DETECTION OF GSM AND GSSK SIGNALS WITH SOFT-OUTPUT DEMODULATORS

A. A. Tijani*, N. Pillay and H. Xu*****

**School of Engineering, University of KwaZulu-Natal, Durban 4041, Republic of South Africa, E-mail: adekilekunmajeed@gmail.com*
*** Senior Lecturer, School of Engineering, University of KwaZulu-Natal, Durban 4041, Republic of South Africa, E-mail: pillayn@ukzn.ac.za*
****Professor, School of Engineering, University of KwaZulu-Natal, Durban 4041, Republic of South Africa, E-mail: xuh@ukzn.ac.za*

Abstract: A special case of conventional spatial modulation (SM) is demonstrated in space shift keying (SSK) modulation, where the amplitude and/or phase modulation symbols are eliminated from the transmission process so as to reduce system complexity. However, the spectral efficiencies of both schemes increase only logarithmically. Hence, large antenna arrays are required to achieve high spectral efficiencies. To reduce the transceiver overhead, while achieving improved spectral efficiencies in both SM and SSK, generalised SM (GSM) and generalised SSK (GSSK), respectively, were proposed. Typically, in coded channels, soft-output detection coupled with soft-input channel decoding yields significant signal-to-noise ratio (SNR) gain. Hence, in this paper, we propose soft-output maximum-likelihood (ML) detectors (SOMLDs) for the GSM and GSSK schemes, with an aim to further improve the error performance of the systems. Monte Carlo simulation results demonstrate that the error performance of the proposed SOMLD schemes closely match with that of their hard-decision ML detector counterparts in uncoded channels; while significant SNR gains are yielded in coded channels.

Key words: Generalised space shift keying, generalised spatial modulation, soft-output detection, space shift keying, spatial modulation.

1. INTRODUCTION

Conventional multiple-input multiple-output (MIMO) systems achieve improved capacity and diversity, but with challenges, such as the need for inter-antenna synchronization (IAS), large system computational complexity overhead, inter-channel interference (ICI), and increased form-factor [1, 2]. A number of schemes have recently been proposed to address these challenges. For instance, ideal spacing of transmit and receive antennas is suggested as a means to achieve IAS, while ICI is avoided by compressing an amplitude/phase modulation (APM) symbol prior to transmission, in order to exploit its spatial information at the receiver input [3, 4]. Research also demonstrated that data rates of wireless communication systems can be considerably increased, without a corresponding increase in transmit power, modulation order or bandwidth, while retaining the other advantages of MIMO [5, 6].

Spatial modulation (SM) was proposed [7] with an idea of employing the index of a single transmit antenna as an extra means to convey information. Original information is divided into two parts: the first part is mapped to a chosen symbol from an APM signal constellation, while the remaining part determines the transmit antenna that is to be activated for transmission of the symbol. The mappings are said to be performed in the in-phase and quadrature (IQ) domain and spatial domain, respectively. This setup results in increased spectral efficiency by base two logarithm of the number of transmit antennas and, in a novel way, overcomes ICI at the receiver. It should be noted that the dormant antennas transmit zero power during each and every transmission. As a result, SM needs no IAS, and requires a single radio frequency (RF) chain, which translates into a relatively low-complexity receiver [8, 9]. An optimal detector [10] for SM jointly estimates the active antenna index and the modulated data symbol at the receiver.

Extensive research has been conducted on SM over the years and has shown that the scheme is simple in concept [7-10]. In [11], a practical implementation of SM is considered for high millimetre-wave (mmWave) frequency. A simpler and more energy-efficient SM transmission was proposed for indoor environments, which is dominated by line-of-sight (LOS) components. A novel transmitter architecture which features a parallel shunt switching structure and a flexible power delivering network was proposed. This selects power amplifiers according to the link power budget, such that hybrid digital processing is employed together with a separate detection of the spatial and IQ streams at the receiver. The setup was shown to be promising as it further optimizes the error performance of SM, especially for short-range indoor mmWave communications.

Based on the above-mentioned merits, the SM scheme proves to be a good candidate for deployment in next generation wireless communication systems. However, further studies reveals that options exist for decreasing its system complexity; thus, a variant of SM in the form of space shift keying (SSK) modulation was proposed [12]. In SSK modulation, only the spatial domain of SM is exploited to relay information. The elimination of the APM results in lowered detection complexity, less stringent transceiver requirements, and simplicity, while exploiting other advantages of SM [12]. Because of its simplicity, the authors of [13] employed SSK modulation to present the SM idea in inherently uncorrelated LOS conditions. High performance, as well as capacity, is achieved by proper placement of transceivers in mmWave MIMO communication.

However, a common criticism of both schemes (SM and SSK) lies in their spectral efficiencies, which increase logarithmically, such that large antenna arrays are required to achieve large efficiencies [14]. Alternative approaches, to innovatively circumvent the constraints that the number of transmit antennas must be a power of two in SM, were proposed as generalised SM (GSM) modulation [15] and generalised SSK (GSSK) modulation [16], respectively. These flexible forms of SM activate more than one transmit antenna at an instance of time.

In GSM modulation [15], a combination of transmit antennas is activated to convey replicas of the chosen constellation signal over the wireless channel to the receiver. Transmitting the same data symbol from more than one antenna at a time, retains the key advantage of SM, which is the complete avoidance of ICI at the receiver. The reliability of the wireless channel is also increased by making available replicas of the transmitted signal at the receiver, in addition to offering diversity gains. As a result, the number of transmit antennas required to achieve a certain spectral efficiency is typically reduced by more than a half in GSM modulation as compared to SM [15]. In the same vein, GSSK modulation represents a fundamental component of SM, which inherently exploits fading in wireless communication to provide better performance over conventional APM techniques. In GSSK modulation, only antenna indices (and not a single antenna index as in the case of SM) are exploited to modulate information. The transmitted GSSK symbols (similar to the case of SSK symbols) are just a means of identifying the activated antennas. In doing so, the aforementioned advantages of SM are maintained, while reducing the transceiver overhead and achieving greater design flexibility [8, 16].

Generally, SM and SSK modulation are generalised by sending the same symbol from more than one transmit antenna at a time to yield GSM modulation and GSSK modulation, respectively; hence, they are no longer limited to a number of transmit antennas, which strictly have to follow a power of two. Instead, an arbitrary number of transmit antennas can be used. Moreover, higher spectral efficiencies can be achieved, in both GSM modulation and GSSK modulation, with a reduced number of transmit antennas. It should be noted that these enhancements are achieved at the cost of a slight increase in complexity - which largely depends on the number of active transmit antennas. However, the increase in complexity is outweighed by the significant reduction in the number of transmit antennas. The schemes in [11] and [13] have also been investigated for GSM, to achieve improved error performance in an indoor mmWave communications setup.

Nowadays, and in practice, the majority of communication systems employ channel coding, in order to achieve coding gains [17]. In [18], a trellis coded modulation (TCM) scheme was applied to the antenna constellation points of SM to enhance performance in correlated channel conditions. TCM is a well-known technique that reduces power requirements without any bandwidth expansion. Moreover, it has been demonstrated that a combination of soft-output detection with soft-input channel decoding results in a net coding gain compared to the conventional hard decision detection/decoding [19-22]. On this note, soft-output detection algorithms for orthogonal frequency division multiplexing based SM (SM-OFDM) and space-time block coded SM (STBC-SM) were proposed, and significant signal-to-noise ratio (SNR) gains were demonstrated [19-21]. Similar detectors have also been proposed for Bi-SSK in [23]. However, no such detectors have been proposed for GSM modulation and GSSK modulation. Hence, in this paper, we are motivated to propose soft-output detectors for GSM and GSSK modulated signals.

The remainder of the paper is organized as follows: System models of the GSM modulation and GSSK modulation schemes are presented in Section 2. The proposed soft-output detectors are presented in Section 3, simulation results and discussions are given in Section 4, and finally, conclusions are drawn in Section 5.
Notation: Throughout the paper, matrices are denoted with bold italics uppercase letters, vectors with bold italics lowercase letters and scalars with regular letters. $(\cdot)^{\mathrm{T}}$ is used for transpose, $\|\cdot\|_{\mathrm{F}}$ for the Frobenius norm of a vector or matrix and $\lfloor\cdot\rfloor$ is the floor operation and $\underset{x}{\operatorname{argmin}} f(x)$ for argument of the minimum, which returns a set of values of x for which the function $f(x)$ attains its smallest value.

2. SYSTEM MODELS

2.1 Generalised spatial modulation (GSM)

Transmission GSM modulation involves activating more than a single transmit antenna to simultaneously send the same complex APM symbol over a wireless communication link. A group of $m = m_\ell + m_\mathrm{s} = \left\lfloor \log_2 \binom{N_\mathrm{t}}{N_\mathrm{a}} \right\rfloor + \log_2(M)$ data bits is transmitted per symbol interval; where M represents the M-*ary* quadrature amplitude modulation (MQAM) constellation order, N_t is the number of available transmit antennas and $N_\mathrm{a} \in [1,2 \ldots N_\mathrm{t}]$ is the number of active transmit antennas out of the total N_t. One of the complex constellation symbols ($\boldsymbol{\chi}$)

is selected by $\log_2(M)$ bits and $\left\lfloor \log_2 \binom{N_\mathrm{t}}{N_\mathrm{a}} \right\rfloor$ bits determines N_a out of the total N_t. That is, the vector of the m data bits is grouped and mapped to form a constellation vector $x^q_{\ell(N_\mathrm{a})}$ of size $N_t \times 1$, where q and $\ell_{(N_\mathrm{a})}$ represent the selected constellation symbol and active transmit antenna combination, respectively.

The GSM modulation signal vector, $\boldsymbol{x}^q_{\ell(N_\mathrm{a})}$ can be written as $\boldsymbol{x}^q_{\ell_{(i,j)}}$. From the foregoing, we assume $N_\mathrm{a} = 2$ active antennas and are represented as i and j, during which all other antennas remain idle. Hence, a typical GSM modulation signal vector can be written as [15]:

$$\boldsymbol{x}_{\mathrm{GSM}} = \boldsymbol{x}^q_{\ell_{(N_\mathrm{a})}} = \boldsymbol{x}^q_{\ell_{(i,j)}} = [\,0 \quad \underbrace{x^q}_{\substack{i^{th} \\ Tx\ position}} \quad 0 \ \ldots \quad \underbrace{x^q}_{\substack{j^{th} \\ Tx\ position}} \quad 0\,]^T \qquad (1)$$

where $q \in [1{:}\,M]$ and $(i,j) \in [1{:}\,N_t]$, such that only $N_\mathrm{a} = 2$ of the $\boldsymbol{x}^q_{\ell(N_\mathrm{a})}$ resulting vector is non-zero.

It is to be noted that a set of $N' = \binom{N_\mathrm{t}}{N_\mathrm{a}}$ antenna combinations can be formed in total, out of which $N_\mathrm{c} = 2^{m_\ell}$ combinations can be used as spatial constellation points [15]. For illustration, GSM modulation system model is depicted in Figure 1 and a data mapping and transmission example is tabulated in Table 1, where $N_\mathrm{t} = 4$, $N_\mathrm{a} = 2$, and binary phase shift keying (BPSK) modulation is assumed. For the stipulated specifications, spectral efficiency of the transmission is 3 b/s/Hz; such that the first $m_\ell = 2$ bits selects one of the $N_\mathrm{c} = 4$ usable antenna combinations, while the remaining $m_\mathrm{s} = 1$ bit modulates a BPSK signal. Note that there are a total $N' = 6$ possible antenna combinations but (2,4) and (3,4), which are not shown in Table 1, are not useful in our example and are simply omitted [15].

The activated antenna pair, therefore, transmits the same complex constellation symbol, x^q, as selected. The GSM modulated signal is transmitted over an $N_\mathrm{r} \times N_\mathrm{t}$ MIMO Rayleigh frequency flat-fading channel, $\boldsymbol{H}$, and thus, the entries of $\boldsymbol{H}$ are modeled as complex independent and identically distributed (i.i.d.) Gaussian random variables with zero-mean and unit-variance, where N_r is the number of receive antennas. The received signal at any given time is [15]:

$$\boldsymbol{y} = \sqrt{\rho/2}\boldsymbol{H}\boldsymbol{x}^q_{\ell_{(i,j)}} + \boldsymbol{w} \qquad (2)$$

where $\boldsymbol{w}$ is an additive white Gaussian noise (AWGN) vector with zero-mean and variance σ^2 at the receiver input and ρ is the average SNR at each receive antenna.

The spatial and data symbols are jointly detected using the maximum-likelihood (ML) principle, at the receiver. This is mathematically given as [15]:

$$[\hat{\ell}_{(i,j)}, \hat{s}] = \underset{\ell_{(i,j)},\ s}{\operatorname{argmin}} \left\| \boldsymbol{y} - \sqrt{\rho/2}\boldsymbol{h}'_{\ell_{(i,j)}} x^q \right\|^2_\mathrm{F} \qquad (3)$$

where x_q is an MQAM transmit symbol, where $q \in [1{:}\,M]$ and $\boldsymbol{h}'_{\ell_{(i,j)}} = \boldsymbol{h}_{\ell_{(i)}} + \boldsymbol{h}_{\ell_{(j)}}$ is the column index of $\boldsymbol{H}$, and it represents the sum of the i and j distinct columns in $\boldsymbol{H}$ for all possible antenna pairs such that $(i,j) \in [1,2 \ldots N_\mathrm{t}]$. It therefore contains the summation of the active antenna channel vectors and is being referred to as an effective column.

2.2 Generalised space shift keying (GSSK) modulation

The underlying transmission concept in GSSK modulation is similar to that of GSM modulation in using more than one antenna at each transmission instance. A total of $N' = \binom{N_t}{N_a}$ antenna combinations are possible out of which $N_\mathrm{c} = \lfloor N' \rfloor$ combinations can be employed for transmission. However, GSSK symbols carry information only in the spatial domain but not in the IQ domain. In other words, APM symbols have been eliminated in GSSK transmission, such that only the activated combination of transmit antennas is used to relay information.

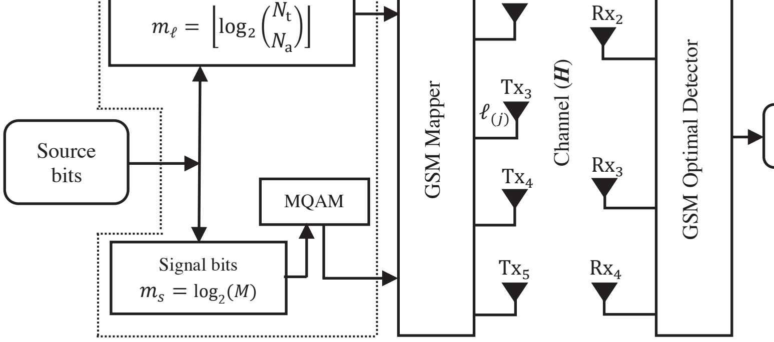

Figure 1: System model of GSM transmission ($N_t = 5, N_a = 2, N_r = 4$)

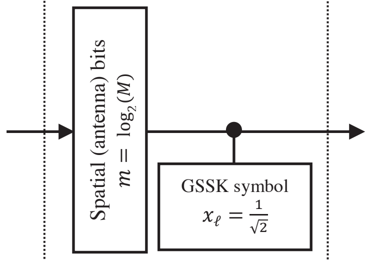

Figure 2: GSSK modulator

Table 1: Mapping illustrations for GSM and GSSK modulations

Input bits (m)=$[b_2 b_1 b_0]$	GSM			GSSK	
	Antenna bits $(m_\ell = b_2 b_1)$ **/ Tx Combination** $(\ell_{(i,j)})$	**Symbol bit** $(m_s = b_0)$ / **Symbol** (s)	$\boldsymbol{x}_{\mathrm{GSM}}$	**Antenna Combination** $(\ell_{(i,j)})$	$\boldsymbol{x}_{\mathrm{GSSK}}$
[0 0 0]	0 0 (1,2)	0 (-1)	$[-1\ -1\ 0\ 0]^{\mathrm{T}}$	(1,2)	$[\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\,0\,0\,0]$
[0 0 1]	0 0 (1,2)	1 (+1)	$[1\ 1\ 0\ 0]^{\mathrm{T}}$	(1,3)	$[\frac{1}{\sqrt{2}}\,0\,\frac{1}{\sqrt{2}}\,0\,0]$
[0 1 0]	0 1 (1,3)	0 (-1)	$[-1\ 0\ -1\ 0]^{\mathrm{T}}$	(1,4)	$[\frac{1}{\sqrt{2}}\,0\,0\,\frac{1}{\sqrt{2}}\,0]$
[0 1 1]	0 1 (1,3)	1 (+1)	$[1\ 0\ 1\ 0]^{\mathrm{T}}$	(1,5)	$[\frac{1}{\sqrt{2}}\,0\,0\,0\,\frac{1}{\sqrt{2}}]$
[1 0 0]	1 0 (1,4)	0 (-1)	$[-1\ 0\ 0\ -1]^{\mathrm{T}}$	(2,3)	$[0\,\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\,0\,0]$
[1 0 1]	1 0 (1,4)	1 (+1)	$[1\ 0\ 0\ 1]^{\mathrm{T}}$	(2,4)	$[0\,\frac{1}{\sqrt{2}}\,0\,\frac{1}{\sqrt{2}}\,0]$
[1 1 0]	1 1 (2,3)	0 (-1)	$[0\ -1\ -1\ 0]^{\mathrm{T}}$	(2,5)	$[0\,\frac{1}{\sqrt{2}}\,0\,0\,\frac{1}{\sqrt{2}}]$
[1 1 1]	1 1 (2,3)	1 (+1)	$[0\ 1\ 1\ 0]^{\mathrm{T}}$	(3,4)	$[0\,0\,\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\,0]$

The modulator for GSSK modulation, illustrated in Figure 2, can be inserted in place of the GSM modulator, shown within the dotted line in Figure 1, to obtain a simple model of GSSK transmission; in addition to replacing the GSM mapper and decoder with the GSSK mapper and decoder, respectively.

Random sequence of independent bits b = $[b_1 b_2 \dots b_k]$ are grouped into $m = \log_2(M)$ information bits, for M GSSK constellation size, which is in multiple of 2. These m bits are mapped onto one of the useful $N_c (= M)$ antenna combinations for transmission. Note that for N_a active antennas in every combination; only N_a position in $\boldsymbol{x}_{\mathrm{GSSK}}$ signal vector are non-zero; and because only spatial symbols are transmitted in GSSK modulation; $\boldsymbol{x}_\ell = \frac{1}{\sqrt{N_a}}$ for all the N_a active antennas

The GSSK signal vector, $\boldsymbol{x}_{\ell_{N_a}}$, indicates the activated N_a antennas, during which all other antennas remain idle, as represented [16]:

$$\boldsymbol{x}_{\mathrm{GSSK}} = \boldsymbol{x}_{\ell_{N_a}} = \boldsymbol{x}_{\ell_{(i,j)}} = \underbrace{[0 \quad {}^{1}/_{\sqrt{N_a}} \quad 0 \quad \dots \quad {}^{1}/_{\sqrt{N_a}} \quad 0]}_{\substack{N_a\ (i\ and\ j)\ active\ antennas\ (out\ of\ N_t) \\ have\ non-zero\ values}}^{\mathrm{T}} \tag{4}$$

A GSSK mapping example is also shown in Table 1 for the configuration: $M = 8$, $N_t = 5$, $N_a = 2$, and a spectral efficiency of 3 b/s/Hz. In this example, $N' = \binom{5}{2} = 10$ antenna combinations are possible, while only $N_c = \lfloor \binom{5}{2} \rfloor = 8$ combinations can be used; hence the unusable antenna combinations/pairs (3,5) and (4,5), are not shown.

The $\boldsymbol{x}_{\mathrm{GSSK}}$ signal in (4) is transmitted over an $N_r \times N_t$ wireless channel $\boldsymbol{H}$, and experiences an N_r- dim AWGN, $\boldsymbol{w} = [w_1 w_2 \dots w_{N_t}]^{\mathrm{T}}$. The $N_r \times 1$ received signal vector is given as [14]:

$$\boldsymbol{y} = \sqrt{\rho}\boldsymbol{H}\boldsymbol{x}_{\ell_{(i,j)}} + \boldsymbol{w} \tag{5}$$

where $\rho = {}^{1}/_{N_a}$ is the average SNR at each receive antenna, and the entries of both $\boldsymbol{H}$ and $\boldsymbol{w}$ are i.i.d. according to $CN(0,1)$.

From (4) and (5), and based on the example given in Table 1; it can be deduced that only $N_a = 2$ columns of $\boldsymbol{H}$ are activated, and these columns change according to the activated antennas, which depends on the transmitted information bits. Effectively, the output of the channel is written as [16]:

$$\boldsymbol{y} = \sqrt{\rho}\boldsymbol{h}'_{\ell_{N_a}} + \boldsymbol{w} \tag{6}$$

As earlier noted, the effective column $\mathbf{h}'_{\ell_{(i,j)}} = \boldsymbol{h}_{\ell_{(i)}} + \boldsymbol{h}_{\ell_{(j)}}$ is the column index of $\boldsymbol{H}$ that represents the sum of the i and j distinct columns for all possible antenna combinations such that $(i,j) \in [1,2 \dots N_t]$. Essentially, $\boldsymbol{h}'_{\ell_{(i,j)}}$ (the scaled version of $\boldsymbol{H}$) acts as random constellation points for GSSK modulation.

At the receiver side, the GSSK ML detector searches over all possible $\boldsymbol{h}'_{\ell_{(i,j)}}$ to obtain the estimates of the antenna indices (i.e. $\hat{\ell}_{N_a}$). This is given by [16]:

$$\hat{\ell}_{N_a} = \underset{\ell_{N_a}}{\operatorname{argmin}} \left\| \boldsymbol{y} - \sqrt{\rho}\boldsymbol{h}'_{\ell_{(i,j)}} \right\|_{\mathrm{F}}^2 \tag{7}$$

3. PROPOSED SOFT-OUTPUT DEMODULATORS

The expectations of users in the next generation communication systems, such as high data rates, reliability and power efficiency are achievable in practice by employing channel coding as a way to reduce errors induced by noise and unreliable channels. Furthermore, literature has demonstrated that detectors with soft-output demodulators coupled with soft-input channel decoders maximize the coding gain achievable [22]. In [16-19, 21] soft-output detection has been investigated for a number of SM variants; whereas, no such investigation has been performed for GSM modulation and GSSK modulation schemes, which maintain several advantages over SM.

Based on this motivation, we propose soft-output ML demodulators (SOMLDs) for the GSM and GSSK modulation schemes. To arrive at the targeted improvements in the error performances of the respective systems due to coding gain, the system model of Figure 1 is extended to include channel coding and decoding. For each of the proposed detection schemes, we assume the following: (i) antenna indices and data symbols (if applicable) are uncorrelated; (ii) data symbols (if applicable) are independent and generated with equal probability; (iii) antenna bits are independent and generated with equal probability, and; (iv) full channel knowledge is available at the receiver, in all instances.

3.1. SOMLD for GSM modulation

The codewords from the channel encoder are transmitted by GSM modulation, such that the input to our proposed demodulator is given as (2). From this, the proposed SOMLD, therefore, calculates the a-posteriori log-likelihood ratio (LLR) [19] for the a^{th} bit of the transmit antenna pair/combination and b^{th} bit of the symbol, independently. These a-posteriori LLRs may be formulated as follows:

Considering demodulation of the a^{th} bit of the transmit antenna pair:

$$LLR(\ell_{N_\mathrm{a}}^a) = LLR(\ell_{(i,j)}^a) = \log\frac{P(\ell_{N_\mathrm{a}}^a = 1 \mid \boldsymbol{y})}{P(\ell_{N_\mathrm{a}}^a = 0 \mid \boldsymbol{y})} \tag{8}$$

$$= \log\left[\frac{\sum_{\hat{\ell}_{N_\mathrm{a}} \in \boldsymbol{\ell}_1^a} \sum_{\hat{x}^q \in \chi} P(\boldsymbol{y} \mid \ell_{N_\mathrm{a}} = \hat{\ell}_{N_\mathrm{a}}, x^q = \hat{x}^q) P(\ell_{N_\mathrm{a}} = \hat{\ell}_{N_\mathrm{a}})}{\sum_{\hat{\ell}_{N_\mathrm{a}} \in \boldsymbol{\ell}_0^a} \sum_{\hat{x}^q \in \chi} P(\boldsymbol{y} \mid \ell_{N_\mathrm{a}} = \hat{\ell}_{N_\mathrm{a}}, x^q = \hat{x}^q) P(\ell_{N_\mathrm{a}} = \hat{\ell}_{N_\mathrm{a}})}\right] \tag{9}$$

where $\boldsymbol{\ell}_1^a$ and $\boldsymbol{\ell}_0^a$ are vectors of the antenna pair indices with '1' and '0', respectively, at the a^{th} antenna bit position and $\boldsymbol{\chi}$ represents the set of MQAM constellation symbols.

On application of Bayes' theorem [21], the demodulator output of (9) can be defined as:

$$LLR(\ell_{N_\mathrm{a}}^a) = \log\left[\frac{\sum_{\hat{\ell}_{N_\mathrm{a}} \in \boldsymbol{\ell}_1^a} \sum_{\hat{x}^q \in \chi} \exp\frac{-\left\|y - \sqrt{\rho/2}\,\mathbf{h}'_{\ell_{(i,j)}} x^q\right\|_\mathrm{F}^2}{2\sigma^2}}{\sum_{\hat{\ell}_{N_\mathrm{a}} \in \boldsymbol{\ell}_0^a} \sum_{\hat{x}^q \in \chi} \exp\frac{-\left\|y - \sqrt{\rho/2}\,\mathbf{h}'_{\ell_{(i,j)}} x^q\right\|_\mathrm{F}^2}{2\sigma^2}}\right] \tag{10}$$

where σ^2 is the variance of the AWGN.

Furthermore, the a-posteriori LLR for the b^{th} bit of the data symbol is formulated as:

$$LLR(\boldsymbol{x}_b^q) = \log\frac{P(\boldsymbol{x}_b^q = 1 \mid \boldsymbol{y})}{P(\boldsymbol{x}_b^q = 0 \mid \boldsymbol{y})} \tag{11}$$

$$= \log\left[\frac{\sum_{\hat{x}^q \in \chi_1^b} \sum_{\hat{\ell}_{N_\mathrm{a}} \in \ell_{N_\mathrm{c}}} P(\boldsymbol{y} \mid \ell_{N_\mathrm{a}} = \hat{\ell}_{N_\mathrm{a}}, x^q = \hat{x}^q) P(x^q = \hat{x}^q)}{\sum_{\hat{x}^q \in \chi_0^b} \sum_{\hat{\ell}_{N_\mathrm{a}} \in \ell_{N_\mathrm{c}}} P(\boldsymbol{y} \mid \ell_{N_\mathrm{a}} = \hat{\ell}_{N_\mathrm{a}}, x^q = \hat{x}^q) P(x^q = \hat{x}^q)}\right] \tag{12}$$

where $\boldsymbol{\chi}_1^b$ and $\boldsymbol{\chi}_0^b$ are vectors of the data symbols with '1' and '0', respectively, at the b^{th} data bit position and ℓ_{N_c}represents the set of all useful antenna combinations.

Similarly, the b^{th} symbol bit is computed as:

$$LLR(\boldsymbol{x}_b^q) = \log\left[\frac{\sum_{\hat{x}^q \in \chi_1^b} \sum_{\hat{\ell}_{N_\mathrm{a}} \in \ell_{N_\mathrm{c}}} \exp\frac{-\left\|y - \sqrt{\rho/2}\,\mathbf{h}'_{\ell_{(i,j)}} x^q\right\|_\mathrm{F}^2}{2\sigma^2}}{\sum_{\hat{x}^q \in \chi_0^b} \sum_{\hat{\ell}_{N_\mathrm{a}} \in \ell_{N_\mathrm{c}}} \exp\frac{-\left\|y - \sqrt{\rho/2}\,\mathbf{h}'_{\ell_{(i,j)}} x^q\right\|_\mathrm{F}^2}{2\sigma^2}}\right] \tag{13}$$

3.2. SOMLD for GSSK Modulation

The coded GSSK modulated signals are represented as (6). At the receiver, the proposed demodulator independently calculates the LLR for antenna pair index bits using the received coded GSSK modulation; as there exist no APM in the GSSK transmission. Therefore, the a-posteriori LLR for the a^{th} transmit antenna bit can be expressed mathematically as:

$$LLR(\ell_{N_\mathrm{a}}^a) = LLR(\ell_{(i,j)}^a) = \log\frac{P(\ell_{N_\mathrm{a}}^a = 1 \mid \boldsymbol{y})}{P(\ell_{N_\mathrm{a}}^a = 0 \mid \boldsymbol{y})} \tag{14}$$

$$= \log\left[\frac{\sum_{\hat{\ell}_{N_\mathrm{a}} \in \boldsymbol{\ell}_1^a} P(\boldsymbol{y} \mid \ell_{N_\mathrm{a}} = \hat{\ell}_{N_\mathrm{a}}) P(\ell_{N_\mathrm{a}} = \hat{\ell}_{N_\mathrm{a}})}{\sum_{\hat{\ell}_{N_\mathrm{a}} \in \boldsymbol{\ell}_0^a} P(\boldsymbol{y} \mid \ell_{N_\mathrm{a}} = \hat{\ell}_{N_\mathrm{a}}) P(\ell_{N_\mathrm{a}} = \hat{\ell}_{N_\mathrm{a}})}\right] \tag{15}$$

where $\boldsymbol{\ell}_1^a$ and $\boldsymbol{\ell}_0^a$ are vectors of the antenna pair indices with '0' and '1', respectively, at the a^{th} antenna bit position.

On application of the Bayes' theorem, the demodulator output in (15) can be defined as:

$$LLR(\ell_{N_\mathrm{a}}^a) = \log\left[\frac{\sum_{\hat{\ell}_{N_\mathrm{a}} \in \boldsymbol{\ell}_1^a} \exp\left(\frac{-\left\|y - \sqrt{\rho}\,\mathbf{h}'_{\ell_{N_\mathrm{a}}}\right\|_\mathrm{F}^2}{2\sigma^2}\right)}{\sum_{\hat{\ell}_{N_\mathrm{a}} \in \boldsymbol{\ell}_0^a} \exp\left(\frac{-\left\|y - \sqrt{\rho}\,\mathbf{h}'_{\ell_{N_\mathrm{a}}}\right\|_\mathrm{F}^2}{2\sigma^2}\right)}\right] \tag{16}$$

Meanwhile, the proposed SOMLDs, as presented in (10), (13) and (16), achieve an improvement in error performance when their outputs are fed into a soft-input Viterbi channel decoder [19-21] and estimates of the transmitted messages are obtained. Considering computational complexity (floating point operations) for the proposed SOMLDs, since a look-up table based method can be employed to compute the logarithm functions [24], no additional computational complexity is imposed compared to its hard-decision counterparts [21].

4. NUMERICAL RESULTS AND DISCUSSION

Monte Carlo simulations of the proposed detectors were executed in the Matlab environment and are in terms of the average bit error rate (BER) as a function of the average SNR. The termination criterion for simulations was the number of bit errors, set at 1,200. Simulations were run until a BER of 10^{-6}. For all simulations, Rayleigh frequency-flat fading channels and the presence of AWGN is assumed. For coded cases, a ½-rate convolutional encoder was employed to encode the information bits under the constraint length 9 with code generator matrices $g_1 = (561)_{octal}$; $g_2 = (753)_{octal}$ [17, 19]. At the respective receivers, the proposed detectors are employed and their outputs are fed into a soft-input Viterbi channel decoder [19, 21], in order to obtain estimates of the transmitted messages.

In both schemes under consideration, we assume the optimal condition that $N_\mathrm{a} = 2$, for all simulations. This is based on the fact that, an attempt to increase N_a beyond 2 increases the possibility of having the same antenna in different antenna combinations, which consequently degrades the performances of GSM and GSSK modulation [13, 15].

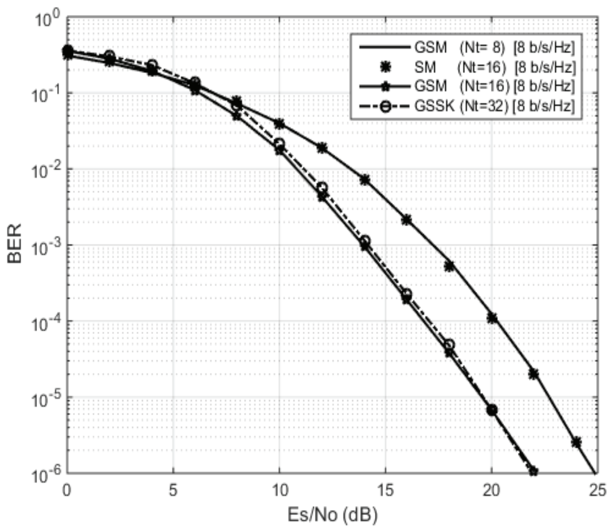

Figure 3: Comparison of error performances for SM, GSM and GSSK modulation systems using $N_r = 4$

In Figure 3, the BER performance versus SNR for SM, GSM and GSSK modulation systems are depicted. At a transmission rate of 8 b/s/Hz, the error performance of GSM modulation is nearly identical to the error performance of SM, when considering the same QAM order of 4. However, number of transmit antennas needed to achieve this is reduced by half in GSM modulation. For the same transceiver configuration and spectral efficiencies, GSM modulation outperforms its SM counterpart by not less than 3 dB SNR gains at a BER of 10^{-6}. Apart from this, the GSM modulation system uses half power due to two active antennas and employs lower QAM symbols.

In the same figure, we compare the BER performance of SM to that of GSSK modulation system at spectral efficiencies of 8 b/s/Hz. At a BER of 10^{-6}, GSSK modulation outperforms the SM system by approximately 3.0 dB SNR gain. To achieve the 8 b/s/Hz spectral efficiency, the GSSK modulation makes use of 32 transmit antennas, while the SM employs 16; however, the detection of the APM symbols in SM is a contributing factor to the poor error performance of the SM.

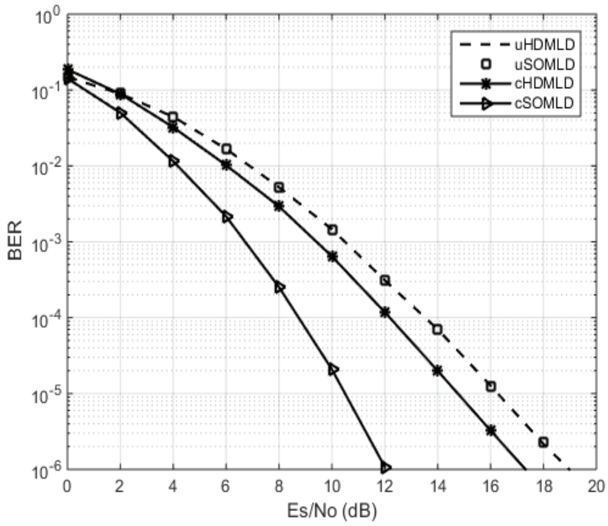

Figure 4**:** Comparison of error performances of 4 b/s/Hz GSM modulation systems in coded and uncoded channels

In Figure 4 and Figure 5, respectively, we present the BER performances of GSM and GSSK modulation signals under coded and uncoded channels as detected by the conventional hard-decision ML detectors (HDMLDs) and the proposed soft-output ML detectors (SOMLDs). In identifying the curves in both figures, “c” and “u” are used to indicate “coded” and “uncoded” channels, respectively; thus, we show 4 different curves labelled by the following abbreviations: uHDMLD, uSOMLD, cHDMLD and cSOMLD.

For example, by “uHDMLD”, we refer to the curve showing the BER performance of a particular system detected using the HDMLD under uncoded channel. This is arrived at when no channel coding is introduced into the original bit streams before transmission takes place, and in turn detected using the conventional HDMLD at the receiver. When channel coding is introduced at the transmitter end and the conventional HDMLD is maintained at the receiver, we have the “cHDMLD”. If the channel coding is maintained and the HDMLD is replaced by the proposed SOMLD, we obtain the “cSOMLD”. Finally, when transmission is performed without channel coding and the SOMLD is maintained at the receiver, we have the “uSOMLD”.

In Figure 4, error performances of 4 b/s/Hz 4×4 4QAM GSM modulation systems are depicted for the two channels and detectors. In the presented results; the systems are equipped with $N_t = 4$ transmit antennas out of which the number of active transmit antennas, N_a, is 2. A set of $N' = \binom{4}{2} = 6$ antenna combinations can be formed in total, but $N_c = 4$ combinations can be used as spatial constellation points. In addition to these, $M = 4$ and $N_r = 4$ are considered. For uncoded channels, the GSM modulation system with HDMLD shows a performance that is matching closely with the proposed SOMLD. Therefore, detecting GSM signals with the proposed soft-output demodulator has no superior effects as compared to the conventional HDMLD under uncoded channel. This is because both detectors are based on the ML principle and there is no additional coding gain that may be exploited, hence reducing to the same solution [17, 22]. We consider this reason a good means of validating the results.

For coded channels, it is evident that the SOMLD yields significant SNR gains. For example, at a BER of 10^{-6}, cSOMLD achieves an SNR gain of approximately 5.2 dB over cHDMLD. It also outperforms the uncoded systems by approximately 7.0 dB SNR gains, at the same BER. These are due, not only to the effectiveness of the soft-decision over hard-decision technique, but also to the coding introduced.

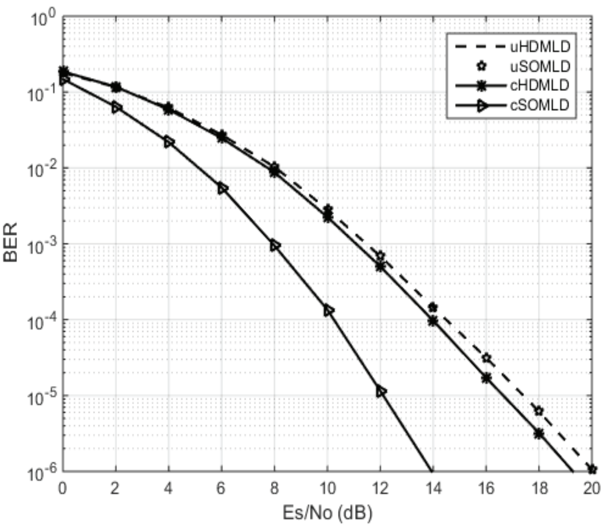

Figure 5: Comparison of error performances of 3 b/s/Hz GSSK modulation systems in coded and uncoded channels

Figure 5 show the results obtained for the performances of 3 b/s/Hz GSSK modulation systems with the configuration: $M = 8, N_t = 5, N_a = 2$, $N_r = 4$, $N' = 10$ and $N_c = 8$, as detected with the conventional HDMLD and the proposed SOMLD in coded and uncoded channels. The GSSK with uSOMLD would demonstrate the same error performance as the conventional uHDMLD. Again, this shows that the soft-output demodulator has no effect as no additional coding gain could be exploited.

Meanwhile, it is evident that in coded channels, the proposed SOMLD for GSSK modulation system performs better than the HDMLD, with an SNR gain of approximately 5.2 dB at a BER of 10^{-6}. Furthermore, it is clear that the cSOMLD outperforms the uHDMLD by an SNR gain of 6.0 dB at the same BER. It should be mentioned that, under coded channel conditions, the proposed soft-output demodulators have been coupled with a soft-input decoder at the receiver so as to reverse the effects of channel coding in obtaining the estimates of the originally transmitted messages.

5. CONCLUSION

The contributions of this paper included the development of new SOMLDs for GSM and GSSK modulation schemes. The developed demodulators have been derived mathematically and their performances, under coded and uncoded channels, were evaluated numerically. In summary, the presented results show that; the proposed SOMLDs for the GSM and GSSK modulation systems match the optimal error performances of their respective HDMLDs in uncoded channel conditions. In coded channels, the proposed SOMLDs yield significant SNR gains over their corresponding conventional HDMLDs. At least a 5.2 dB SNR gain is achieved in both cases of GSM and GSSK modulation, and is significant. In comparison to the HDMLDs, the proposed SOMLDs impose no additional computational complexity, since a look-up table can be employed to compute the logarithms. Hence, the proposed detectors would fall among the important models suitable for deployment in the emerging and existing communication systems.

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