$Q_d^{(r,n-1)}\frac{\partial P_{d,n|\gamma,\theta}}{\partial\lambda_n}\bigg|_{\lambda_n^{opt}}=\beta Q_f^{(r,n-1)}\frac{\partial P_{f,n|\gamma,\theta}}{\partial\lambda_n}\bigg|_{\lambda^{opt}}, \tag{38}$$

where the optimal threshold is given in this scenario as

$$\lambda_n^{opt}=\frac{\sigma_s^2}{2}+\sigma_s^2\sqrt{\frac{1}{4}+\frac{\gamma_n}{2}+\frac{4\gamma_i+2}{M\gamma_n}\log\left(\beta\sqrt{2\gamma_i+1}*B\right)}, \tag{39}$$

where $B=\frac{Q_f^{(r-1,n-1)}-Q_f^{(r,n-1)}}{Q_d^{(r-1,n-1)}-Q_d^{(r,n-1)}}$ is the detection factor, γ_n is the SNR for the *n-th* SU, σ_s^2 is the noise variance and M is the signal data samples. The decremented detection error is expressed as

$$\begin{aligned}Q_e^{(r,n)}=&P(H_1)P_{d,n|\gamma,\theta}\left(Q_d^{(r-1,n-1)}-Q_d^{(r,n-1)}\right)\\ &-P(H_0)P_{f,n|\gamma,\theta}\left(Q_f^{(r-1,n-1)}-Q_f^{(r,n-1)}\right),\end{aligned} \tag{40}$$

where the $P(H_0)$ and $P(H_1)$ are the weights for probability of false $(P_{f,n|\gamma,\theta})$ and probability of detection $(P_{d,n|\gamma,\theta})$ respectively, n is the number of SUs participating in detection of the presence or absence of the PU on the channel, γ is the SNR and θ is the uniformly distributed phase angle.

5.3 Second stage optimal strategy

At the FC, a specific *k out of n* strategy is employed to process the SUs' received decisions at the FC. Where k is number of SUs in the range of $(1\le k\le n)$ and n is the total number of SUs selected from a total of N as realized in the first optimization stage. The idea behind this rule is to find the number of SUs whose local binary decisions is 1. If this number is larger than or equal k, then the spectrum is said to be used otherwise the spectrum is unused. An iterative numerical search determined in algorithm 4 is carried out to find an optimal number of k SUs in *k out of n* combinatorial order is done at the FC. To achieve this an upper-threshold of global probability false alarm (Q_f) of less than utilization level (ε) is set.
The maximization problem can be formulated as [7,15,16]

$$\begin{aligned}&\max_{1\le k\le n}\left(Q_d(k)\right)\\ &\text{Subject to } Q_f(k)<\varepsilon.\end{aligned} \tag{41}$$

The global probability of false alarm Q_f based on *k out of n* counting rule is formulated in algorithm 3 and mathematically derived as

$$Q_f(k)=\sum_{j=k}^{n}\binom{n}{j}\left(P_{f,i|\gamma,\theta}^k\right)\left(1-P_{f,i|\gamma,\theta}\right)^{n-k}=\varepsilon, \tag{42}$$

where ε is the utilization level, k is number of SUs selected to participate in the *k out of n* fusion process, n is number of SUs iteratively found in the first optimization stage in section 5.2. The derivative of global probability of false alarm (Q_f) as function of (P_f) is derived as

$$\begin{aligned}\frac{\partial Q_f(P_f)}{\partial(P_f)}=&n\binom{n-1}{k-1}P_{f,i|\gamma,\theta}^k\left(1-P_{f,i|\gamma,\theta}\right)^{n-k-1}\\ =&n\varphi(k-1,n-1,P_{f,i|\lambda,\theta})>0.\end{aligned} \tag{43}$$

From equation (43) it follows that φ is the binomial cumulative function give as

$$\varphi = \binom{n-1}{k-1}\left(P^{k}_{f,i|\gamma,\theta}\right)\left(1-P_{f,i|\gamma,\theta}\right)^{n-k}. \quad (44)$$

Subsequently the global probability of detection in *k out of n* case is given as

$$Q_d(k) = \sum_{j=k}^{n}\binom{n}{j}\left(P^{k}_{d,i|\gamma,\theta}\right)\left(1-P_{d,i|\gamma,\theta}\right)^{n-k} > 0. \quad (45)$$

To optimize equation (45), we differentiate by parts the function as follows

$$\frac{\partial Q_d(P_d)}{\partial(P_d)} = n\binom{n-1}{k-1}P^{k}_{d,i|\gamma,\theta}\left(1-P_{d,i|\gamma,\theta}\right)^{n-k-1} > 0. \quad (46)$$

From equations (25) and (26) the following probabilities must hold true.

$$\frac{P_{d,i|\gamma,\theta}}{P_{f,i|\gamma,\theta}} > \frac{\partial(P_{d,i|\gamma,\theta})}{\partial(P_{f,i|\gamma,\theta})} > \frac{1-P_{d,i|\gamma,\theta}}{1-P_{f,i|\gamma,\theta}}. \quad (47)$$

Similarly the above equation can be further formulated as follows

$$\frac{Q_d(k)}{Q_f(k)} = \frac{\partial Q_d(k)}{\partial P_f(k)} * \frac{\partial P_f}{\partial Q_f} = \frac{\frac{\partial Q_d(k)}{\partial P_d(k)} * \frac{\partial P_d}{\partial P_f}}{\frac{\partial Q_f}{\partial P_f}}. \quad (48)$$

From the above equation it is true to say $Q_d(k)$ is linearly increasing function of $Q_f(k)$. For all $k \in [1,n]$ then the roots of $Q_f(k,P_f)$ are formulated in Bisection algorithm 3. The algorithm is broken down as follows, for each $P_{f,i|\gamma,\theta}$ determine the corresponding $P_{d,i|\gamma,\theta}$ and $Q_d(k,P_f)$, select the highest global probability, the value of k is the optimal number of SUs.

6. ENERGY EFFICIENCY

Energy efficiency is the ratio of throughput to average energy consumed during the cooperative spectrum sensing time. The throughput ($\overline{THR}$) is formulated as [21, 23]

$$\overline{THR} = P(H_0)(1-Q_f)Rt, \quad (49)$$

where R is the data rate, t is the transmission time length, $P(H_0)$ is the probability that the spectrum is not being used, Q_f is the global probability of false alarm. The average energy consumed in the network by all SUs E_c is derived as

$$E_c = n\,e_{su} + P_u\,e_{st}, \quad (50)$$

where n is the total number of SUs selected from first optimization stage, e_{su} is the energy consumed during CSS by all the SUs, e_{st} is the energy consumed during data transmission, P_u is the probability of identifying if the spectrum is idle, given as

$$P_u = P(H_0)(1-Q_f) + P(H_1)(1-Q_d), \quad (51)$$

where $P(H_1) = 1 - P(H_0)$ is the probability of the spectrum being used, Q_f is the global probability of false alarm and Q_d is the probability of detection. Note that the

Algorithm 4 Second Stage Bisection Algorithm

Input: $P_f = P_{f,i|\gamma,\theta}$, $\varepsilon = 0.001$
Output: k, $Q_d(k)$
  n ← from algorithm 3
  intialize: $endpoints \leftarrow P_{f,L} = 0.01, P_{f,U} = 0.1$
  for i = length (P_f) and k = length(n)
  while $Q_f(k) \leq \varepsilon$, $k \leftarrow 1 \leftarrow$ from eqn.(42) **do**
    if $P_{f,U} \leq P_{f,L}$, $Q_f(P_{f,L}) \leq 0$ and $Q_f(P_{f,L}) > 0$ **then**
      $mid\ (P_f) = \frac{P_{f,L} - P_{f,U}}{2}$
      condition: if $Q_f(mid\ (P_f)) = 0$ **then**
      solution is found else
      Determine the following;
      $P_{d,1|\gamma,\theta}$ ← cal. detection probability, eqn. (25)
      $Q_f(1)$ ← cal.the false alarm, eqn. (42)
      $Q_d(1)$ ← cal. detection probability, eqn. (45)
    else $\{Q_f(P_{f,L}) > 0$ and $Q_f(P_{f,U}) < 0\}$
      mid $(P_f) = \frac{P_{f,U} - mid\ (P_f)}{2}$
      if $sign\ Q_f(\text{mid}\ (P_f)) = sign\ Q_f(P_{f,U})$ **then**
      $P_{f,L}$ ← mid (P_f)
      else
      $P_{f,U}$ ← mid(P_f)
      increment counter ← $k = k+1$ and $i = i + 0.01$
      untill $|Q_f(k)| < \varepsilon$
      determine the biggest $Q_d(k)$
      optimal value of (k) found.
      Determine η
      for $k = 1, ..n$
      η ← cal. the effeciency eqn.(52)
    end if
  end while

energy consumption during transmission occurs only if the spectrum is identified as unused. The efficiency (η) can be formulated as [20, 21]

$$\eta = \frac{\overline{THR}}{E_c} = \frac{P(H_0)(1-Q_f)Rt}{n\,e_{su} + (1 - P_0 Q_f - P_1 Q_d)e_{st}}, \quad (52)$$

where n is number of SUs in equation (52), computed as

$$n = ln\left(\frac{P(H_1)(1-Q_f)e_{st}}{Ne_{su} + P(H_1)(1-Q_d)e_{st}}\right) - k\,ln\left(\frac{P_y(1-P_x)}{P_x(1-P_y)}\right), \quad (53)$$

where N is the total SUs in CSS network, k is the number of SUs in the *k out of n* counting rule. A noisy channel is modeled as binary symmetric channel with error probability (P_e) and it is the same among all SUs. $p_x = P_{d,i|\gamma,\theta}(1-P_e) + (1-P_{d,i|\gamma,\theta})P_e$ is the probability of receiving a local binary decision of 1 when the spectrum is busy and $p_y = P_{f,i|\gamma,\theta}(1-P_e) + (1-P_{f,i|\gamma,\theta})P_e$ is the probability of receiving a local binary decision of 1 when the spectrum is idle.

Table 1: Modulation constants

Parameters	Actual values used
a_{00}, a_{10}, a_{20}, a_{21}	$\frac{24\rho_n^8}{M}$, $\frac{96\rho_n^8}{M}$, $\frac{46\rho_n^8}{M}$, $\frac{-48.96\rho_n^8}{M}$
a_{30}, a_{31}, a_{40}, a_{41}	$\frac{33.28\rho_n^8}{M}$, $\frac{128.64\rho_n^8}{M}$, $\frac{10.33\rho_n^8}{M}$, $\frac{-1.93\rho_n^8}{M}$
a_{42}, σ_0, μ_0	$\frac{1.74\rho_n^8}{M}$, $\frac{24\rho_n^8}{M}$, 1

7. SIMULATION RESULTS

In order to evaluate the HOS test for cooperative spectrum sensing capability, we considered a cognitive radio network with 15 SUs transmitting on 16 QAM constellation modulated signal built in matlab software for simulation and analysis. It should be noted that any other modulation scheme can be used to model the PU signal. In all subsequent figures, the numerical results are plotted on receiver operating characteristics curves (ROC). Simulation results are denoted with discrete marks on the curves. The simulation parameters are give in table 2.

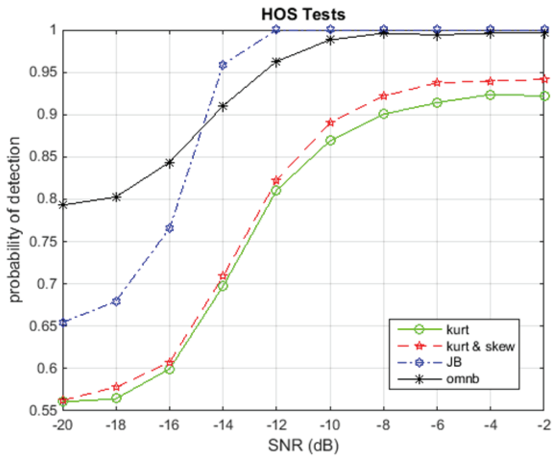

Figure 3: Detection probability for HOS tests against a range of SNR in 2048 FFT data points

In figure 3, the ROC curves show the performance of omnibus ($omnb$), Jarque-Bera (JB), kurtosis & skewness ($kurt\,\&\,skew$) and kurtosis ($kurt$) test statistics as function of the SNR. In this scheme 2048 FFT sample points were considered. From the plot, as expected the probability of detection increased with increase in SNR starting from a low SNR. The *omnb* test displayed the highest probability of detection progressively from a low SNR up to about -16 dB. The plot shows that *omnb* perform better at low SNR. This was followed by JB, then *kurt & skew*. The results of the other HOS tests are close to those in [9, 10, 20].

In figure 4, the graph illustrates performance of the four HOS test considered under a smaller data sample of 512 FFT points. The plot shows *omnb* still has higher detection probability for all ranges of SNR and even better under extremely low SNR (-30dB). The *omnb* test technique therefore tends to suppress the Gaussian noise showing an improved performance. From the two results displayed in

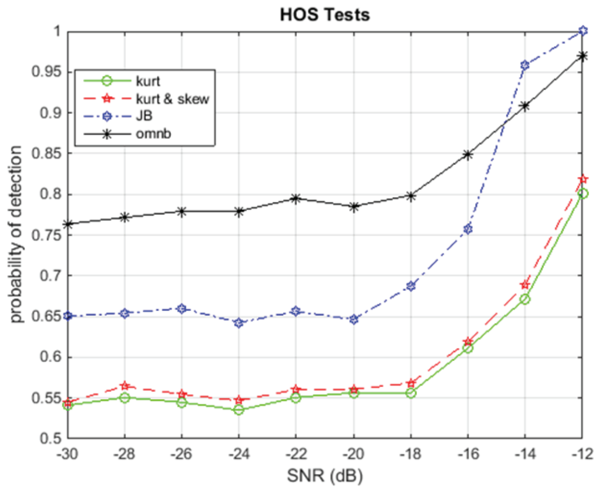

Figure 4: Detection probability for HOS tests against a range of SNR in 512 FFT data points

figures (3) and (4), it can be concluded that omnibus is a superior statistical test for both small and big data sample at low SNRs.

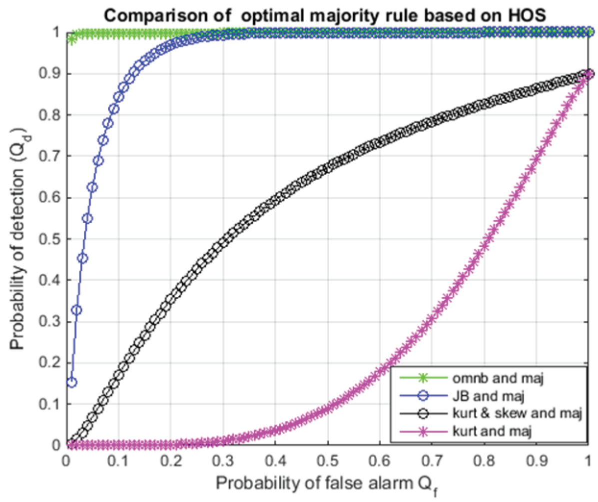

Figure 5: Global probability of detection against false alarm for HOS tests

In figure 5, the performance of optimal *k out of n* counting rule based on all HOS tests is displayed. The rules are for omnibus and majority rule ($omnb\ and\ maj$), Jarque-Bera and majority ($JB\ and\ maj$), kurtosis & skewness and majority ($kurt\ \&\ skew\ and\ maj$). The optimal number of 8 out of 10 SUs was realized through a two stage optimization as given in algorithms (3) and (4). From ROC curves it can deduced that a combination of $omnb\ and\ maj$ displayed a higher probability of detection for a false alarm of less than 0.1. This is as per the requirement of IEE 802.22 standards [17]. The performance was then followed by $JB\ and\ maj$ and lastly $kurt\ \&\ skew\ and\ maj$.

Figure 6, shows a comparative performance HOS based optimal majority rules; $omnb\ and\ maj$, $JB\ and\ maj$, $kurt\,\&\,skew\ and\ maj$ and lastly $kurt\ and\ maj$. The

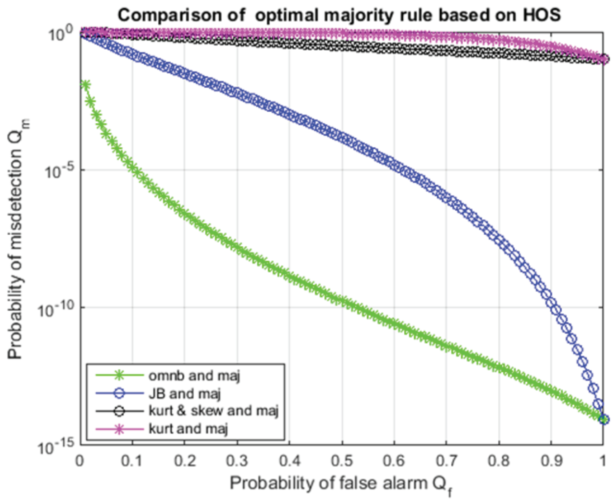

Figure 6: Global probability of misdetection against false alarm for HOS tests

optimal number of 8 *out of* 10 SUs was realized in the algorithm 3. From the plot, it can be deduced that *omnb and maj* combination strategy displayed the lowest probability of misdetection for all values of probability of false alarm as compared to the three other combinations. In conclusion, based on the figure (5) and (6), *omnb and maj* rule showed the highest probability of detection and the lowest misdetection as compared to all the other HOS based majority rule for all ranges of false alarm.

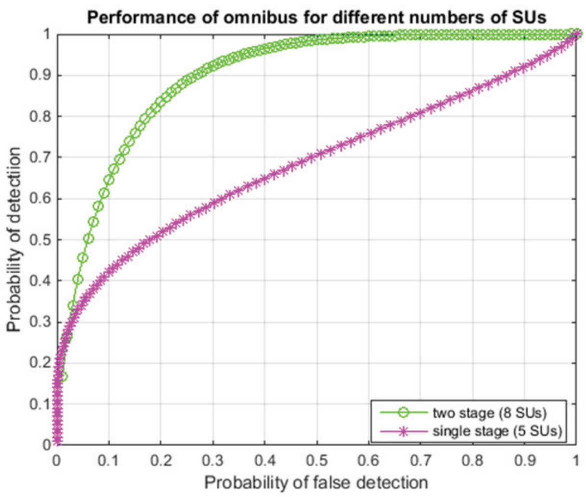

Figure 7: Comparative analysis on single and two stage optimization

Figure 7, shows the performance of hybrid sensing scheme of *k out of n* counting rule based omnibus test for different numbers of SUs was investigated. The plot showed the comparative performance of different numbers of SUs as selected in single stage compared to two stage optimization. Where $n = 10$, $k = 5$ and $k = 8$ respectively. From this plot, it can be deduced that omnibus a local detection test based on a two stage optimization global detection scheme displayed higher probability of detection to that of single stage optimization for all ranges of false alarm.

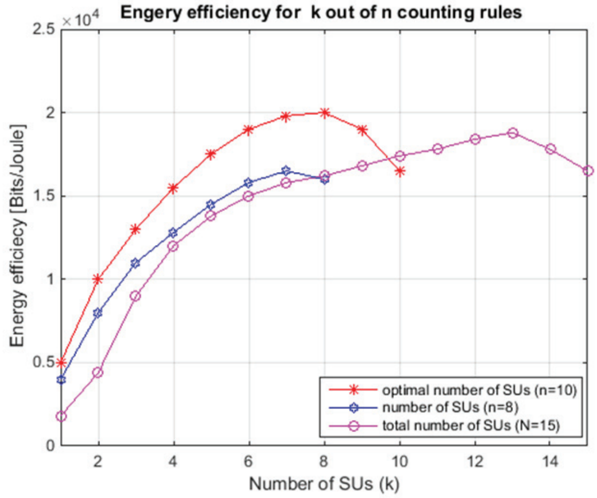

Figure 8: Energy efficiency in *k out of n* counting rule.

Figure 8, shows the energy efficiency for the different *k out of n* counting rules representing three scenarios. The first case is when all the SUs in the cooperative spectrum sensing $N = 15$ participate in the detection of the PU. The second case is when an optimal number of SUs as found in the first optimization stage $n = 10$ and the third case is when $n = 8$ just for the purpose of benchmarking. From this plot the optimal case showed the greatest energy efficiency of about $2 * 10^4$ Bits per Joule. This was achieved when $k = 8$ SUs in the combinatorial order of 8 *out of* 10 counting rule. Note that due to the *k out of n* rule the number of k can only go up to n number of SUs.

Table 2: Simulation parameters

Simulation parameters	Actual values used
$P(H_0)$ *and* $P(H_1)$	0.5
Frequency range	0-800
Monte Carlo trials	10^3 to 10^4
Noise variance σ_n	1
phase angle, Range of δ	$0 \leq \theta \leq 2\pi$, $0 \leq \delta \leq 1$
mean (μ_0)	0
e_{st}, e_{su}	1 Joule, 100 mJoule
Tran.time (t), Data rate (R)	0.5 sec, 100 kbps
SNR	$-30 \leq \gamma \leq 0$

8. CONCLUSION

In our proposed hybrid model, an optimal *k out of n* based omnibus (K^2) statistics test was shown to be more superior to the other HOS tests. This model would be preferred to detect the PU in cognitive radio networks operating under noisy conditions. Another advantage of this model is the overall reduction in energy consumption in the network due to the two stage optimization. Fewer SUs make the final decision on the status of the PU on the channel but still maintain reliable decision outcomes.

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